

# GEOMETRY HOMEWORK 4

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## Problem 3.

- (a) 假設  $\kappa(s) \neq 0, \tau(s) \neq 0$ , 由四點決定一球, 討論空間曲線  $\gamma(s)$  的密切球, 並決定球心與半徑。
- (b) 討論螺線  $(a \cos t, a \sin t, bt)$  的密切球,  $a > 0$ 。

*Proof.*

□

**Problem 4.**  $\kappa \neq 0, \tau \neq 0$  為兩常數, 請決定  $\kappa(s) = \kappa, \tau(s) = \tau$  的曲線方程式。(長度參數  $s$ )

*Proof.* Up to translations and rotations, all space curves  $\alpha(s)$  satisfying the condition are of the following form

$$\begin{aligned}\alpha(s) &= \left( \frac{\kappa}{\kappa^2 + \tau^2} \sin \sqrt{\kappa^2 + \tau^2} s, \frac{\kappa}{\kappa^2 + \tau^2} \cos \sqrt{\kappa^2 + \tau^2} s, \frac{\tau}{\sqrt{\kappa^2 + \tau^2}} s \right) \\ T(s) &= \left( \frac{\kappa}{\sqrt{\kappa^2 + \tau^2}} \cos \sqrt{\kappa^2 + \tau^2} s, -\frac{\kappa}{\sqrt{\kappa^2 + \tau^2}} \sin \sqrt{\kappa^2 + \tau^2} s, \frac{\tau}{\sqrt{\kappa^2 + \tau^2}} \right) \\ \|T(s)\| &= 1 \quad (\text{arc-length}) \\ T'(s) &= (-\kappa \sin \sqrt{\kappa^2 + \tau^2} s, -\kappa \cos \sqrt{\kappa^2 + \tau^2} s, 0) \\ \kappa(s) &= \|T'(s)\| = \kappa \\ N(s) &= (-\sin \sqrt{\kappa^2 + \tau^2} s, -\cos \sqrt{\kappa^2 + \tau^2} s, 0) \\ B(s) &= T(s) \times N(s) = \left( \frac{\tau}{\sqrt{\kappa^2 + \tau^2}} \cos \sqrt{\kappa^2 + \tau^2} s, -\frac{\tau}{\sqrt{\kappa^2 + \tau^2}} \sin \sqrt{\kappa^2 + \tau^2} s, -\frac{\kappa}{\sqrt{\kappa^2 + \tau^2}} \right) \\ B'(s) &= (-\tau \sin \sqrt{\kappa^2 + \tau^2} s, -\tau \cos \sqrt{\kappa^2 + \tau^2} s, 0) \\ \tau(s) &= B'(s)/N(s) = \tau\end{aligned}$$

□

## Problem 5 (Darboux vector). $\gamma(s)$ arc length

- (a) 說明  $\exists$  vector  $\omega(s)$  (called Darboux vector) such that

$$\begin{cases} T' &= \omega \times T \\ N' &= \omega \times N \\ B' &= \omega \times B \end{cases}$$

(b)  $V(s)$  is a vector along  $\gamma(s)$  且 w.r.t  $(T, N, B)$ ,  $V(s) = (v_1(s), v_2(s), v_3(s))$   
 $\Rightarrow V' = (v'_1, v'_2, v'_3) + \omega \times V$

(c) 說明  $\omega = \frac{1}{2}(T \times T' + N \times N' + B \times B')$

**Problem 8.**

(a) 令函數  $x_i : \mathbb{R}^n \rightarrow \mathbb{R}, (x_1, \dots, x_n) \mapsto x_i$ 。計算  $[dx_i]$ ，在不同的  $a \in \mathbb{R}^n$ ， $dx_i$  如何隨  $a$  變化。

(b) 由上題微分式  $df = \frac{\partial f}{\partial x_1} dx_1 + \dots + \frac{\partial f}{\partial x_n} dx_n$  與映射  $df$  結合起來。

(c)  $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ ，怎麼利用上題幫你計算  $df$