HW 1

Problem 2:

(a) $T(n) = a \cdot T(\frac{n}{b}) + f(n)$

i. $T(n) = 2 \cdot T(\frac{n}{4}) + 1$

$a = 2$, $b = 4$, $f(n) = 1$

$n^{\log_b a} = n^{\log_4 2} = n^{\frac{1}{2}}$

$f(n) = 1 = O(n^{\frac{1}{2} - \frac{1}{2}})$ by case (1) of master theorem

$T(n) = \Theta(n^{\log_4 2}) = \Theta(n^{\frac{1}{2}})$

ii. $T(n) = 2T(\frac{n}{4}) + \sqrt{n}$

$a = 2$ $b = 4$ $f(n) = n^{\frac{1}{2}}$

$n^{\log_b a} = n^{\frac{1}{2}}$, $f(n) = O(n^{\log_4 b})$, we apply case 2

$T(n) = \Theta(n^{\log_b a} \cdot \log n) = \Theta(n^{\frac{1}{2}} \cdot \log n)$

iii. $T(n) = 2T(\frac{n}{4}) + n$

$a = 2$ $b = 4$ $f(n) = n$

$f(n) = \Omega(n^{\log_b a}) = \Omega(n^{\frac{1}{2}})$, $2 \cdot f(\frac{n}{4}) \le \frac{1}{2} \cdot f(n)$, $\because 2 \cdot \frac{n}{4} \le \frac{1}{2} \cdot n$

case (3) applies, $T(n) = \Theta(f(n)) = \Theta(n)$

iv. $T(n) = 2T(\frac{n}{4}) + n^2$

$a = 2$, $b = 4$, $f(n) = n^2$ $n^{\log_b a} = n^{\frac{1}{2}}$

$f(n) = \Omega(n^{\frac{1}{2}})$ and $2 f(\frac{n}{4}) \le \frac{1}{2} f(n)$ $\because 2 \cdot (\frac{n}{4})^2 \le \frac{1}{2} \cdot n^2$

case (3) applies, $T(n) = \Theta(f(n)) = \Theta(n^2)$

(b) Binary Search for question (2)

$T(n) = T(\frac{n}{2}) + 1$

$a = 1$ $b = 2$ $f(n) = 1$

$n^{\log_2 1} = n^0 = 1$ $\Rightarrow$ $f(n) = O(n^{\log_2 1})$

case (2) applies, $T(n) = \Theta(n^{\log_2 1} \cdot \log n) = \Theta(\log n)$