

GEOMETRY HOMEWORK 13

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Problem 4. *Helicoid* $\mathbb{X}(u, v) = (v \cos u, v \sin u, u)$, $\gamma(t) = \mathbb{X}(t, 1)$, $p = \mathbb{X}(0, 1) = (1, 0, 0)$, $V(0) = \gamma'(0)$ 求解平行向量場 $V(t)$ along $\gamma(t)$

Problem 6. 如圖考慮一旋轉體上的緯圈 γ , 已知其 *generating curve* (經線) 切線與中心軸夾角為 θ 。

- (a) 求一向量沿 γ 平行移動, 繞一圈後與原向量的夾角 (不妨假設起始向量與緯圈同向)
- (b) 將該 *surface* 放大或縮小, 相對應問題的夾角有何變化
- (c) 計算此緯圈之 $\oint_{\gamma} \kappa_g ds$, 值與 *surface* 的縮放有關嗎?

Problem 10 (Ex P282 4.).

- (a) *Compute the Euler-Poincaré characteristic of (1) An ellipsoid. (2) The surface $S = \{(x, y, z) \in \mathbb{R}^3; x^2 + y^2 + z^6 = 1\}$.*
- (b) 如圖, 將一圓盤的邊界如圖「黏」起來 (也可以想成將對稱點「黏」起來), 找一個三角分割, 計算此 *projective space* 的 *Euler characteristic*.

Problem 12 (Ex P283 6.). *Show that $(0, 0)$ is an isolated singular point and compute the index at $(0, 0)$ of the following vector fields in the plane:*

- (a) $v = (x, y)$.
- (b) $v = (-x, y)$.
- (c) $v = (x, -y)$.
- (d) $v = (x^2 - y^2, -2xy)$.
- (e) $v = (x^3 - 3xy^2, y^3 - 3x^2y)$.