Geometry Homework 13

B96201044 黃上恩, *B98901182* 時丕勳, *K0020100x* 劉士瑋

January 4, 2012

Problem 4. *Helicoid* $\mathbb{X}(u, v) = (v\cos u, v\sin u, u)$, $\gamma(t) = \mathbb{X}(t, 1)$, $p = \mathbb{X}(0, 1) = (1, 0, 0)$, $V(0) = \gamma'(0)$ 求解平行向量場 $V(t)$ *along* $\gamma(t)$

Problem 6. 如圖考慮一旋轉體上的緯圈 γ，已知其 *generating curve(經線)* 切線與中心軸夾角為 θ。

 (a) 求一向量沿 γ 平行移動，繞一圈後與原向量的夾角 *(不妨假設起始向量 與緯圈同向)*

 (b) 將該 *surface* 放大或縮小，相對應問題的夾角有何變化

 (c) 計算此緯圈之 $\oint_\gamma \kappa_g \mathrm{d}s$，值與 *surface* 的縮放有關嗎？

Problem 10 (Ex P282 4.)**.**

 (a) *Compute the Euler-Poincaré characteristic of (1) An ellipsoid. (2) The surface* $S = \{(x, y, z) \in \mathbb{R}^3; x^2 + y^{10} + z^6 = 1\}$.

 (b) 如圖，將一圓盤的邊界如圖「黏」起來 *(也可以想成將對稱點「黏」起 來)*，找一個三角分割，計算此 *projective space* 的 *Euler characteristic*。

Problem 12 (Ex P283 6.)**.** *Show that* $(0, 0)$ *is an isolated singular point and compute the index at* $(0, 0)$ *of the following vector fields in the plane:*

 (a) $v = (x, y)$.

 (b) $v = (-x, y)$.

 (c) $v = (x, -y)$.

 (d) $v = (x^2 - y^2, -2xy)$.

 (e) $v = (x^3 - 3xy^2, y^3 - 3x^2y)$.