Homework for convex optimization

KEEP
CALM
AND
DO YOUR
HOMEWORK

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Dantzig selector

Objective function:

$$\begin{aligned} \min_{\beta} \quad & \sum_{j=1}^{p} |\beta_j| \\ \text{such that} \quad & ||X^T(X\beta - y)||_\infty \leq \lambda \end{aligned}$$

- Recast the problem into a linear programming.
- Use R interface of a linear programming solver (such as rmosek, rcplex, Rglpk, clpAPI).
- Follow the examples in Convex_demo.pdf.

Lasso

Objective function:

$$\hat{\beta}^{\text{lasso}} = \arg\min ||y - X\beta||_2^2 + \lambda||\beta||_1^2$$

- Implement any algorithm (from class or from the literature) using a convex optimization package.
- Benchmark the performance with lasso functions in *glmnet* package and benchmark with your Dantzig selector implementation. You can use one or more simulated datasets with at least $n = 200$ and $p > 500$. Define your problem and define the metrics that you use to evaluate performance.

My glmnet in python doesn't work, so I use Sklearn.linear_mode

- Write a reproducible report with R markdown.

Markowitz portfolio optimization

- Suppose there are p assets held over a fixed period and let β_j denote the amount invested in the j-th asset. Let R be a p-variate random variable denoting the relative price change of each asset. We assume R has mean μ_R and covariance Σ_R.
- Markowitz (1952) proposed choosing the asset allocation which solves

$$\begin{aligned} \min \beta \qquad & \beta^T \Sigma_R \beta \\ & \beta^T \mu_R \geq l \\ & \mathbf{1}^T \beta = 1 \\ & \beta \geq 0 \end{aligned}$$

- Can you implement Markowitz portfolio optimization using R interface of a quandratic programming solver?