Geometry Homework 8

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Problem 2. 考慮直線族 $L_\lambda : \frac{x}{\lambda} + \frac{y}{1-\lambda} = 1$，令 *ruled surface* $\mathbb{X}$ 為 $(L_\lambda, \lambda) \subset \mathbb{R}^2 \times \mathbb{R}$

(a) 求出 *line of striction(*龍骨*)* $\beta(\lambda) \in \mathbb{R}^3$

(b) 令 $\gamma(\lambda)$ 為 $\beta(\lambda)$ 在 $\mathbb{R}^2$ 上的投影，說明 L_λ 為 $\gamma(\lambda)$ 的切線

(c) $\gamma(\lambda)$ 是圓嗎？其方程式為何（以 $f(x, y) = c$ 的方式表示）*?*

Proof. □

Problem 4 (Ex p.210 6)**.** *Let*

$$\mathbf{X}(t, v) = \alpha(t) + vw(t)$$

be a developable surface. Prove that at a regular point we have

$$\langle N_v, \mathbf{X}_v\rangle = \langle N_v, \mathbf{X}_t\rangle = 0.$$

Conclude that the tangent plane of a developable surface is constant along *(the regular points of)* a fixed ruling.

Proof.

$$\begin{aligned}
\mathbf{X}_{vv} =&0 \Rightarrow g = \langle N, \mathbf{X}_{vv}\rangle = 0;\\
K =&\det(-dN) = 0 \Rightarrow eg = f^2 \Rightarrow f = 0;\\
N_v =&dN(\mathbf{X}_v) = \begin{bmatrix} E & F \\ F & G \end{bmatrix}^{-1} \begin{bmatrix} e & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};\\
\Rightarrow \langle N_v, \mathbf{X}_v\rangle =&\langle N_v, \mathbf{X}_t\rangle = 0.
\end{aligned}$$

Thus N, the normal vector of the tangent plane, is independent of v and hence the conclusion follows. □

Problem 5 (Ex p.210 8)**.** *Show that if* $C \subset S^2$ *is a parallel of a unit sphere* S^2*, then the envelope of tangent planes of* S^2 *along* C *is either a cylinder, if* C *is an equator, or a cone, if* C *is not an equator.*

Proof. WLOG, let the unit sphere's centre be the origin and let the plane on which the C is be parallel to the xy-plane. If C is an equator, that is, on the xy-plane, the tangent plane of each point is therefore parallel to the z-axis and thus the envelope form a cylinder. Hence consider that C is not on the xy plane. By the symmetry of S and C, the intersection of the envelope and any plane containing z-axis is identical up to rotation along z-axis. Picking such a plane and observing that the intersection being a line should intersect z-axis at exactly one point since $\alpha \neq 0$, we conclude that each intersection passes through the very point in z-axis. Let the point in z-axis be the generator of the envelope. Since each ruler should pass through exactly one point in C, the envelope therefore forms a cone.

$\square$