Assignment 1

Deadline: Mar. 26

1. Given two strings $x = x_1x_2\ldots x_n$ and $y = y_1y_2\ldots y_m$, we wish to find the length of their longest common substring, that is, the largest k for which there are indices i and j with $x_ix_{i+1}\ldots x_{i+k-1} = y_jy_{j+1}\ldots y_{j+k-1}$. Show how to do this in time $O(mn)$.

2. STING SAT is the following problem: given a set of clauses (each a disjunction of literals) and an integer k, find a satisfying assignment in which at most k variables are true, if such an assignment exists. Prove that STING SAT is NP-complete.

3. Given a directed graph $G = (V, E)$ with weights w_e on its edges $e \in E$. The weights can be negative or positive. The ZERO-WEIGHT-CYCLE PROBLEM is to decide if there is a simple cycle in G so that the sum of the edge weights on this cycle is exactly 0. Prove that this problem is NP-complete.

4. There are N villages, which are numbered from 1 to N, and you should build some roads such that every two villages can connect to each other. We say two villages A and B are connected, if and only if there is a road between A and B, or there exists a village C such that there is a road between A and C, and C and B are connected.

The distance between every two villages is known. Furthermore, there are already some roads between some villages and your job is to build some roads such that all the villages are connect and the distance of all roads newly built is minimum.

5. Let G be a n vertices graph. Show that if every vertex in G has degree at least $n/2$, then G contains a Hamiltonian path.

6. Show that INDEPENDENT SET PROBLEM is NP-hard even graphs of maximum degree 3.

7. For your new startup company, *Uber for Algorithms*, you are trying to assign projects to employees. You have a set P of n projects and a set E of m employees. Each employee e can only work on one project, and each project $p \in P$ has a subset $E_p \subseteq E$ of employees that must be assigned to p to complete p. The decision problem we want to solve is whether we can assign the employees to projects such that we can complete (at least) k projects.

- Give a straightforward algorithm that checks whether any subset of k projects can be completed to solve the decisional problem. Analyze its time complexity in terms of m, n, and k.
- Show that the problem is NP-hard via a reduction from 3D matching.

8. Let $d \in N$. The d-COLORABILITY problem is to decide whether a given graph $G = (V, E)$ can be colored by d colors. i.e., whether there exists a function $f : V \to \{1, \ldots, d\}$ such that for every

$u, v \in V$ with $\{u, v\} \in E$ we have $f(u) \neq f(v)$. Formulate d-COLORABILITY as a search problem. Give a reduction from 4-COLORABILITY to 7-COLORABILITY.