

## GEOMETRY HOMEWORK 8

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**Problem 2.** 考慮直線族  $L_\lambda : \frac{x}{\lambda} + \frac{y}{1-\lambda} = 1$ , 令 ruled surface  $\mathbb{X}$  為  $(L_\lambda, \lambda) \subset \mathbb{R}^2 \times \mathbb{R}$

- (a) 求出 line of striction(龍骨)  $\beta(\lambda) \in \mathbb{R}^3$
- (b) 令  $\gamma(\lambda)$  為  $\beta(\lambda)$  在  $\mathbb{R}^2$  上的投影, 說明  $L_\lambda$  為  $\gamma(\lambda)$  的切線
- (c)  $\gamma(\lambda)$  是圓嗎? 其方程式為何 (以  $f(x, y) = c$  的方式表示)?

*Proof.*

□

**Problem 4** (Ex p.210 6). Let

$$\mathbf{X}(t, v) = \alpha(t) + vw(t)$$

be a developable surface. Prove that at a regular point we have

$$\langle N_v, \mathbf{X}_v \rangle = \langle N_v, \mathbf{X}_t \rangle = 0.$$

Conclude that the tangent plane of a developable surface is constant along (the regular points of) a fixed ruling.

**Problem 5** (Ex p.210 8). Show that if  $C \subset S^2$  is a parallel of a unit sphere  $S^2$ , then the envelope of tangent planes of  $S^2$  along  $C$  is either a cylinder, if  $C$  is an equator, or a cone, if  $C$  is not an equator.