

GEOMETRY HOMEWORK 8

B96201044 黃 恩, B98901182 時 勳, K0020100x 劉士瑋

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Problem 2. 考慮直線族 $L_\lambda : \frac{x}{\lambda} + \frac{y}{1-\lambda} = 1$, 令 ruled surface \mathbb{X} 為 $(L_\lambda, \lambda) \subset \mathbb{R}^2 \times \mathbb{R}$

- (a) 求出 line of striction(龍骨) $\beta(\lambda) \in \mathbb{R}^3$
- (b) 令 $\gamma(\lambda)$ 為 $\beta(\lambda)$ 在 \mathbb{R}^2 上的投影, 說明 L_λ 為 $\gamma(\lambda)$ 的切線
- (c) $\gamma(\lambda)$ 是圓嗎? 其方程式為何 (以 $f(x, y) = c$ 的方式表示)?

Proof.

□

Problem 4 (Ex p.210 6). Let

$$\mathbf{X}(t, v) = \alpha(t) + v w(t)$$

be a developable surface. Prove that at a regular point we have

$$\langle N_v, \mathbf{X}_v \rangle = \langle N_v, \mathbf{X}_t \rangle = 0.$$

Conclude that the tangent plane of a developable surface is constant along (the regular points of) a fixed ruling.

Proof.

$$\mathbf{X}_{vv} = 0 \Rightarrow g = \langle N, \mathbf{X}_{vv} \rangle = 0;$$

$$K = \det(-dN) = 0 \Rightarrow eg = f^2 \Rightarrow f = 0;$$

$$N_v = dN(\mathbf{X}_v) = \begin{bmatrix} E & F \\ F & G \end{bmatrix}^{-1} \begin{bmatrix} e & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\Rightarrow \langle N_v, \mathbf{X}_v \rangle = \langle N_v, \mathbf{X}_t \rangle = 0.$$

Thus N , the normal vector of the tangent plane, is independent of v and hence the conclusion follows. □

Problem 5 (Ex p.210 8). Show that if $C \subset S^2$ is a parallel of a unit sphere S^2 , then the envelope of tangent planes of S^2 along C is either a cylinder, if C is an equator, or a cone, if C is not an equator.