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A novel approach based on heuristics and a neural network to solve a capacitated location routing problem

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ABSTRACT

In this work, we describe a method to solve the capacitated location-routing problem (CLRP) to minimize the delivery distance for a vehicle. The CLRP consists of locating depots, assigning each customer to one depot, and determining routes. The objective is to minimize the cost (distance). In the locating problem, we use a self-organizing map (SOM) to determine the depots and assign customers to depots. The SOM is an unsupervised learning method with two layers and has proven effective in several research areas, such as clustering. In the routing problem, we use the Clarke and Wright technique to determine routes. In the present work, we propose an improvement of the capacitated self-organizing map (CSOM) to optimize the location of depots and the Or-Opt algorithm to ameliorate the routes obtained by Clarke and Write (CSOM&CW). The numerical results show that the proposed method can meet many benchmarks of small and medium instances. Computational results assess the higher performance of our approach and demonstrate its efficiency in solving large-size instances.

1. Introduction

One of the most important problems in supply chain management is the location routing problem (LRP). It contains both the facility location problem (FLP) and the vehicle routing problem (VRP) since both problems have been studied independently in several papers. In fact, the effect of ignoring routes when locating a depot may lead to suboptimal solutions. Boventer first proposed the simultaneous optimization of the FLP and VRP [1]. Watson-Gandy and Dohrn [2] proposed the basic LRP in 1973. They considered customers' visits while locating depots. Salhi and Rand [3] and Nagy and Salhi [4] showed the potential benefits found when vehicle routing decisions are considered in a location problem. As a result, the LRP has become an active research area in the field of logistics optimization. Several extensions and different applications of LRP were developed, such as dynamic LRP [5] and the LRP with time windows [6,7]. This problem consists of finding the placement of a depot and routing between the customer and facilities within allowable times, and LRP with pickup and delivery [8]. In addition, many authors have studied the integration of energy and environment problems [9]. In the [10] the authors proposed a LRP with electric vehicle (EV). An introduction of the pollution routing problem (PRP) in classical LRP in which the CO2 emissions are considered [11].

Some researchers consider either capacitated routes or capacitated depots [5,12,13]. A synthesis of LRP studies without capacity constraints can also be found in List and Mirchandani [14]. However, some authors studied the LRP according to time windows [6,15], limited heterogeneous fleets [16] or the uncertainty of some parameters [17,18]. Lopes et al. [19], Prodhon and Prins [20],

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and Drexl and Schneider [21] analysed and wrote literature updates for the LRP. Xie et al. [22] proposed a mixed-integer formulation for the periodic location routing problem. Their objective was to minimize the fixed facility costs, routing costs, and customer penalty costs. Schiffer and Walther [23] addressed a robust LRP combining two decisions, routing vehicles and locating charging stations for designing electric logistic fleet networks. Rabbani et al. [24] studied a new industrial hazardous waste LRP with three (multi) objective functions, including total cost, hazardous waste total transportation risk, and the total site risk.

Solution approaches for the LRP can be divided approximately into heuristics and exact methods [25]. Xie et al. [22] proposed the first exact method branch and bound algorithm to solve the problem. Akca et al. [26] described a set-partitioning formulation of LRP basing in a branch-and-price algorithm. Ponboon et al. [27] developed a branch-and-price method to solve the LRP with Time Windows (LRPTW). These LRPs combined two problems, location problems, and vehicle routing problems. In such complicated combinatorial problems, exact methods are inefficient, especially on large LRP instances [28,29].

As expected, most of the LRP literature has successfully used several heuristics and meta-heuristic methods to solve various optimization models. Albareda-Sambola et al. [30] presented a tabu search heuristic for the LRP with a single-vehicle associated with each depot. The simulated annealing approach was applied to capacitated LRP [31]. Hemmelmayer et al. [32] developed an adaptive large neighborhood search heuristic. Lopes et al. [33] and Oudouar and El Fellahi [34] proposed a hybrid genetic method for the problem. Also, Engine et al. 2019 [z] proposed a hybrid algorithm that combines variable neighborhood search (VNS) and evolutionary local search (ELS) for solving CLRP.

2. Literature review

In recent years, most studies consider capacity impacts within the LRP, this type of problem called CLRP. Wu et al. [16] are the authors who present this problem with capacity constraints for both the depots and the routes. The first exact method was a lower bound proposed by Barreto 2004 to solve the CLRP. In [35], the solution to an CLRP was obtained using a branch-and-cut method. [36] present a branch-and-price-and-cut- for the capacitated location-routing problem (CLRP). In [37] used branch and price to solve the heterogeneous fleet green vehicle routing problem with time windows. In light of its complexity, the exact algorithms have been proposed to solve the instances with up to 200 customers.

The authors have developed heuristic and metaheuristic for finding an optimal solution for large-sized CLRP instances. In [38] a hybridized greedy randomized adaptive search procedure (HGRASP) for solving the two sub-problem of CLRP. A simulated annealing algorithm in sense to test a new set of benchmark instances with capacity constraint for each depot [39]. Moreover, many authors elaborate a heuristic method, which divided the CLRP into phases and solved them sequentially or iteratively. In [30,40] the authors developed a two-phase tabu-search algorithm, this algorithm solved the facility location problem (FLP) and the vehicle routing problem (VRP) separately, also an Hybride genetic algorithm was used for the CLRP [41]. Engine and Selin presented a heuristic that combines variable neighborhood search (VNS) and evolutionary local search (ELS) [42].

This work aims to solve a CLRP using an approximate algorithm. It alternates between a facility-location phase, solved using a proposed algorithm for a capacitated self-organizing map approach (CSOM) and a routing phase, handled by a Clarke and Wright heuristic. To improve the obtained routes, we use the Or-Opt method. The objective is to minimize the total cost of distribution, the depot opening cost and the cost of using vehicles. This paper is organized as follows. Section 2 describes the mathematical formulation. Section 3 presents self-organizing maps in the first sub-section and in second, the improved SOM algorithm is reported. In Section 4 and Section 5, the Clarke and Write (CW) and Or-Opt (respectively) methods are respectively presented, Section 6 describes our new approach for solving the LRP using an improved CSOM, named CSOM&CW, in the clustering phase. In the last section, the numerical results and discussion are reported.

3. Mathematical formulation

In this paper, we are studying the CLRP, since it is an extension of the LRP. The CLRP studied in our work can be defined on a complete, weighted and directed graph, $G = (V, A)$, where V is the set of nodes comprising a subset $I$of m possible depots locations and a subset $J = V \neq \text{I}$ of n customers. A is a set of arcs, each one associating two nodes in V, except depots are not connected. The related cost for each arc $a = (i, j)$ is given by C_{ij}. Each depot $i \in I$ has a fixed opening cost O_i and a non-negative capacity W_i that must be respected when serving clients demands. Each customer $j \in J$ is characterized by a non-negative demand d_j. In addition, A set K of homogeneous vehicles available for all depots, each one with a capacity Q that must be respected when collecting customers' demands. Each vehicle has a fixed usage cost F_k. The objective of the CLRP is to determine which facilities must be opened, the allocation to open warehouses, and the design of optimal routes for visiting customers from their corresponding facilities.

The total cost of a route includes the travel costs and the fixed costs to set up a route. The objective is to determine the set of depots to open and to find the good routes to be constructed to minimize the total cost (fixed costs of depots plus operating costs of the vehicles). The cost C_{ij} of edge (i,j) is the Euclidian distance between vertices i and j. The model chosen to solve the CLRP is a zero-one linear program. The proposed approach extends the formulation introduced by Prin et al. [43]. The following constraint must hold:

- Each demand d_j should be served by a single vehicle.
- Each route must begin and end at the same depot, and its total load must be less than its capacity.
- The total load of the routes assigned to a depot must not exceed the capacity of that depot.

We use following decision variables in order to formulate CLRP.

$$y_i = \begin{cases} 1 \ \textit{if the depot } i \textit{ is opened} \\ \quad 0 \ \textit{else} \end{cases}$$

$$x_{ijk} = \begin{cases} 1 \ \textit{if edge } (i, j) \textit{is traversed from } i \textit{ to } j \textit{ in route } k \\ \quad 0 \ \textit{else} \end{cases}$$

$$f_{ij} = \begin{cases} 1 \ \textit{is cusomer } j \textit{ is affected to depot } i \\ \quad 0 \ \textit{else} \end{cases}$$

The flow-based formulation of CLRPMB is given as follows:
Minimize:

$$z = \sum_{i\in I} O_i y_i + \sum_{i\in V}\sum_{j\in V}\sum_{k\in K} C_{ij} x_{ijk} + \sum_{k\in K}\sum_{i\in I}\sum_{j\in I} F_k x_{ijk} \tag{1}$$

Subject to:

$$\sum_{k\in K}\sum_{i\in V} x_{ijk} = 1 \quad \forall \ j \in J \tag{2}$$

$$\sum_{j\in J}\sum_{i\in V} d_j x_{ijk} \le Q \quad \forall \ k \in K \tag{3}$$

$$\sum_{j\in V} x_{ijk} + \sum_{j\in V} x_{jik} = 0 \quad \forall \ i \in V, \ k \in K. \tag{4}$$

$$\sum_{i\in I}\sum_{j\in J} x_{ijk} \le 1 \quad \forall \ k \in K, \tag{5}$$

$$\sum_{i\in S}\sum_{J\in S} x_{ijk} \ \le \ |S| - 1 \quad \forall \ S \subseteq J, \ k \in K \tag{6}$$

$$\sum_{u\in J} x_{iuk} + \sum_{u\in V/\{J\}} x_{ujk} \le 1 + f_{ij} \quad \forall \ i \in I, \ j \in J, \ k \in k \tag{7}$$

$$\sum_{j\in J} d_j f_{ij} \le W_i y_i \quad \forall \ i \in I \tag{8}$$

$$x_{ijk} \in \{0, \ 1\} \quad \forall \ i \in I, \ j \in V, \ k \in K \tag{9}$$

$$y_i \in \{0, \ 1\} \quad \forall \ i \in I \tag{10}$$

$$f_{ij} \in \{0, \ 1\} \quad \forall \ i \in I, \ j \in V \tag{11}$$

The objective function (1) defines the total system cost, which is assigned to open depots, using fleets and performing the routes. Constraint (2) guarantee that each customer is visited exactly once. Constraint (3) state the total delivery demands of each route must not exceed vehicle's capacity. Route continuity and return to the depot of origin are guaranteed through constraints (4). Constraint (5) assigns only one tour to each vehicle. Sub-tour elimination is ensured in constraint (6). Constraint (7) ensures that a customer can be assigned to a depot only if a route linking them is opened. Constraint (8) guarantees the total demands of all customers assigned to a depot must not exceed depot's capacity. Finally, constraints (9), and –(11) specify the binary requirements used in the formulation.

4. Self-organizing map

An SOM or Kohonen network is a popular type of artificial neural network. It is a very powerful tool in many areas [44]. The SOM is an unsupervised neural network learning algorithm introduced by Kohonen. This unsupervised learning method is an excellent data-clustering tool as well [45]. The goal of a SOM is to project all input patterns in a high-dimensional space onto a prototype in a low-dimensional space, such that the distance and topology are preserved as much as possible [46,47].

The SOM has two layers; the first input layer distributes the input data to the output layer. The number of units on the input layer is equal to the dimension of the data vector. Each neuron [47] is connected with their neighbours according to topological connections. The SOM permits projecting high-dimensional data into lower dimension data. This projection produces a map with lower dimensionality that can be useful in analysing and discovering patterns in the input space. Consequently, the SOM can be used to identify groups for similar input data. Each neuron network is identified by its location and weight vector and is connected to a subset of its neighbours [48].

In the training process, the customers (input data) consist of n-dimensional vectors $X = \{x^1, x^2, ..., x^n\}$. The SOM contains N neurons. Each neuron has an associated vector $W = \{w_1, w_2, ..., w_N\}$. In the training phase, one customer x is compared with all w^i to

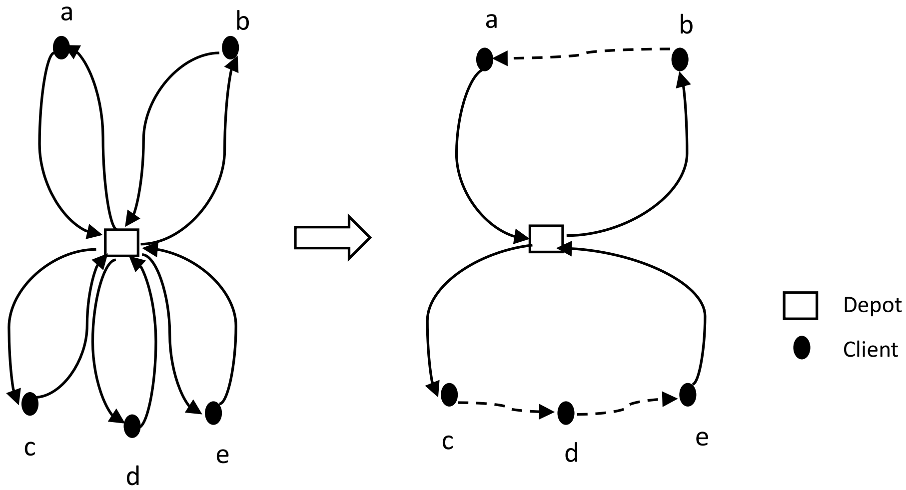

Fig. 1. Illustration of the Clarke and Wright concept.

find the closest vector w^k that satisfies a maximum similarity criterion using the Euclidean distance (Eq. (12)):

$$k = \arg\min_{i}^{N} ||x - w_i|| \tag{12}$$

where k is the index of the neuron winner, N is the number of units on the map and x is an input vector of the customers.

The following equation is used to update the weight of neuron neighbours:

$$w_i(t) = w_i(t-1) + \beta(t, k)(x - w_i(t-1)) \tag{13}$$

where $\beta = (t,\ k)$ is a decreased function between 0 and 1.

5. Clarke and Wright algorithm

The Clarke and Wright (CW) algorithm was first proposed by Clarke and Wright [49]. This method is one of the most used classical heuristics in VRPs. The purpose of this heuristic is to produce good solutions very fast. Deif and Bodin [50] developed the first heuristic for the VRP with backhauling VRPB, based on the classical well-known CW heuristic.

The CW algorithm applies to problems for which the number of routes is a decision variable. Two versions (variants) of the algorithm exist, a parallel version (variant, type) in which all routes are built simultaneously and a sequential version (variant, type) in which one route is built at a time. The parallel Clarke and Wright (PCW) heuristic is most often used.

The CW algorithm consistes of two stages. At the first stage, every customer is visited in a single route (one vehicle per customer) as shown in Fig. 1 presented in [51] . In the second stage, for every two customers i and j, the algorithm calculates the savings S_{ij}. Then, customers with larger saving values are merged in the same routes without violating the capacity constraint of the vehicle. Assuming that C_{i0} is the traveling cost between depot 0 and customer i and C_{ij} is the traveling cost between customer i and j, the algorithm works as follows:

- Step 1: Calculate the savings $S_{ij} = C_{i0} + C_{0j} - C_{ij}$ for $i, j, i \neq j$. Sort the savings S_{ij} in decreasing order.
- Step 2: Beginning from the top of the saving list (the largest S_{ij}), both customers i and j are combined into the same route if their total demand respects the capacity of the vehicle and no route constraint will be violated by including customer i and j in that route. The following route constraints need to be considered:

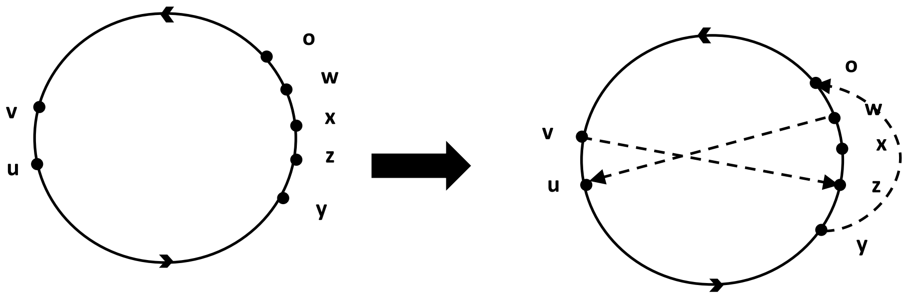

Fig. 2. An example of a three-city Or-Opt local improvement.

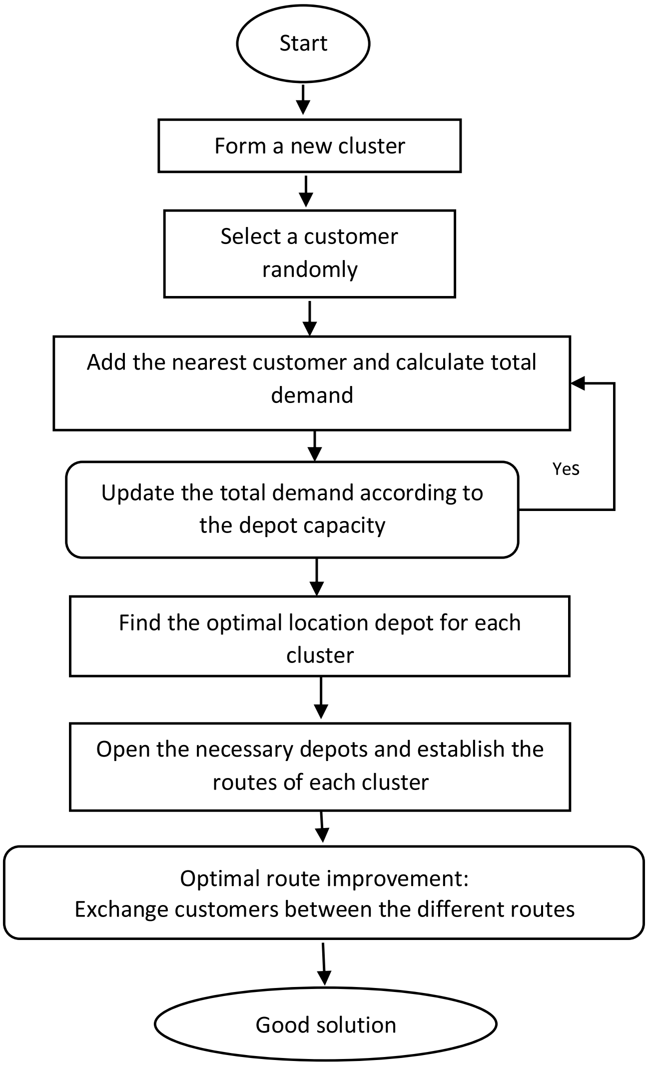

Fig. 3. Different steps of the proposed approach to solve the CLRP.

Algorithm 1
Improved capacitated Self Organizing Map algorithm (CSOM).

1 Calculate the number of centres (depots) using the customers' demand (di) and the depot's capacity (C): $N = \sum_{i=1}^{n} d_i / C$
2 Initialize the weights $w(0)$ using an auxiliary phase to generate starting points. We choose the first weight $w_1$from the input data (customers). For second weight w_2, we choose the customer further from the first weight (centre) using Euclidean distance. From weights $w_1$and w_2, we calculate the distance between these centres and all customers and we select the furthest customer as the third weight w_3, etc.
3 While not converged
a For each customer x
i Calculate the Euclidean distance x and all depots.
ii Find the nearest centre $k(t)$ using Eq. (12). If the constraint $\sum_i d_{i,k} \leq C_k$ is satisfied, the selected customer x will be assigned to center k, else customer x will be assigned to next nearest centre using Eq. (12).
iii Update the centre coordinate using Eq. (13).
b End For
The convergence condition of the learning process can be performed according to one of the following criteria:
Choosing a fixed number of steps.
The learning process continues until the adjusting quantity $\|w_i(t) - w_i(t-1)\|$ falls under some specified value, i.e. $\|w_i(t) - w_i(t-1)\| < \varepsilon$ where ε is the threshold.
4 End While

i If neither i nor j have already been assigned to a route, then a new route is constructed including both i and j.
ii If exactly one of two customers (i or j) has already been covered in an existing route and that customer is not interior to that route (a client is interior to a route if it is not adjacent to depot 0 in order of traversal of clients), then edge (i, j) is added to that same route.

Table 1
Computational results from Barreto's instances.

		Barreto	GRASP			MA/PM			LRGTS			CSOM&CW		
	BKR	*Cost*	*Cost*	n. d	*n. v*	*Cost*	*n. d*	*n. v*	*Cost*	*n. d*	*n. v*	*Cost*	*n. d*	*n. v*
Christofides69-50 × 5	565.6	582.7	599.1	3	8	565.6	2	6	586.4	2	6	591.7	2	6
Christofides69-75 × 10	861.6	886.3	861.6	3	9	866.1	3	9	863.5	3	10	837.98	3	11
Christofides69-100 × 10	842.9	889.4	861.6	3	9	850.1	2	8	842.9	2	8	848.1	3	9
Daskin95-88 × 8	355.8	384.9	356.9	2	8	355.8	2	7	368.7	2	6	348.82	2	6
Daskin95-150 × 10	44,011.7	46,642.7	44,625.2	3	13	44,011.7	3	11	44,386.3	3	14	45,163.28	3	11
Gaskell67-21 × 5	424.9	435.9	429.6	2	5	424.9	2	4	424.9	2	4	457.06	2	4
Gaskell67-22 × 5	585.1	591.5	585.1	1	3	611.8	2	3	587.4	1	3	610.32	1	3
Gaskell67-29 × 5	512.1	512.1	515.1	2	4	512.1	2	4	512.1	2	4	498.64	2	4
Gaskell67-32 × 5	571.7	571.7	571.9	2	4	571.9	2	4	584.6	2	4	578.31	2	5
Gaskell67-32 × 5	504.3	511.4	504.3	1	3	534.7	2	3	504.8	1	3	490.79	1	3
Gaskell67-36 × 5	460.4	470.7	460.4	1	4	485.4	2	4	476.5	1	4	475.22	1	4
Min92-27 × 5	3062.0	3062.0	3062.0	2	4	3062.0	2	4	3065.2	2	4	3104.54	2	4
Min92-134 × 8	5809	6238	5965.1	4	11	5950.1	4	10	5809	3	11	5740.05	4	12

Table 2
Computational results from Prins et al. instances.

Instances		GRASP			MAPM					LRGTS	CSOM&CW		
	BKR	*Cost*	*n. d*	*n. v*	*Cost*	*n. d*	*n. v*	*Cost*	*n. d*	*n. v*	*Cost*	*n. d*	*n. v*
2051a	54,793	55,021	3	5	54,793	3	5	55,131	3	5	53,785	3	6
2051	39,104	39,104	2	3	39,104	2	3	39,104	2	3	39,802	2	3
2052a	48,908	48,908	3	5	48,908	3	5	48,908	3	5	48,735	3	6
2052b	37,542	37,542	2	3	37,542	2	3	37,542	2	3	37,833	2	3
5051	90,111	90,632	3	12	90,160	3	12	90,160	3	12	87,670	3	14
5051b	63,242	64,761	2	6	63,242	2	6	63,256	2	6	63,372	2	6
5052	88,298	88,786	3	12	88,298	3	12	88,715	3	12	84,695	3	12
5052b	67,340	68,042	3	6	67,893	3	6	67,698	3	6	65,692	3	7
5052bis	84,055	84,055	3	12	84,055	3	12	84,181	3	12	85,576	3	13
5052bbis	51,822	52,059	3	6	51,822	3	6	51,992	3	6	51,919	3	6
5053	86,203	87,380	2	12	86,203	2	12	86,203	2	12	86,166	2	13
5053b	61,830	61,890	2	6	61,830	2	6	61,830	2	6	65,234	2	6
10051	275,993	279,437	3	24	281,944	3	24	277,935	3	24	273,223	3	25
10051b	214,392	216,159	3	12	216,656	3	12	214,885	3	11	206,744	3	11
10052	194,598	199,520	2	24	195,568	2	24	196,545	2	24	192,914	2	25
10052b	157,173	159,550	2	11	157,325	2	11	157,792	2	11	157,842	2	12
10053	200,246	203,999	2	25	201,749	2	24	201,952	2	24	201,259	2	25
10053b	152,586	154,596	2	11	153,322	2	11	154,709	2	12	154,871	2	12
100101	290,429	323,171	4	26	316,575	4	25	291,887	3	26	301,679	3	26
100101b	234,641	271,477	4	12	270,251	4	11	235,532	3	12	229,960	3	12
100102	244,265	254,087	3	25	245,123	3	24	246,708	3	24	239,856	3	24
100102b	203,988	206,555	3	11	205,052	3	11	204,435	3	11	204,095	3	11
100103	253,344	270,826	3	25	253,669	3	24	258,656	3	25	247,052	3	26
100103b	204,597	216,173	3	11	204,815	3	11	205,883	3	11	202,027	3	13
200101	479,425	490,820	3	48	483,497	3	47	481,676	3	47	492,482	3	49
200101b	378,773	416,753	3	22	380,044	3	22	380,613	3	22	382,266	3	23
200102	450,468	512,679	3	49	451,840	3	48	453,353	3	48	440,347	3	48
200102b	374,435	379,980	3	23	375,019	3	22	377,351	3	23	379,383	3	23
200103	472,898	496,694	3	46	478,132	3	46	476,684	3	47	463,518	3	47
200103b	364,178	389,016	3	22	364,834	3	22	365,250	3	22	357,064	3	22

iii If both clients *i* and *j* have already been inserted in two different existing routes and neither client is interior to its route, then the two routes are combined by connecting *i* and *j*. The route merging procedure is repeated until either the capacity does not allow more merging or the saving list is handled.

- Step 3: Finally, in the case of non-routed customers, consider a route that begins at depot 0, visits the unsigned customer and returns to depot 0.

6. Or-Opt

Or-Opt is one of the most famous chain exchange (CE) methods and was proposed by Ilhan Or in 1976 [52]. The complexity of Or-Opt is $O(n^2)$. The heuristic is very effective and is easy to implement. The Or-Opt heuristic improves the r-opt algorithm by reducing the number of exchanges to be tested. It moves a sequence of three, two and one consecutive cities to improve the current routes, as shown in Fig. 2 proposed in [53].

Table 3
Computational results on Tuzun-Burke's instances.

Instances	Tuzun & Burke*		GRASP			MAPM			LRGTS			CSOM&CW		
	BKR	*Cost*	*Cost*	*n. d*	*n. v*	*Cost*	*n. d*	*n. v*	*Cost*	*n. d*	*n. v*	*Cost*	*n. d*	*n. v*
111112	1468,4	1556,64	1525,25	3	11	1493,92	3	11	1490,82	3	11	1464,23	3	11
111122	1449,2	1531,88	1526,9	3	11	1471,36	2	11	1471,76	3	11	1443,37	3	12
111212	1396,46	1443,43	1423,54	2	11	1418,83	3	10	1412,04	3	10	1449,41	3	12
111222	1432,29	1511,39	1482,29	2	11	1492,46	2	11	1443,06	2	11	1453,63	2	11
112112	1167,53	1231,11	1200,24	2	11	1173,22	2	11	1187,63	2	11	1160,07	2	11
112122	1102,7	1132,02	1123,64	3	11	1115,37	3	11	1115,95	3	11	1124,1	3	11
112212	793,97	825,12	814	3	12	793,97	2	11	813,28	3	12	750,12	3	12
112222	728,3	740,54	747,84	3	11	730,51	2	11	742,96	3	11	716,3	3	12
113112	1238,49	1316,98	1273,1	3	11	1262,32	3	11	1267,93	3	11	1260,62	3	12
113122	1246,34	1274,5	1272,94	2	11	1251,32	3	11	1256,12	3	11	1289,76	3	12
113212	902,38	920,75	912,19	3	12	903,82	3	12	913,06	3	12	890,22	3	12
113222	1021,31	1042,21	1022,51	3	11	1022,93	4	11	1025,51	3	11	1005,86	3	12
131112	1866,75	2000,97	2006,7	3	16	1959,39	3	16	1946,01	3	16	1899,89	3	17
131122	1841,86	1892,84	1888,9	4	16	1881,67	3	16	1875,79	3	16	1844,61	3	17
131212	1981,37	2022,11	2033,93	3	17	1984,25	3	16	2010,53	3	16	2021,67	3	17
131222	1809,25	1854,97	1856,07	4	16	1855,25	3	16	1819,89	3	16	1840,64	3	16
132112	1448,27	1555,82	1508,33	3	16	1448,27	2	16	1448,65	2	16	1424,19	2	16
132122	1444,25	1478,8	1456,82	2	16	1459,83	2	16	1492,86	3	16	1419,18	2	16
132212	1206,73	1231,34	1240,4	2	16	1207,41	3	17	1211,07	3	17	1199,64	3	17
132222	931,94	948,28	940,8	3	17	934,79	3	17	936,93	3	17	902,05	3	17
133112	1699,92	1762,45	1736,9	3	17	1720,3	3	16	1729,31	3	16	1776,62	3	17
133122	1401,82	1488,34	1425,74	3	16	1429,34	4	16	1424,59	3	16	1397,94	3	16
133212	1199,51	1264,63	1223,7	3	17	1203,44	3	17	1216,32	3	16	1204,29	3	17
133222	1152,86	1182,28	1231,33	4	17	1158,54	3	17	1162,16	3	17	1132,19	3	17
121112	2281,78	2379,47	2384,01	3	21	2293,99	3	21	2296,52	3	22	2274,86	3	22
121122	2185,55	2211,74	2288,09	4	22	2277,39	3	21	2207,5	4	22	2213,62	4	21
121212	2234,78	2288,17	2273,19	3	21	2274,57	3	21	2260,87	4	21	2242,9	3	22
121222	2259,52	2355,81	2345,1	3	22	2376,25	3	21	2259,52	3	21	2261,23	3	21
122112	2101,9	2158,6	2137,08	3	22	2106,26	3	21	2120,76	3	21	2206,23	3	22
122122	1709,56	1787,02	1807,29	4	21	1771,53	2	20	1737,81	3	21	1717,28	3	21
122212	1467,54	1549,79	1496,75	2	21	1467,54	2	21	1488,55	2	21	1441,84	2	21
122222	1084,78	1112,96	1095,92	3	22	1088	3	22	1090,59	3	22	1045,87	3	22
123112	1973,28	2056,11	2044,66	4	23	1973,28	4	22	1984,06	4	22	1976,74	4	23
123122	1957,23	2002,42	2090,95	4	22	1979,05	5	21	1986,49	4	22	1914,78	4	22
123212	1771,06	1877,3	1788,7	2	21	1782,23	3	22	1786,79	3	22	1859,48	3	22
123222	1393,62	1414,83	1408,63	5	22	1396,24	5	22	1401,16	5	22	1488,26	3	22

The Or-Opt procedure can be divided into four steps. The first one is to generate an initial route. The second is to exchange or reverse three consecutive cities to improve the current route. The third is to exchange or reverse two consecutive cities to improve the current route. Finally, exchange a single city to improve the current route.

Fig. 2 shows an example of a three-vertex Or-Opt improvement. In this example, a three vertices subtour (z, x, w) is being considered for insertion (relocation) between a pair of adjacent cities y and o if $d_{yo} + d_{vz} + d_{wu} > d_{vu} + d_{yz} + d_{wo}$, will be inserted in reverse order between cities y and o, and cities v and u will be connected, resulting in a shorter length tour.

7. Proposed solution

In this section, we propose a hybrid approach based on a neural network to construct the depot using capacitated, Clarke and Wright to solve the routing problem and Or-Opt for improving the routes. In general, the proposed approach has three phases.

Fig. 3 illustrates the different steps of the proposed approach to solve our problem. In the first phase, the customers are clustered using a special breed of unsupervised neural networks, the SOM. To demonstrate the idea of our proposed model CSOM, we have introduced a constraint to control the capacity for each cluster (depot) and a new technique to choose the initial weights and number of neurons (Algorithm 1). These parameters impact the convergence of the training methods and network stabilization after the training stage. After the training stage, the total demand of each cluster should be less than or equal to the capacity of the depot. Then, the depot location for each cluster is constructed. The second phase aims to solve the VRP for each cluster. Each route should include as many customers as possible while its total demand remains less than the capacity of the vehicle. The routes are constructed using the CW approach. Finally, we improved the good routes by exchanging customers between the different routes using the Or-Opt technique.

In our study, the sum of the squared-error of SOM can be defined as:

$$SSE = \sum_{j=1}^{n} \sum_{i=1}^{N} \beta_{i,k} x_j - w_i^2$$

Table 4
Comparison of the proposed approach with algorithms from the literature for Barreto's instances.

		Barreto LB*		Barreto		GRASP		MAPM		LRGTS		CSOM&CW	
	BKR	*Cost*	*Gap(%)*	*Cost*	*Gap(%)*	*Cost*	*Gap(%)*	*Cost*	*Gap(%)*	*Cost*	*Gap(%)*	*Cost*	*Gap(%)*
Christofides69-50 × 5	565.6	549.4	−0.30	582.7	3.02	599.1	5.92	565.6	0.00	586.4	3.68	591.7	4.61
Christofides69-75 × 10	861.6	744.7	−5.90	886.3	2.87	861.6	0.00	866.1	0.52	863.5	0.22	837.98	−2.74
Christofides69-100 × 10	842.9	788.6	−3.60	889.4	5.52	861.6	2.22	850.1	0.85	842.9	0.00	848.1	0.62
Daskin95-88 × 8	355.8	356.4	0.00	384.9	8.18	356.9	0.31	355.8	0.00	368.7	3.63	348.82	−1.96
Daskin95-150 × 10	44,011.7	43,406	0.00	46,642.7	5.98	44,625.2	1.39	44,011.7	0.00	44,386.3	0.85	45,163.28	2.62
Gaskell67-21 × 5	424.9	424.9	0.00	435.9	2.59	429.6	1.11	424.9	0.00	424.9	0.00	457.06	7.56
Gaskell67-22 × 5	585.1	585.1	0.00	591.5	1.09	585.1	0.00	611.8	4.56	587.4	0.39	610.32	4.31
Gaskell67-29 × 5	512.1	512.1	0.00	512.1	0.00	515.1	0.59	512.1	0.00	512.1	0.00	498.64	−2.60
Gaskell67-32 × 5	571.7	556.5	−1.00	571.7	0.00	571.9	0.03	571.9	0.03	584.6	2.26	578.31	1.16
Gaskell67-32 × 5	504.3	504.3	0.00	511.4	1.41	504.3	0.00	534.7	6.03	504.8	0.10	490.79	−2.68
Gaskell67-36 × 5	460.4	460.4	0.00	470.7	2.24	460.4	0.00	485.4	5.43	476.5	3.50	475.22	3.22
Min92-27 × 5	3062	3062	0.00	3062	0.00	3062	0.00	3062	0.00	3065.2	0.10	3104.54	1.38
Min92-134 × 8	5809	–	-	6238	7.39	5965.1	2.69	5950.1	2.43	5809	0.00	5740.05	−1.19
Average	4505.2	4329.2	−0.90	4752.3	3.10	4569.1	1.10	4523.3	1.53	4539.41	1.13	4595.7	1.10

Table 5
Comparison of the proposed approach with algorithms from the literature for Prins et al. instances.

Instances		GRASP		MAPM		LRGTS		CSOM&CW	
	BKR	*Cost*	*Gap(%)*	*Cost*	*Gap(%)*	*Cost*	*Gap(%)*	*Cost*	*Gap(%)*
2051a	54,793	55,021	0.42	54,793	0	55,131	0.62	53,785	−1.84
2051b	39,104	39,104	0	39,104	0	39,104	0	39,802	1.78
2052a	48,908	48,908	0	48,908	0	48,908	0	48,735	−0.35
2052b	37,542	37,542	0	37,542	0	37,542	0	37,833	0.77
5051	90,111	90,632	0.58	90,160	0.05	90,160	0.05	87,670	−2.71
5051b	63,242	64,761	2.4	63,242	0	63,256	0.02	63,372	0.21
5052	88,298	88,786	0.55	88,298	0	88,715	0.47	84,695	−4.08
5052b	67,340	68,042	1.04	67,893	0.82	67,698	0.53	65,692	−2.44
5052bis	84,055	84,055	0	84,055	0	84,181	0.15	85,576	1.8
5052bbis	51,822	52,059	0.46	51,822	0	51,992	0.33	51,919	0.18
5053	86,203	87,380	1.37	86,203	0	86,203	0	86,166	−0.04
5053b	61,830	61,890	0.1	61,830	0	61,830	0	65,234	5.5
10051	275,993	279,437	1.25	281,944	2.16	277,935	0.7	273,223	−1
10051b	214,392	216,159	0.82	216,656	1.06	214,885	0.23	206,744	−3.56
10052	194,598	199,520	2.53	195,568	0.5	196,545	1	192,914	−0.87
10052b	157,173	159,550	1.51	157,325	0.1	157,792	0.39	157,842	0.43
10053	200,246	203,999	1.87	201,749	0.75	201,952	0.85	201,259	0.51
10053b	152,586	154,596	1.32	153,322	0.48	154,709	1.39	154,871	1.5
100101	290,429	323,171	11.27	316,575	9	291,887	0.5	301,679	3.87
100101b	234,641	271,477	15.7	270,251	15.18	235,532	0.38	229,960	−1.99
100102	244,265	254,087	4.02	245,123	0.35	246,708	1	239,856	−1.8
100102b	203,988	206,555	1.26	205,052	0.52	204,435	0.22	204,095	0.05
100103	253,344	270,826	6.9	253,669	0.13	258,656	2.1	247,052	−2.48
100103b	204,597	216,173	5.66	204,815	0.11	205,883	0.63	202,027	−1.25
200101	479,425	490,820	2.38	483,497	0.85	481,676	0.47	492,482	2.72
200101b	378,773	416,753	10.03	380,044	0.34	380,613	0.49	382,266	0.92
200102	450,468	512,679	13.81	451,840	0.3	453,353	0.64	440,347	−2.84
200102b	374,435	379,980	1.48	375,019	0.16	377,351	0.78	379,383	1.32
200103	472,898	496,694	5.03	478,132	1.11	476,684	0.8	463,518	−1.98
200103b	364,178	389,016	6.82	364,834	0.18	365,250	0.29	357,064	−1.95
Average	197,322.57	207,322.40	3.35	200,308.83	1.14	198,552.20	0.50	196,568.70	−0.32

where n is the number of customers and N is the number of depots. Neighbourhood kernel $\beta_{i,\ k}$ is centred at depot k, which is the best matching unit of input vector x_j, and is evaluated for unit i.

8. Numerical results and discussion

8.1. Instances

The first set was proposed by Prins et al. [54] and contains 30 instances with capacitated routes and depots. The number of depots is either 5 or 10. The number of customers is set to 20, 50, 100 or 200; the vehicle capacity Q is set to $a = 70$ or $b = 150$. The number of clusters is in {1, 2, 3}, where 1 cluster means that all nodes scatter on a Euclidean plane. The demand follows a uniform distribution in the interval [11,20]. The traveling cost corresponds to the distance multiplied by 100 and rounded up to the next integer. Finally, the other data (demand, depot capacities and fixed cost) are also selected. The second set of 36 instances was designed by Tuzun and Burke [55], with capacitated routes and incapacitated depots. The number of customers is 100, 150 or 200 and the number of potential depots is either 10 or 20. The vehicle capacity is set to 150 and the demand is assumed to be uniformly distributed in the range [1,20]. In this set, distances are not rounded. The third set of 13 benchmark instances was created by Barreto [56], with capacity restricted vehicles and depots; the number of depots is between 5 and 10 and the number of customers is varied between 21 and 150.

8.2. Simulation and discussion

The tables in this section present the information for the three sets of benchmark instances and a comparison of the solutions obtained using the SOM&CW algorithm with other heuristics from the literature. In Tables 1, 2 and 3, Column 2 shows the best-known solutions (BKR) that are reported in the literature. Columns 3–5 report the computational results obtained by GRASP [57], the solution value found (cost), the number of depots used and the number of vehicles used, respectively. Columns 6–8 present the results obtained by MA/PM [43], the solution value found (cost), the number of depots used and the number of vehicles used, respectively. Columns 9–11 show the results obtained by LRGTS [54], the solution value found (cost), the number of depots used and the number of vehicles used, respectively. Finally, the solutions obtained from the proposed approach are included in the last three columns.

Table 4 presents the computational results obtained for Barreto's set. It includes a comparison in terms of percentage gaps. In this

Table 6
Comparison of the proposed approach with algorithms from the literature for Tuzun-Burke's instances.

Instances		Tuzun & Burke*		GRASP		MAPM		LRGTS		CSOM&CW	
	BKR	*Cost*	*Gap(%)*	*Cost*	*Gap(%)*	*Cost*	*Gap(%)*	*Cost*	*Gap(%)*	*Cost*	*Gap(%)*
111112	1468.4	1556.64	6.01	1525.25	3.87	1493.92	1.74	1490.82	1.53	1464.23	−0.28
111122	1449.2	1531.88	5.71	1526.9	5.36	1471.36	1.53	1471.76	1.56	1443.37	−0.4
111212	1396.46	1443.43	3.36	1423.54	1.94	1418.83	1.6	1412.04	1.12	1449.41	−1.02
111222	1432.29	1511.39	5.52	1482.29	3.49	1492.46	4.2	1443.06	0.75	1453.63	1.49
112112	1167.53	1231.11	5.45	1200.24	2.8	1173.22	0.49	1187.63	1.72	1160.07	−0.64
112122	1102.7	1132.02	2.66	1123.64	1.9	1115.37	1.15	1115.95	1.2	1124.1	1.94
112212	793.97	825.12	3.92	814	2.52	793.97	0	813.28	2.43	750.12	−5.52
112222	728.3	740.54	1.68	747.84	2.68	730.51	0.3	742.96	2.01	716.3	−1.64
113112	1238.49	1316.98	6.34	1273.1	2.79	1262.32	1.92	1267.93	2.38	1260.62	1.78
113122	1246.34	1274.5	2.26	1272.94	2.13	1251.32	0.4	1256.12	0.78	1289.76	3.48
113212	902.38	920.75	2.04	912.19	1.09	903.82	0.16	913.06	1.18	890.22	−1.34
113222	1021.31	1042.21	2.05	1022.51	0.12	1022.93	0.16	1025.51	0.41	1005.86	−1.51
131112	1866.75	2000.97	7.19	2006.7	7.5	1959.39	4.96	1946.01	4.25	1899.89	1.78
131122	1841.86	1892.84	2.77	1888.90	2.55	1881.67	2.16	1875.79	1.84	1844.61	0.15
131212	1981.37	2022.11	2.06	2033.93	2.65	1984.25	0.15	2010.53	1.47	2021.67	2.03
131222	1809.25	1854.97	2.53	1856.07	2.59	1855.25	2.54	1819.89	0.59	1840.64	1.73
132112	1448.27	1555.82	7.43	1508.33	4.15	1448.27	0.00	1448.65	0.03	1424.19	−1.66
132122	1444.25	1478.80	2.39	1456.82	0.87	1459.83	1.08	1492.86	3.37	1419.18	−1.73
132212	1206.73	1231.34	2.04	1240.40	2.79	1207.41	0.06	1211.07	0.36	1199.64	0.58
132222	931.94	948.28	1.75	940.80	0.95	934.79	0.31	936.93	0.54	902.05	−3.20
133112	1699.92	1762.45	3.68	1736.90	2.18	1720.30	1.20	1729.31	1.73	1776.62	3.29
133122	1401.82	1488.34	6.17	1425.74	1.71	1429.34	1.96	1424.59	1.62	1397.94	−0.27
133212	1199.51	1264.63	5.43	1223.70	2.02	1203.44	0.33	1216.32	1.40	1204.29	0.39
133222	1152.86	1182.28	2.55	1231.33	6.81	1158.54	0.49	1162.16	0.81	1132.19	−1.79
121112	2281.78	2379.47	4.28	2384.01	4.48	2293.99	0.54	2296.52	0.65	2274.86	−0.30
121122	2185.55	2211.74	1.20	2288.09	4.69	2277.39	4.20	2207.50	1.00	2213.62	1.28
121212	2234.78	2288.17	2.39	2273.19	1.72	2274.57	1.78	2260.87	1.17	2242.90	0.36
121222	2259.52	2355.81	4.26	2345.10	3.79	2376.25	5.17	2259.52	0.00	2261.23	0.07
122112	2101.90	2158.60	2.70	2137.08	1.67	2106.26	0.21	2120.76	0.90	2206.23	4.96
122122	1709.56	1787.02	4.53	1807.29	5.72	1771.53	3.62	1737.81	1.65	1717.28	0.45
122212	1467.54	1549.79	5.60	1496.75	1.99	1467.54	0.00	1488.55	1.43	1441.84	−1.75
122222	1084.78	1112.96	2.60	1095.92	1.03	1088.00	0.30	1090.59	0.54	1045.87	−3.58
123112	1973.28	2056.11	4.20	2044.66	3.62	1973.28	0.00	1984.06	0.55	1976.74	0.17
123122	1957.23	2002.42	2.31	2090.95	6.83	1979.05	1.11	1986.49	1.49	1914.78	−2.16
123212	1771.06	1877.30	6.00	1788.70	1.00	1782.23	0.63	1786.79	0.89	1859.48	4.99
123222	1393.62	1414.83	1.52	1408.63	1.08	1396.24	0.19	1401.16	0.54	1488.26	0.59
123222	1509.79	1566.77	3.74	1556.51	2.92	1532.19	1.30	1528.75	1.27	1517.42	0.11

table, we compare our algorithm CSOM&CW with the results found by GRASP, MA/PM and LRGTS. It can be observed that our algorithm has been able to achieve 5 to 13 BKSs with an average gap of 1.1% with respect to BKS. It must be noted that the performance of the proposed algorithm improved for different instances (29 to 134 customers).

The detailed results for the second data subset are shown in Table 5. The results indicate that the proposed algorithm has average gap values smaller than those of GRASP, MA/PM and LRGTS. Moreover, our algorithm was able to achieve 16 of the 30 BKS, while its average gap was (−0.32%.). Similarly, Table 6 summarizes the results for the third data subset. The proposed approach found 17 new best solutions from 36 instances. For the other 19 instances, the improvement in the solution is between 0.07% and 3.48%. The experiments indicated that the proposed method is a computationally effective approach to solve the CLRP.

9. Conclusion

In this work, we aim to optimize the capacitated location routing problem (CLRP). This problem integrates three decisions in supply chain management: location selection, client assignment, and route distribution. The constraint capacity of the depots and vehicles are considered as deterministic. Due to its computational complexity, we decompose the problem into facility location problem (FLP) and multi depot vehicle routing problem (MDVRP).To solve the CLRP, we have developed a hybrid approach based on two steps. In the first-step, placement of depots and customer assignment decisions are obtained by using a capacitated self-organizing map (CSOM) algorithm. The second one consists of solving the vehicle routing problem for each depot using a heuristic method that combines Clarke and Wright and Or-opt.

We compared our approach with the three most effective published algorithms for the CLRP on three well-known sets of benchmark instances. In addition, our approach can provide competitive results when compared to the most effective published heuristics for the CLRP in terms of solution quality. It provides 38 new best solutions, respectively, in 79 instances considered in this study. This result confirms the main advantages of using our heuristic approach to find a good solution for small, medium and large size instances.

The good solutions obtained suggest that our approach could be applied to a variant of LRP as the location-inventory-routing problem, the periodic location routing problem, and green location routing problem. In our future works, we will look to introduce the stochastic demand in the CLRP, and study how the stochastic constraint could affect the total cost and warehouse placement.

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