Geometry Homework 2

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Problem 3 (P26: 16). *Show that the knowledge of the vector function $n = n(s)$ (normal vector) of a curve α, with nonzero torsion everywhere, determines the curvature $\kappa(s)$ and the torsion τ of α. ($\vec{n}$ 能決定曲線嗎？說明題目錯誤並找反例。)*

Proof. □

Problem 4 (P26: 17，另一種描述 Helix 的方式). *In general, a curve α is called a* helix *if the tangent lines of α make a constant angle with a fixed direction. Assume that $\tau(s) \neq 0$, $s \in I$, and prove that:*

(a) α is a helix if and only if $\kappa/\tau =$constant.

(b) α is a helix if and only if the lines containing $n(s)$ and passing through $\alpha(s)$ are parallel to a fixed plane.

(c) α is a helix if and only if the lines containing $b(s)$ and passing through $\alpha(s)$ make a constant angle with a fixed direction.

(d) The curve

$$\alpha(s) = \left(\frac{a}{c} \int \sin\theta(s) ds, \frac{a}{c} \int \cos\theta(s) ds, \frac{b}{c} s \right),$$

where $c^2 = a^2 + b^2$, is a helix, and that $\kappa/\tau = a/b$.

Problem 6. $\gamma(s)$ 長度參數。若將 $T(s)$ 寫成 $(\sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi)$，$\phi, \theta$ 是 s 的函數。說明 $\kappa(s) = \sqrt{\phi'^2 + \theta'^2 \sin^2\phi}$。

Problem 7. $\gamma : \mathbb{R} \to \mathbb{R}^3$，不妨假設是長度參數。

(b) 若 $M^t M = I$，$\det(M) = -1$ 且 $\overline{\gamma} = M\gamma$，討論 κ, τ 變化。

(c) $\overline{\gamma}(s) = \gamma(-s)$，說明 κ, τ 變化。

Problem 8. 說明 $\overline{\gamma}(u) = \gamma(-s)$ 時，在對應點

$$\frac{\det(\overline{\gamma}', \overline{\gamma}'', \overline{\gamma}''')}{|\overline{\gamma}' \times \overline{\gamma}''|^2}(u) = \frac{\det(\gamma', \gamma'', \gamma''')}{|\gamma' \times \gamma''|^2}(t)$$

再用 *chain rule* 直接說明。

Proof. □