Geometry Homework 12

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Problem 3 (Ex p294 3.). *If p is a point of a regular surface S, prove that*

$$K(p) = \lim_{r \to 0} \frac{12}{\pi} \frac{\pi r^2 - A}{r^4},$$

where $K(p)$ is the Gaussian curvature of S at p, r is the radius of a geodesic circle $S_r(p)$ centered in p, and A is the area of the region bounded by $S_r(p)$.

Proof.

$$\begin{aligned}
A_R &= \int_0^R \int_0^{2\pi} \sqrt{EG - F^2} d\theta dr \\
&= \int_0^R \int_0^{2\pi} \sqrt{G} d\theta dr \\
&\approx \int_0^R \int_0^{2\pi} r - \frac{K}{6} r^3 d\theta dr \\
&= \int_0^{2\pi} \frac{1}{2} R^2 - \frac{K}{24} R^4 d\theta \\
&= \pi R^2 - \frac{R^4}{24} \int_0^{2\pi} K d\theta \\
\rightarrow \frac{1}{2\pi} \int_0^{2\pi} K d\theta &= \frac{12}{r^4}(r^2 - \frac{1}{\pi} A_r) \\
\rightarrow K(p) &= \lim_{r \to 0} \frac{12}{r^4}(r^2 - \frac{1}{\pi} A_r) \\
&= \lim_{r \to 0} \frac{12}{\pi} \frac{\pi r^2 - A_r}{r^4}
\end{aligned}$$

□

Problem 4 (Ex p295 4.). *Show that in a system of normal coordinates centered in p, all the Christoffel symbols are zero at p.*

Problem 5 (Ex p295 5.). *For which of the pair of surfaces given below does there exist a local isometry?*

(a) *Torus of revolution and cone.*

(b) *Cone and sphere.*

(c) *Cone and cylinder.*

Problem 8.

(a) 在半徑 R 的球面上，計算 *geodesic circle* 的長度，並驗證 *P292* 課文中間 $K(p)$ 的公式。

(b) 用一樣的精神，檢驗 *P294 3.* 的公式。