2010 Geometry: Homework 1

due 2011/09/23 with items marked ✠

1. P7: 2
2. P7: 3
3. ✠P7: 4. (There is something wrong in the graph.)
4. P11: 10
5. ✠P47: 6
6. s is the arc-length parametr of $X(s)$. Let $Y(\bar{s}) \equiv X(\bar{s}+c)$, c is a constant.
   (a) Is $\bar{s}$ the arc length parameter of Y.
   (b) What is the relation between $\kappa(s)$ and $\kappa(\bar{s})$
7. (a) Let $Z(\bar{s}) = X(-\bar{s})$. Answer the two questions in 6.
   (b) Let $W(\bar{s}) = \lambda . X(-\bar{s})$, $\lambda \neq 0$. Answer the two questions in 6.
8. ✠(Curvature is a geometric object I)

   $X(s) = (x(s), y(s))$, where s is the arc-length parameter.

   $$M = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, M^t = M^{-1}, \text{i.e. } M \text{ is orthogonal.}$$

   Let $\bar{X}(s) = M \cdot \begin{bmatrix} x(s) \\ y(s) \end{bmatrix} + \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$, $\alpha, \beta \in \mathbf{R}$. What is the relation between $\kappa_X(s)$ and $\kappa_{\bar{X}}(s)$.
9. ✠(Curvature is a geometric object II)

   $X(t) = (x(t), y(t))$ be a regular curve. Let

   $$\kappa(x(t), y(t)) \equiv \kappa(t) = \frac{\begin{vmatrix} x' & y' \\ x'' & y'' \end{vmatrix}}{(x'^2 + y'^2)^{\frac{3}{2}}}$$

   Let $Y(u) = X(t(u)), t'(u) \neq 0$. Discuss the relation of $\kappa(x(t), y(t))$ and $\kappa(x(t(u)), y(t(u)))$ at the corresponding points.
10. ✠Let $F(x, y) = c$ defines a plane curve. Prove that the curvature of the curve satisfies

   $$|\kappa| = \left| \frac{\begin{bmatrix} F_y, -F_x \end{bmatrix} \begin{bmatrix} F_{xx} & F_{xy} \\ F_{xy} & F_{yy} \end{bmatrix} \begin{bmatrix} F_y \\ -F_x \end{bmatrix}}{(F_x^2 + F_y^2)^{\frac{3}{2}}} \right|$$

   Where $F_x^2 + F_y^2 \neq 0$.