

GEOMETRY HOMEWORK 4

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Problem 3.

- (a) 假設 $\kappa(s) \neq 0, \tau(s) \neq 0$, 由四點決定一球, 討論空間曲線 $\gamma(s)$ 的密切球, 並決定球心與半徑。
- (b) 討論螺線 $(a \cos t, a \sin t, bt)$ 的密切球, $a > 0$ 。

Proof.

□

Problem 4. $\kappa \neq 0, \tau \neq 0$ 為兩常數, 請決定 $\kappa(s) = \kappa, \tau(s) = \tau$ 的曲線方程式。(長度參數 s)

Proof. If $\kappa \neq 0$, upto translations and rotations, all space curves $\alpha(s)$ satisfying the condition are of the following form

$$\begin{aligned}\alpha(s) &= \left(\frac{\kappa}{\kappa^2 + \tau^2} \sin \sqrt{\kappa^2 + \tau^2} s, \frac{\kappa}{\kappa^2 + \tau^2} \cos \sqrt{\kappa^2 + \tau^2} s, \frac{\tau}{\sqrt{\kappa^2 + \tau^2}} s \right) \\ T(s) &= \left(\frac{\kappa}{\sqrt{\kappa^2 + \tau^2}} \cos \sqrt{\kappa^2 + \tau^2} s, -\frac{\kappa}{\sqrt{\kappa^2 + \tau^2}} \sin \sqrt{\kappa^2 + \tau^2} s, \frac{\tau}{\sqrt{\kappa^2 + \tau^2}} \right) \\ \|T(s)\| &= 1 \quad (\text{arc-length}) \\ T'(s) &= (-\kappa \sin \sqrt{\kappa^2 + \tau^2} s, -\kappa \cos \sqrt{\kappa^2 + \tau^2} s, 0) \\ \kappa(s) &= \|T'(s)\| = \kappa \\ N(s) &= (-\sin \sqrt{\kappa^2 + \tau^2} s, -\cos \sqrt{\kappa^2 + \tau^2} s, 0) \\ B(s) &= T(s) \times N(s) = \left(\frac{\tau}{\sqrt{\kappa^2 + \tau^2}} \cos \sqrt{\kappa^2 + \tau^2} s, -\frac{\tau}{\sqrt{\kappa^2 + \tau^2}} \sin \sqrt{\kappa^2 + \tau^2} s, -\frac{\kappa}{\sqrt{\kappa^2 + \tau^2}} \right) \\ B'(s) &= (-\tau \sin \sqrt{\kappa^2 + \tau^2} s, -\tau \cos \sqrt{\kappa^2 + \tau^2} s, 0) \\ \tau(s) &= B'(s)/N(s) = \tau\end{aligned}$$

□

Problem 5 (Darboux vector). $\gamma(s)$ arc length

- (a) 說明 \exists vector $\omega(s)$ (called Darboux vector) such that

$$\begin{cases} T' &= \omega \times T \\ N' &= \omega \times N \\ B' &= \omega \times B \end{cases}$$

(b) $V(s)$ is a vector along $\gamma(s)$ 且 w.r.t (T, N, B) , $V(s) = (v_1(s), v_2(s), v_3(s))$
 $\Rightarrow V' = (v'_1, v'_2, v'_3) + \omega \times V$

(c) 說明 $\omega = \frac{1}{2}(T \times T' + N \times N' + B \times B')$

Problem 8.

(a) 令函數 $x_i : \mathbb{R}^n \rightarrow \mathbb{R}, (x_1, \dots, x_n) \mapsto x_i$ 。計算 $[dx_i]$ ，在不同的 $a \in \mathbb{R}^n$ ， dx_i 如何隨 a 變化。

(b) 由上題微分式 $df = \frac{\partial f}{\partial x_1} dx_1 + \dots + \frac{\partial f}{\partial x_n} dx_n$ 與映射 df 結合起來。

(c) $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ ，怎麼利用上題幫你計算 df