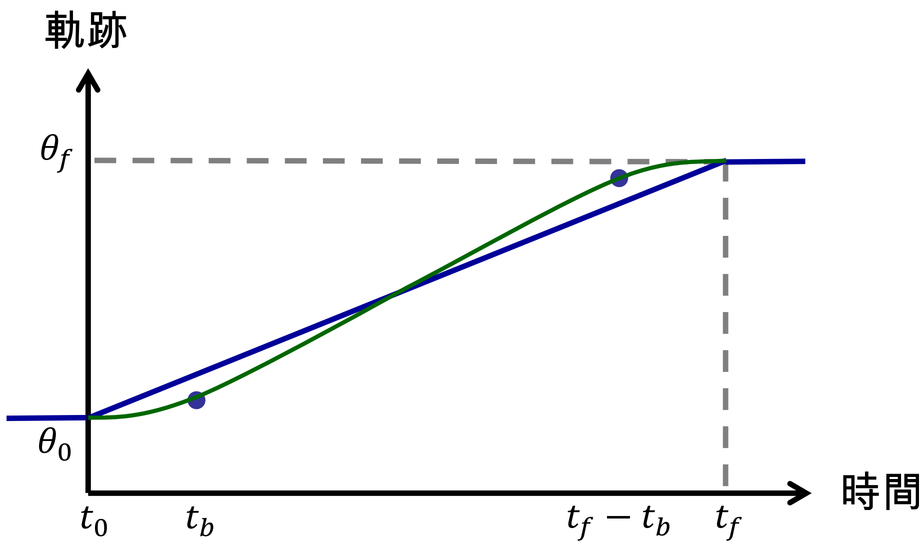
Linear Function with Parabolic Blends說明-1

- 緣由
  - 在許多類型的任務上均需要使用 直線軌跡
  - 軌跡中若包含多個直線段軌跡，線段間轉折點速度不連續
  - 解決方式：將直線段兩端修正為二次方程式，讓速度軌跡smooth



$(assume\ \dot{\theta}_0 = \dot{\theta}_f = 0)$

Linear Function with Parabolic Blends說明-2

- 規劃方式
  - linear（直線，一次多項式）
    - 等速

$$\dot{\theta} = \frac{\theta_h - \theta_b}{t_h - t_b} = \dot{\theta}_{t_b} \qquad \dots 1$$

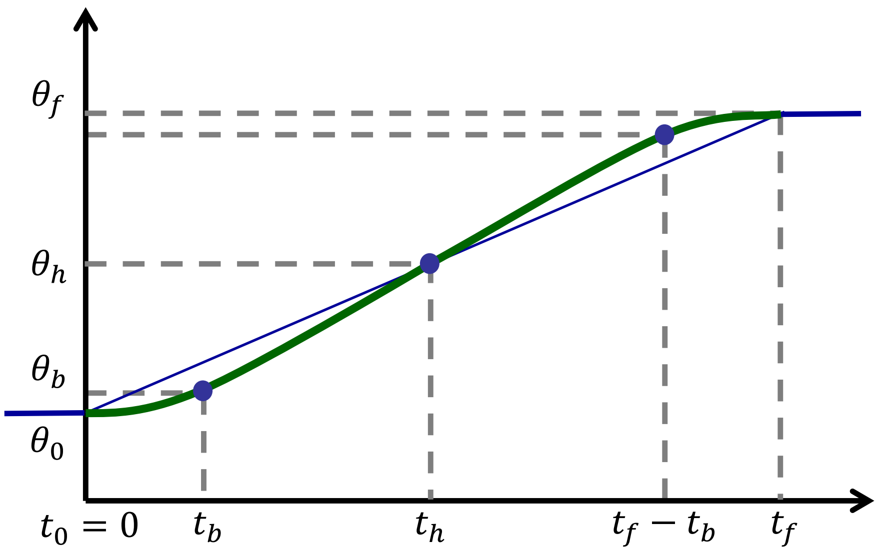
  - Parabolic（二次多項式）
    - 等加速度

$$\theta(t) = \theta_0 + \dot{\theta}_0 t + \frac{1}{2}\ddot{\theta}t^2$$

加速度

$$\dot{\theta}(t) = \dot{\theta}_0 + \ddot{\theta}t$$

$$\dot{\theta}(t_b) = \ddot{\theta}t_b \qquad \dots 2$$



$$t_h = \frac{1}{2}t_f \quad \theta_h = \frac{\theta_f + \theta_0}{2}$$

$$assume\ \dot{\theta}_0 = \dot{\theta}_f = 0$$

Linear Function with Parabolic Blends說明-3

- 交界處速度需要連續

②＝①

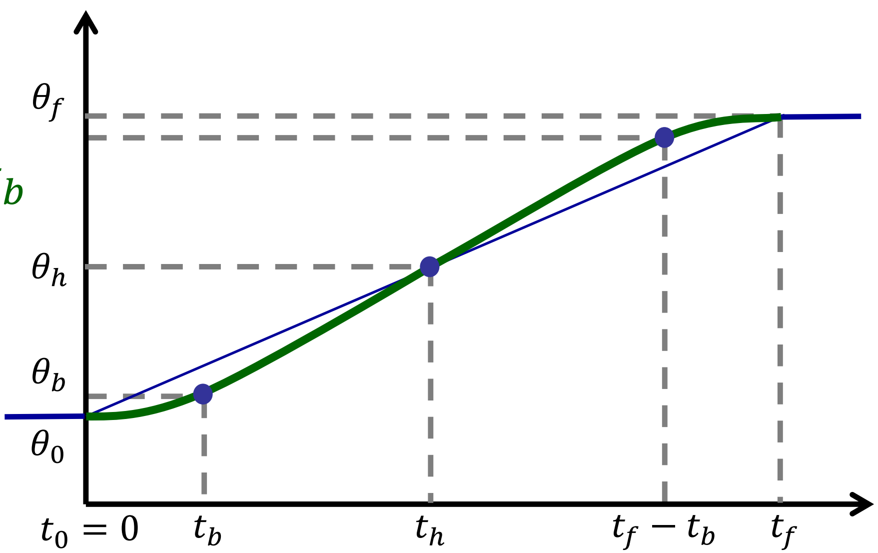
$$\ddot{\theta} t_b = \dot{\theta}_{t_b} = \frac{\theta_h - \theta_b}{t_h - t_b} = \frac{\frac{\theta_f + \theta_0}{2} - (\theta_0 + \frac{1}{2}\ddot{\theta} t_b{}^2)}{\frac{t_f}{2} - t_b} = \frac{\theta_f - \theta_0 - \ddot{\theta} t_b{}^2}{t_f - 2t_b}$$

$$\ddot{\theta} t_b{}^2 - \ddot{\theta} t_f t_b + (\theta_f - \theta_0) = 0$$

$$\Rightarrow \quad t_b = \frac{\ddot{\theta} t_f - \sqrt{\ddot{\theta}^2 t_f{}^2 - 4\ddot{\theta}(\theta_f - \theta_0)}}{2\ddot{\theta}}$$

判別式內需為正數或0，所得出t_b才為實數：$\ddot{\theta} \geq \frac{4(\theta_f - \theta_0)}{t_f{}^2}$

$$\ddot{\theta}_{min} = \frac{4(\theta_f - \theta_0)}{t_f{}^2}$$



Linear Function with Parabolic Blends說明-4

- 加速度$\ddot{\theta}$狀態討論
  - If $\ddot{\theta} = \ddot{\theta}_{min}$

    $t_b = \frac{t_f}{2} = t_h$ ⇒無linear線段,兩個parabolic曲線段相連

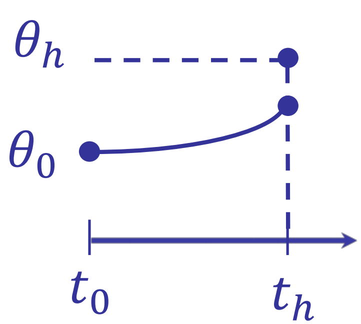
    at t_b：$\dot{\theta}(t_b) = \ddot{\theta} t_b = \frac{4(\theta_f - \theta_0)}{{t_f}^2}\frac{t_f}{2} = 2\frac{\theta_f - \theta_0}{t_f}$

    速度為和「原本無規劃直接相連的速度：$\dot{\theta} = \frac{\theta_f - \theta_0}{t_f}$」相比為2倍

  - If $\ddot{\theta} < \ddot{\theta}_{min}$

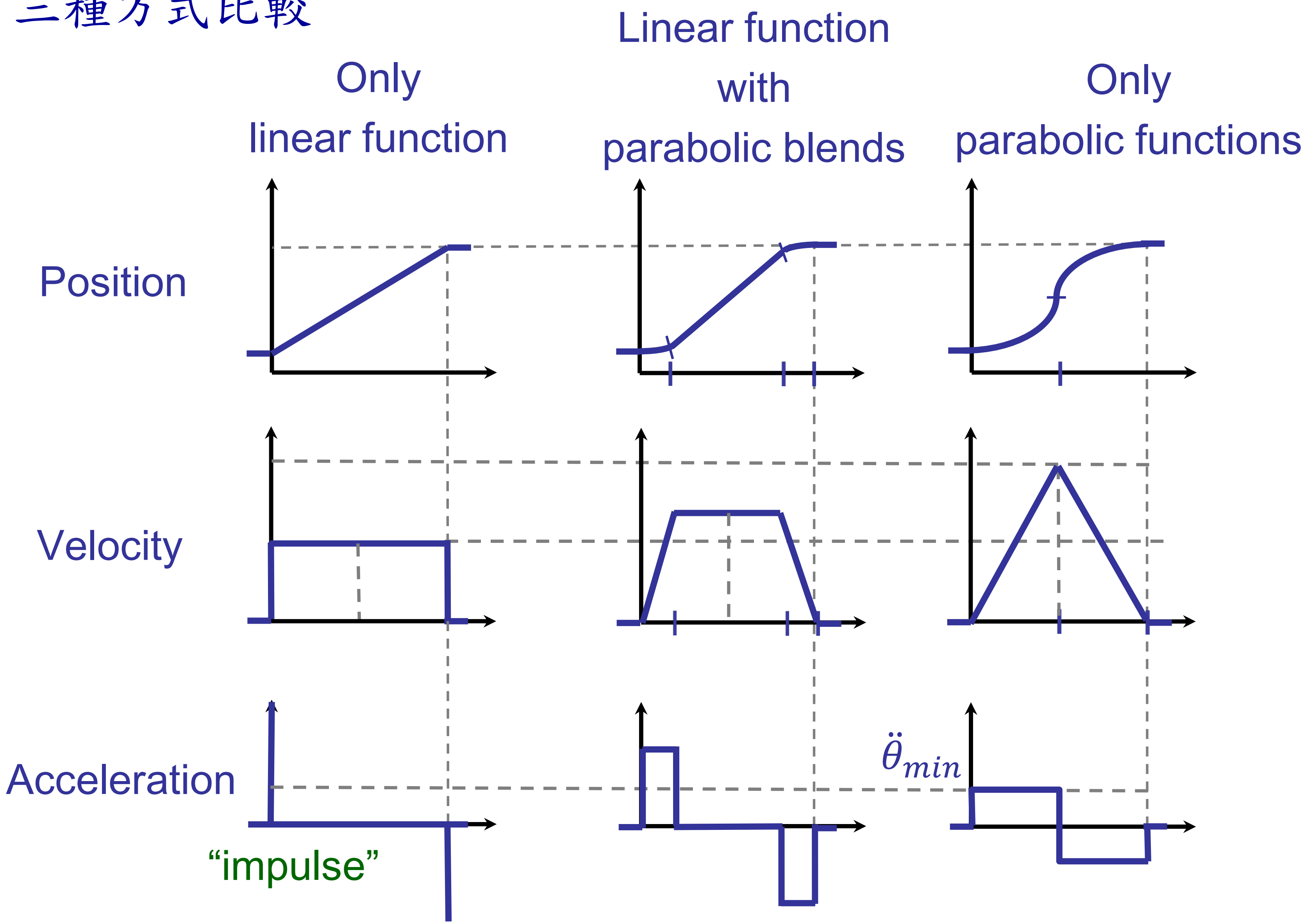
    加速度不足

    at $t_b = t_h, \quad \theta < \theta_h$



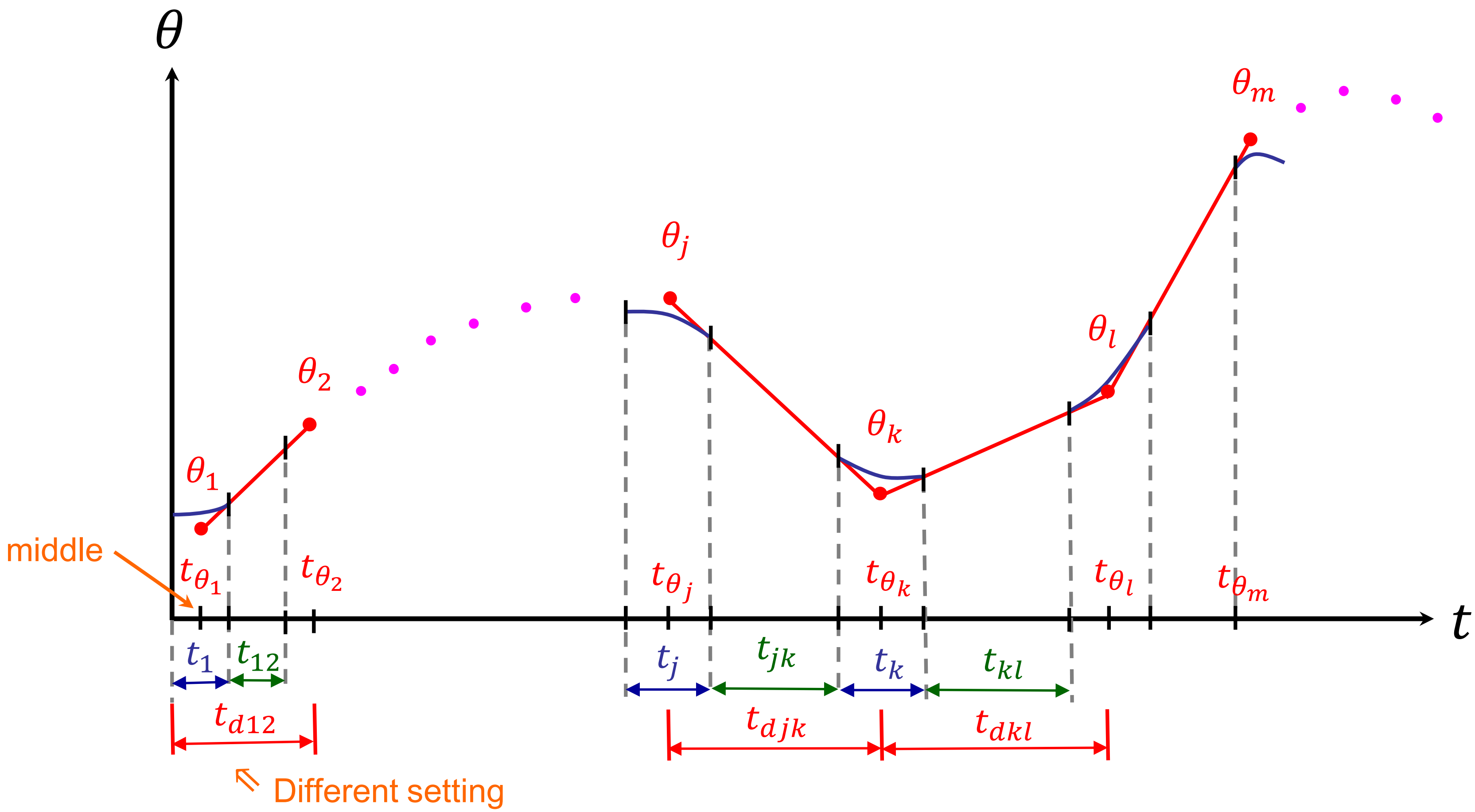
Linear Function with Parabolic Blends說明-5

- 三種方式比較



多段Linear Function with Parabolic Blends -1

- 設定：A path with n via points
  - 將每一個區段[θ_i θ_{i+1}]各自等效到之前舉例單一linear線段[θ_0 θ_f]，但與此線段前後相連接線段的速度不為0



多段Linear Function with Parabolic Blends -2

- 對任一線段 $[\,\theta_i\ \theta_{i+1}\,]$
  - linear（直線，一次多項式）

$$\dot{\theta}_{jk}=\frac{\theta_k-\theta_j}{t_{djk}}\qquad \dot{\theta}_{kl}=\frac{\theta_l-\theta_k}{t_{dkl}}$$

  - Parabolic（二次多項式）

方法一：設定加速度解時間

$$\ddot{\theta}_k=sgn(\dot{\theta}_{kl}-\dot{\theta}_{jk})|\ddot{\theta}_k|$$

$$t_k=\frac{\dot{\theta}_{kl}-\dot{\theta}_{jk}}{\ddot{\theta}_k}$$

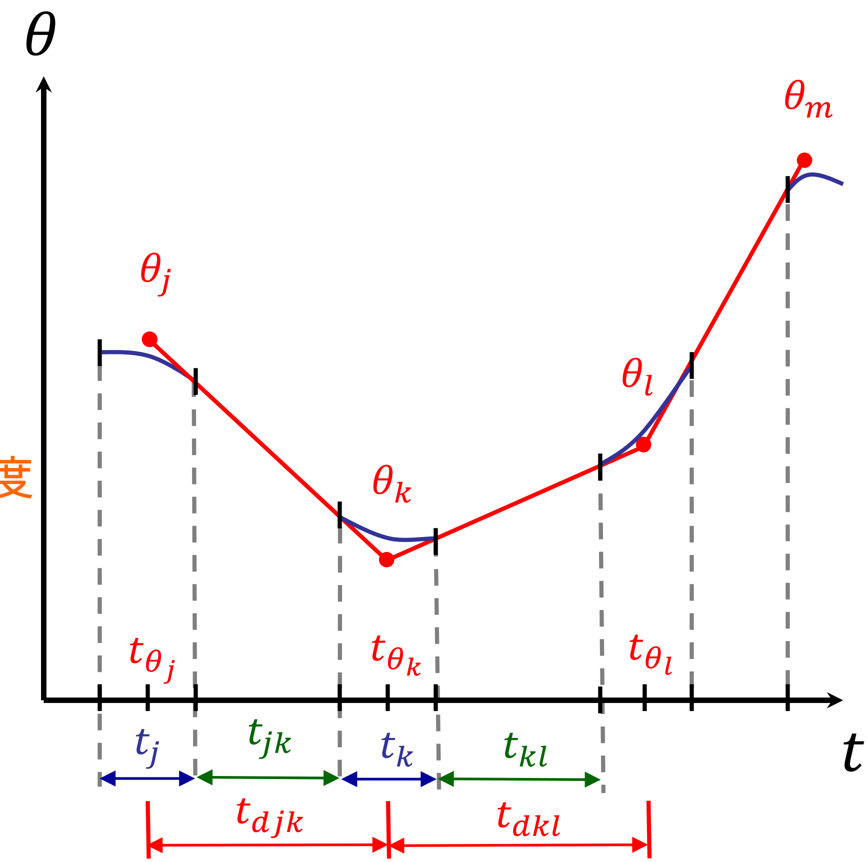
設定加速度

方法二：設定時間解加速度

$$\ddot{\theta}_k=\frac{\dot{\theta}_{kl}-\dot{\theta}_{jk}}{t_k}$$

設定時間

$$t_{jk}=t_{djk}-\frac{1}{2}t_j-\frac{1}{2}t_k$$



多段Linear Function with Parabolic Blends -3

- 第一個線段
  - θ_1可視為整段軌跡起始點θ_0在時間上往後移（parabolic曲線段所需時間t_1的一半），以導入parabolic曲線段，讓速度由起始點開始可以連續

方法一：設定加速度解時間

設定加速度

$$\ddot{\theta}_1 = sgn(\theta_2 - \theta_1)|\ddot{\theta}_1|$$

$$\dot{\theta}_{12} = \frac{\theta_2 - \theta_1}{t_{d12} - \frac{1}{2}t_1} = \ddot{\theta}_1 t_1$$

$$\frac{1}{2}\ddot{\theta}_1 t_1^2 - t_{d12}\ddot{\theta}_1 t_1 + (\theta_2 - \theta_1) = 0$$

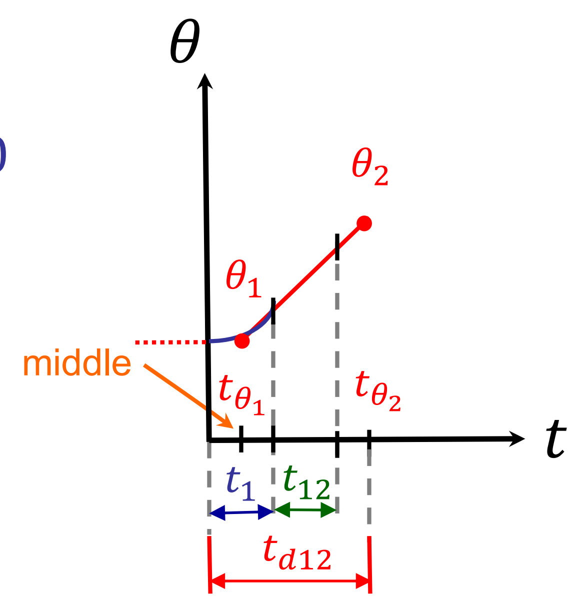
$$t_1 = t_{d12} - \sqrt{{t_{d12}}^2 - \frac{2\,(\theta_2 - \theta_1)}{\ddot{\theta}_1}}$$

方法二：設定時間解加速度

$$\ddot{\theta}_1 = \frac{\theta_2 - \theta_1}{(t_{d12} - \frac{1}{2}t_1)}\frac{1}{t_1}$$

設定時間

$$t_{12} = t_{d12} - t_1 - \frac{1}{2}t_2$$



多段Linear Function with Parabolic Blends -4

- 最後一個線段
  - θ_n可視為整段軌跡起始點θ_f在時間上往前移（parabolic曲線段所需時間t_n的一半），以導入parabolic曲線段，讓速度由起始點開始可以連續

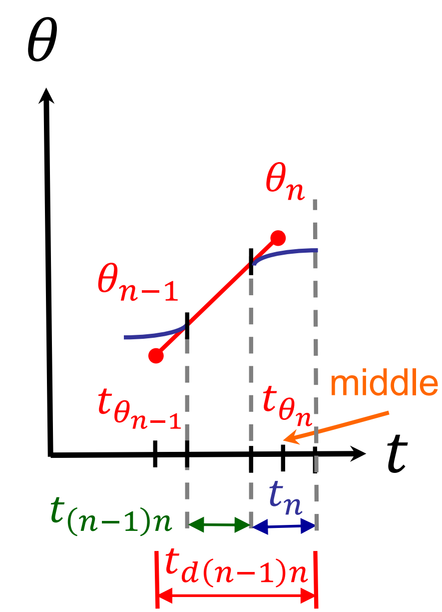
方法一：設定加速度解時間　設定加速度

$$\ddot{\theta}_n = sgn(\theta_n - \theta_{n-1})|\ddot{\theta}_n|$$

$$\dot{\theta}_{(n-1)n} = \frac{\theta_n - \theta_{n-1}}{t_{d(n-1)n} - \frac{1}{2}t_n} = \ddot{\theta}_n(-t_n)$$

$$\frac{1}{2}\ddot{\theta}_n t_n^2 - t_{d(n-1)n}\ddot{\theta}_n t_n + (\theta_n - \theta_{n-1}) = 0$$

$$t_n = t_{d(n-1)n} - \sqrt{{t_{d(n-1)n}}^2 - \frac{2(\theta_n - \theta_{n-1})}{\ddot{\theta}_n}}$$



方法二：設定時間解加速度

$$\ddot{\theta}_n = \frac{\theta_n - \theta_{n-1}}{(t_{d(n-1)n} - \frac{1}{2}t_n)} \frac{1}{-t_n}$$

設定時間

$$t_{(n-1)n} = t_{d(n-1)n} - t_n - \frac{1}{2}t_{n-1}$$

Linear Function with Parabolic Blends註解 -1

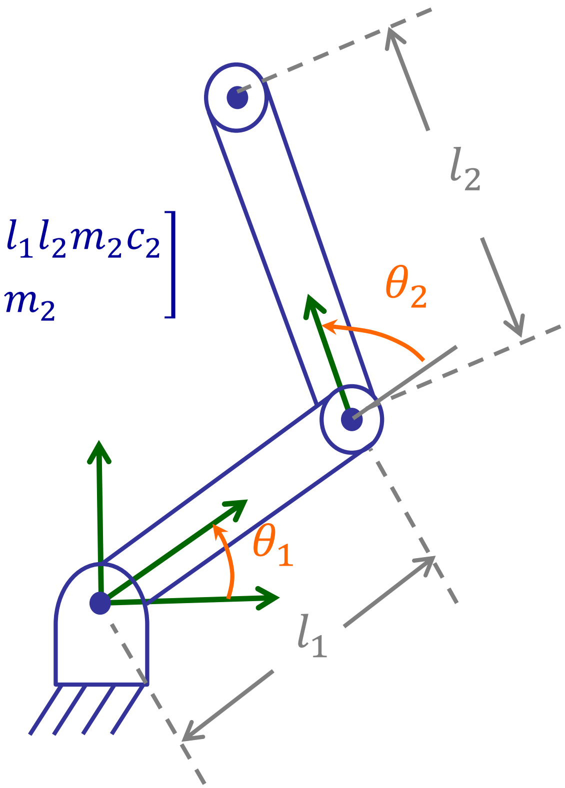
- 真實系統中可達到的加速度$\ddot{\theta}$取決於許多因素
  - 馬達規格
  - 手臂姿態：手臂在不同姿態下，各軸所需承載（如重力）的扭力不同
  - 手臂動態狀態：手臂在不同動態下，各軸需承載慣性力不同

$$\tau = M(\Theta)\ddot{\Theta} + V\left(\Theta,\dot{\Theta}\right) + G(\Theta)$$

$$M(\Theta) = \begin{bmatrix} {l_2}^2 m_2 + 2l_1 l_2 m_2 c_2 + {l_1}^2(m_1+m_2) & {l_2}^2 m_2 + l_1 l_2 m_2 c_2 \\ {l_2}^2 m_2 + l_1 l_2 m_2 c_2 & {l_2}^2 m_2 \end{bmatrix}$$

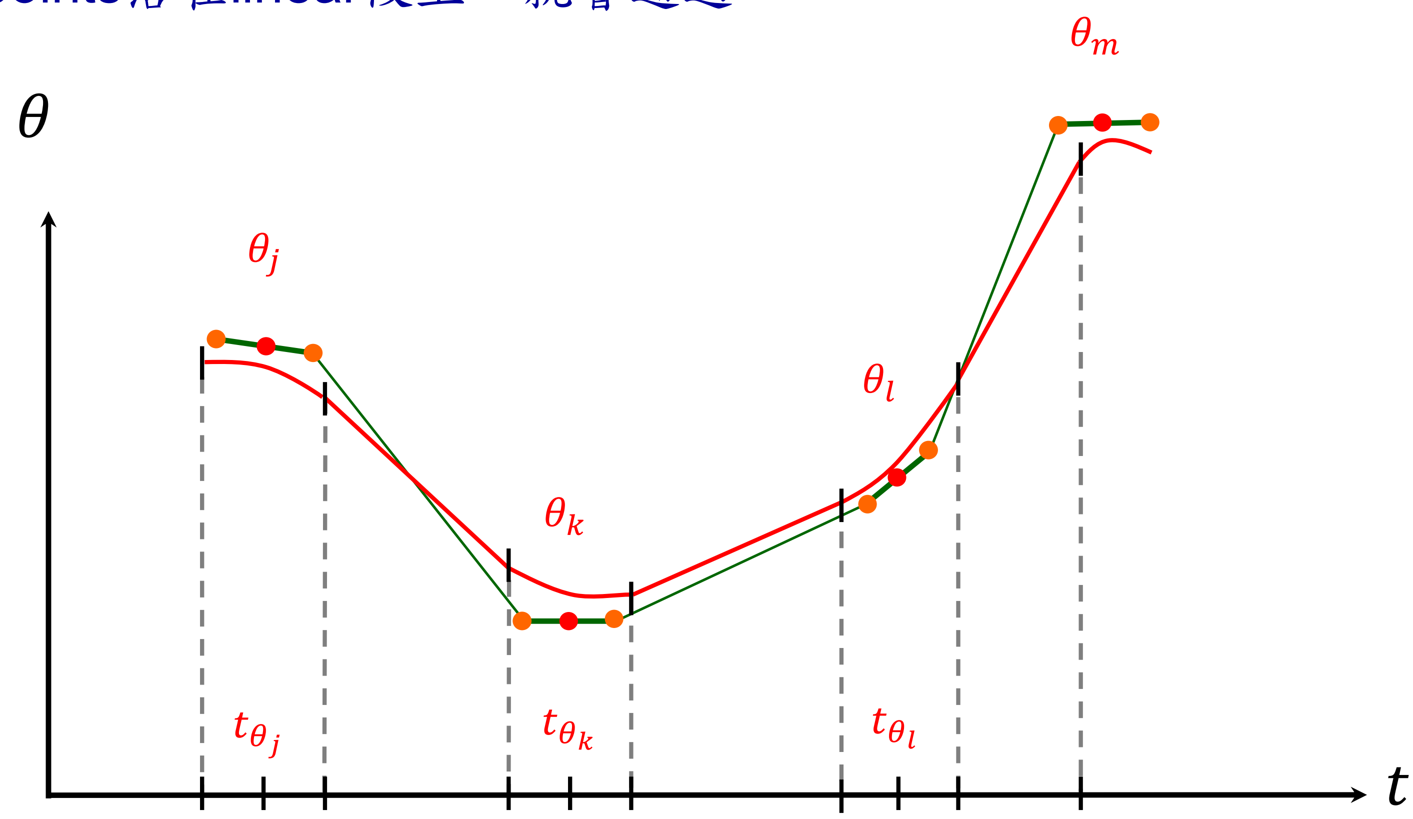
$$V\left(\Theta,\dot{\Theta}\right) = \begin{bmatrix} -m_2 l_1 l_2 s_2 {\dot{\theta}_2}^2 - 2m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 \\ m_2 l_1 l_2 s_2 {\dot{\theta}_1}^2 \end{bmatrix}$$

$$G(\Theta) = \begin{bmatrix} m_2 l_2 g c_{12} + (m_1+m_2) l_1 g c_1 \\ m_2 l_2 g c_{12} \end{bmatrix}$$



Linear Function with Parabolic Blends註解 -2

- 規劃後軌跡並未通過via points
  - 僅加速度→ ∞的軌跡有通過via points
  - 如果通過via points為必須 ⇒建立pseudo via points，讓原本via points落在linear段上，就會通過



Linear Function with Parabolic Blends註解 -3

- 若有Cartesian space下直線軌跡的需求，軌跡規劃需在Cartesian space下進行
- Programming裡，仔細定義好某時間t所屬的線段或曲線段
  - Linear: $t \in [\, t_{\theta_j} + \frac{1}{2}t_j \quad t_{\theta_k} - \frac{1}{2}t_k \,]$

$$\theta(t) = \theta_j + \dot{\theta}_{jk}\Delta t = \theta_j + \dot{\theta}_{jk}(t - t_{\theta_j})$$

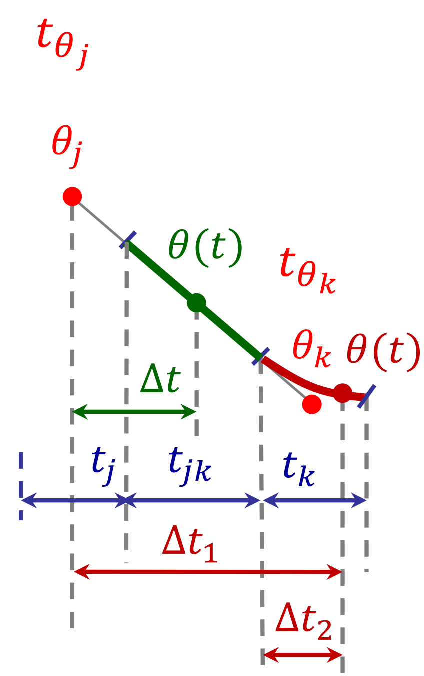
$$\dot{\theta}(t) = \dot{\theta}_{jk} \qquad \ddot{\theta} = 0$$

  - Parabolic: $t \in [\, t_{\theta_k} - \frac{1}{2}t_k \quad t_{\theta_k} + \frac{1}{2}t_k \,]$

$$\theta(t) = \theta_j + \dot{\theta}_{jk}\Delta t_1 + \frac{1}{2}\ddot{\theta}_k\,\Delta t_2{}^2$$

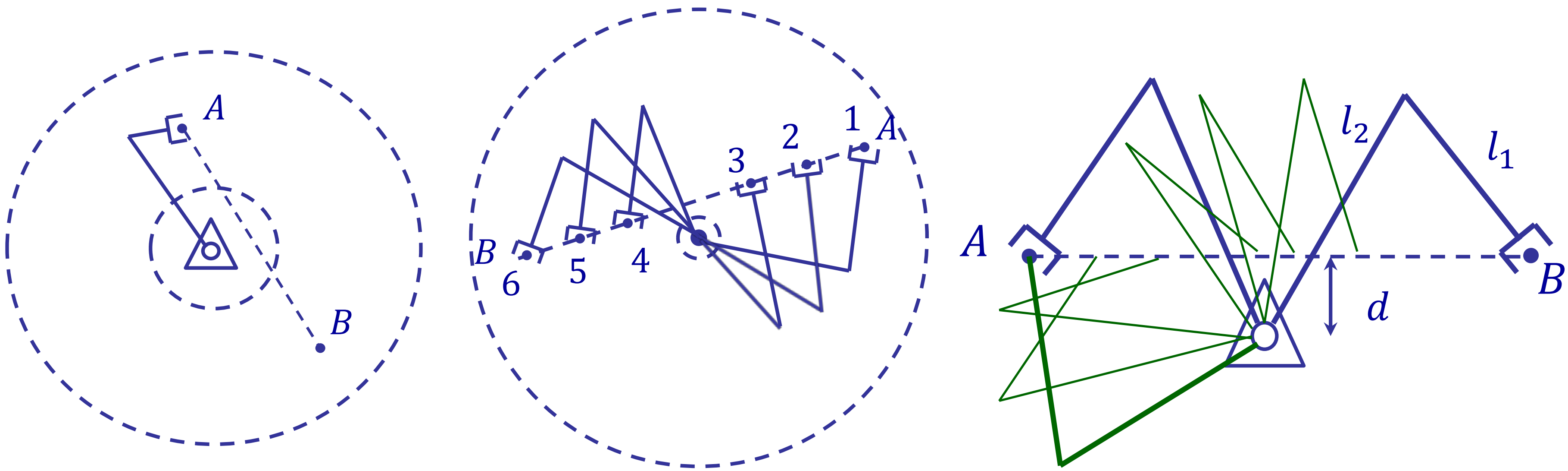
$$= \theta_j + \dot{\theta}_{jk}\left(t - t_{\theta_j}\right) + \frac{1}{2}\ddot{\theta}_k\,(t - t_{\theta_k} + \frac{1}{2}t_k)^2$$

$$\dot{\theta}(t) = \dot{\theta}_{jk} + \ddot{\theta}_k\,(t - t_{\theta_k} + \frac{1}{2}t_k) \qquad \ddot{\theta}(t) = \ddot{\theta}_k$$



Cartesian Space下軌跡幾何限制

- 中間某些區段無法到達（i.e., 在work space之外）
- 軌跡需要高加減速（i.e., 接近singular configuration）
- 針對特定起使和終點姿態，無法產生連續軌跡
    - 如下圖，除非 $l_1 - l_2 = d$



Example: Revisit the RRR Manipulator -1

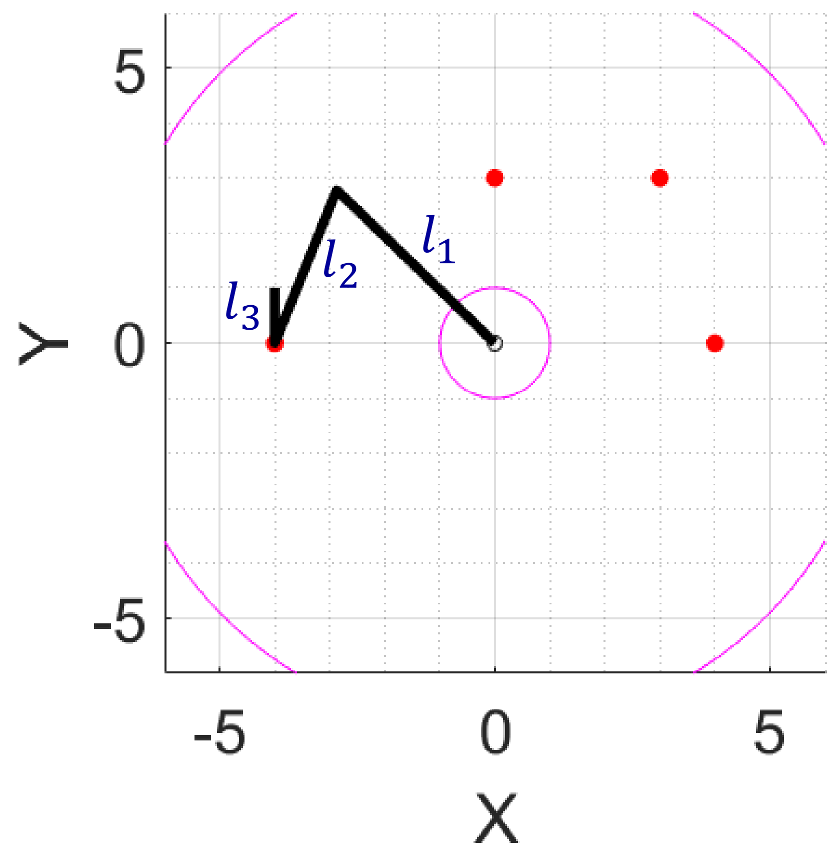
- 平面RRR手臂長度：$l_1 = 4, l_2 = 3$, and $l_3 = 1$

  下表定義 initial, via, via, 和 final points的位置

- 方法：以linear function with parabolic blends在Cartesian-space下規劃軌跡

i	t_i	x_i	y_i	θ_i
0	0	-4	0	90
1	2	0	3	45
2	4	3	3	30
3	7	4	0	0

(X,Y) 定義在第二桿件的末端
θ為第三桿件對X座標軸的夾角

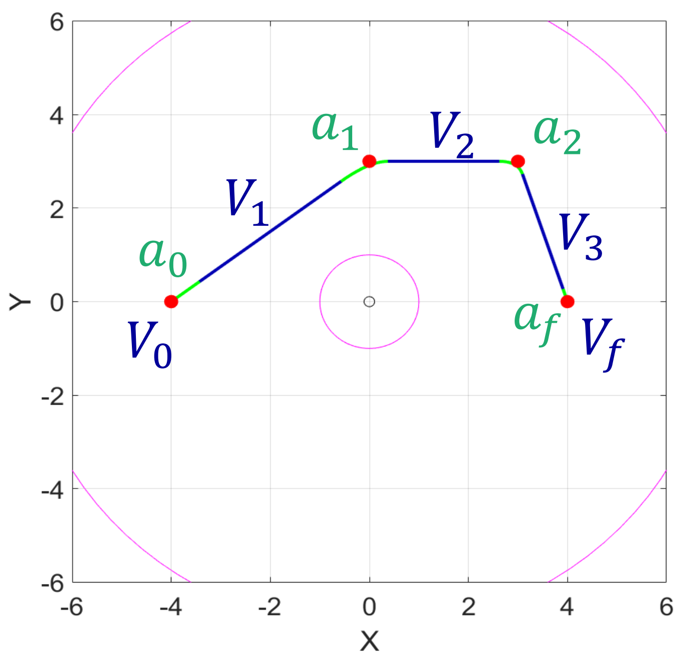


Example: Revisit the RRR Manipulator -2

1. 求出各DOF (X, Y, θ) 在每段的速度及加速度

(每段Parabolic function區間長0.5秒)

i	t_i	x_i	y_i	θ_i
0	0	-4	0	90
1	2	0	3	45
2	4	3	3	30
3	7	4	0	0



	X	Y	θ(deg/s)
V_0	0	0	0
V_1	2.29	1.71	-25.71
V_2	1.5	0	-7.5
V_3	0.36	-1.09	-10.9
V_f	0	0	0

中間線段：

$$V_2 = \frac{DOF_2 - DOF_1}{4-2}$$

頭尾線段：

$$V_1 = \frac{DOF_1 - DOF_0}{2 - 0 - \frac{0.5}{2}}$$

$$V_3 = \frac{DOF_3 - DOF_2}{7 - 4 - \frac{0.5}{2}}$$

	X	Y	θ(deg/s²)
a_0	4.57	3.43	-51.4
a_1	-1.57	-3.43	36.4
a_2	-2.27	-2.18	-6.82
a_f	-0.72	2.18	21.8

$$a_0 = \frac{V_1 - V_0}{0.5}$$

$$a_1 = \frac{V_2 - V_1}{0.5}$$

$$a_2 = \frac{V_3 - V_2}{0.5}$$

$$a_3 = \frac{V_f - V_3}{0.5}$$

Example: Revisit the RRR Manipulator -3

2. 建立各DOF (X, Y, θ) 在每段的equation (Linear/Parabolic 共7段)

以X為例:

$$X_{eq1}(t) = X_0 + V_0\Delta t + \frac{1}{2}a_0\Delta t^2 = -4 + 0(t-0) + \frac{1}{2}4.57(t-0)^2 \qquad t \in [0, 0.5]$$

$$X_{eq2}(t) = X_0 + V_1\Delta t = -4 + 2.29(t-0.25) \qquad t \in [0.5, 1.75]$$

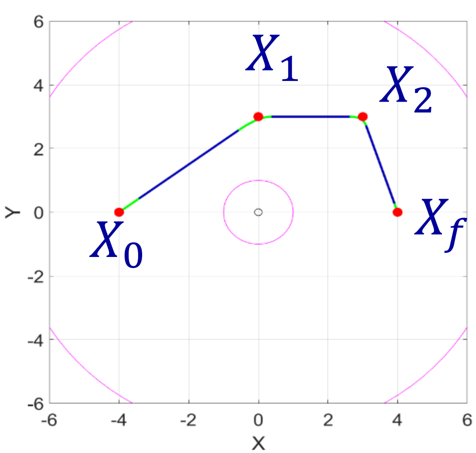
$$X_{eq3}(t) = X_0 + V_1\Delta t_1 + \frac{1}{2}a_1\Delta t_2{}^2 = -4 + 2.29(t-0.25) + \frac{1}{2}(-1.57)(t-1.75)^2 \qquad t \in [1.75, 2.25]$$

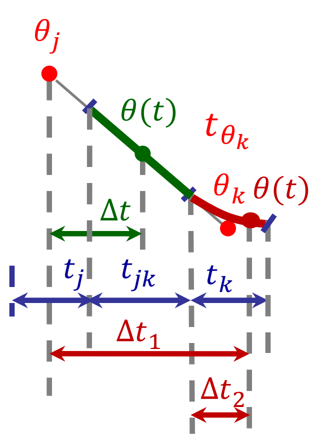
$$X_{eq4}(t) = X_1 + V_2\Delta t = 0 + 1.5(t-2) \qquad t \in [2.25, 3.75]$$

$$X_{eq5}(t) = X_1 + V_2\Delta t_1 + \frac{1}{2}a_2\Delta t_2{}^2 = 0 + 1.5(t-2) + \frac{1}{2}(-2.27)(t-3.75)^2 \qquad t \in [3.75, 4.25]$$

$$X_{eq6}(t) = X_2 + V_3\Delta t = 3 + 0.36(t-4) \qquad t \in [4.25, 6.5]$$

$$X_{eq7}(t) = X_2 + V_3\Delta t_1 + \frac{1}{2}a_3\Delta t_2{}^2 = 3 + 0.36(t-4) + \frac{1}{2}(-0.73)(t-6.5)^2 \qquad t \in [6.5, 7]$$





Recall:

Linear:

$$\theta(t) = \theta_j + \dot{\theta}_{jk}\Delta t = \theta_j + \dot{\theta}_{jk}(t - t_{\theta_j})$$

Parabolic:

$$\theta(t) = \theta_j + \dot{\theta}_{jk}\Delta t_1 + \frac{1}{2}\ddot{\theta}_k\Delta t_2{}^2$$

$$= \theta_j + \dot{\theta}_{jk}\left(t - t_{\theta_j}\right) + \frac{1}{2}\ddot{\theta}_k\left(t - t_{\theta_k} + \frac{1}{2}t_k\right)^2$$

Example: Revisit the RRR Manipulator -4

3. 繪出各DOFs (X, Y, θ) 之規劃軌跡
4. 以IK計算3軸轉角，將手臂參數帶入FK，畫出手臂的空間運動軌跡，以確認軌跡規劃的正確性