Geometry Homework 11

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Problem 4. *Show that if all the geodesics of a connected surface are plane curves, then the surface is contained in a plane or a sphere.*

Problem 5. *Let $\alpha : I \to S$ be a curve parametrized by arc length s, with nonzero curvature. Consider the parametrized surface*

$$\mathbf{x}(s,v) = \alpha(s) + vb(s), \quad s \in I, -\epsilon < v < \epsilon, \epsilon > 0,$$

where b is the binormal vector of α. Prove that if ϵ is small, $\mathbf{x}(I \times (-\epsilon, \epsilon)) = S$ is a regular surface over which $\alpha(I)$ is geodesic. (thus, every curve is a geodesic on the surface generated by its binormals).

Proof.

$$\begin{aligned} \mathbf{x}_s &= \alpha'(s) + vb'(s) \\ &= t(s) + v\tau(s)n(s) \\ \mathbf{x}_v &= b(s) \\ \to \mathbf{x}_s \times \mathbf{x}_v &= -n(s) + v\tau(s)t(s) \\ &\neq 0 \end{aligned}$$

So $\mathbf{x}$ is a regualr surface.
Since $\alpha''(s) = n(s)$, and at $v = 0$, $N \parallel \mathbf{x}_s \times \mathbf{x}_v = -n(s)$. So $\alpha'(s) \parallel N$, and $\kappa_g = 0$. So $\alpha(I)$ is geodesic. □

Problem 8. 用 *(A)* 表示在座標變換下不變、用 *(B)* 表示在 *isometry* 下不變 *(*保 E, F, G*)* 下的性質

	line of curvature	*geodesic*	*asymptotic curve*	Γ_{ij}^k	H	K
(A)	*Yes*	*Yes*	*Yes*	*No*	*Yes*	*Yes*
(B)	*(2)*	*Yes*	*(6)*	*Yes*	*(10)*	*Yes*

Problem 9. 考慮 *p221, p222* 中 *helicoid* Y 和 *catenoid* X 的 *parametrization*。
$X(u,v) = (a\cosh v \cos u, a\cosh v \sin u, av)$

(a) X 中的 *geodesics* 相對應映到 Y 中也是 *geodesics* 嗎？

(b) 已知 X 的經線 *(*$u =$ *const)* 與 $v = 0$ 都是 *geodesics*。描述他們在 Y 中的對應曲線？他們都是 *geodesics* 嗎？