Homework 8

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```r
library(tidyverse)
library(survey)

hw8_data = read.csv("./data_hw8.csv")
head(hw8_data)
```

```
##   X id t0 L1         L2 L3 A  Y
## 1 1  1  0  3  0.5774203  8 1 NA
## 2 2  1  1  3 -1.6196027  8 1 NA
## 3 3  1  2  3 -1.3912145  8 1 NA
## 4 4  1  3  2 -1.3814392  8 0 NA
## 5 5  1  4  1  0.4641491  8 1 NA
## 6 6  1  5  3 -1.5809908  8 1 NA
```

Question 1

1. Time point 1: L1, L2, L3, A
2. Time point 2: L1, L2, A
3. Time point 3: L1, L2, A
4. Time point 4: L1, L2, A
5. Time point 5: L1, L2, A
6. Time point 6: L1, L2, A, Y

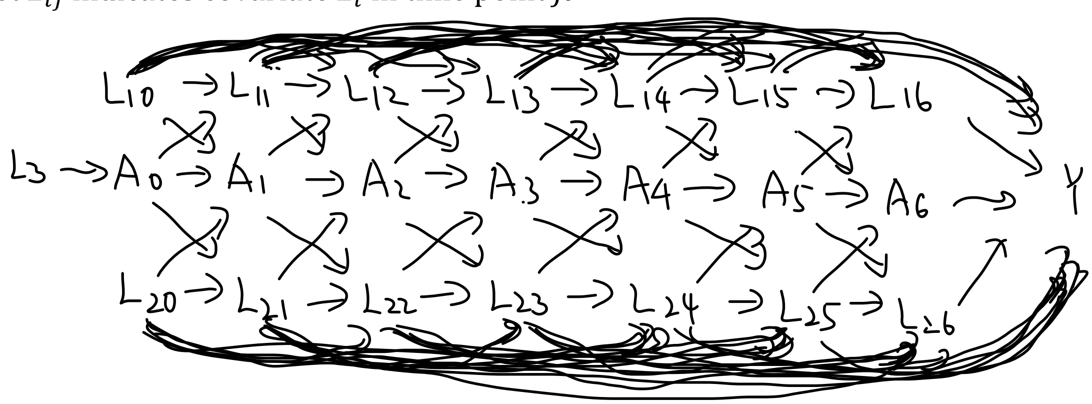
Question 2

The causal contrast is:

$E[Y|A_i = 1] - E[Y|A_i = 0]$, for $i = 0,1,2,3,4,5,6$.

Question 3

Let L_{ij} indicates covariate L_i in time point j.



Question 4

(i) No unmeasured exposure-outcome confounding given C

(ii) No unmeasured mediator-outcome confounding given C

(iii) No unmeasured exposure-mediator confounding given C

(iv) No effect of exposure that confounds the mediator-outcome relationship

Question 5

```r
# create wide data
AC_data = hw8_data %>%
  gather(key = "Lt", value = "value", c(L1,L2,A)) %>%
  arrange(id) %>%
  mutate(Lt = str_c(Lt, t0)) %>%
  select(id, Lt, value) %>%
  spread(key = Lt, value = value)
Y_data = hw8_data %>%
  select(id, L3, Y) %>%
  na.omit()
wide_data = merge(AC_data, Y_data)

# Time point 0
glm.model0 = glm(A0~L3, data = wide_data, family = binomial)
p0 = predict(glm.model0, type = "response")
w0 = ifelse(wide_data$A0==1, 1/p0, 1/(1-p0))
# Time point 1
glm.model1 = glm(A1~L3+A0+L10+L20, data = wide_data, family = binomial)
p1 = predict(glm.model1, type = "response")
w1 = ifelse(wide_data$A1==1, 1/p1, 1/(1-p1))
# Time point 2
glm.model2 = glm(A2~A1+L11+L21+L3+A0+L10+L20, data = wide_data, family =
binomial)
p2 = predict(glm.model2, type = "response")
w2 = ifelse(wide_data$A2==1, 1/p2, 1/(1-p2))
# Time point 3
glm.model3 = glm(A3~A2+L12+L22+A1+L11+L21+L3+A0+L10+L20, data = wide_data,
family = binomial)
p3 = predict(glm.model3, type = "response")
w3 = ifelse(wide_data$A3==1, 1/p3, 1/(1-p3))
# Time point 4
glm.model4 = glm(A4~A3+L13+L23+A2+L12+L22+A1+L11+L21+L3+A0+L10+L20, data =
wide_data, family = binomial)
p4 = predict(glm.model4, type = "response")
w4 = ifelse(wide_data$A4==1, 1/p4, 1/(1-p4))
# Time point 5
glm.model5 =
glm(A5~A4+L14+L24+A3+L13+L23+A2+L12+L22+A1+L11+L21+L3+A0+L10+L20, data =
wide_data, family = binomial)
p5 = predict(glm.model5, type = "response")
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```
w5 = ifelse(wide_data$A5==1, 1/p5, 1/(1-p5))
# Time point 6
glm.model6 =
glm(A6~A5+L15+L25+A4+L14+L24+A3+L13+L23+A2+L12+L22+A1+L11+L21+L3+A0+L10+L20,
data = wide_data, family = binomial)
p6 = predict(glm.model6, type = "response")
w6 = ifelse(wide_data$A6==1, 1/p6, 1/(1-p6))
w = w0*w1*w2*w3*w4*w5*w6
```

Question 6

Marginal model: $E[Y_a] = \beta + \beta_0 A_0 + \beta_1 A_1 + \beta_2 A_2 + \beta_3 A_3 + \beta_4 A_4 + \beta_5 A_5 + \beta_6 A_6$

```
set.seed(123)
nboots = 1000
n_sample = nrow(wide_data)

beta = rep(NA, nboots)
beta0 = rep(NA, nboots)
beta1 = rep(NA, nboots)
beta2 = rep(NA, nboots)
beta3 = rep(NA, nboots)
beta4 = rep(NA, nboots)
beta5 = rep(NA, nboots)
beta6 = rep(NA, nboots)

for (i in 1:nboots) {
  S.b <- sample(1:n_sample, size = n_sample, replace = TRUE)
  data.b <- wide_data[S.b, ]
  # Time point 0
  glm.model0 = glm(A0~L3, data = data.b, family = binomial)
  p0 = predict(glm.model0, type = "response")
  w0 = ifelse(data.b$A0==1, 1/p0, 1/(1-p0))
  # Time point 1
  glm.model1 = glm(A1~L3+A0+L10+L20, data = data.b, family = binomial)
  p1 = predict(glm.model1, type = "response")
  w1 = ifelse(data.b$A1==1, 1/p1, 1/(1-p1))
  # Time point 2
  glm.model2 = glm(A2~A1+L11+L21+L3+A0+L10+L20, data = data.b, family =
binomial)
  p2 = predict(glm.model2, type = "response")
  w2 = ifelse(data.b$A2==1, 1/p2, 1/(1-p2))
  # Time point 3
  glm.model3 = glm(A3~A2+L12+L22+A1+L11+L21+L3+A0+L10+L20, data = data.b,
family = binomial)
  p3 = predict(glm.model3, type = "response")
  w3 = ifelse(data.b$A3==1, 1/p3, 1/(1-p3))
  # Time point 4
  glm.model4 = glm(A4~A3+L13+L23+A2+L12+L22+A1+L11+L21+L3+A0+L10+L20, data =
```

```
data.b, family = binomial)
  p4 = predict(glm.model4, type = "response")
  w4 = ifelse(data.b$A4==1, 1/p4, 1/(1-p4))
  # Time point 5
  glm.model5 =
glm(A5~A4+L14+L24+A3+L13+L23+A2+L12+L22+A1+L11+L21+L3+A0+L10+L20, data =
data.b, family = binomial)
  p5 = predict(glm.model5, type = "response")
  w5 = ifelse(data.b$A5==1, 1/p5, 1/(1-p5))
  # Time point 6
  glm.model6 =
glm(A6~A5+L15+L25+A4+L14+L24+A3+L13+L23+A2+L12+L22+A1+L11+L21+L3+A0+L10+L20,
data = data.b, family = binomial)
  p6 = predict(glm.model6, type = "response")
  w6 = ifelse(data.b$A6==1, 1/p6, 1/(1-p6))
  w = w0*w1*w2*w3*w4*w5*w6

  data.b$w = w
  design = svydesign(ids = ~id, weights = ~w, data = data.b)
  msm = svyglm(Y ~ A0 + A1 + A2 + A3 + A4 + A5 + A6, family = gaussian(link =
"identity"), design = design)
  beta[i] = msm$coef[1]
  beta0[i] = msm$coef[2]
  beta1[i] = msm$coef[3]
  beta2[i] = msm$coef[4]
  beta3[i] = msm$coef[5]
  beta4[i] = msm$coef[6]
  beta5[i] = msm$coef[7]
  beta6[i] = msm$coef[8]
}

beta_est = mean(beta)
CIL_beta = mean(beta) - 1.96*sqrt(var(beta))
CIU_beta = mean(beta) + 1.96*sqrt(var(beta))
beta0_est = mean(beta0)
CIL_beta0 = mean(beta0) - 1.96*sqrt(var(beta0))
CIU_beta0 = mean(beta0) + 1.96*sqrt(var(beta0))
beta1_est = mean(beta1)
CIL_beta1 = mean(beta1) - 1.96*sqrt(var(beta1))
CIU_beta1 = mean(beta1) + 1.96*sqrt(var(beta1))
beta2_est = mean(beta2)
CIL_beta2 = mean(beta2) - 1.96*sqrt(var(beta2))
CIU_beta2 = mean(beta2) + 1.96*sqrt(var(beta2))
beta3_est = mean(beta3)
CIL_beta3 = mean(beta3) - 1.96*sqrt(var(beta3))
CIU_beta3 = mean(beta3) + 1.96*sqrt(var(beta3))
beta4_est = mean(beta4)
CIL_beta4 = mean(beta4) - 1.96*sqrt(var(beta4))
CIU_beta4 = mean(beta4) + 1.96*sqrt(var(beta4))
beta5_est = mean(beta5)
```

```
CIL_beta5 = mean(beta5) - 1.96*sqrt(var(beta5))
CIU_beta5 = mean(beta5) + 1.96*sqrt(var(beta5))
beta6_est = mean(beta6)
CIL_beta6 = mean(beta6) - 1.96*sqrt(var(beta6))
CIU_beta6 = mean(beta6) + 1.96*sqrt(var(beta6))
```

The estimated value of β is -2.25 and 95% confidence interval is (-2.706, -1.794).

The estimated value of $\beta 0$ is -0.052 and 95% confidence interval is (-0.231, 0.128).

The estimated value of $\beta 1$ is 0.107 and 95% confidence interval is (-0.05, 0.264).

The estimated value of $\beta 2$ is -0.213 and 95% confidence interval is (-0.401, -0.026).

The estimated value of $\beta 3$ is 0.049 and 95% confidence interval is (-0.107, 0.205).

The estimated value of $\beta 4$ is -0.048 and 95% confidence interval is (-0.284, 0.188).

The estimated value of $\beta 5$ is -0.329 and 95% confidence interval is (-0.5, -0.157).

The estimated value of $\beta 6$ is -2.231 and 95% confidence interval is (-2.398, -2.064).

Question 7

Interpretation:

β: The estimated value of outcome on average if the subject is always in control group is -2.25 and we have 95% confidence that the ture value lies between -2.706 and -1.794.

β_0: On average, the difference of estimated value of outcome between treatment group and control group in time point 0 is -0.052 and we have 95% confidence that the ture value lies between -0.231 and 0.128.

β_1: On average, the difference of estimated value of outcome between treatment group and control group in time point 1 is 0.107 and we have 95% confidence that the ture value lies between -0.05 and 0.264.

β_2: On average, the difference of estimated value of outcome between treatment group and control group in time point 2 is -0.213 and we have 95% confidence that the ture value lies between -0.401 and -0.026.

β_3: On average, the difference of estimated value of outcome between treatment group and control group in time point 3 is 0.049 and we have 95% confidence that the ture value lies between -0.107 and 0.205.

β_4: On average, the difference of estimated value of outcome between treatment group and control group in time point 4 is -0.048 and we have 95% confidence that the ture value lies between -0.284 and 0.188.

β_5: On average, the difference of estimated value of outcome between treatment group and control group in time point 5 is -0.329 and we have 95% confidence that the ture value lies between -0.5 and -0.157.

β_6: On average, the difference of estimated value of outcome between treatment group and control group in time point 6 is -2.231 and we have 95% confidence that the ture value lies between -2.398 and -2.064.

Question 8

Assumptions:

1. Consistency;
2. Stable Unit Treatment Value Assumption(SUTVA);
3. Exchangeability;
4. Positivity.

Question 9

According to estimated values of coefficients and 95% CI, we can find that $\beta, \beta_2, \beta_5, \beta_6$ are significant, which means treatments in time point 2, 5, 6 bring significant difference in outcome.