機械手臂 軌跡規劃

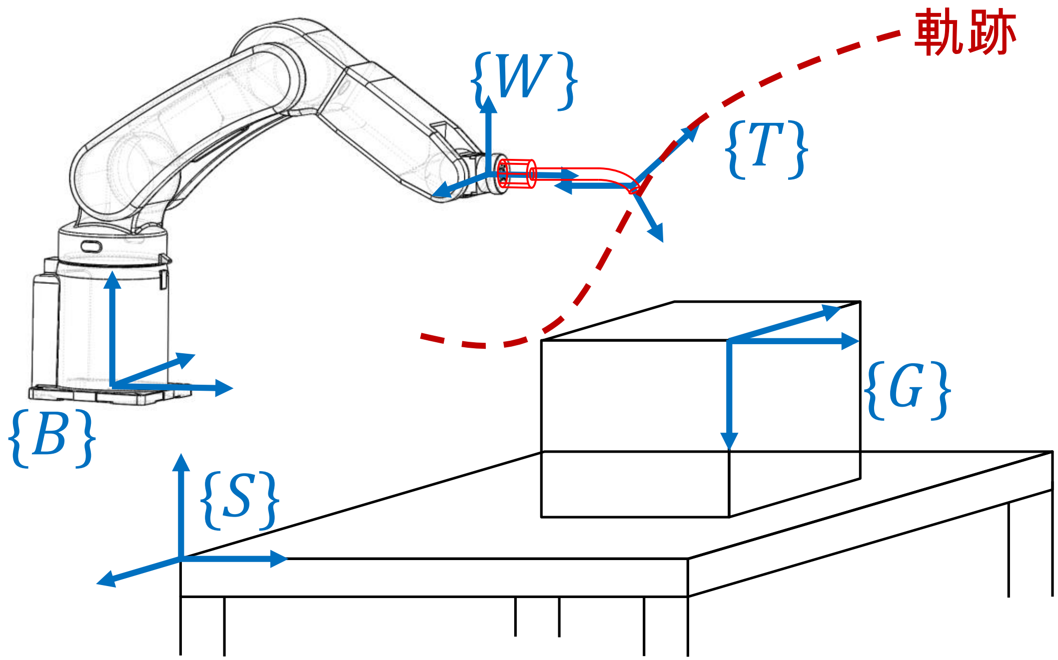
Manipulator Trajectory Planning

林沛群
國立台灣大學
機械工程學系

引言 -1

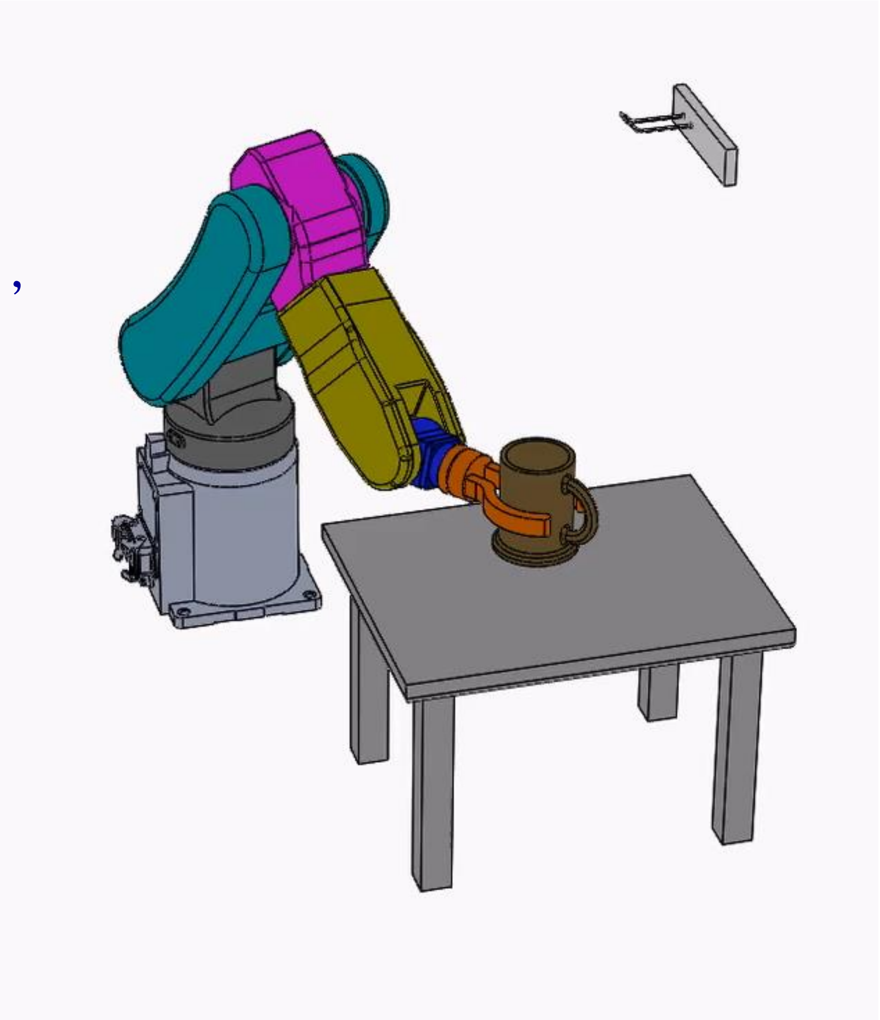
- 軌跡：機械手臂（的末端點或操作點）的位置、速度、加速對對時間的歷程
- 可進一步定義成 $\{T\}$相對$\{G\}$的狀態歷程

和手臂種類無關，$\{G\}$也可隨時間變動（如輸送帶）



引言 -2

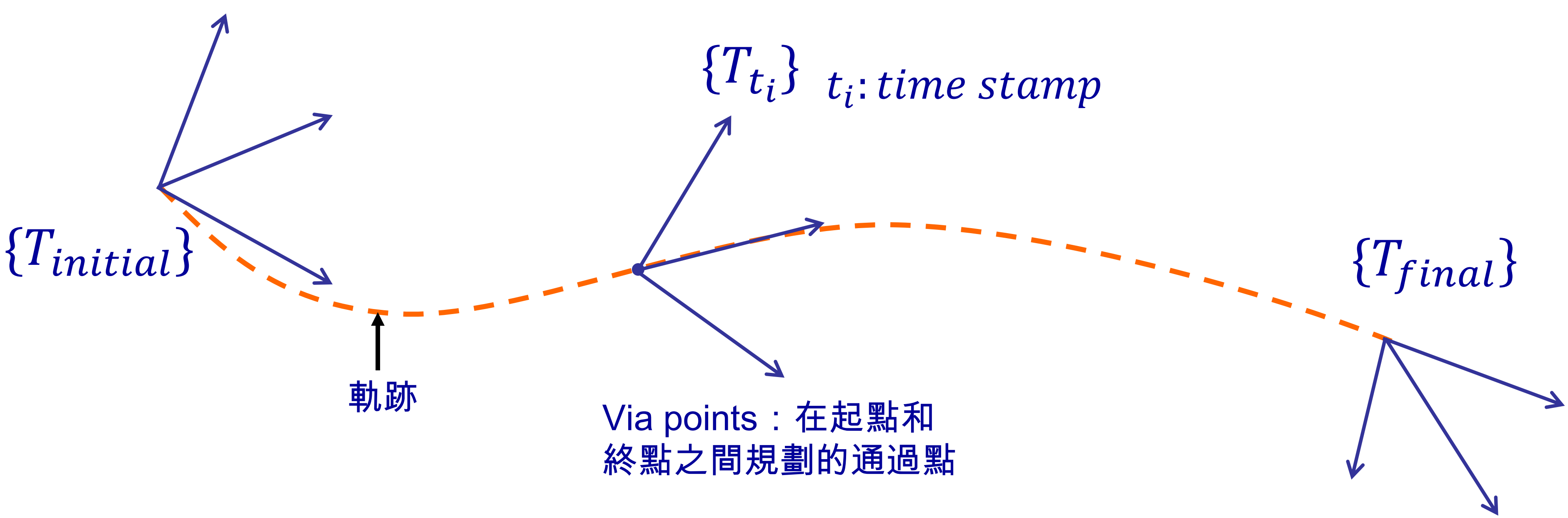
- 物件取放任務中手臂末端點（或物件）的運動歷程，即是軌跡規劃所進行的內容
  - Example情境：
  - 機械手臂夾住放在桌上的杯子，移動手臂將杯子掛到牆上的杯架



引言 -3

- 理想軌跡：Smooth path (i.e., continuous with continuous first derivative)

常常為 ${}^{G}_{T}T$



Joint-space下的軌跡規劃 -1

- 步驟
  - 定義{T}相對於{G}的initial, via, & final points，${}^{G}_{T}T_i$

    （包含移動和轉動自由度）

    i=1 initial

    i=2~N-1 via points

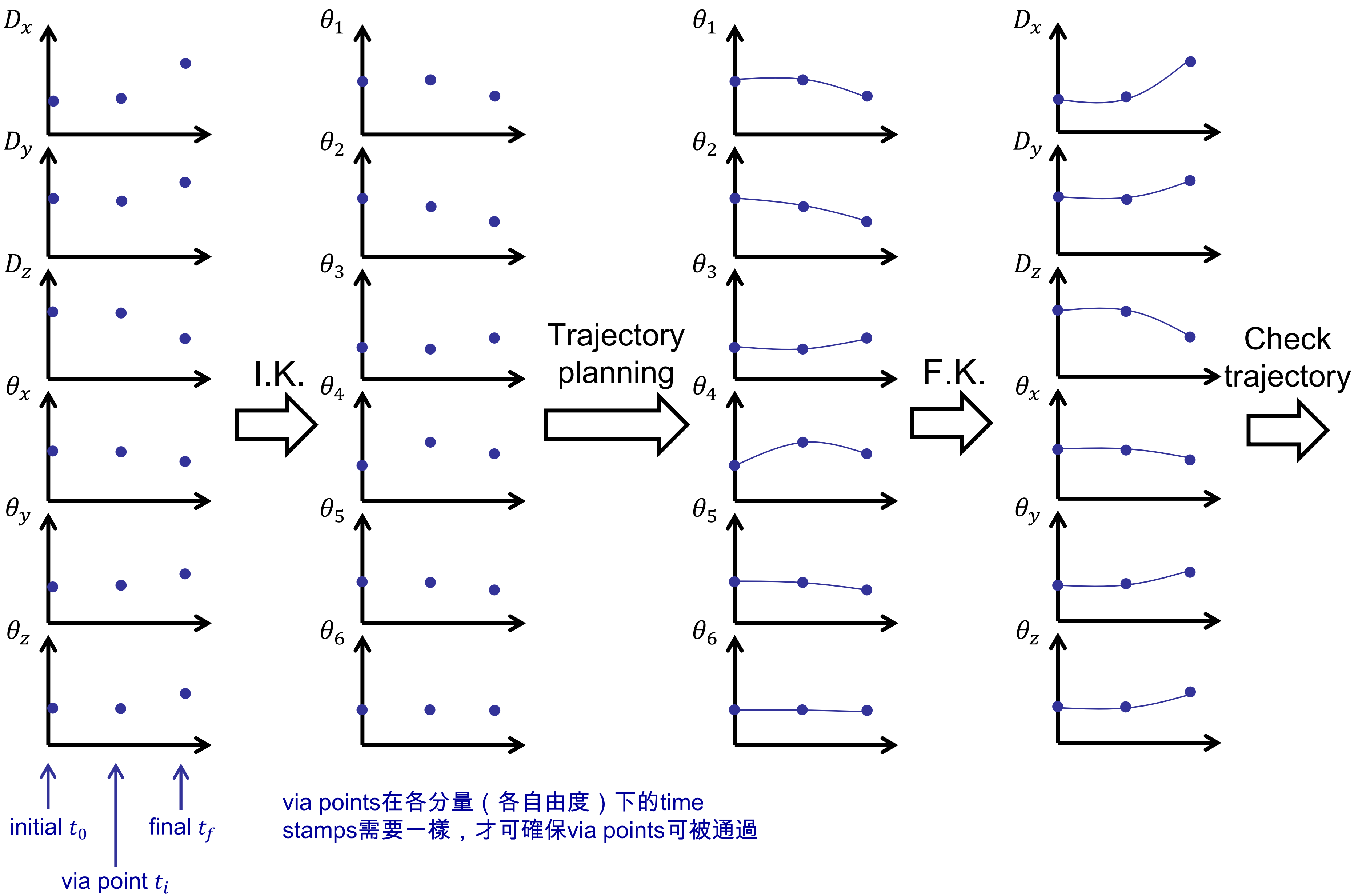
    i=N+1 final

    將 ${}^{G}_{T}T_i$ 以6個參數方式來表達 $${}^{G}X_T = \begin{bmatrix} {}^{G}P_{T\,org} \\ \underline{ROT({}^{G}\hat{K}_T, \theta)} \end{bmatrix}$$

    非rotation matrix的方式表達

  - Inverse kinematics: 將手臂末端點狀態轉換到joint狀態： ${}^{G}X_T \rightarrow \Theta_i$
  - 對所有joints規劃smooth trajectories
  - Forward Kinematics:將joint狀態轉換到手臂末端點狀態，檢查莫短點在Cartesian-space下軌跡的可行性

Joint-space下的軌跡規劃 -2



via points在各分量（各自由度）下的time stamps需要一樣，才可確保via points可被通過

Cartesian-space下的軌跡規劃 -1

- 步驟
  - 定義{T}相對於{G}的initial, via, & final points，${}^{G}_{T}T_i$

    （包含移動和轉動自由度）

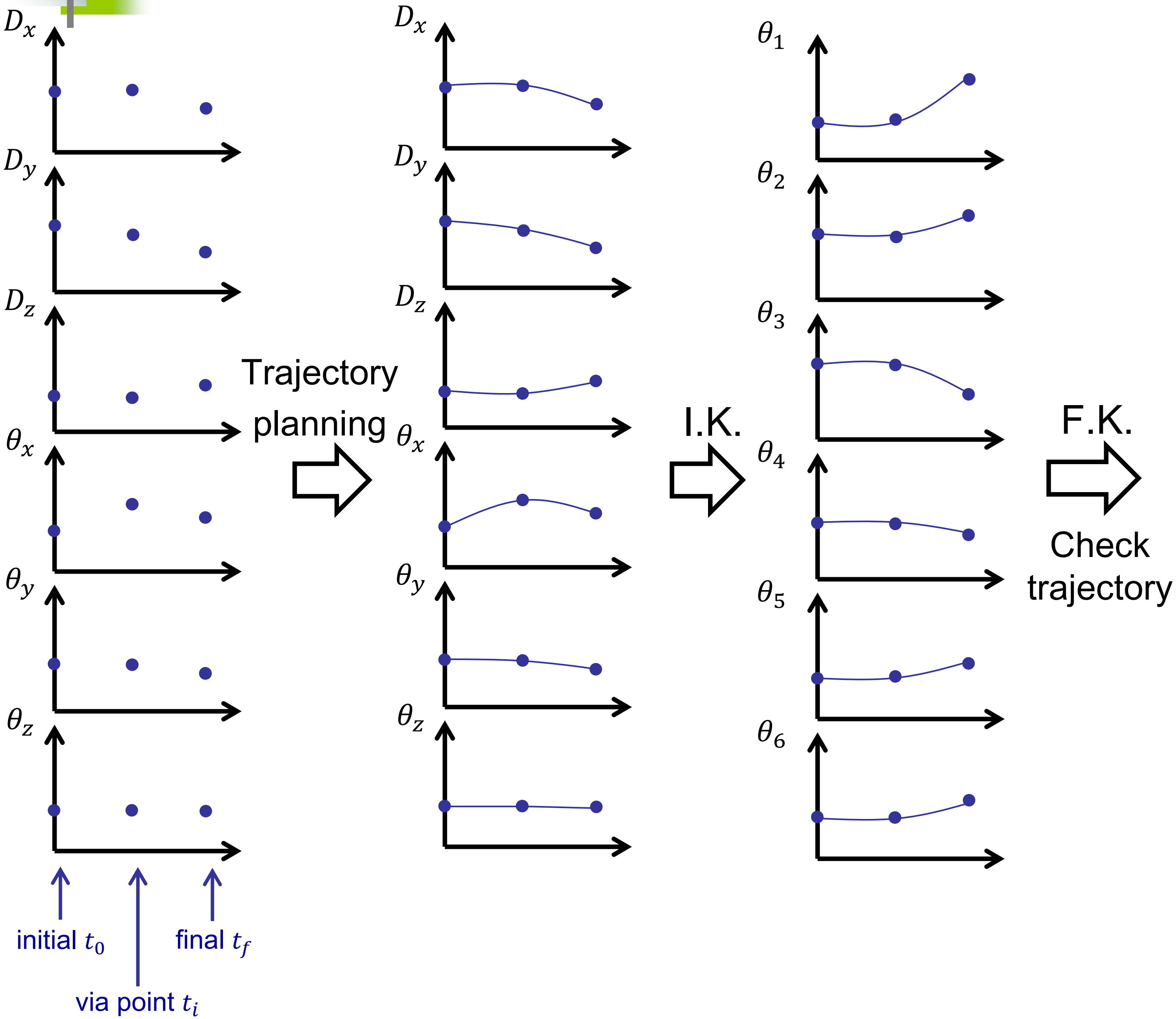
    將 ${}^{G}_{T}T_i$ 以6個參數方式來表達 ${}^{G}X_T = \begin{bmatrix} {}^{G}P_{T\,org} \\ ROT({}^{G}\hat{K}_T, \theta) \end{bmatrix}$

    i=1 initial

    i=2~N-1 via points

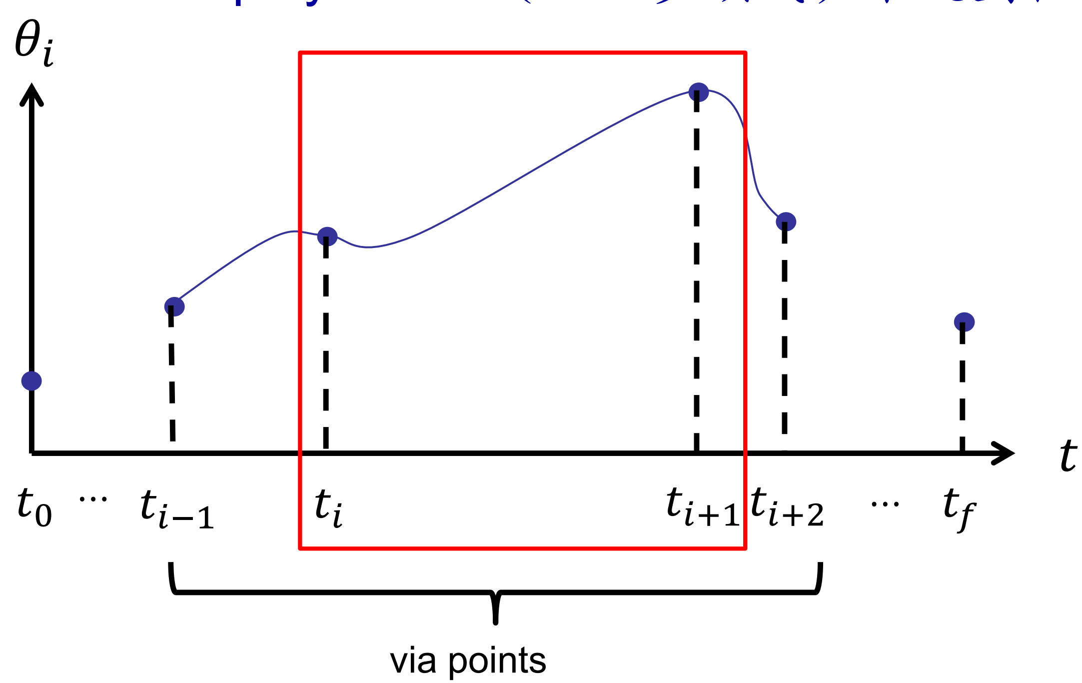
    i=N+1 final
  - 對所有手臂末端點狀態規劃smooth trajectories
  - 將規劃好手臂末端點狀態的軌跡點轉換到joint狀態：${}^{G}X_T \rightarrow \Theta_i$
  - 檢查joint狀態在Joint-space下軌跡的可行性
- Comments
  - 較具物理直觀的軌跡
  - 較高的運算負載 (i.e., IK)

Cartesian-space下的軌跡規劃 -2



Cubic Polynomials說明 -1

- 原則
  - 軌跡：不同軌跡區段 $[t_i \quad t_{i+1}]$ 以不同參數的函數來規劃
  - Smooth: 需定義各函數的邊界條件（包含位置和速度 $\theta(t_i), \theta(t_{i+1}), \dot{\theta}\,(t_i), \dot{\theta}\,(t_{i+1})$，有4個條件）
  - 以cubic polynomial（三次多項式）來規劃軌跡



1 linear
2 quadratic
3 cubic
4 quartic
5 quintic
6 hexic (sextic)
7 heptic (septic)
8 octic
9 nonic
10 decic

Cubic Polynomials說明-2

- 解cubic polynomial
  - 通式

$$\theta(\tilde{t}) = a_0 + a_1\tilde{t} + a_2\tilde{t}^2 + a_3\tilde{t}^3 \qquad 4\ unknowns\colon a_j \quad {}_{j=0\sim3}$$

  - 對每一個區段： $t \in [t_i, t_{i+1}]$

$$\tilde{t} = t - t_i \qquad so\ \tilde{t}|_{t=t_i} = 0 \ \text{ and } \ \tilde{t}|_{t=t_{i+1}} \equiv \Delta t = t_{i+1} - t_i > 0$$

每一個區段$[t_i, t_{i+1}]$的Δt可以不同，取決於via points的設定

邊界條件

$$\theta\left(\tilde{t}|_{t=t_i}\right) = \theta_i = a_0 \qquad ①$$

$$\theta\left(\tilde{t}|_{t=t_{i+1}}\right) = \theta_{i+1} = a_0 + a_1\Delta t + a_2\Delta t^2 + a_3\Delta t^3 \qquad ②$$

$$\dot{\theta}\left(\tilde{t}|_{t=t_i}\right) = \dot{\theta}_i = a_1 \qquad ③$$

$$\dot{\theta}\left(\tilde{t}|_{t=t_{i+1}}\right) = \dot{\theta}_{i+1} = a_1 + 2a_2\Delta t + 3a_3\Delta t^2 \qquad ④$$

Cubic Polynomials說明-3

- 解聯立方程式 ① ② ③ ④

$$a_2 = \frac{3}{\Delta t^2}(\theta_{i+1} - \theta_i) - \frac{2}{\Delta t}\dot{\theta}_i - \frac{1}{\Delta t}\dot{\theta}_{i+1}$$

$$a_3 = -\frac{2}{\Delta t^3}(\theta_{i+1} - \theta_i) + \frac{1}{\Delta t^2}(\dot{\theta}_{i+1} + \dot{\theta}_i)$$

Cubic Polynomials說明-4

- 以Matrix operation方式運算

$$\begin{bmatrix} \theta_i \\ \theta_{i+1} \\ \dot{\theta}_i \\ \dot{\theta}_{i+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \Delta t & \Delta t^2 & \Delta t^3 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2\Delta t & 3\Delta t^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$\Theta = T_{4\times 4}(\Delta t) \cdot A$$

$$\det(T_{4\times 4}) = -\Delta t^4 \neq 0 \; as \; long \; as \; \Delta t \neq 0$$

所以

$$\mathrm{A} = T_{4\times 4}^{-1} \cdot \Theta$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\dfrac{3}{\Delta t^2} & \dfrac{3}{\Delta t^2} & -\dfrac{2}{\Delta t} & -\dfrac{1}{\Delta t} \\ \dfrac{2}{\Delta t^3} & -\dfrac{2}{\Delta t^2} & \dfrac{1}{\Delta t^2} & \dfrac{1}{\Delta t^2} \end{bmatrix} \begin{bmatrix} \theta_i \\ \theta_{i+1} \\ \dot{\theta}_i \\ \dot{\theta}_{i+1} \end{bmatrix}$$

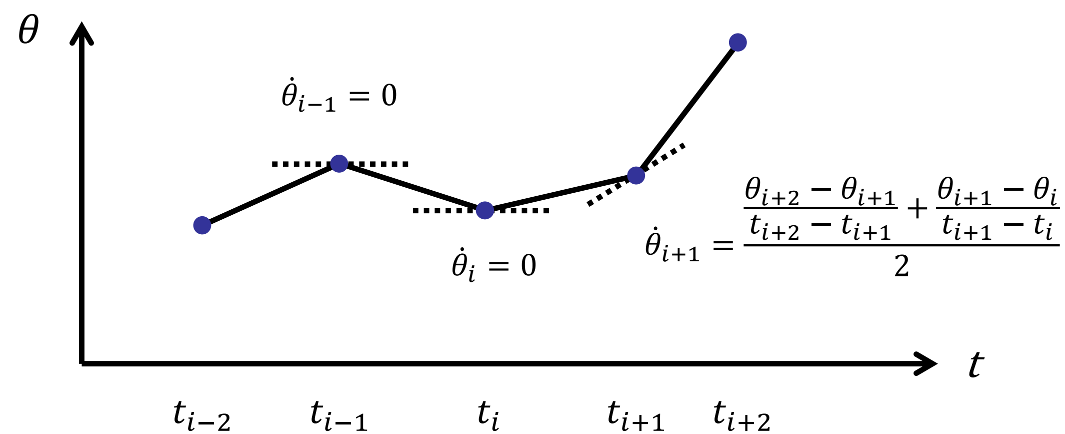
多段Cubic Polynomials -1

- 如何選擇速度條件 $\dot{\theta}_i$ 和 $\dot{\theta}_{i+1}$？
  - 直接定義，不論在 Cartesian space 或 joint space

    不建議，過於複雜，尤其軌跡落在singular points附近時
  - 自動生成

    Ex: 如果 $\dot{\theta}_i$ 在 t_i 前後變號，選擇 $\dot{\theta}_i = 0$

    如果 $\dot{\theta}_i$ 在 t_i 前後同號，選擇平均



以此兩個方式，不同區段的Cubic polynomials可以分開求解

多段Cubic Polynomials -2

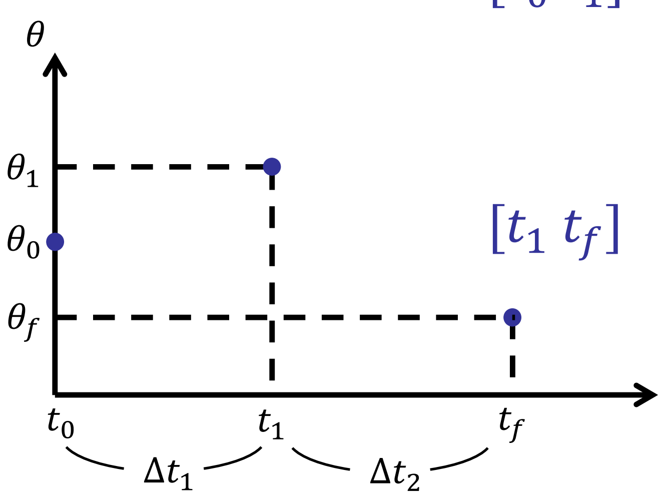
- 規劃速度使過程中加速度也CONTINUOUS

  （有效使用此可調控的變數）

  不同區段的Cubic polynomials需要整合一起，一併求解

- Example: A trajectory with one via point

各段在運算時將時間平移到0開始



$[t_0\ t_1]$ $\quad \tilde{t} = t_1 - t_0 \quad [\,0\ \ \Delta t_1\,]$

$$\theta_I(\tilde{t}) = a_{10} + a_{11}\tilde{t} + a_{12}\tilde{t}^2 + a_{13}\tilde{t}^3$$

$[t_1\ t_f]$ $\quad \tilde{t} = t_f - t_1 \quad [\,0\ \ \Delta t_2\,]$

$$\theta_{II}(\tilde{t}) = a_{20} + a_{21}\tilde{t} + a_{22}\tilde{t}^2 + a_{23}\tilde{t}^3$$

$$\Rightarrow 8\ unknowns$$

多段Cubic Polynomials -3

- Example: A trajectory with one via point (cont.)

4 position B.C.s

2 for each $\theta_j(t)$ $_{j\,=\,\mathrm{I},\,\mathrm{II}}$

$$\begin{cases} \theta_0 = a_{10} \\ \theta_1 = a_{10} + a_{11}\Delta t_1 + a_{12}\Delta t_1^2 + a_{13}\Delta t_1^3 \\ \theta_1 = a_{20} \\ \theta_f = a_{20} + a_{21}\Delta t_2 + a_{22}\Delta t_2^2 + a_{23}\Delta t_2^3 \end{cases}$$

2 velocity B.C.s

$$\begin{cases} \dot{\theta}_0 = \boxed{0} = a_{11} \\ \dot{\theta}_f = \boxed{0} = a_{21} + 2a_{22}\Delta t_2 + 3a_{23}\Delta t_2^2 \end{cases}$$

$not\ necessary$ "0"

Via point

velocity continuity

acceleration continuity

$$\begin{cases} \dot{\theta}_1 = a_{11} + 2a_{12}\Delta t_1 + 3a_{13}\Delta t_1^2 = a_{21} \\ \ddot{\theta}_1 = 2a_{12} + 6a_{13}\Delta t_1 = 2a_{22} \end{cases}$$

$$\Rightarrow 8\ equations$$

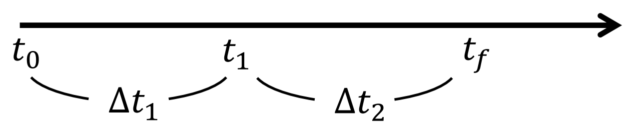
多段Cubic Polynomials -4

- Example: A trajectory with one via point (cont.)

8 equations, 8 unknowns

代數解法

(when $\Delta t_1 = \Delta t_2 = \Delta t$)



$$a_{10} = \theta_0$$

$$a_{11} = 0$$

$$a_{12} = \frac{12\theta_1 - 3\theta_f - 9\theta_0}{4\Delta t^2}$$

$$a_{13} = \frac{-8\theta_1 + 3\theta_f + 5\theta_0}{4\Delta t^3}$$

$$a_{20} = \theta_1$$

$$a_{21} = \frac{3\theta_f - 3\theta_0}{4\Delta t}$$

$$a_{22} = \frac{-12\theta_1 + 6\theta_f + 6\theta_0}{4\Delta t^2}$$

$$a_{23} = \frac{8\theta_1 - 5\theta_f - 3\theta_0}{4\Delta t^3}$$

多段Cubic Polynomials -5

- Example: A trajectory with one via point (cont.)

矩陣解法

$$\Theta_{8\times1} = T_{8\times8}\,A_{8\times1}$$

$[\,0\ \ \Delta t_1\,]$ $\quad \theta_I(\tilde{t}) = a_{10} + a_{11}\tilde{t} + a_{12}\tilde{t}^2 + a_{13}\tilde{t}^3$

$[\,0\ \ \Delta t_2\,]$ $\quad \theta_{II}(\tilde{t}) = a_{20} + a_{21}\tilde{t} + a_{22}\tilde{t}^2 + a_{23}\tilde{t}^3$

$$\begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_1 \\ \theta_f \\ \dot{\theta}_0 \\ \dot{\theta}_f \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \Delta t_1 & {\Delta t_1}^2 & {\Delta t_1}^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta t_2 & {\Delta t_2}^2 & {\Delta t_2}^3 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2\Delta t_2 & 3{\Delta t_2}^2 \\ 0 & 1 & 2\Delta t_1 & 3{\Delta t_1}^2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 6\Delta t_1 & 0 & 0 & -2 & 0 \end{bmatrix} \begin{bmatrix} a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \end{bmatrix}$$

$$A_{8\times1} = T_{8\times8}^{-1}\,\Theta_{8\times1}$$

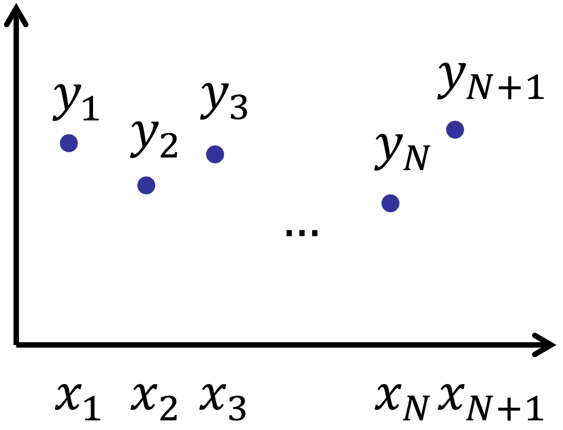
$$\det(T_{8\times8}) = 4{\Delta t_1}^4{\Delta t_2}^3 + 4{\Delta t_1}^3{\Delta t_2}^4$$

$$\neq 0\ as\ long\ as\ \Delta t_1 \neq 0, \Delta t_2 \neq 0, \Delta t_1 \neq -\Delta t_2)$$

General Cubic Polynomials -1

- General cubic spline function

N+1 set points (x_i, y_i) $\begin{cases} 1 & \text{initial} \\ N\text{-}1 & \text{via} \\ 1 & \text{final} \end{cases}$



N cubic functions

$$s_j(x) = a_j + b_j x + c_j x^2 + d_j x^3 \qquad \begin{matrix} x_j \le x \le x_{j+1} \\ j = 1 \ldots N \end{matrix}$$

$\Rightarrow$ total 4N unknown coefficients

General Cubic Polynomials -2

Position conditions at both ends of each $s_j(x)$

$\Rightarrow$ 2N conditions

Velocity & acceleration continuity conditions at via points

$\Rightarrow$ 2(N-1) conditions

還需要 2 CONDITIONS 以求解

Revisit example: A trajectory with one via point

$$\left[\begin{array}{l} y_1 = s_1(x_1) \\ y_2 = s_1(x_2) \end{array}\right. \qquad \left[\begin{array}{l} y_2 = s_2(x_2) \\ y_3 = s_2(x_3) \end{array}\right.$$

$$\dot{y}_2 = \dot{s}_1(x_2) = \dot{s}_2(x_2)$$
$$\ddot{y}_2 = \ddot{s}_1(x_2) = \ddot{s}_2(x_2)$$

共 6 conditions

General Cubic Polynomials -3

最後 2 conditions 的選擇方法

(1) $\ddot{s}_1(x_1) = \ddot{s}_N(x_{N+1}) = 0$ 　定義加速度，Natural cubic spline

(2) $\dot{s}_1(x_1) = u \quad \dot{s}_N(x_{N+1}) = v$ 　定義速度，Clamped cubic spline

(3) $if \quad s_1(x_1) = s_N(x_{N+1})$

$use \quad \dot{s}_1(x_1) = \dot{s}_N(x_{N+1})$

$\ddot{s}_1(x_1) = \ddot{s}_N(x_{N+1})$

週期運動的連續性，Periodic cubic spline

Note: Matlab® command $spline$

$$[YY] = spline(x, y, XX)$$

Example: A RRR Manipulator -1

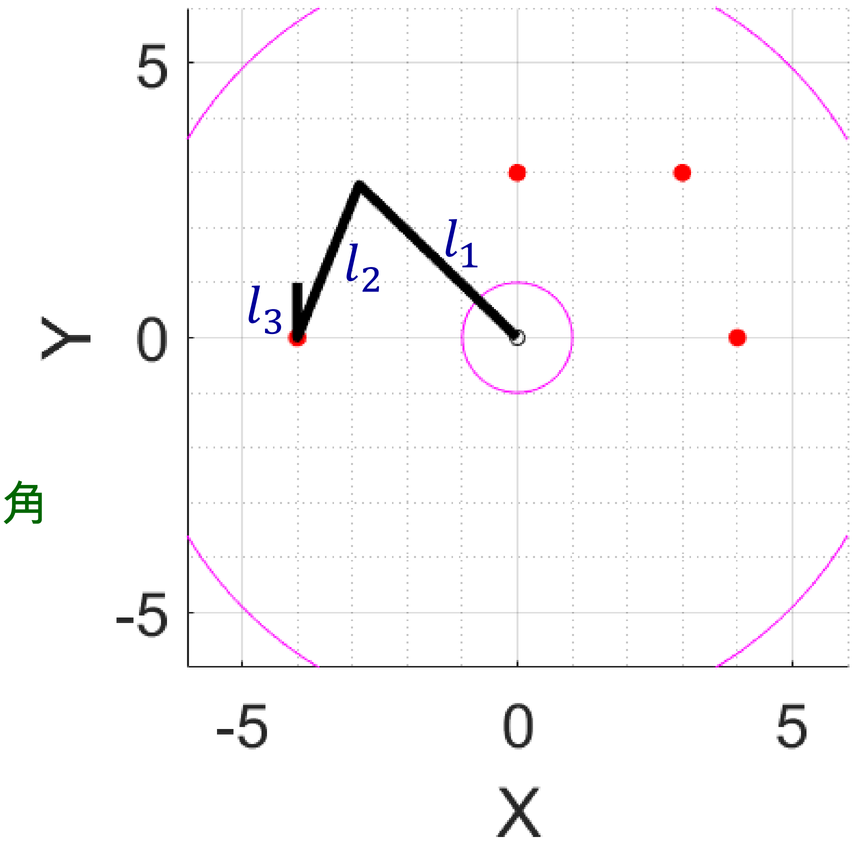
- 平面RRR手臂長度： $l_1 = 4$, $l_2 = 3$, and $l_3 = 1$

  下表定義 initial, via, via, 和 final points的位置

t	x	y	theta
0	-4	0	90
2	0	3	45
4	3	3	30
7	4	0	0

(X,Y) 定義在第二桿件的末端

Theta為第三桿件對X座標軸的夾角



Example: A RRR Manipulator -2

- 方法一：以cubic polynomials在Cartesian-space下規劃軌跡

1. 求出3個DOF (X, Y, θ) 各自cubic polynomials的coefficients

   需通過4個點：每個DOF有3個cubic polynomials，共12個未知數

$$\Theta_{12\times 1} = \underbrace{T_{12\times 12}}_{\text{為}(\Delta t_1, \Delta t_2, \Delta t_3)\text{函數}} A_{12\times 1}$$

$X/Y/\theta$

$$\begin{array}{l} \text{6 position B.C.s} \\ \\ \text{2 init., final vel.} \\ \\ \text{4 vel. \& acl. continuity} \end{array}
\begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_1 \\ \theta_2 \\ \theta_2 \\ \theta_f \\ \dot{\theta}_0 \\ \dot{\theta}_f \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & \Delta t_1 & {\Delta t_1}^2 & {\Delta t_1}^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & \Delta t_2 & {\Delta t_2}^2 & {\Delta t_2}^3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \Delta t_3 & {\Delta t_3}^2 & {\Delta t_3}^3 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2\Delta t_3 & 3{\Delta t_3}^2 \\
0 & 1 & 2\Delta t_1 & 3{\Delta t_1}^2 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 6\Delta t_1 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 2\Delta t_2 & 3{\Delta t_2}^2 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 6\Delta t_2 & 0 & 0 & -2 & 0
\end{bmatrix}
\begin{bmatrix} a_{10} \\ a_{11} \\ a_{12} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ a_{31} \\ a_{32} \\ a_{33} \end{bmatrix}$$

Example: A RRR Manipulator -3

- 方法一：以cubic polynomials在Cartesian-space下規劃軌跡

  將三個DOF在各點的參數代入matlab後將會是:

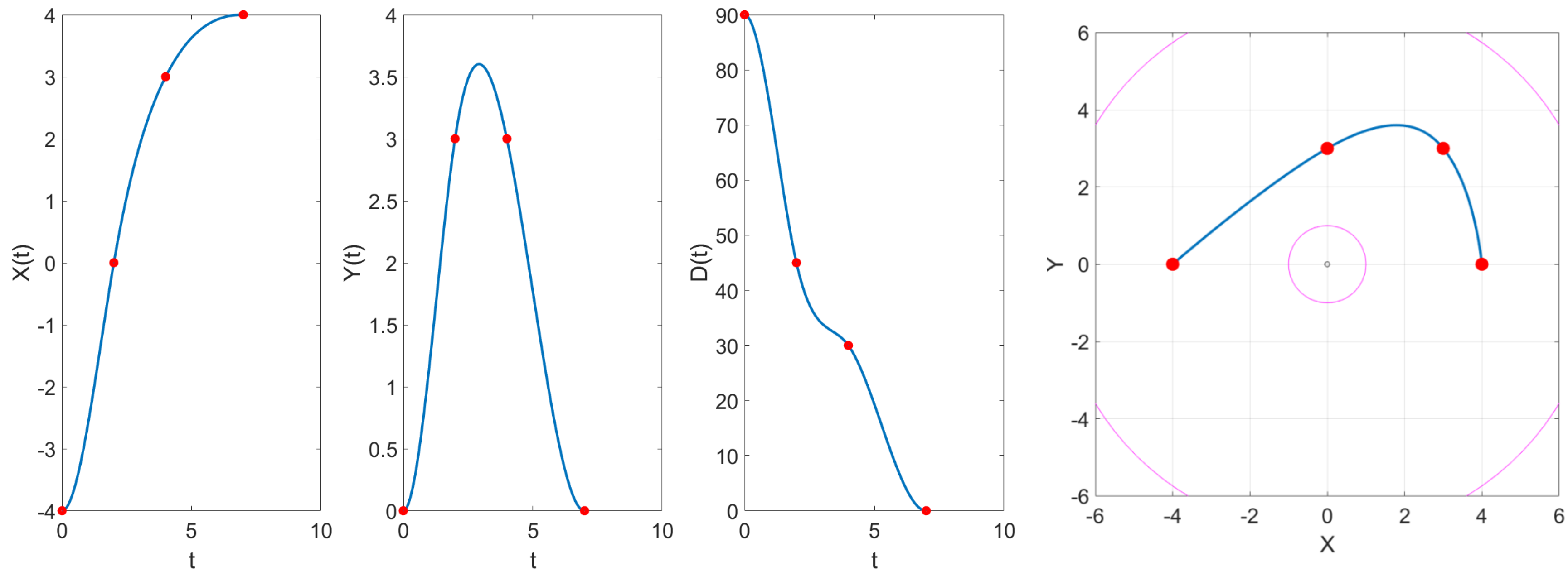
$$\Theta_{12\times 3} = T_{12\times 12}A_{12\times 3}$$

$$\begin{array}{c}
\begin{array}{ccc} X & Y & \theta \end{array} \\
\begin{bmatrix}
-4 & 0 & 90 \\
0 & 3 & 45 \\
0 & 3 & 45 \\
3 & 3 & 30 \\
3 & 3 & 30 \\
4 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\end{array}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 2 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 2 & 4 & 8 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & 9 & 27 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 6 & 27 \\
0 & 1 & 4 & 12 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 12 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 4 & 12 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 12 & 0 & 0 & -2 & 0
\end{bmatrix}
\begin{bmatrix}
\\ \\ \\ \\ \\ A_{12\times 3} \\ \\ \\ \\ \\ \\
\end{bmatrix}$$

Example: A RRR Manipulator -4

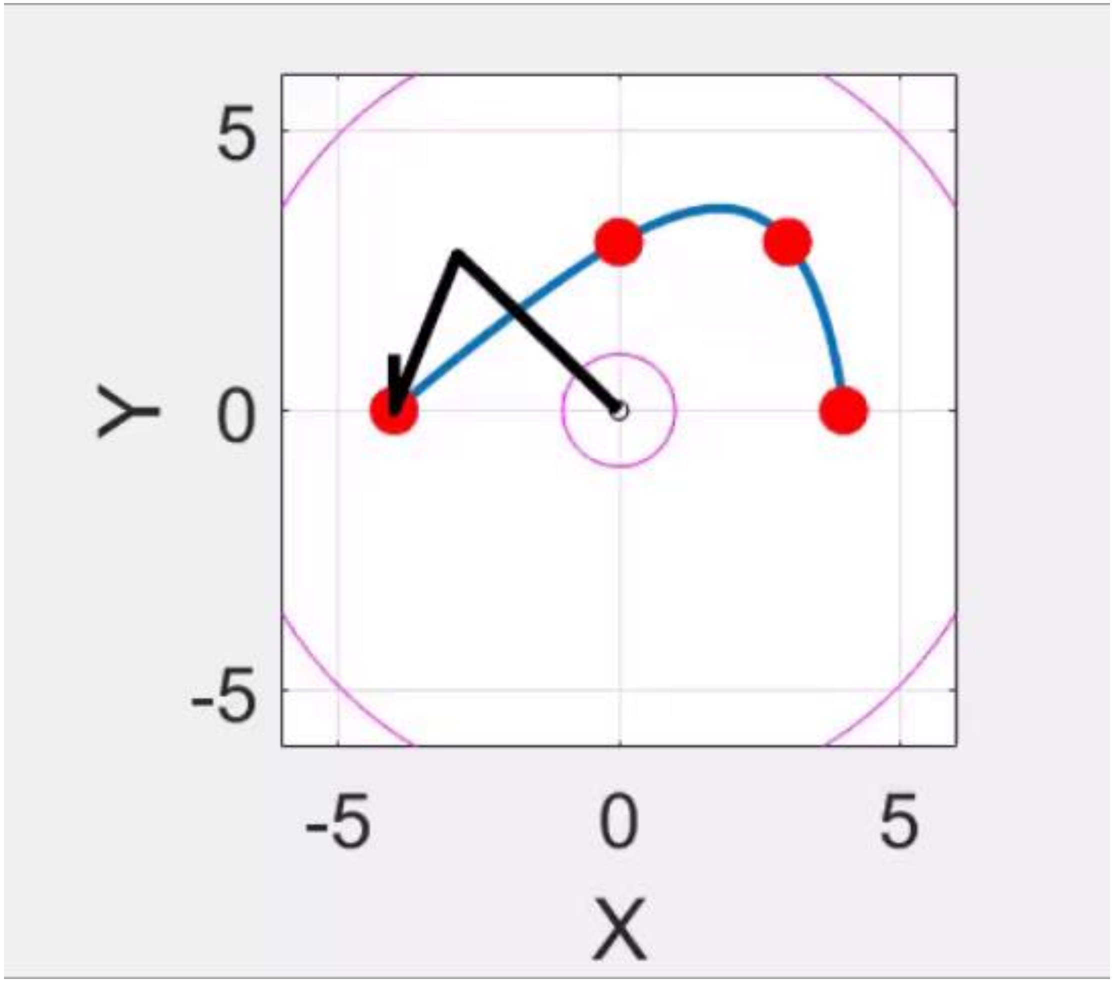
2. 對所有DOF規劃smooth trajectories

t	x	y	theta
0	-4	0	90
2	0	3	45
4	3	3	30
7	4	0	0



Example: A RRR Manipulator -5

3. I.K.，找出各Joints的對應軌跡
4. 將Joints帶入手臂模擬動作，確認手臂末端點軌跡如規劃運作



Example: A RRR Manipulator -6

- 方法二：以cubic polynomials在Joint-space下規劃軌跡

    1. I.K.，求出initial、via、final points的Joint angles $(\theta_1, \theta_2, \theta_3)$
    2. 求出各$(\theta_1, \theta_2, \theta_3)$ cubic polynomials的coefficients
       需通過4個點：每個Joint angle有3個cubic polynomials，共12個未知數

$$\Theta_{12\times3} = T_{12\times12} A_{12\times3}$$

6 position B.C.s

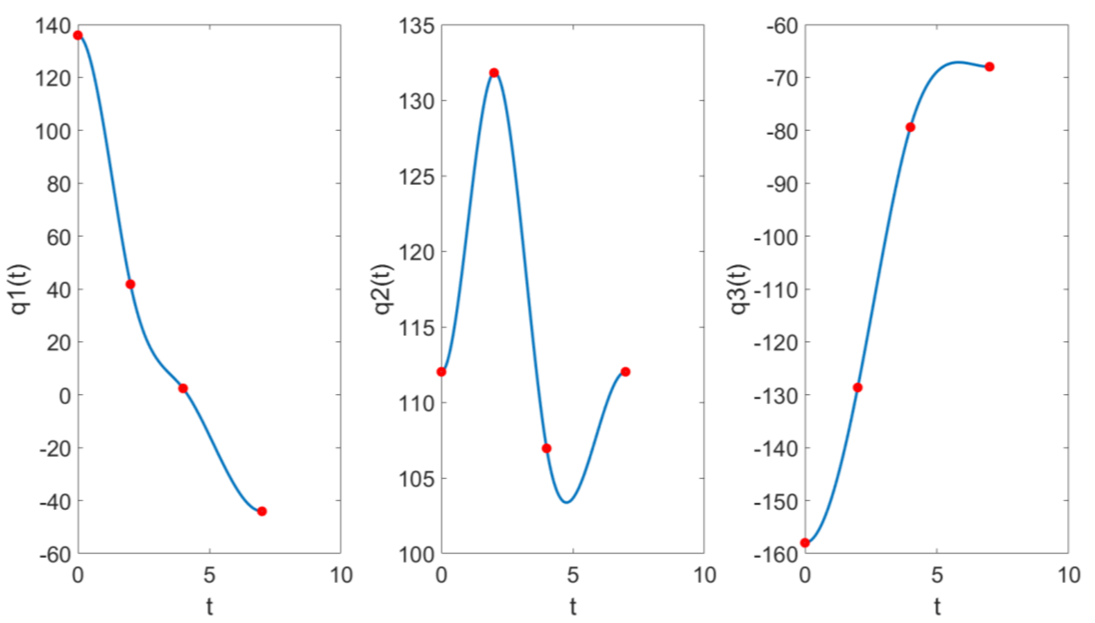
4 vel. & acl. continuity

$$\begin{array}{c} \begin{array}{ccc} \theta_1 & \theta_2 & \theta_3 \end{array} \\ \begin{bmatrix} 2.3728 & 1.9552 & -2.7572 \\ 0.7297 & 2.3005 & -2.2449 \\ 0.7297 & 2.3005 & -2.2449 \\ 0.0426 & 1.8668 & -1.3858 \\ 0.0426 & 1.8668 & -1.3858 \\ -0.7688 & 1.9552 & -1.1864 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 4 & 8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & 9 & 27 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 6 & 27 \\ 0 & 1 & 4 & 12 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 12 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 & 12 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 12 & 0 & 0 & -2 & 0 \end{bmatrix} \begin{bmatrix} \\ A_{12\times3} \\ \\ \end{bmatrix}$$

Example: A RRR Manipulator -7

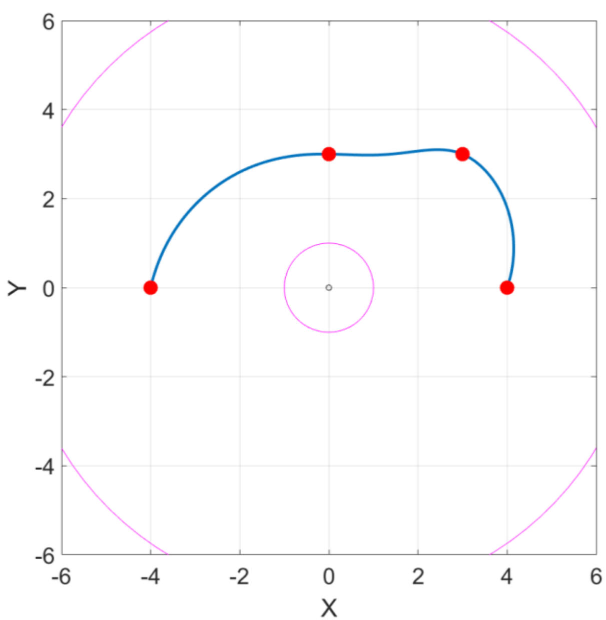
3. 對所有DOF規劃smooth trajectories

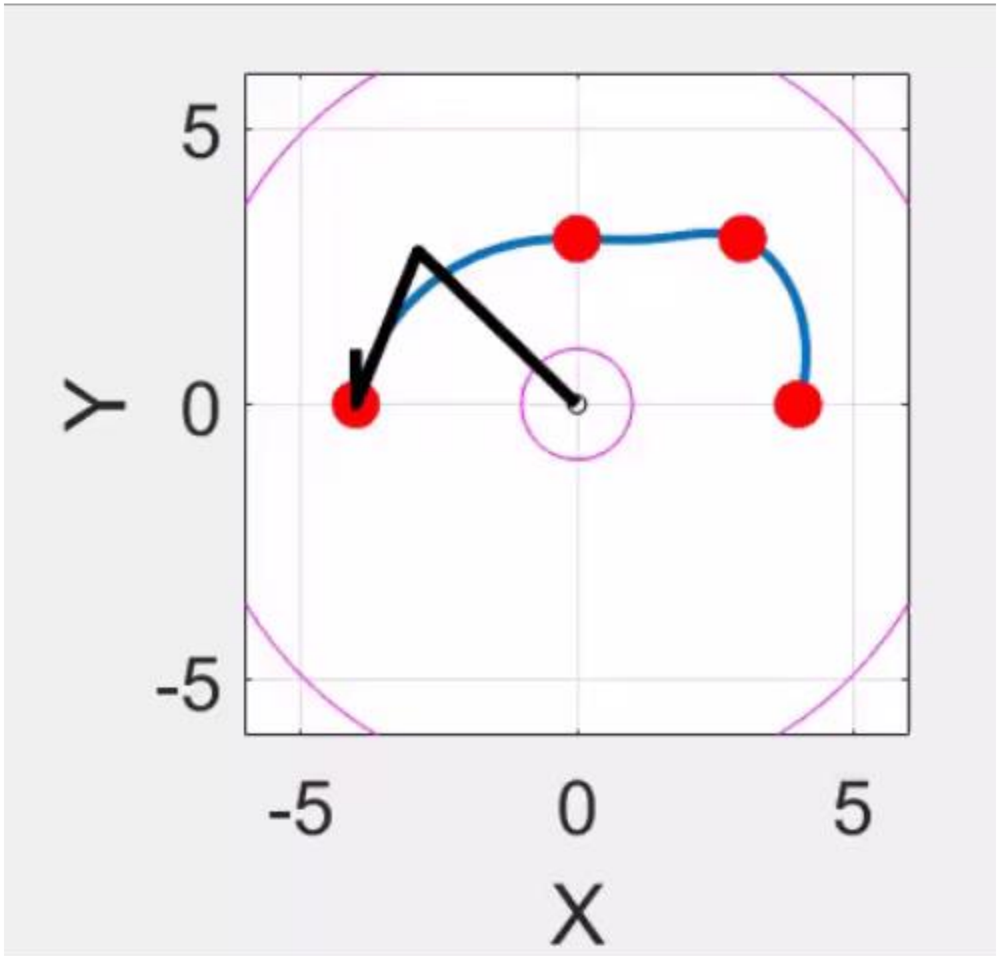
t	x	y	theta
0	-4	0	90
2	0	3	45
4	3	3	30
7	4	0	0



Example: A RRR Manipulator -8

4. 將Joints帶入手臂模擬動作，確認手臂末端點軌跡如規劃運作





High-order Polynomials

- 如果位置、速度、和加速度都必須要規劃

⇒ Quintic polynomial $\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 = \sum_{i=0}^{5} a_i t^i$

有6個條件，quintic polynomial有6個未知數

$$
\begin{cases}
\theta_0 = a_0 \\
\theta_f = a_0 + a_1 \Delta t + a_2 \Delta t^2 + a_3 \Delta t^3 + a_4 \Delta t^4 + a_5 \Delta t^5 \\
\dot{\theta}_0 = a_1 \\
\dot{\theta}_f = a_1 + 2a_2 \Delta t + 3a_3 \Delta t^2 + 4a_4 \Delta t^3 + 5a_5 \Delta t^4 \\
\ddot{\theta}_0 = 2a_2 \\
\ddot{\theta}_f = 2a_2 + 6a_3 \Delta t + 12a_4 \Delta t^2 + 20a_5 \Delta t^3
\end{cases}
$$

$$a_0 = \theta_0 \qquad a_3 = \frac{20(\theta_f - \theta_0) - (8\dot{\theta}_f + 12\dot{\theta}_0)\Delta t - (3\ddot{\theta}_0 - \ddot{\theta}_f)\Delta t^2}{2\Delta t^3}$$

$$a_1 = \dot{\theta}_0 \qquad a_4 = \frac{30(\theta_0 - \theta_f) + (14\dot{\theta}_f + 16\dot{\theta}_0)\Delta t - (3\ddot{\theta}_0 - 2\ddot{\theta}_f)\Delta t^2}{2\Delta t^4}$$

$$a_2 = \frac{1}{2}\ddot{\theta}_0 \qquad a_5 = \frac{12(\theta_f - \theta_0) - (6\dot{\theta}_f + 6\dot{\theta}_0)\Delta t - (\ddot{\theta}_0 - \ddot{\theta}_f)\Delta t^2}{2\Delta t^5}$$

1 linear
2 quadratic
3 cubic
4 quartic
5 quintic
6 hexic (sextic)
7 heptic (septic)
8 octic
9 nonic
10 decic