

GEOMETRY HOMEWORK 11

B96201044 黃上恩, B98901182 時丕勳, K0020100x 劉士璋

December 15, 2011

Problem 4. Show that if all the geodesics of a connected surface are plane curves, then the surface is contained in a plane or a sphere.

Problem 5. Let $\alpha : I \rightarrow S$ be a curve parametrized by arc length s , with nonzero curvature. Consider the parametrized surface

$$\mathbf{x}(s, v) = \alpha(s) + vb(s), \quad s \in I, -\epsilon < v < \epsilon, \epsilon > 0,$$

where b is the binormal vector of α . Prove that if ϵ is small, $\mathbf{x}(I \times (-\epsilon, \epsilon)) = S$ is a regular surface over which $\alpha(I)$ is geodesic. (thus, every curve is a geodesic on the surface generated by its binormals).

Problem 8. 用 (A) 表示在座標變換下不變、用 (B) 表示在 *isometry* 下不變 (保 E, F, G) 下的性質

	line of curvature	geodesic	asymptotic curve	Γ_{ij}^k	H	K
(A)						
(B)						

Problem 9. 考慮 p221, p222 中 *helicoid* Y 和 *catenoid* X 的 parametrization.

- (a) X 中的 *geodesics* 相對應映到 Y 中也是 *geodesics* 嗎?
- (b) 已知 X 的經線 ($u = \text{const}$) 與 $v = 0$ 都是 *geodesics*。描述他們在 Y 中的對應曲線? 他們都是 *geodesics* 嗎?