Homework 4

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Problem 1

Question 1

$$b_1 = \frac{n\sum x_i Y_i - \sum x_i \sum Y_i}{n\sum x_i^2 - (\sum x_i)^2} = \frac{\sum x_i Y_i - n\hat{Y}\,\overline{x}}{\sum x_i^2 - n\overline{x}^2}$$

$$b_0 = \hat{Y} - b_1\overline{x}$$

Since

$$\sum x_i Y_i - n\overline{Y}\overline{x} = \sum x_i Y_i - \overline{x}\sum Y_i = \sum (x_i - \overline{x})Y_i$$

So, the expectation of b_1's numerator is

$$\begin{aligned} E\{\sum (x_i - \overline{x})Y_i\} &= \sum (x_i - \overline{x})E(Y_i) \\ &= \sum (x_i - \overline{x})(\beta_0 + \beta_1 x_i) \\ &= \beta_0 \sum x_i - n\overline{x}\beta_0 + \beta_1 \sum {x_i}^2 - n\overline{x}^2\beta_1 \\ &= \beta_1(\sum x_i^2 - n\overline{x}^2) \end{aligned}$$

So

So b_1 and b_0 are unbiased estimators of β_1 and β_0.

Question 2

As $\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1$, $b_1 = \frac{n\sum x_i Y_i - \sum x_i \sum Y_i}{n\sum x_i^2 - (\sum x_i)^2} = \frac{\sum x_i Y_i - n\hat{Y}\overline{x}}{\sum x_i^2 - n\overline{x}^2}$ and estimated regression model $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1\, x_i$,

when $x_i = \overline{x}$,

$$\hat{Y}_i = \overline{Y} - \hat{\beta}_1\,\overline{x} + \hat{\beta}_1\,\overline{x} = \overline{Y}$$

so regression model always goes through the point $(\overline{x}, \overline{y})$.

```r
library(tidyverse)
library(patchwork)
```

Problem 2

First, we need to import data

```
HeartDisease_df = read_csv("./data/HeartDisease.csv")

## Parsed with column specification:
## cols(
##   id = col_integer(),
##   totalcost = col_double(),
##   age = col_integer(),
##   gender = col_integer(),
##   interventions = col_integer(),
##   drugs = col_integer(),
##   ERvisits = col_integer(),
##   complications = col_integer(),
##   comorbidities = col_integer(),
##   duration = col_integer()
## )

head(HeartDisease_df)

## # A tibble: 6 x 10
##      id totalcost   age gender interventions drugs ERvisits complications
##   <int>     <dbl> <int>  <int>         <int> <int>    <int>         <int>
## 1     1      179.    63      0             2     1        4             0
## 2     2      319     59      0             2     0        6             0
## 3     3     9311.    62      0            17     0        2             0
## 4     4      281.    60      1             9     0        7             0
## 5     5    18727.    55      0             5     2        7             0
## 6     6      453.    66      0             1     0        3             0
## # ... with 2 more variables: comorbidities <int>, duration <int>
```

Question 1

This dataset includes 788 observations and 10 variables. Among variables, main outcome is `totalcost` and main predictor is `ERvisits`.

Then, we show descriptive statistics for all variables of interest.

```
mean_and_sd = function(x) {

  if (!is.numeric(x)) {
    stop("Argument x should be numeric")
  } else if (length(x) == 1) {
    stop("Cannot be computed for length 1 vectors")
  }

  mean_x = mean(x)
  sd_x = sd(x)
```

```r
  list(mean = mean_x,
       sd = sd_x)
}
```

totalcost

```r
mean_and_sd(HeartDisease_df$totalcost)

## $mean
## [1] 2799.956
##
## $sd
## [1] 6690.26
```

ERvisits

```r
mean_and_sd(HeartDisease_df$ERvisits)

## $mean
## [1] 3.425127
##
## $sd
## [1] 2.637474
```

age

```r
mean_and_sd(HeartDisease_df$age)

## $mean
## [1] 58.71827
##
## $sd
## [1] 6.754118
```

gender

```r
summary(as.factor(HeartDisease_df$gender))

##   0   1
## 608 180
```

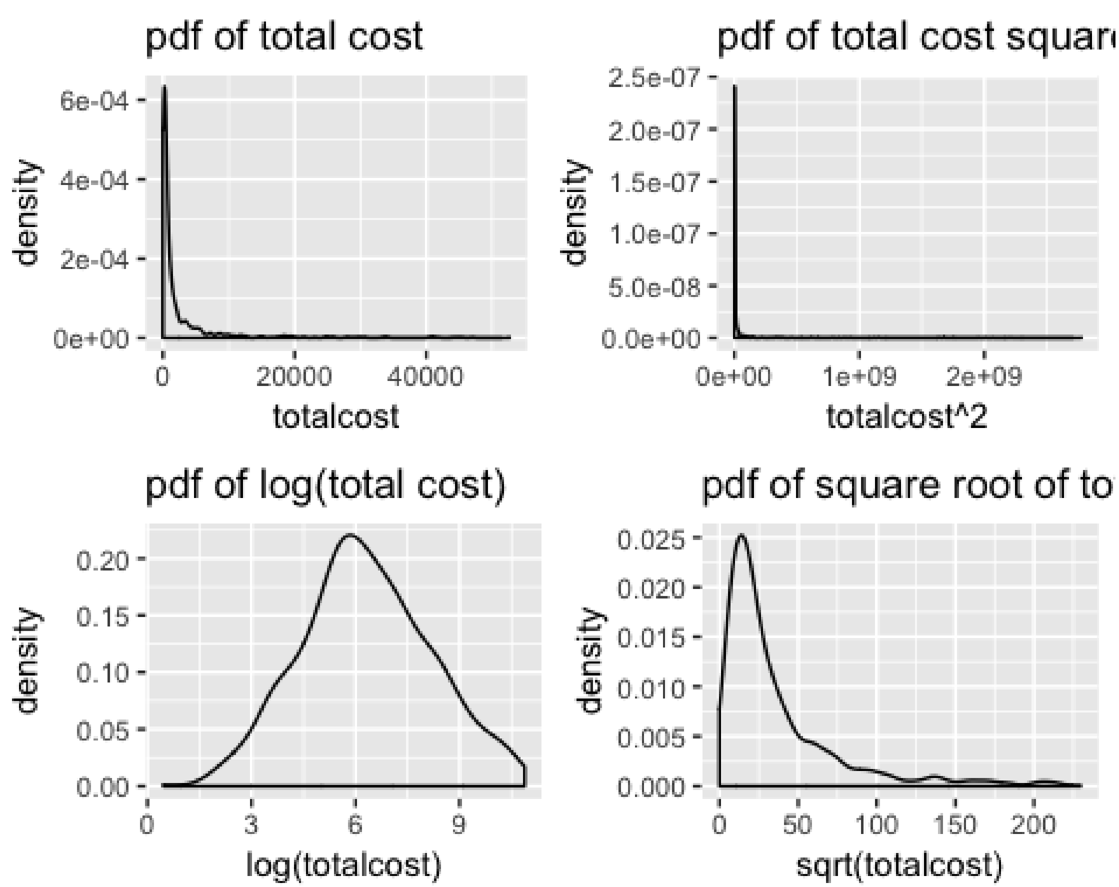
complications

```r
summary(as.factor(HeartDisease_df$complications))

##   0   1   3
## 745  42   1
```

Question 2

```r
total_plot =
  HeartDisease_df %>%
  ggplot(aes(x = totalcost)) +
```

```
  geom_density() +
  labs(title = "pdf of total cost")

log_plot =
  HeartDisease_df %>%
  ggplot(aes(x = log(totalcost))) +
  geom_density() +
  labs(title = "pdf of log(total cost)")

sqrt_plot =
  HeartDisease_df %>%
  ggplot(aes(x = sqrt(totalcost))) +
  geom_density() +
  labs(title = "pdf of square root of total cost")

square_plot =
  HeartDisease_df %>%
  ggplot(aes(x = totalcost^2)) +
  geom_density() +
  labs(title = "pdf of total cost square")

(total_plot + square_plot)/(log_plot + sqrt_plot)

## Warning: Removed 3 rows containing non-finite values (stat_density).
```



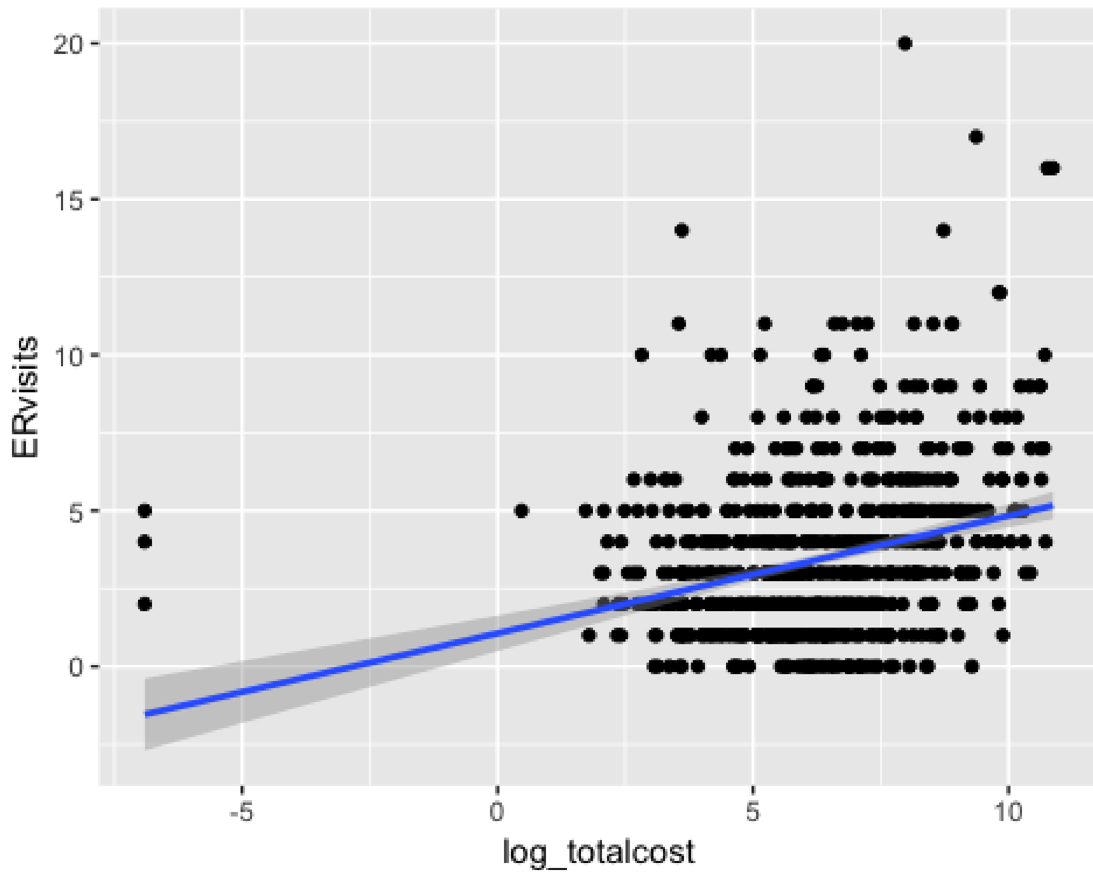
Above are distribution of total cost, log(totalcost), suqre root of totalcost and totalcost square. We can find that apply log to total cost is the best transformations.

Question 3

```
HeartDisease_df =
  HeartDisease_df %>%
  mutate(comp_bin = ifelse(complications == 0, 0, 1)) %>%
  mutate(totalcost = ifelse(totalcost == 0, 0.001, totalcost))

head(HeartDisease_df)

## # A tibble: 6 x 11
##      id totalcost   age gender interventions drugs ERvisits complications
##   <int>     <dbl> <int>  <int>         <int> <int>    <int>         <int>
## 1     1      179.    63      0             2     1        4             0
## 2     2      319     59      0             2     0        6             0
## 3     3     9311.    62      0            17     0        2             0
## 4     4      281.    60      1             9     0        7             0
## 5     5    18727.    55      0             5     2        7             0
## 6     6      453.    66      0             1     0        3             0
## # ... with 3 more variables: comorbidities <int>, duration <int>,
## #   comp_bin <dbl>
```

Question 4

```
HeartDisease_df %>%
  mutate(log_totalcost = log(totalcost)) %>%
  ggplot(aes(x = log_totalcost, y = ERvisits)) +
  geom_point() +
  geom_smooth(method = 'lm',formula = y~x)
```




```
reg_Heart =
  HeartDisease_df %>%
  mutate(log_totalcost = log(totalcost)) %>%
  #filter(is.finite(log_totalcost)) %>%
  lm(formula = log_totalcost ~ ERvisits, data = .)

reg_Heart %>%
  broom::tidy()

## # A tibble: 2 x 5
##   term        estimate std.error statistic   p.value
##   <chr>          <dbl>     <dbl>     <dbl>     <dbl>
## 1 (Intercept)    5.49     0.114      48.2  3.56e-237
## 2 ERvisits       0.225    0.0263      8.53 7.39e- 17

summary(reg_Heart)

##
## Call:
## lm(formula = log_totalcost ~ ERvisits, data = .)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -13.5255  -1.0922   0.0608   1.3147   4.3314
```

```
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  5.49384    0.11387  48.248   <2e-16 ***
## ERvisits     0.22477    0.02635   8.531   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.949 on 786 degrees of freedom
## Multiple R-squared:  0.08475,    Adjusted R-squared:  0.08359
## F-statistic: 72.78 on 1 and 786 DF,  p-value: < 2.2e-16
```

According to the results, we can find that adjusted R-squared is 0.1014 which is very closed to 0 and means this simple linear model is not a proper model. However, p-value is lower than 2.2e-16, which means the slope is significant and there are positive relationship between log of total cost and number of emergency room visits.

Interpretation: The slop of model is 0.227 which means if the number of emergency room vistis increases by 1 unit, the log of total cost will increase 0.452 units.

Question 5

Test if `comp_bin` is an effect modifier

```
reg_modifier_Heart =
  HeartDisease_df %>%
  mutate(log_totalcost = log(totalcost)) %>%
  #filter(is.finite(log_totalcost)) %>%
  lm(formula = log_totalcost ~ ERvisits + comp_bin + ERvisits*comp_bin, data
= .)

reg_modifier_Heart %>%
  broom::tidy()

## # A tibble: 4 x 5
##   term              estimate std.error statistic   p.value
##   <chr>                <dbl>     <dbl>     <dbl>     <dbl>
## 1 (Intercept)         5.46      0.114     47.8   1.12e-234
## 2 ERvisits            0.208     0.0271     7.70  4.01e- 14
## 3 comp_bin            2.22      0.602      3.69  2.39e-  4
## 4 ERvisits:comp_bin  -0.0964    0.105     -0.921 3.57e-  1

summary(reg_modifier_Heart)

##
## Call:
## lm(formula = log_totalcost ~ ERvisits + comp_bin + ERvisits *
##     comp_bin, data = .)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
```

```
## -13.4051  -1.0559   0.0325   1.2269   4.4353
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)        5.45548    0.11406  47.828  < 2e-16 ***
## ERvisits           0.20837    0.02705   7.703 4.01e-14 ***
## comp_bin           2.22320    0.60233   3.691 0.000239 ***
## ERvisits:comp_bin -0.09639    0.10461  -0.921 0.357103
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.911 on 784 degrees of freedom
## Multiple R-squared:  0.1227, Adjusted R-squared:  0.1193
## F-statistic: 36.55 on 3 and 784 DF,  p-value: < 2.2e-16
```

Since the corresponding p-value of 'ERvisits*comp_bin' is 0.357 which is bigger than 0.05, we can conclude that there is no interaction between `ERvisits` and `comp_bin` and `comp_bin` is not a modifier.

Test if `comp_bin` is a confounder.

```
reg_confounder_Heart =
  HeartDisease_df %>%
  mutate(log_totalcost = log(totalcost)) %>%
  #filter(is.finite(log_totalcost)) %>%
  lm(formula = log_totalcost ~ ERvisits + comp_bin, data = .)

reg_confounder_Heart %>%
  broom::tidy()
```

```
## # A tibble: 3 x 5
##   term        estimate std.error statistic   p.value
##   <chr>          <dbl>     <dbl>     <dbl>     <dbl>
## 1 (Intercept)    5.48     0.112      49.1  2.79e-241
## 2 ERvisits       0.202    0.0261      7.73 3.33e- 14
## 3 comp_bin       1.74     0.303       5.75 1.27e-  8
```

```
summary(reg_confounder_Heart)
```

```
##
## Call:
## lm(formula = log_totalcost ~ ERvisits + comp_bin, data = .)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -13.3943  -1.0451   0.0252   1.2191   4.4397
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  5.47693    0.11165  49.054  < 2e-16 ***
## ERvisits     0.20193    0.02613   7.728 3.33e-14 ***
```

```
## comp_bin     1.74365    0.30321   5.751 1.27e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.911 on 785 degrees of freedom
## Multiple R-squared:  0.1218, Adjusted R-squared:  0.1195
## F-statistic: 54.41 on 2 and 785 DF,  p-value: < 2.2e-16
```

When adding `comp_bin` in model the association between `log_totalcost` and `ERvisits` becomes smaller but still significant and the regression coefficient decreased by 10.2%, so `comp_bin` is a confounder.

Since `comp_bin` is a confounder but not a modifier, we use 'Partial' F-test to test whether we should include `comp_bin` as a factor.

Model 1: $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \varepsilon_i$

Model 2: $Y_i = \beta_0 + \beta_1 X_{i1} + \varepsilon_i$

Among which, X_1 represents `ER_visits`, X_2 represents `comp_bin`.

Null hypothesis $H_O: \beta_2 = 0$, alternative hypothesis $H_1: \beta_2 \neq 0$

```
anova(reg_confounder_Heart, reg_Heart) %>%
  broom::tidy()

## Warning: Unknown or uninitialised column: 'term'.

## # A tibble: 2 x 6
##   res.df   rss    df  sumsq statistic       p.value
## *  <dbl> <dbl> <dbl> <dbl>     <dbl>         <dbl>
## 1    785 2866.    NA    NA        NA   NA
## 2    786 2987.    -1 -121.      33.1  0.0000000127
```

According to results, p-value is smaller than 0.01 so we reject H_0 and conclude that Model 1 is 'superior'.As a resuit, we should include `comp_bin`.

Question 6

```
reg_added_Heart =
  HeartDisease_df %>%
  mutate(log_totalcost = log(totalcost)) %>%
  #filter(is.finite(log_totalcost)) %>%
  lm(formula = log_totalcost ~ ERvisits + comp_bin + age + gender + duration,
data = .)

reg_added_Heart %>%
  broom::tidy()

## # A tibble: 6 x 5
##   term          estimate std.error statistic  p.value
##   <chr>            <dbl>     <dbl>     <dbl>    <dbl>
```

```
## 1 (Intercept)  5.80     0.556        10.4  5.91e-24
## 2 ERvisits     0.173    0.0246        7.05 4.07e-12
## 3 comp_bin     1.53     0.282         5.45 6.89e- 8
## 4 age         -0.0193   0.00945      -2.05 4.10e- 2
## 5 gender      -0.323    0.151        -2.14 3.26e- 2
## 6 duration     0.00606  0.000533     11.4  6.76e-28
```

```r
summary(reg_added_Heart)
```

```
##
## Call:
## lm(formula = log_totalcost ~ ERvisits + comp_bin + age + gender +
##     duration, data = .)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -12.1885  -0.9962  -0.0838   1.0099   4.3499
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  5.8016080  0.5559910  10.435  < 2e-16 ***
## ERvisits     0.1732359  0.0245897   7.045 4.07e-12 ***
## comp_bin     1.5335773  0.2815738   5.446 6.89e-08 ***
## age         -0.0193389  0.0094493  -2.047   0.0410 *
## gender      -0.3234418  0.1510875  -2.141   0.0326 *
## duration     0.0060629  0.0005325  11.386  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.769 on 782 degrees of freedom
## Multiple R-squared:  0.2502, Adjusted R-squared:  0.2454
## F-statistic: 52.18 on 5 and 782 DF,  p-value: < 2.2e-16
```

According to results, we can find all p-value of covariates are smaller than 0.01, so all covariates have significant influence in total cost.

We use 'Partial' F-test to compare SLR and MLR models.

Model 1: $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i4} + \beta_5 X_{i5} + \varepsilon_i$

Model 2: $Y_i = \beta_0 + \beta_1 X_{i1} + \varepsilon_i$

Among which, X_1 represents `ERvisits`, X_2 represents `comp_bin`, X_3 represents `age`, X_4 represents `gender`, X_5 represents `duration`.

Null hypothesis $H_O: \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$, alternative hypothesis H_1: at least one of β is not zero.

```r
anova(reg_Heart, reg_added_Heart) %>% broom::tidy()
```

```
## Warning: Unknown or uninitialised column: 'term'.
```

```
## # A tibble: 2 x 6
##   res.df   rss    df sumsq statistic   p.value
## *  <dbl> <dbl> <dbl> <dbl>     <dbl>     <dbl>
## 1    786 2987.    NA    NA        NA   NA
## 2    782 2447.     4   540.      43.1  1.00e-32
```

According to the ANOVA results, p-value is smaller than 0.01 so we reject H_0 and conclude that Model 1 is 'superior'.As a resuit, we should use MLR model.

Problem 3

First, we import data

```
PatSatisfaction_df = readxl::read_xlsx("./data/PatSatisfaction.xlsx") %>%
  janitor::clean_names() %>%
  reshape::rename(c(safisfaction = "satisfaction"))

head(PatSatisfaction_df)

## # A tibble: 6 x 4
##   satisfaction   age severity anxiety
##          <dbl> <dbl>    <dbl>   <dbl>
## 1           48    50       51     2.3
## 2           57    36       46     2.3
## 3           66    40       48     2.2
## 4           70    41       44     1.8
## 5           89    28       43     1.8
## 6           36    49       54     2.9
```

Question 1

```
PatSatisfaction_df %>%
  cor()

##              satisfaction        age   severity    anxiety
## satisfaction    1.0000000 -0.7867555 -0.6029417 -0.6445910
## age            -0.7867555  1.0000000  0.5679505  0.5696775
## severity       -0.6029417  0.5679505  1.0000000  0.6705287
## anxiety        -0.6445910  0.5696775  0.6705287  1.0000000
```

According to the correlation matrix, we can find that all `age`, `severity`, `anxirty` have negative relationship with satisfaction and the relationship between `age` and `satisfaction` is stronger than `severity` and `anxiety` .

Question 2

Assuming the model is

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \varepsilon_i$$

Among which, X_1 represents age, X_2 represents severity, X_3 represents anxiety.

Null hypothesis H_0: $\beta_0 = \beta_1 = \beta_2 = \beta_3 = 0$, alternative hypothesis H_1: at least one β is not zero.

Decision rule:

If $F^* = \frac{MSR}{MSE} > F(1-\alpha; p, n-p-1)$, reject H_0,

if $F^* = \frac{MSR}{MSE} \leq F(1-\alpha; p, n-p-1)$, fail to reject H_0.

with a significance level of 0.05, $\alpha = 0.05$

```
reg_all =
  PatSatisfaction_df %>%
  lm(satisfaction ~ age + severity + anxiety, data = .)

summary(reg_all)
```

```
##
## Call:
## lm(formula = satisfaction ~ age + severity + anxiety, data = .)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -18.3524  -6.4230   0.5196   8.3715  17.1601
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 158.4913    18.1259   8.744 5.26e-11 ***
## age          -1.1416     0.2148  -5.315 3.81e-06 ***
## severity     -0.4420     0.4920  -0.898   0.3741
## anxiety     -13.4702     7.0997  -1.897   0.0647 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.06 on 42 degrees of freedom
## Multiple R-squared:  0.6822, Adjusted R-squared:  0.6595
## F-statistic: 30.05 on 3 and 42 DF,  p-value: 1.542e-10
```

According to results, we can find F^* = 30.05 > 2.8270487, so we reject H_0 and conclude that there is a regression relation.

Question 3

```
reg_all %>% broom::tidy()
```

```
## # A tibble: 4 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)  158.       18.1        8.74  5.26e-11
```

```
## 2 age           -1.14      0.215    -5.31  3.81e- 6
## 3 severity      -0.442     0.492    -0.898 3.74e- 1
## 4 anxiety      -13.5       7.10     -1.90  6.47e- 2
```

```r
confint(reg_all)
```

```
##                  2.5 %      97.5 %
## (Intercept) 121.911727 195.0707761
## age          -1.575093  -0.7081303
## severity     -1.434831   0.5508228
## anxiety     -27.797859   0.8575324
```

By using function `confint`, we get 95% CIs of all estimators. The 95% CIs of `severity` is (-1.4348, 0.5508) which means at $\alpha = 0.05$ significant level, we can conclude that the mean value of satisfaction changes somewhere between decreasing 1.4348 and increasing 0.5508 for each additional unit of the severity of the illness given all other values of predictors stay constant. The estimated coefficient of `severity` is -0.442 which means if the value of `severity` increased by 1 units, the mean value of satisfaction will decrease 0.442 given all other values of predictors stay constant.

Question 4

```r
list(age = 35, severity = 42, anxiety = 2.1) %>%
  predict(object = reg_all, newdata = ., interval = "predict")
```

```
##        fit      lwr      upr
## 1 71.68332 50.06237 93.30426
```

By using `predict` function, we can get the prediction interval for the new patient's satisfaction is (50.0624, 93.3042).

Interprest: We are 95% confident that the the new patient's satisfaction fall within (50.0624, 93.3042) given `age` equals 35, `severity` equals 42 and `anxiety` equals 2.1

Question 5

Model 1: $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \varepsilon_i$

Model 2: $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \varepsilon_i$

Among which, X_1 represents `age`, X_2 represents `severity`, X_3 represents `anxiety`.

We use 'Partial' F-test for nested models. Null hypothesis $H_O: \beta_3 = 0$, alternative hypothesis $H_1: \beta_3 \neq 0$

Decision rule:

$$F^* = \frac{(SSR_L - SSR_S)/(df_L - df_S)}{\frac{SSE_L}{df_L}} \sim F_{df_L - df_S, dfL}$$

where $df_S = n - p_S - 1, df_L = n - p_L - 1$.

If $F^* > F(1-\alpha; df_L - df_S, df_L)$, reject H_0;

If $F^* \leq F(1-\alpha; df_L - df_S, df_L)$, fail to reject H_0.

With $\alpha = 0.05$, when $p - value \geq 0.05$, fail to reject H_0, when $p - value < 0.05$, reject H_0.

```
reg_without_anxiety =
  PatSatisfaction_df %>%
  lm(satisfaction ~ age + severity, data = .)

anova(reg_all, reg_without_anxiety) %>%
  broom::tidy()
```

```
## Warning: Unknown or uninitialised column: 'term'.
```

```
## # A tibble: 2 x 6
##   res.df   rss    df sumsq statistic p.value
## *  <dbl> <dbl> <dbl> <dbl>     <dbl>   <dbl>
## 1     42 4249.    NA   NA       NA    NA
## 2     43 4613.    -1 -364.       3.60  0.0647
```

According to the ANOVA results, p-value is 0.0647 which is larger than 0.05, so we fail to reject H_0 and conclude that Model 1 is not 'superior` and we should use Model 2.