

Geometry Homework 1

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Problem 1 (P7: 4.). Let $\alpha : (0, \pi) \rightarrow \mathbf{R}^2$ be given by

$$\alpha(t) = \left(\cos t, \cos t + \log \tan \frac{t}{2} \right),$$

where t is the angle that the y axis makes with the vector $\alpha(t)$. The trace of α is called the tractrix (Fig. 1-9). Show that

- (a) α is a differentiable parametrized curve, regular except at $t = \pi/2$.
- (b) The length of the segment of the tangent of the tractrix between the point of tangency and the y axis is constantly equal to 1.

Proof. this is another test □

Problem 2 (Curvature is a geometric object I). $\alpha X(s) = (x(s), y(s))$, where s is the arc-length parameter.

$$M = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, M^t = M^{-1}, \text{ i.e. } M \text{ is orthogonal.}$$

Let $\bar{X}(s) = M \cdot \begin{bmatrix} x(s) \\ y(s) \end{bmatrix} + \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$, $\alpha, \beta \in \mathbf{R}$. What is the relation between $\kappa_X(s)$ and $\kappa_{\bar{X}}(s)$?

Proof. This is the test. □