Assignment 3

Deadline: May 14

1. Let $G = (V, E)$ be an undirected graph with nonnegative edge costs. S, the senders and R, the receivers, are disjoint subsets of V. The problem is to find a minimum cost subgraph of G such that for every receiver r in R, there is at least one sender s in S such that there is a path connecting r to s in the subgraph. Give a factor 2 approximation algorithm that runs in polynomial time.

2. Use layering to get a factor f approximation algorithm for set cover, where f is the frequency of the most frequent element. Provide a tight example for this algorithm.

3. Given an undirected graph. The problem is to remove a minimum number of edges such that the residual graph contains no triangle. (I.e., there is no three vertices a, b, c such that edges $(a, b), (b, c), (c, a)$ are all in the residual graph.) Give a factor 3 approximation algorithm.

4. Consider the maximum weighted matching problem, where you are given a graph $G = (V, E)$ with nonnegative weights on the edges, and your goal is to find a maximum weight set of edges such that no two edges from the set share a vertex, i.e., they form a matching. It's known that this problem can be solved exactly in polynomial time. Your task here however, is to give a linear time 2-approximation algorithm.

5. Given n points in $\mathbb{R}^2$, define the optimal Euclidean Steiner tree to be a minimum length tree containing all n points and any other subset of points from $\mathbb{R}^2$. Prove that each of the additional points must have degree three, with all three angles being $120°$.

6. Consider a more restricted algorithm than First-Fit, called Next-Fit, which tries to pack the next item only in the most recently started bin. If it does not fit, it is packed in a new bin. Show that this algorithm also achieves factor 2. Give a factor 2 tight example.

7. Consider the following problem: Given an undirected graph and compute the number of matchings in the graph. Show that if we have an α-approximation algorithm for it for some constant α, then we also have a PTAS.

8. Given an undirected complete graph, each edge is assigned with a nonnegative cost by the function c. Find a Hamilton cycle with the largest cost by the greedy approach, and prove the guarantee factor is 2.