Geometry Homework 1

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Problem 3 (P7: 4). Let $\alpha:(0,\pi)\to\mathbf{R}^2$ be given by

$$lpha(t) = \left(\cos t, \cos t + \log an rac{t}{2}
ight)$$
 ,

where t is the angle that the y axis makes with the vector $\alpha(t)$. The trace of α is called the tractrix (Fig. 1-9). Show that

- (a) α is a differentiable parametrized curve, regular except at $t=\pi/2$.
- (b) The length of the segment of the tangent of the tractrix between the point of tangency and the y axis is constantly equal to 1.

Proof. This is the proof.

Problem 5 (P47: 6). If a closed plane curve C is contained inside a disk of radius r, prove that there exists a point $p \in C$ such that the curvature κ of C at p satisfies $|\kappa| \geq 1/r$.

Problem 8 (Curvature is a geometric object I.). X(s) = (x(s), y(s)), where s is the arc-length parameter.

$$M=\left[egin{array}{ccc} a_{11} & a_{12} \ a_{21} & a_{22} \end{array},
ight]M^t=M^{-1}, i.e. \,\,M\,\,\,is\,\,orthogonal.$$

Let $ar{M}(s)=M\cdot\left[egin{array}{c} x(s) \\ y(s) \end{array}
ight]+\left[egin{array}{c} lpha \\ eta \end{array}
ight]$, $lpha,eta\in\mathbf{R}$. What is the relation between $\kappa_X(s)$ and $\kappa_{ar{X}}(s)$?

Problem 9 (Curvature is a geometric object II.). X(t) = (x(t), y(t)) be a regular curve. Let

$$\kappa(x(t),y(t)) \equiv \kappa(t) = rac{\left|egin{array}{cc} x' & y' \ x'' & y'' \end{array}
ight|}{\left(x'^2 + y'^2
ight)^{rac{3}{2}}}$$

Let Y(u) = X(t(u)), $t'(u) \neq 0$. Discuss the relation of $\kappa(x(t), y(t))$ and $\kappa(x(t(u)), y(t(u)))$ at the corresponding points.

Problem 10. Let F(x,y) = c defines a plane curve. Prove that the curvature of the curve satisfies

$$|\kappa| = \left| egin{array}{ccc} \left[& F_y, & -F_x \end{array}
ight] \left[egin{array}{ccc} F_{xx} & F_{xy} \ F_{xy} & F_{yy} \end{array}
ight] \left[egin{array}{c} F_y \ -F_x \end{array}
ight] }{(F_x^2 + F_y^2)^{rac{3}{2}}}
ight|$$

Where $F_x^2 + F_y^2 \neq 0$.