Geometry Homework 4

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Problem 3.

(a) 假設 $\kappa(s) \neq 0, \tau(s) \neq 0$，由四點決定一球，討論空間曲線 $\gamma(s)$ 的密切球，並決定球心與半徑。

(b) 討論螺線 $(a\cos t, a\sin t, bt)$ 的密切球，$a > 0$。

Proof. □

Problem 4. $\kappa \neq 0, \tau \neq 0$ 為兩常數，請決定 $\kappa(s) = \kappa, \tau(s) = \tau$ 的曲線方程式。*(*長度參數 s*)*

Proof. If $\kappa \neq 0$, upto translations and rotations, all space curves $\alpha(s)$ satisfying the condition are of the following form

$$\begin{aligned}
\alpha(s) &= (\frac{\kappa}{\kappa^2+\tau^2}\sin\sqrt{\kappa^2+\tau^2}s, \frac{\kappa}{\kappa^2+\tau^2}\cos\sqrt{\kappa^2+\tau^2}s, \frac{\tau}{\sqrt{\kappa^2+\tau^2}}s) \\
T(s) &= (\frac{\kappa}{\sqrt{\kappa^2+\tau^2}}\cos\sqrt{\kappa^2+\tau^2}s, -\frac{\kappa}{\sqrt{\kappa^2+\tau^2}}\sin\sqrt{\kappa^2+\tau^2}s, \frac{\tau}{\sqrt{\kappa^2+\tau^2}})
\end{aligned}$$

$$\|T(s)\| = 1 \quad \text{(arc-length)}$$

$$\begin{aligned}
T'(s) &= (-\kappa\sin\sqrt{\kappa^2+\tau^2}s), -\kappa\cos\sqrt{\kappa^2+\tau^2}s, 0) \\
\kappa(s) &= \|T(s)\| = \kappa \\
N(s) &= (-\sin\sqrt{\kappa^2+\tau^2}s), -\cos\sqrt{\kappa^2+\tau^2}s, 0) \\
B(s) &= T(s)\times N(s) = (\frac{\tau}{\sqrt{\kappa^2+\tau^2}}\cos\sqrt{\kappa^2+\tau^2}s, -\frac{\tau}{\sqrt{\kappa^2+\tau^2}}\sin\sqrt{\kappa^2+\tau^2}s, -\frac{\kappa}{\sqrt{\kappa^2+\tau^2}}) \\
B'(s) &= (-\tau\sin\sqrt{\kappa^2+\tau^2}s, -\tau\cos\sqrt{\kappa^2+\tau^2}s, 0) \\
\tau(s) &= B'(s)/N(s) = \tau
\end{aligned}$$

If $\kappa = 0$, it's trivially $\alpha(s) = (0, 0, s)$. □

Problem 5 (Darboux vector)**.** $\gamma(s)$ *arc length*

(a) 說明 $\exists$ *vector* $\omega(s)$ *(called* Darboux vector*) such that*

$$\begin{cases} T' &= \omega \times T \\ N' &= \omega \times N \\ B' &= \omega \times B \end{cases}$$

(b) $V(s)$ *is a vector along* $\gamma(s)$ 且 *w.r.t* (T, N, B), $V(s) = (v_1(s), v_2(s), v_3(s))$
$\Rightarrow V' = (v_1', v_2', v_3') + \omega \times V$

(c) 說明 $\omega = \frac{1}{2}(T \times T' + N \times N' + B \times B')$

Problem 8.

(a) 令函數 $x_i : \mathbb{R}^n \to \mathbb{R}.(x_1, \cdots, x_n) \mapsto x_i$。計算 $[dx_i]$，在不同的 $a \in \mathbb{R}^n$，dx_i 如何隨 a 變化。

(b) 由上題微分式 $df = \frac{\partial f}{\partial x_1} dx_1 + \cdots + \frac{\partial f}{\partial x_n} dx_n$ 與映射 df 結合起來。

(c) $f : \mathbb{R}^n \to \mathbb{R}^m$，怎麼利用上題幫你計算 df