Homework_2

Xinyi Lin

2/13/2019

Problem 1

Question 1

Model	Estimate of β	CI of β	Deviance	$\hat{p}$(dying|x=0.01)
logit	1.1618949	(0.806, 1.518)	0.3787	0.0901
probit	0.6863805	(0.497, 0.876)	0.3137	0.0853
c-log-log	0.7468193	(0.532, 0.961)	2.23	0.1282

Comments: According to results, we can find that estimated β of three methods are different. The model given by probit method has smallest deviance, while the model given by complementary log-log method has biggest deviance. Besides, probability of dying when $x = 0.01$ given by logit and probit methods are similar while probability given by complementary log-log method is larger.

Question 2

When using logit method or probit method, as $\pi = 0.5$, $g(0.5) = \log\frac{0.5}{1-0.5} = 0 = \beta_0 + \beta_1 x$, $g(0.5) = \phi(0.5) = 0 = \beta_0 + \beta_1 x$, $x = -\frac{\beta_0}{\beta_1}$

$\frac{\partial x_0}{\partial \beta_0} = -\frac{1}{\beta_1}, \frac{\partial x_0}{\partial \beta_1} = -\frac{\beta_0}{\beta_1^2}$

$Var(\hat{x}_0) = (\frac{\partial x_0}{\partial \beta_0})^2 Var(\hat{\beta}_0) + (\frac{\partial x_0}{\partial \beta_1})^2 Var(\hat{\beta}_1) + 2(\frac{\partial x_0}{\partial \beta_0})(\frac{\partial x_0}{\partial \beta_1}) cov(\hat{\beta}_0, \hat{\beta}_1) = \frac{Var(\hat{\beta}_0)}{\beta_1^2} + \frac{\beta_0^2 Var(\hat{\beta}_1)}{\beta_1^4} + \frac{2\beta_0 cov(\hat{\beta}_0, \hat{\beta}_1)}{\beta_1^3}$

When using complementary log-log method, $\pi = 0.4$, $g(0.5) = \log(-\log\frac{1}{2}) = \beta_0 + \beta_1 x$, $x = \frac{\log(-\log\frac{1}{2}) - \beta_0}{\beta_1}$

$\frac{\partial x_0}{\partial \beta_0} = -\frac{1}{\beta_1}, \frac{\partial x_0}{\partial \beta_1} = -\frac{\log(-\log\frac{1}{2}) - \beta_0}{\beta_1^2}$

$Var(\hat{x}_0) = (\frac{\partial x_0}{\partial \beta_0})^2 Var(\hat{\beta}_0) + (\frac{\partial x_0}{\partial \beta_1})^2 Var(\hat{\beta}_1) + 2(\frac{\partial x_0}{\partial \beta_0})(\frac{\partial x_0}{\partial \beta_1}) cov(\hat{\beta}_0, \hat{\beta}_1) = \frac{Var(\hat{\beta}_0)}{\beta_1^2} + \frac{[\log(-\log\frac{1}{2}) - \beta_0]^2 Var(\hat{\beta}_1)}{\beta_1^4} + \frac{2[\log(-\log\frac{1}{2}) - \beta_0] cov(\hat{\beta}_0, \hat{\beta}_1)}{\beta_1^3}$

By using R to calculate results, we get the point estimate of LD 50 is 7.389 and CI is (5.510, 9.909) based on logit method, the point estimate of LD 50 is 7.436 and CI is (5.582, 9.904) based on probit method and the point estimate of LD 50 is 8.841 and CI is (6.526, 11.977) based on complementary log-log method.

Problem 2

For each offer $Y_i \sim Bin(1, \pi_i)$, so we use logit method to build the link function.

Question 1

By using R to fit model, we get $\beta_0 = -1.648, \beta_1 = 0.031$. Then, we calculate deviance to test how does the model fit data. Deviance equals 10.613 which follows chi-square distribution with 15 degrees of freedom. The corresponding p-value is 0.780, so we fail to reject null hypothesis and the model fits data well.

Question 2

β_0:By using R to fit model, we get $\beta_0 = -1.648$, which means the log odds of enrollment rate is -1.648 given scholarship amount is 0. The 95% CI of β_0 is (-2.474, -0.822).

β_1:By using R to fit model, we get $\beta_1 = 0.031$, which means the log odds ration of enrollment rate is 0.031 when scholarship amount increases 1 thousand dollars. The 95% CI of β_1 is (0.012, 0.050).

Question 3

As yield rate $= \pi = 0.4$, $g(0.4) = \log\frac{0.4}{1-0.4} = \log\frac{2}{3} = \beta_0 + \beta_1 x$, $x = \frac{\log\frac{2}{3} - \beta_0}{\beta_1}$

$\frac{\partial x_0}{\partial \beta_0} = -\frac{1}{\beta_1}, \frac{\partial x_0}{\partial \beta_1} = -(\log\frac{2}{3} - \beta_0)\frac{1}{\beta_1^2}$

$Var(\hat{x}_0) = (\frac{\partial x_0}{\partial \beta_0})^2 Var(\hat{\beta}_0) + (\frac{\partial x_0}{\partial \beta_1})^2 Var(\hat{\beta}_1) + 2(\frac{\partial x_0}{\partial \beta_0})(\frac{\partial x_0}{\partial \beta_1}) cov(\hat{\beta}_0, \hat{\beta}_1) = \frac{Var(\hat{\beta}_0)}{\beta_1^2} + \frac{(\log\frac{2}{3} - \beta_0)^2 Var(\hat{\beta}_1)}{\beta_1^4} + \frac{2(\log\frac{2}{3} - \beta_0) cov(\hat{\beta}_0, \hat{\beta}_1)}{\beta_1^3}$

By using R to calculate, we can get estimated x(scholarship) is 40.134 thousand dollars and 95% CI is (30.583, 49.686) thousand dollars.