Geometry Homework 12

B96201044 黃上恩, *B98901182* 時丕勳, *K0020100x* 劉士瑋

December 21, 2011

Problem 3 (Ex p294 3.). *If p is a point of a regular surface S, prove that*

$$K(p) = \lim_{r \to 0} \frac{12}{\pi} \frac{\pi r^2 - A}{r^4},$$

where $K(p)$ is the Gaussian curvature of S at p, r is the radius of a geodesic circle $S_r(p)$ centered in p, and A is the area of the region bounded by $S_r(p)$.

Problem 4 (Ex p295 4.). *Show that in a system of normal coordinates centered in p, all the Christoffel symbols are zero at p.*

Problem 5 (Ex p295 5.). *For which of the pair of surfaces given below does there exist a local isometry?*

(a) Torus of revolution and cone.

(b) Cone and sphere.

(c) Cone and cylinder.

Problem 8.

(a) 在半徑 R 的球面上，計算 *geodesic circle* 的長度，並驗證 *P292* 課文中間 $K(p)$ 的公式。

(b) 用一樣的精神，檢驗 *P294 3.* 的公式。