Homework 2

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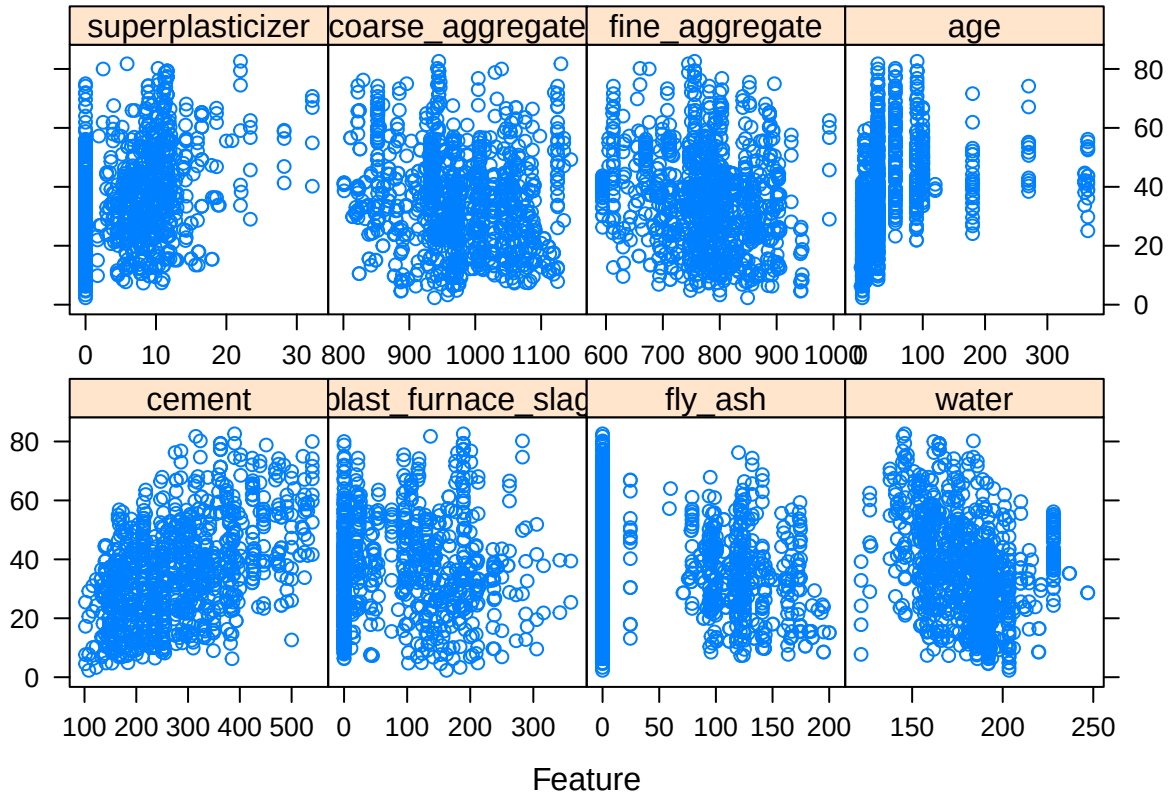
```
library(tidyverse)
library(caret)
library(boot)  # for smooth spline
library(ggplot2)
library(mgcv)   # for gam
library(patchwork)
```

```
concrete_data = read_csv("./concrete.csv") %>%
  janitor::clean_names()
```

Question 1

```
x = concrete_data[,1:8]
y = y = as.numeric(unlist(concrete_data[,9]))
featurePlot(x, y, "scatter")
```



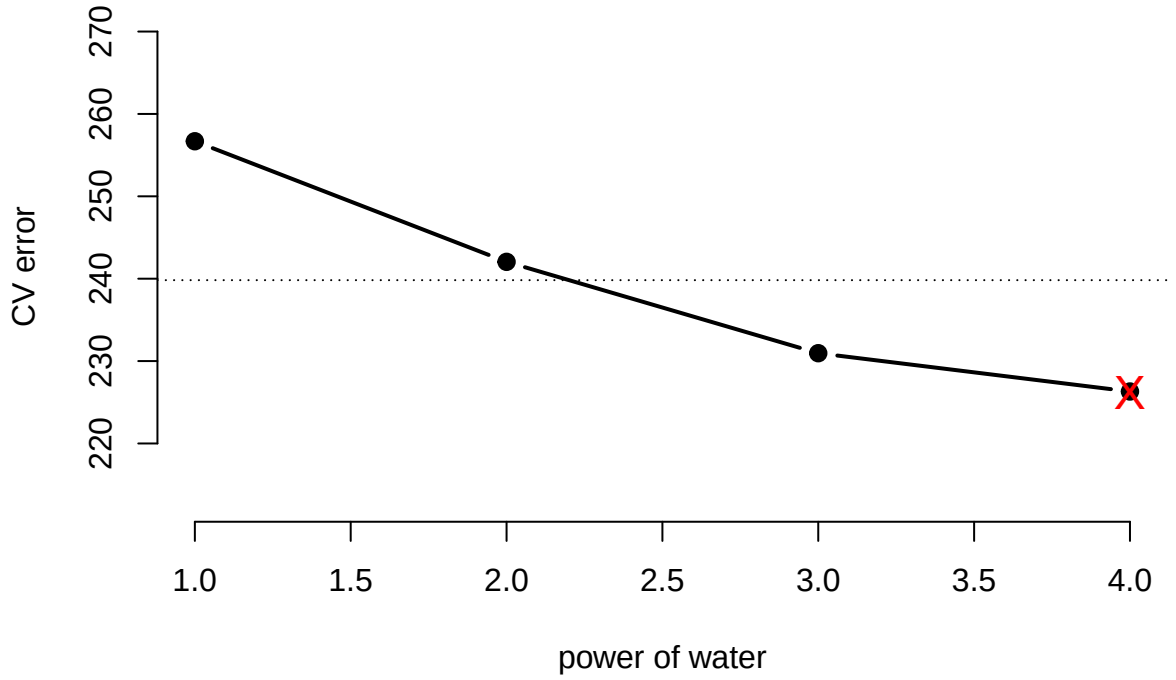
Question 2

Cross validation

```
set.seed(123)

# container of test errors
cv.MSE <- NA

# loop over powers of water
for (i in 1:4) {
  glm.fit <-  glm(compressive_strength ~ poly(water, i), data = concrete_data)
  # we use cv.glm's cross-validation and keep the vanilla cv test error
  cv.MSE[i] <-  cv.glm(concrete_data, glm.fit, K = 10)$delta[1]
}
# inspect results object
cv.MSE
```

```
## [1] 256.6841 242.0471 230.9552 226.3080
```

```
# illustrate results with a line plot connecting the cv.error dots
plot( x = 1:4, y = cv.MSE, xlab = "power of water", ylab = "CV error",
      type = "b", pch = 19, lwd = 2, bty = "n",
      ylim = c( min(cv.MSE) - sd(cv.MSE), max(cv.MSE) + sd(cv.MSE) ) )

# horizontal line for 1se to less complexity
abline(h = min(cv.MSE) + sd(cv.MSE) , lty = "dotted")

# where is the minimum
points( x = which.min(cv.MSE), y = min(cv.MSE), col = "red", pch = "X", cex = 1.5 )
```



According to the result, we should choose degree of freedom equals to 4.

ANOVA

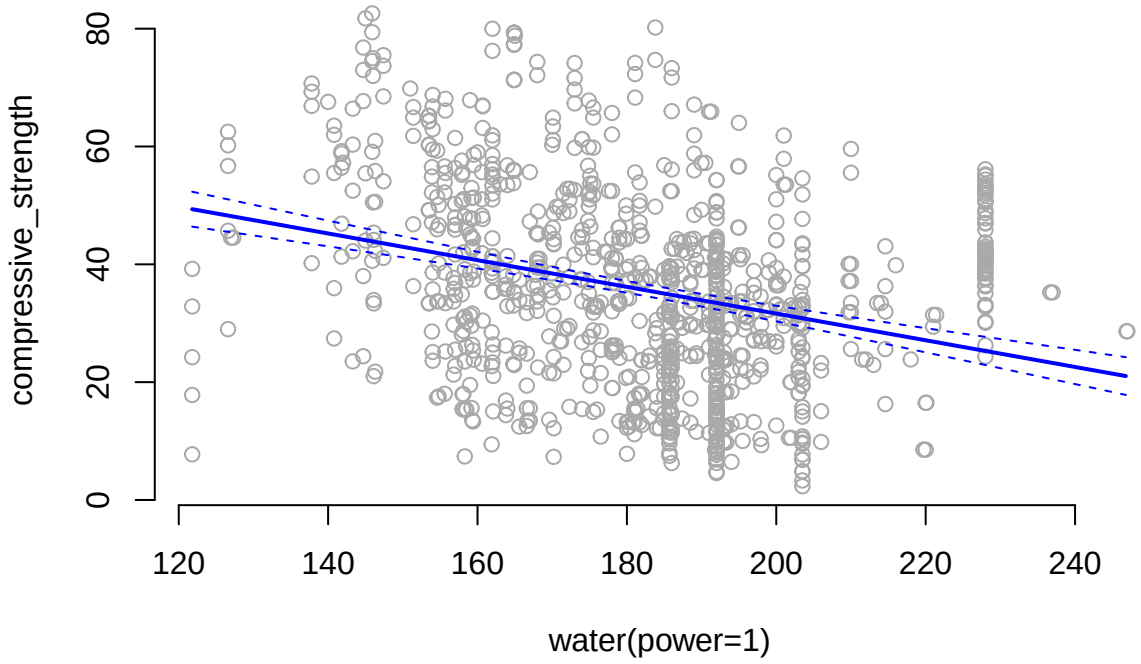
```
# container for the models we will fit
models <- vector("list", length(cv.MSE))
# fit all 15 models
for( a in 1:length(cv.MSE)){
  models[[a]] <- glm(compressive_strength ~ poly(water, a), data = concrete_data)
}
# f-test
anova(models[[1]], models[[2]], models[[3]], models[[4]], test = "F")
```

```
## Analysis of Deviance Table
##
## Model 1: compressive_strength ~ poly(water, a)
## Model 2: compressive_strength ~ poly(water, a)
## Model 3: compressive_strength ~ poly(water, a)
## Model 4: compressive_strength ~ poly(water, a)
##   Resid. Df Resid. Dev Df Deviance      F    Pr(>F)
## 1      1028     263085
## 2      1027     247712  1  15372.8 68.140 4.652e-16 ***
## 3      1026     235538  1  12174.0 53.962 4.166e-13 ***
## 4      1025     231246  1   4291.5 19.022 1.423e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

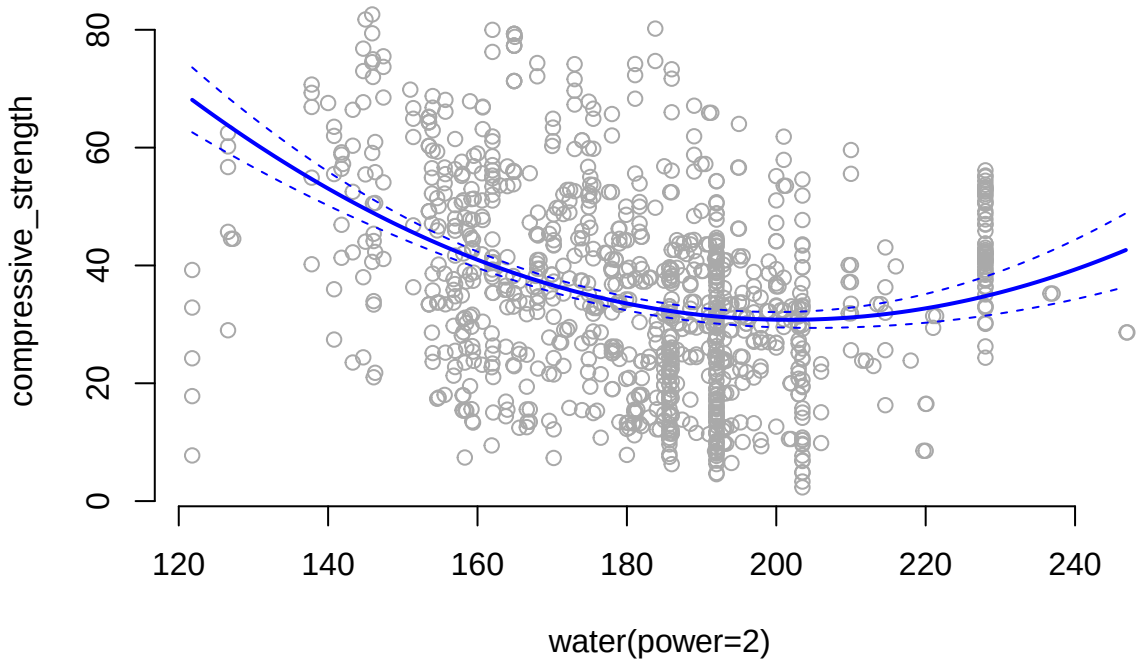
According to the result from F-test, comparing to the model with 3 degrees of freedom, the model with 4 degree of freedom is significant, so we should choose degree equals to 4.

Plots of different polynomial models

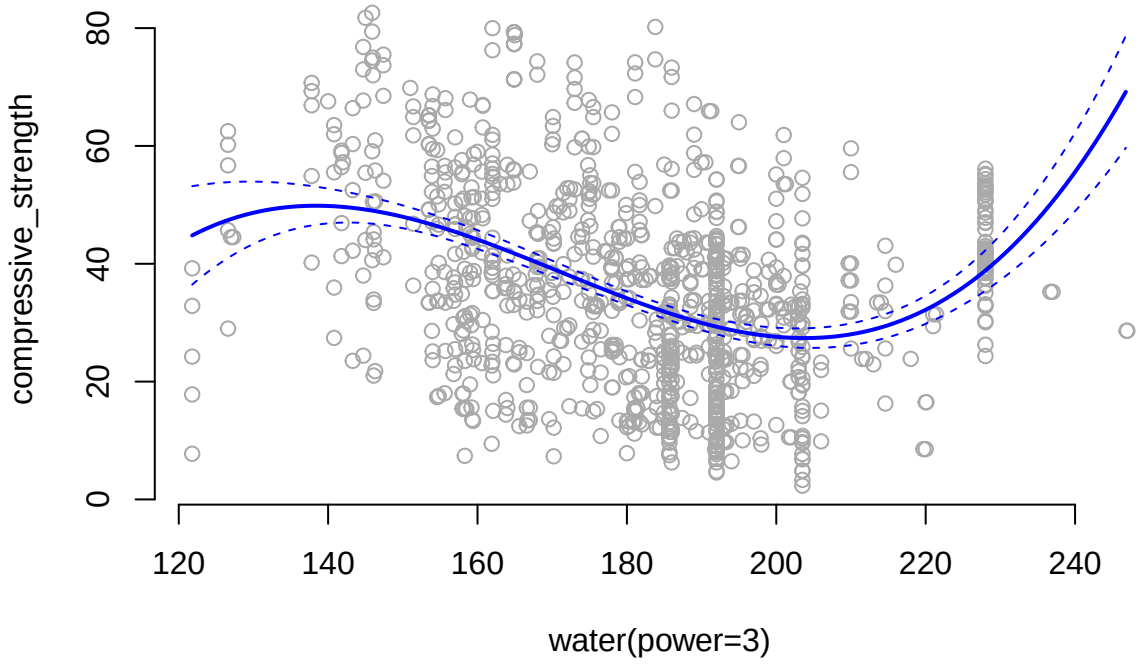
```
plot(compressive_strength ~ water, data = concrete_data, col = "darkgrey",  bty = "n", xlab="water(powe
waterlims <-  range(concrete_data$water)
water.grid <-  seq(from = waterlims[1], to = waterlims[2])
lm.fit <-  lm(compressive_strength ~ poly(water, 1), data = concrete_data)
lm.pred <-  predict(lm.fit, data.frame(water = water.grid), se = TRUE)
# mean prediction
lines(x = water.grid , y = lm.pred$fit, col = "blue", lwd = 2)
# uncertainty bands
matlines( x = water.grid, y = cbind( lm.pred$fit + 2*lm.pred$se.fit, lm.pred$fit - 2*lm.pred$se.fit),
          lty = "dashed", col = "blue")
```



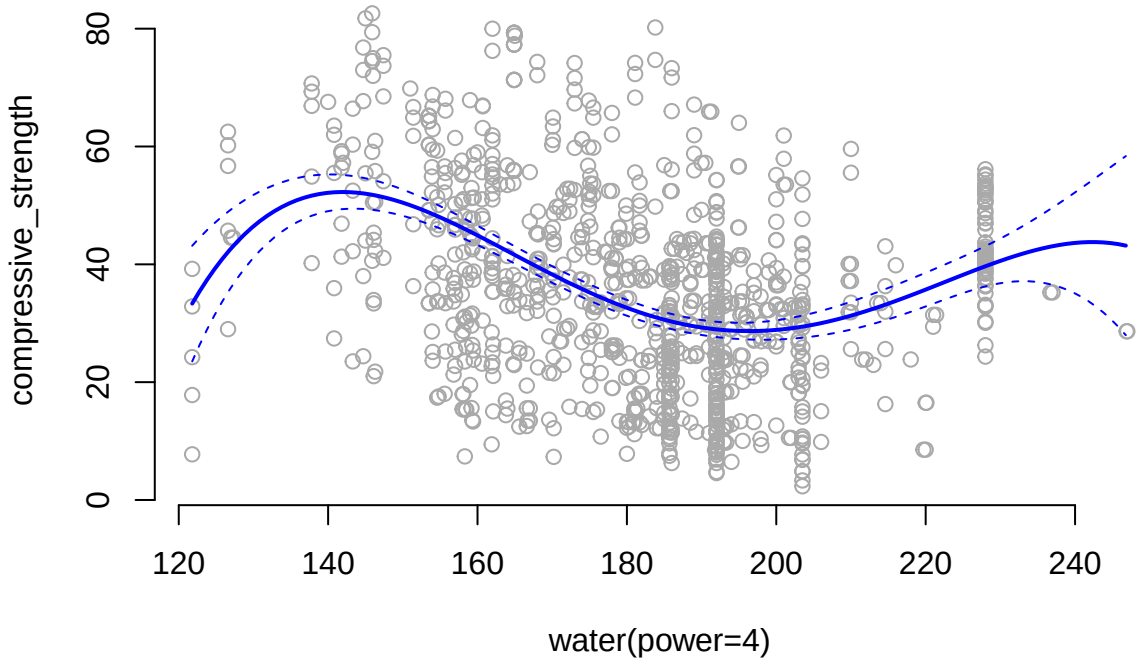

```
plot(compressive_strength ~ water, data = concrete_data, col = "darkgrey",  bty = "n", xlab="water(powe
waterlims <-  range(concrete_data$water)
water.grid <-  seq(from = waterlims[1], to = waterlims[2])
lm.fit <-  lm(compressive_strength ~ poly(water, 2), data = concrete_data)
lm.pred <-  predict(lm.fit, data.frame(water = water.grid), se = TRUE)
# mean prediction
lines(x = water.grid , y = lm.pred$fit, col = "blue", lwd = 2)
# uncertainty bands
matlines( x = water.grid, y = cbind( lm.pred$fit + 2*lm.pred$se.fit, lm.pred$fit - 2*lm.pred$se.fit),
          lty = "dashed", col = "blue")
```



```r
plot(compressive_strength ~ water, data = concrete_data, col = "darkgrey",  bty = "n", xlab="water(powe
waterlims <-  range(concrete_data$water)
water.grid <-  seq(from = waterlims[1], to = waterlims[2])
lm.fit <-  lm(compressive_strength ~ poly(water, 3), data = concrete_data)
lm.pred <-  predict(lm.fit, data.frame(water = water.grid), se = TRUE)
# mean prediction
lines(x = water.grid , y = lm.pred$fit, col = "blue", lwd = 2)
# uncertainty bands
matlines( x = water.grid, y = cbind( lm.pred$fit + 2*lm.pred$se.fit, lm.pred$fit - 2*lm.pred$se.fit),
          lty = "dashed", col = "blue")
```




```r
plot(compressive_strength ~ water, data = concrete_data, col = "darkgrey",  bty = "n", xlab="water(powe
waterlims <-  range(concrete_data$water)
water.grid <-  seq(from = waterlims[1], to = waterlims[2])
lm.fit <-  lm(compressive_strength ~ poly(water, 4), data = concrete_data)
lm.pred <-  predict(lm.fit, data.frame(water = water.grid), se = TRUE)
# mean prediction
lines(x = water.grid , y = lm.pred$fit, col = "blue", lwd = 2)
# uncertainty bands
matlines( x = water.grid, y = cbind( lm.pred$fit + 2*lm.pred$se.fit, lm.pred$fit - 2*lm.pred$se.fit),
          lty = "dashed", col = "blue")
```



Question 3

A range of df

```
p <- ggplot(data = concrete_data, aes(x = water, y = compressive_strength)) +
     geom_point(color = rgb(.2, .4, .2, .5))
```

Degrees of freedom = 30

```
fit.ss <- smooth.spline(concrete_data$water, concrete_data$compressive_strength, df = 30)

pred.ss <- predict(fit.ss,
                   x = water.grid)

pred.ss.df <- data.frame(pred = pred.ss$y,
                         water = water.grid)

p1 = p +
geom_line(aes(x = water, y = pred), data = pred.ss.df,
          color = rgb(.8, .1, .1, 1)) +
  ggtitle("df=30")
```

Degrees of freedom = 40

```
fit.ss <- smooth.spline(concrete_data$water, concrete_data$compressive_strength, df = 40)

pred.ss <- predict(fit.ss,
                   x = water.grid)

pred.ss.df <- data.frame(pred = pred.ss$y,
                         water = water.grid)
```

```
p2 = p +
geom_line(aes(x = water, y = pred), data = pred.ss.df,
          color = rgb(.8, .1, .1, 1)) +
  ggtitle("df=40")
```

Degrees of freedom = 50

```
fit.ss <- smooth.spline(concrete_data$water, concrete_data$compressive_strength, df = 50)

pred.ss <- predict(fit.ss,
                   x = water.grid)

pred.ss.df <- data.frame(pred = pred.ss$y,
                         water = water.grid)

p3 = p +
geom_line(aes(x = water, y = pred), data = pred.ss.df,
          color = rgb(.8, .1, .1, 1)) +
  ggtitle("df=50")
```

Degrees of freedom = 60

```
fit.ss <- smooth.spline(concrete_data$water, concrete_data$compressive_strength, df = 60)

pred.ss <- predict(fit.ss,
                   x = water.grid)

pred.ss.df <- data.frame(pred = pred.ss$y,
                         water = water.grid)

p4 = p +
geom_line(aes(x = water, y = pred), data = pred.ss.df,
          color = rgb(.8, .1, .1, 1)) +
  ggtitle("df=60")
```

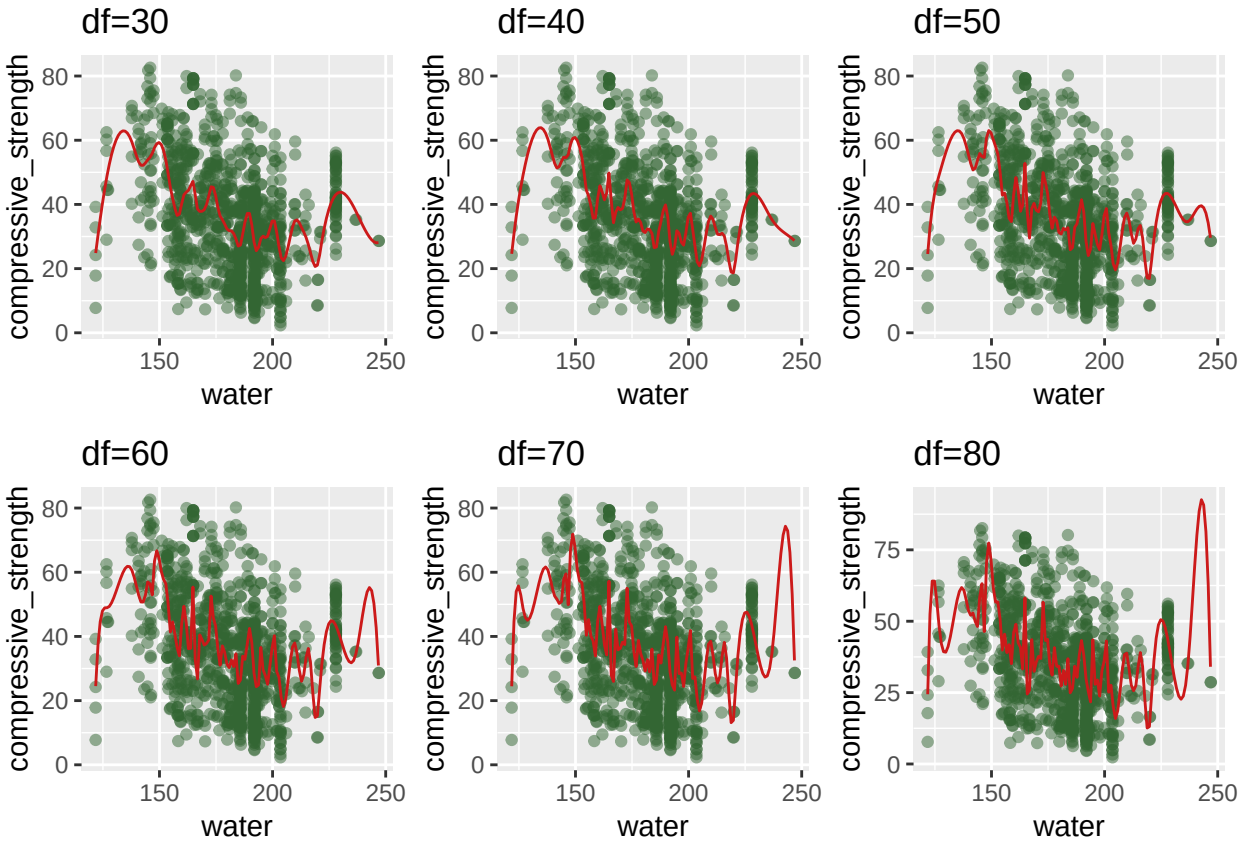
Degrees of freedom = 70

```
fit.ss <- smooth.spline(concrete_data$water, concrete_data$compressive_strength, df = 70)

pred.ss <- predict(fit.ss,
                   x = water.grid)

pred.ss.df <- data.frame(pred = pred.ss$y,
                         water = water.grid)

p5 = p +
geom_line(aes(x = water, y = pred), data = pred.ss.df,
          color = rgb(.8, .1, .1, 1)) +
  ggtitle("df=70")
```

Degrees of freedom = 80

```
fit.ss <- smooth.spline(concrete_data$water, concrete_data$compressive_strength, df = 80)

pred.ss <- predict(fit.ss,
                   x = water.grid)
```

```
pred.ss.df <- data.frame(pred = pred.ss$y,
                         water = water.grid)

p6 = p +
geom_line(aes(x = water, y = pred), data = pred.ss.df,
          color = rgb(.8, .1, .1, 1)) +
  ggtitle("df=80")

(p1+p2+p3)/(p4+p5+p6)
```



With degrees of freedom increase, the fitted model become more flexible.

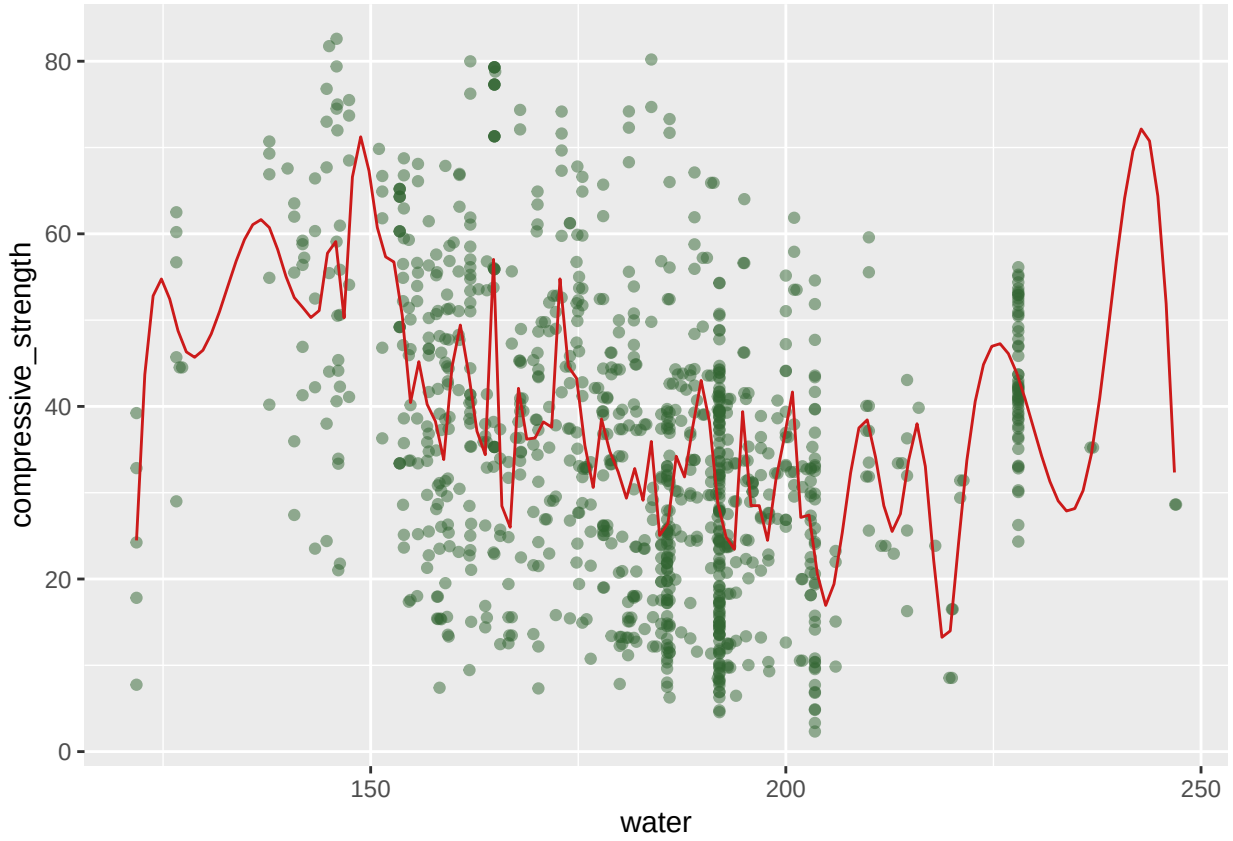
Generalized cross-validation

```
fit.ss <- smooth.spline(concrete_data$water, concrete_data$compressive_strength)
fit.ss$df
```

```
## [1] 68.88205
```

```
pred.ss <- predict(fit.ss,
                   x = water.grid)

pred.ss.df <- data.frame(pred = pred.ss$y,
                         water = water.grid)

p <- ggplot(data = concrete_data, aes(x = water, y = compressive_strength)) +
```

```
    geom_point(color = rgb(.2, .4, .2, .5))

p +
geom_line(aes(x = water, y = pred), data = pred.ss.df,
          color = rgb(.8, .1, .1, 1))
```



The degree of freedom obtained by generalized cross-validation is 68.88 and the fitted model is very flexible.

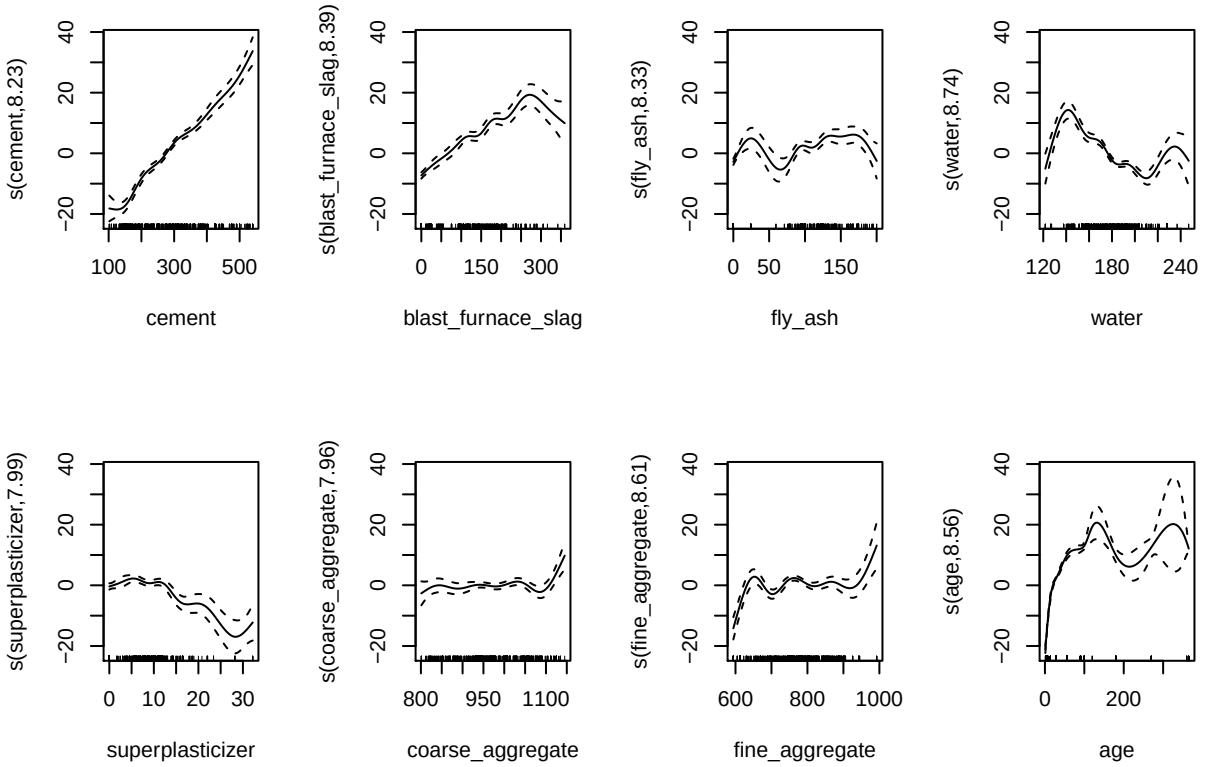
Question 4

```
gam.m1 <- gam(compressive_strength ~ s(cement)+s(blast_furnace_slag)+s(fly_ash)+s(water)+s(superplastic
summary(gam.m1)
```

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## compressive_strength ~ s(cement) + s(blast_furnace_slag) + s(fly_ash) +
##     s(water) + s(superplasticizer) + s(coarse_aggregate) + s(fine_aggregate) +
##     s(age)
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  35.8180     0.1671   214.4   <2e-16 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                           edf Ref.df       F  p-value
## s(cement)               8.228  8.833  48.285  < 2e-16 ***
## s(blast_furnace_slag) 8.388  8.874  24.855  < 2e-16 ***
## s(fly_ash)              8.331  8.851   9.742 3.04e-14 ***
## s(water)                8.742  8.974  26.469  < 2e-16 ***
## s(superplasticizer)    7.989  8.714  10.871 7.77e-16 ***
## s(coarse_aggregate)    7.956  8.702   3.595 0.000305 ***
## s(fine_aggregate)      8.614  8.950  18.405  < 2e-16 ***
## s(age)                  8.561  8.901 366.698  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.897   Deviance explained = 90.4%
## GCV = 30.786  Scale est. = 28.759    n = 1030
```

```
par(mfrow = c(2,4))
plot(gam.m1)
```



According to the result, we can find that with the cement increases, the strength increases. When the blast furnace slag is around 270, the strength is the highest. Fly ash and coarse aggregate do not have huge influence in the strength. When the superplasticizer is around 28, the strength is the lowest. When the fine aggregate is lower than 650, the strength starts to decrease and when it larger than 920, the strength starts to increase. When age is around 120 or 320, the strength is the highest. When water is around 145, the strength is the highest and when water is around 225, the strength is the lowest.