

Solutions to Lab 5

Part I: ESDA Using R

Assignment I

Figure 1 shows the distribution of adjacency connections, indicating the frequency of neighbor numbers. For example, 10 of the 48 census tracts have 4 neighbors; 10 census tracts have 3 neighbors, while another 8 census tracts have 5 neighbors. Some census tracts even have 9 or 10 neighbors. This is because we used queen type adjacency.

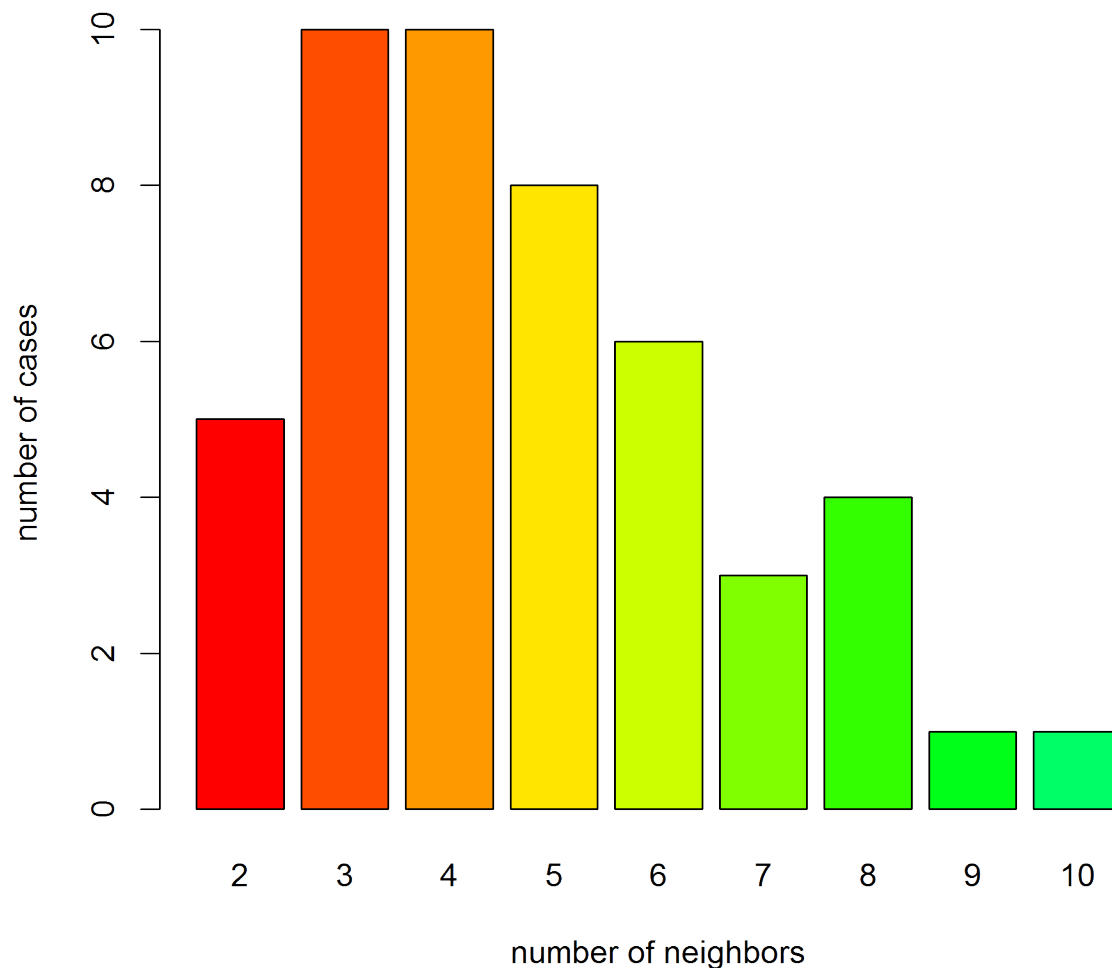


Figure 1 Barplot of the adjacency connections

Assignment II

Figure 2 shows the connections for the 1, 2 nearest neighbors, indicating how they would impact each other.

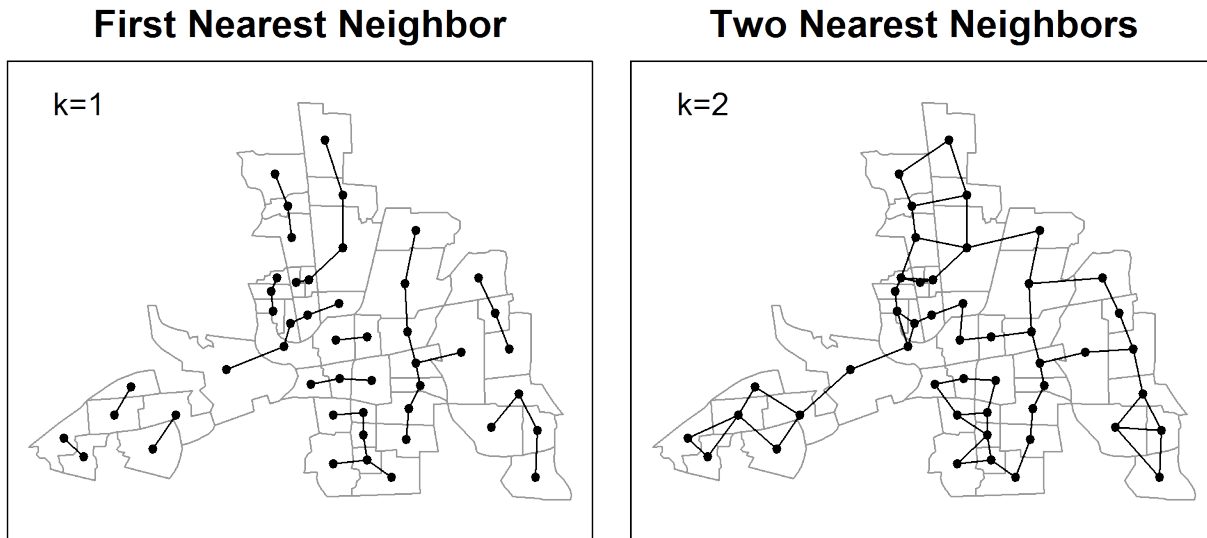


Figure 2 Connections for 1, 2 nearest neighbors

Assignment III

Figure 3 shows the distance based neighbors for two bandwidth: 0.5 and 0.8. The larger the bandwidth, the more connections.

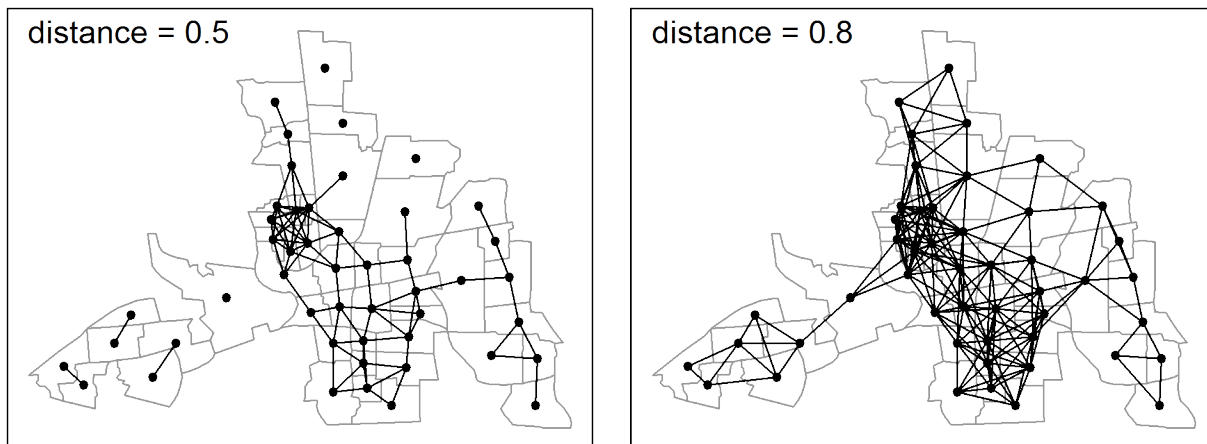


Figure 3 Distance based neighbors

Assignment IV

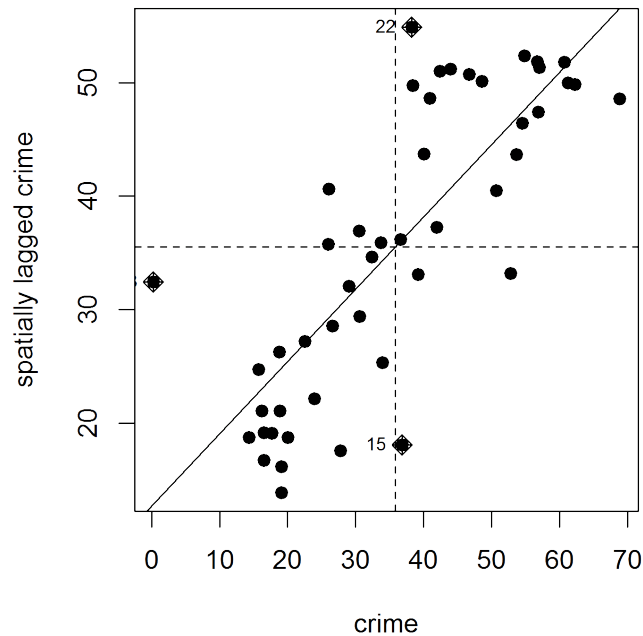


Figure 4 Moran's I

Moran's $I_{\text{crime}} = 0.637$, indicating positive spatial autocorrelation. For Moran's I test, P-value = $2.084\text{e-}12$, so the spatial autocorrelation is significant.

Assignment V

The LISA cluster map (Figure 5): census tracts in the downtown area have high crime rate and are positively correlated, i.e., hot spots. There are two cold spots (low crime rates clusters) in the outer skirts.

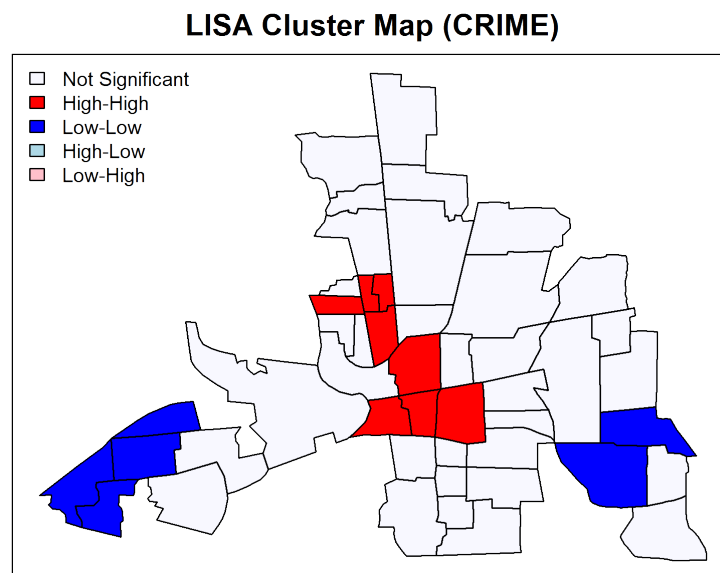


Figure 5 LISA Cluster map

Assignment VI

While the G_i and G_i^* cluster maps (Figure 6) have similar pattern to the LISA map (Figure 5), they are more sensitive to the local cluster effect: more census units are included in the clusters.

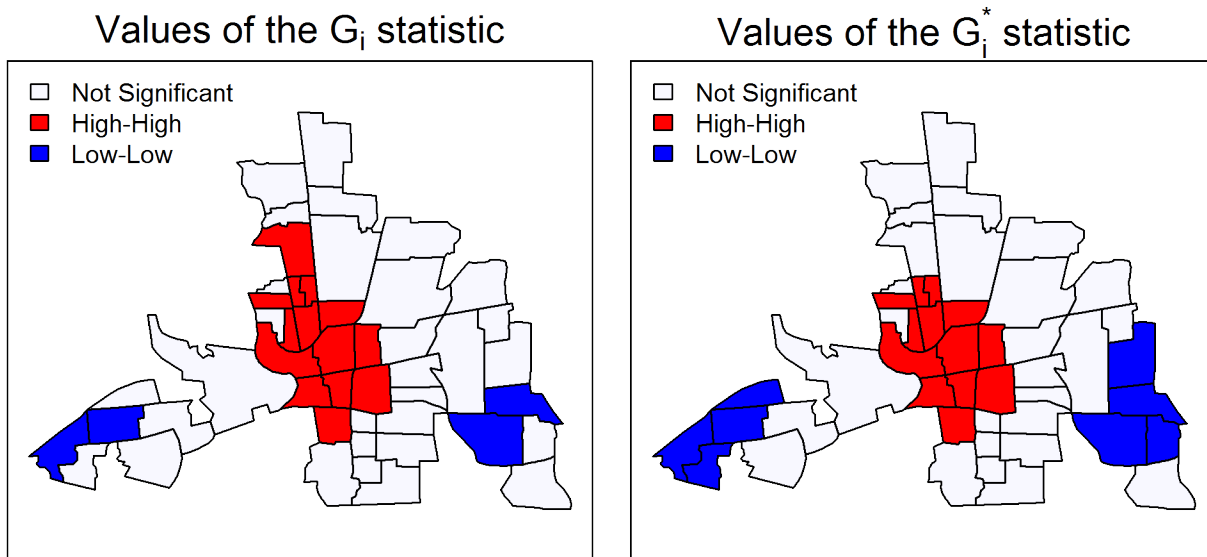


Figure 6 G_i and G_i^* cluster maps

Part II: Spatial Regression Models Using R

Assignment VII

The residual map shows patterns of spatial autocorrelation, and the Moran's I test confirms this statement: Moran's $I_{res} = 0.313 < I_{crime} = 0.637$, $p\text{-value} = 7.761e-05$, significant spatial autocorrelation in the residues. The Moran's I for residual is smaller than that for the CRIME variable itself. We can conclude that the spatial autocorrelation in the CRIME variable has been partially explained by the covariates INC and HOVAL.

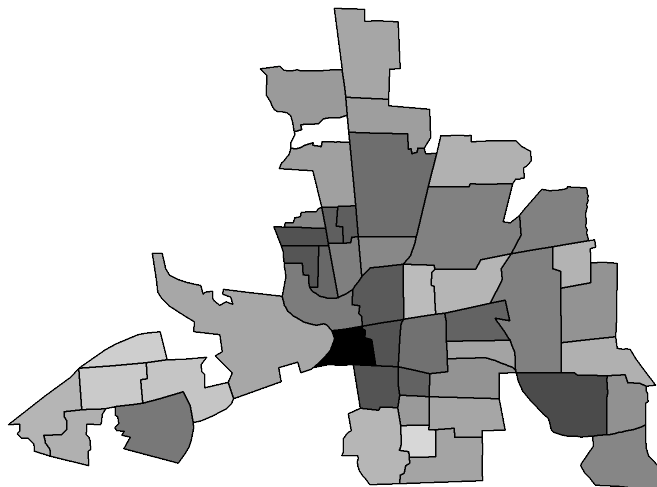


Figure 7 Residual map of OLS regression

Assignment VIII

The fitted model results are shown in Table 1. The results for SAR and CAR models are quite similar.

Table 1 Model fitting results for SAR and CAR models

	SAR Model		CAR Model	
	Estimate/Value	p-value	Estimate/Value	p-value
Intercept	57.813	$< 2.2\text{e-}16$	58.047	$< 2.2\text{e-}16$
INC	-1.183	$9.963\text{e-}06^{***}$	-1.256	0.0047^{**}
HOVAL	-0.135	00.07769^{***}	-0.143	$6.905\text{e-}06^{***}$
Lambda	0.716	$5.3834\text{e-}05^{***}$	0.948	0.0677^*
AIC	347.89	-	348.62	-

* Significance at 0.1 level. *** Significance at 0.001 level.

Figure 8 shows that the residuals are not completely random. There are a few outliers. SAR model is a slightly better than CAR model.

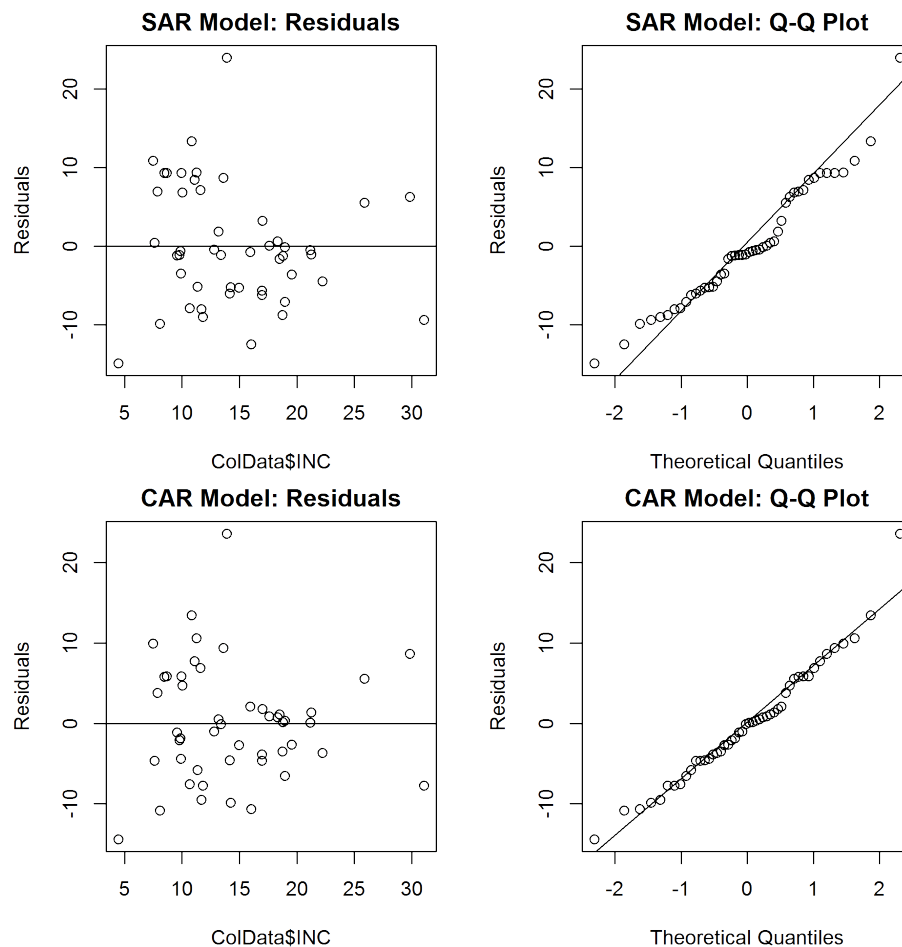


Figure 8 Scatterplot and QQ-plot of Residuals

The Moran's I tests of the residuals are shown Table 2. It can be concluded that there is no spatial autocorrelation in the residuals of both models.

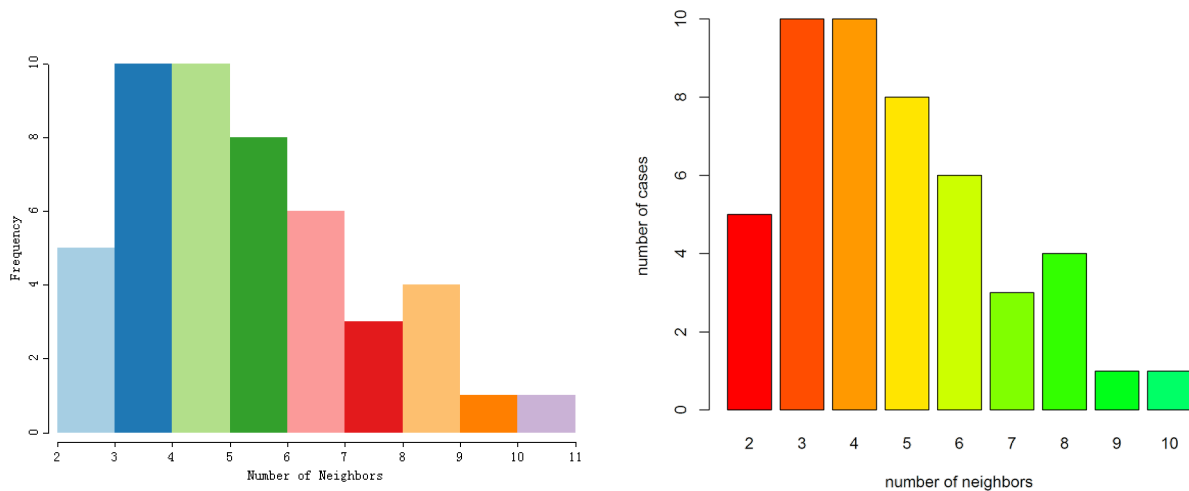
Table 2 Moran's I tests for the residuals of the SAR and CAR models

	Observed Moran's I	Expected Moran's I	p-value
SAR Model	0.0149	-0.0213	0.3493
CAR Model	-0.1499	-0.0213	0.9168

Part III: Lattice Data Analysis using Geoda (Practice ONLY)

Assignment B-I

The graphical output for the connectivity (Figure 9) are the same as that of Assignment I.



(a) Generated using GeoDa

(b) Generated using R

Figure 9 Barplot of the adjacency connections

Assignment B-II

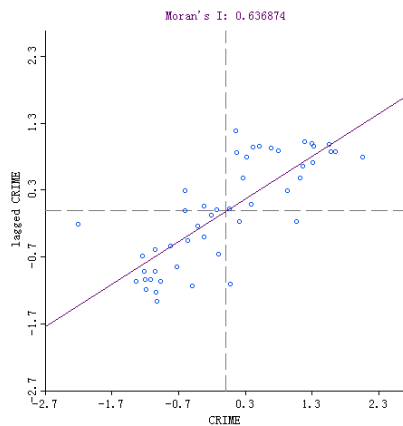


Figure 10 Moran Scatter plot of CRIME

The plot shows a notable clustering of criminality in the center of Columbus and declining rates of crimes as it reaches out into the suburban area.

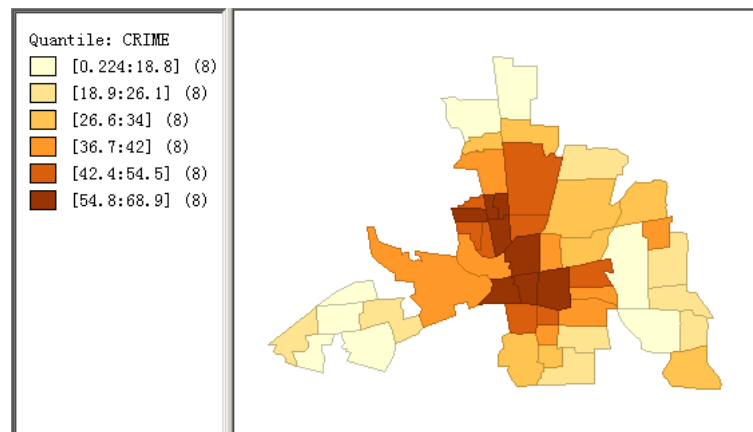


Figure 11 Choropleth map for CRIME

Assignment B-III

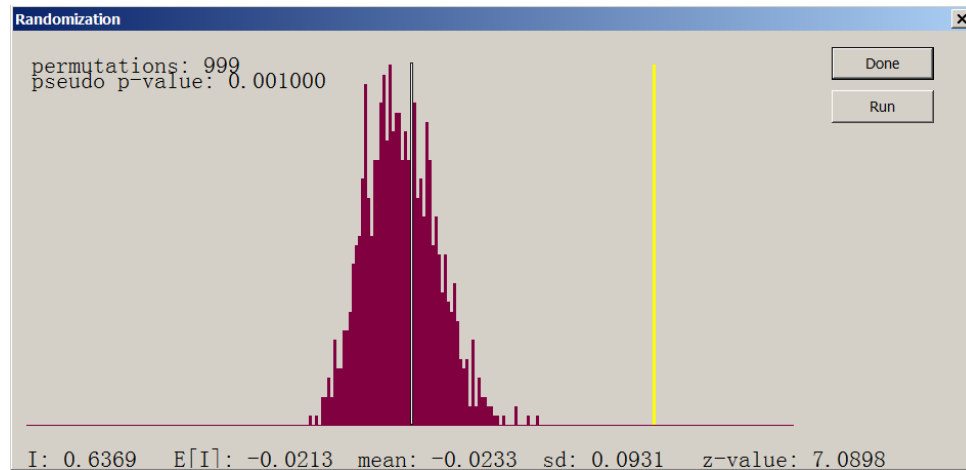


Figure 12 Moran's I and permutation for CRIME

The permutation test for Moran's I of CRIME shows that there is strong spatial dependence. The pseudo p-value = 0.001, significant at $\alpha = 0.001$ level, of course also significant $\alpha = 0.05$.

Assignment B-IV

1. Regression results: $\text{CRIME} = 69.41 - 0.11 * \text{HOVAL} - 2.03 * \text{INC} + \varepsilon$
(Note: ε is random error)

2. Moran's I = 0.313. There is spatial autocorrelation in the residuals

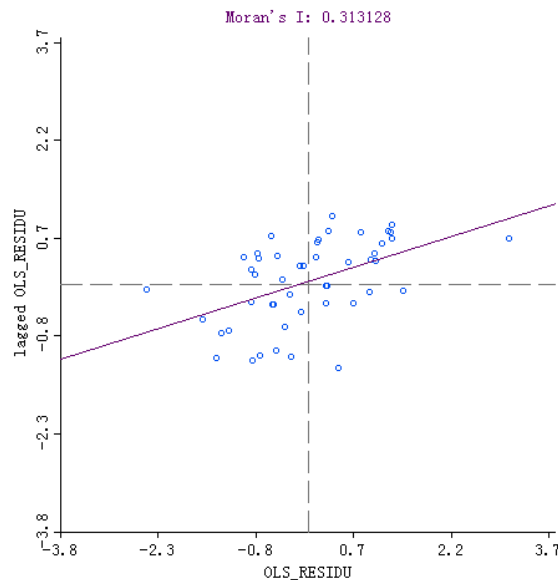


Figure 13 Moran's I for the residuals of the OLS model

3. The permutation test: p-value = 0.001, the spatial autocorrelation in the residuals is significant.

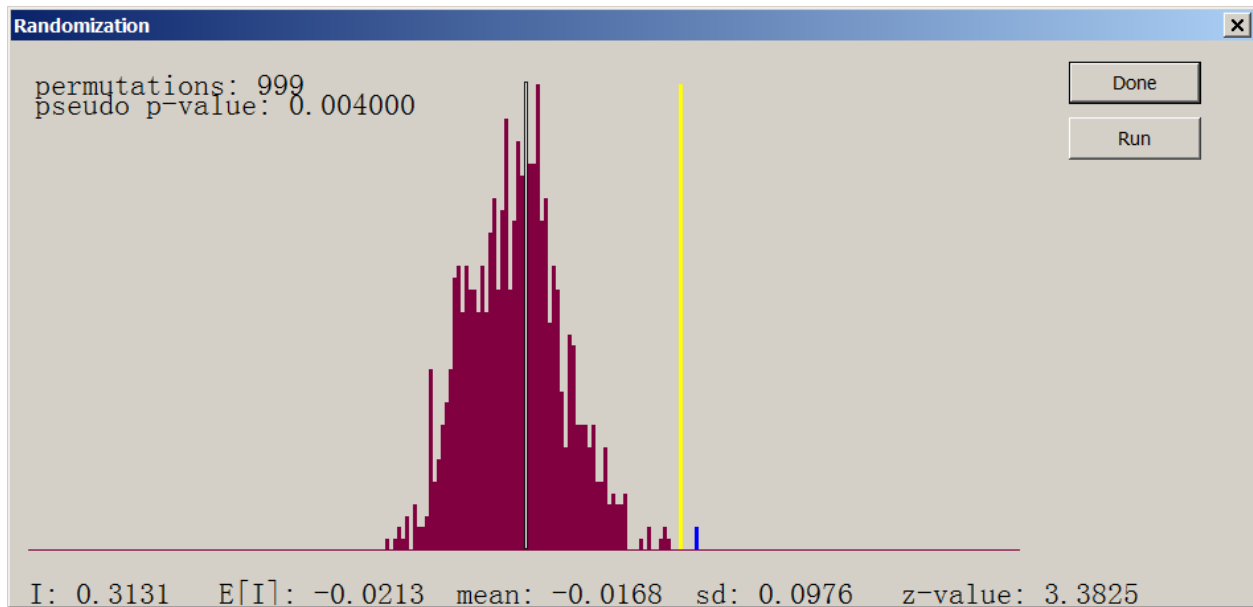


Figure 14 Permutation of Moran's I for the residuals of the OLS model

4. The spatial clustering is weaker in the choropleth map of Residuals than that of CRIME, since the regression model has explained some of the spatial pattern in CRIME.

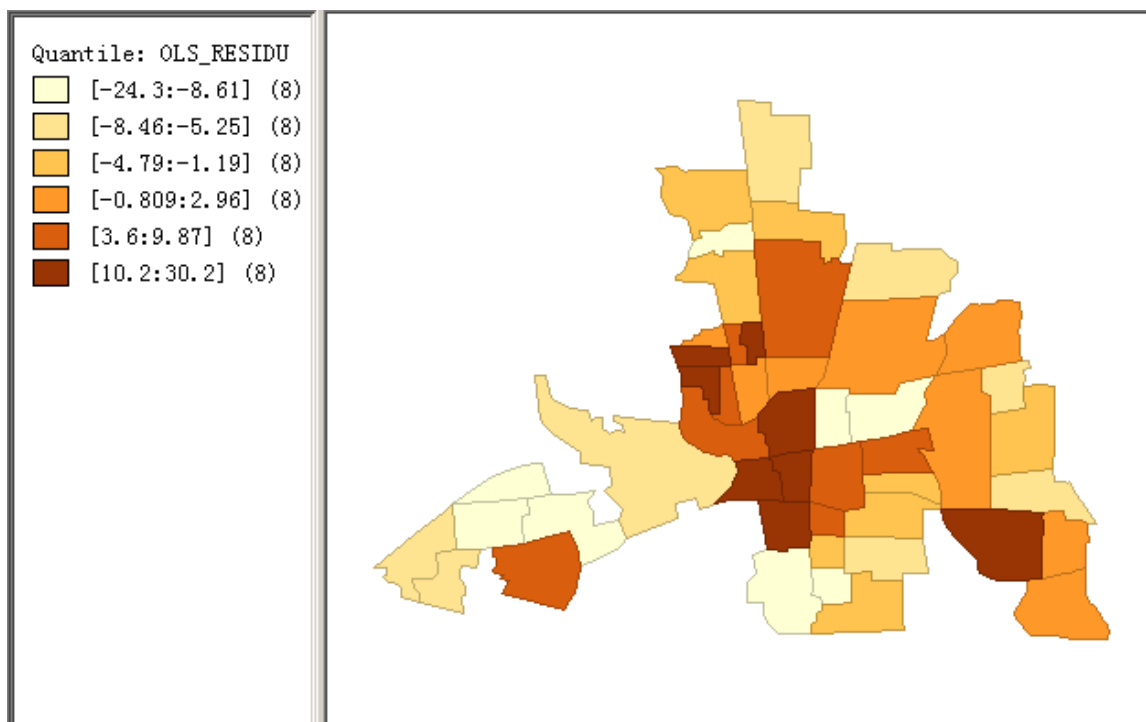


Figure 15 Choropleth map for the residuals of the OLS model

Assignment B-V

1. The equation for the spatial lag model is:

$$\text{CRIME} = 39.83 - 1.32 * \text{INC} - 0.09 * \text{HOVAL} + 0.53 * \text{W} * \text{CRIME} + \varepsilon$$
 (Note: W is the spatial proximity matrix, ε is random error)

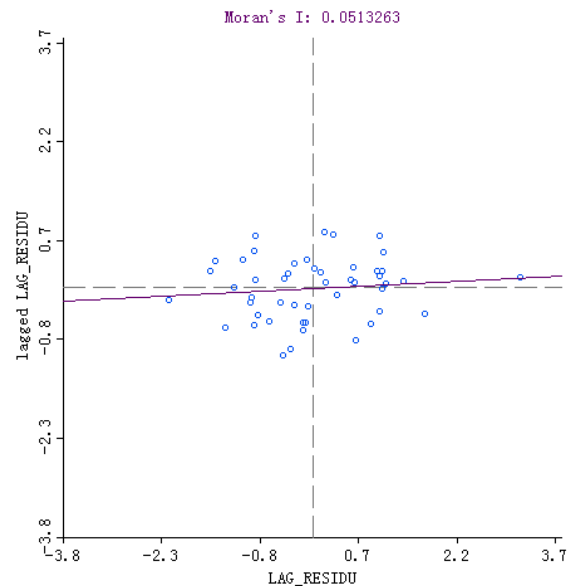


Figure 16 Moran's I for residuals of the spatial lag model

2. The Moran's I for the residuals of the spatial lag model is 0.05. The permutation test (Figure 17) shows p-value = 0.212, not significantly different from random. Therefore, no spatial autocorrelation in the residuals.

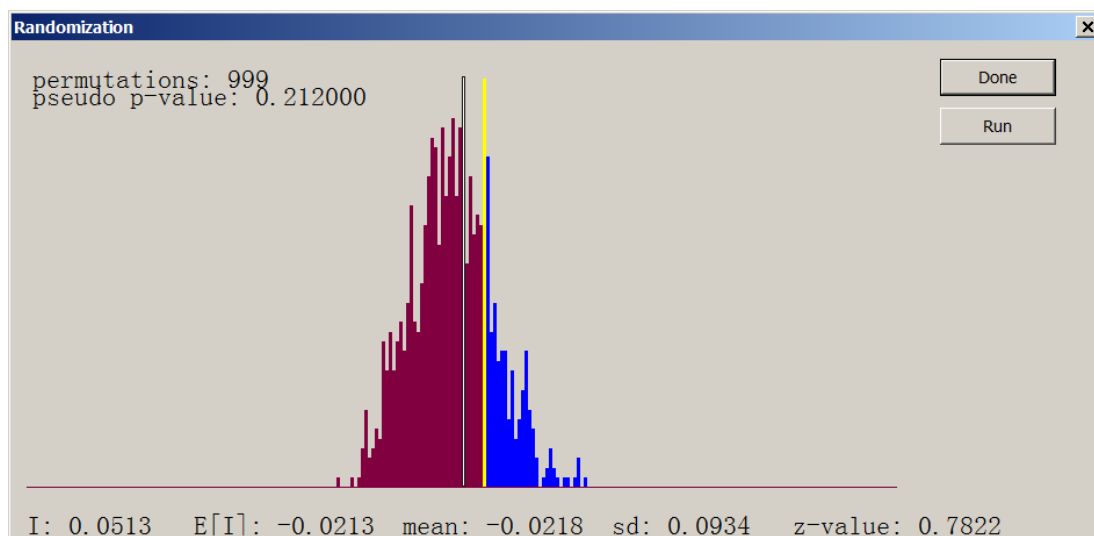


Figure 17 Permutation test for the residuals of the spatial lag model

3. The choropleth map for the residuals of spatial lag model shows less spatial clustering compared to the residuals of OLS model although the visual comparison is difficult to judge. Moran's I and permutation is a better method.

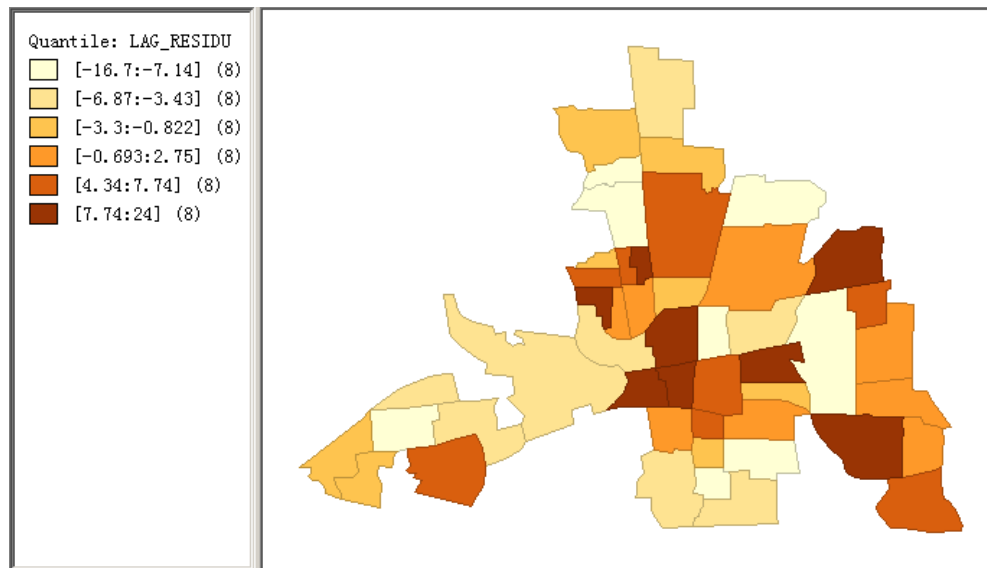


Figure 18 Choropleth map for the residuals of spatial lag model

Assignment B-VI

The equation for the spatial error model is:

$$\text{CRIME} = 57.81 - 1.18 * \text{INC} - 0.13 * \text{HOVAL} + 0.72 * W * (\text{CRIME} + 1.18 * \text{INC} + 0.13 * \text{HOVAL}) + \varepsilon$$

(Note: W is the spatial proximity matrix, ε is random error)

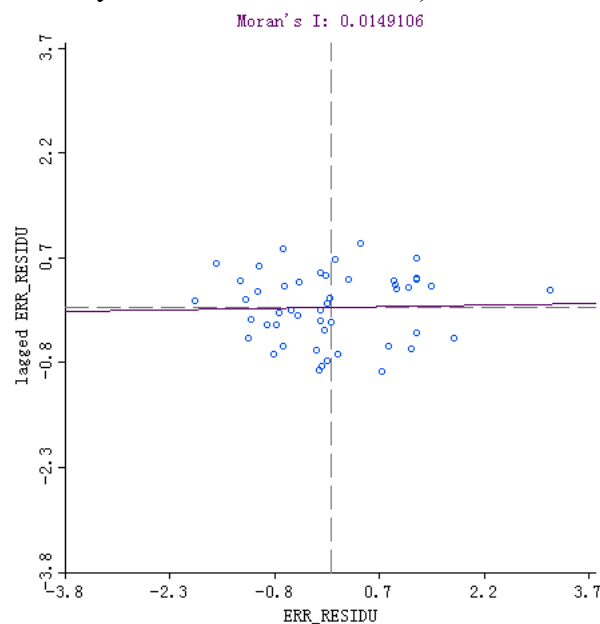


Figure 19 Moran's I for the residuals of spatial error model

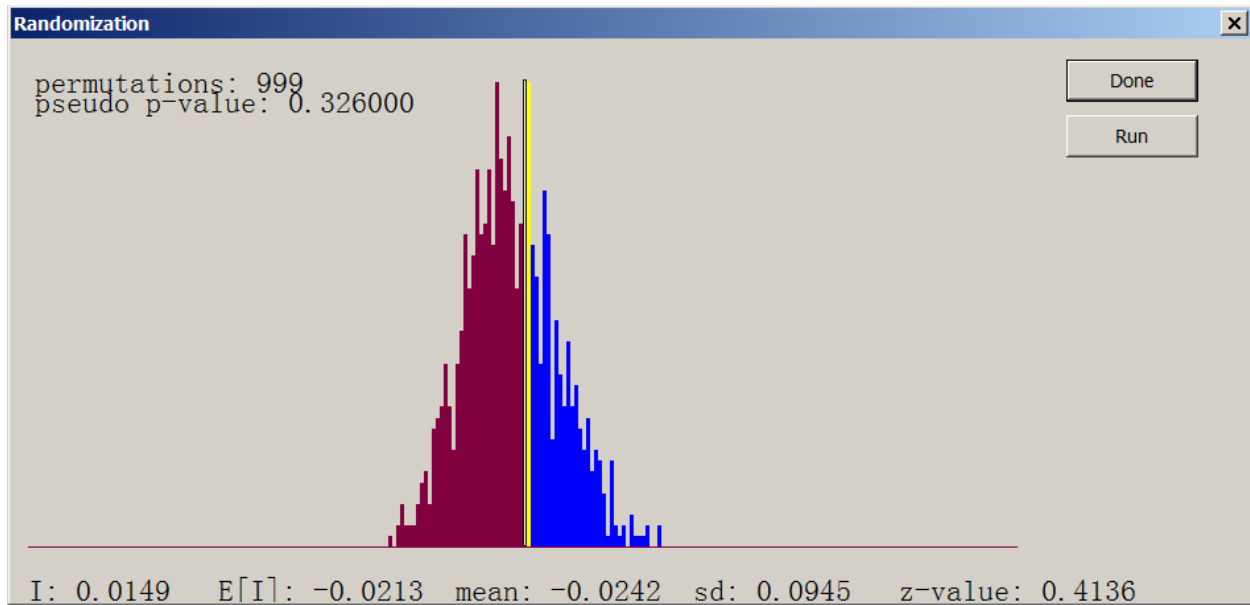


Figure 20 Permutation test for the residuals of the spatial error model

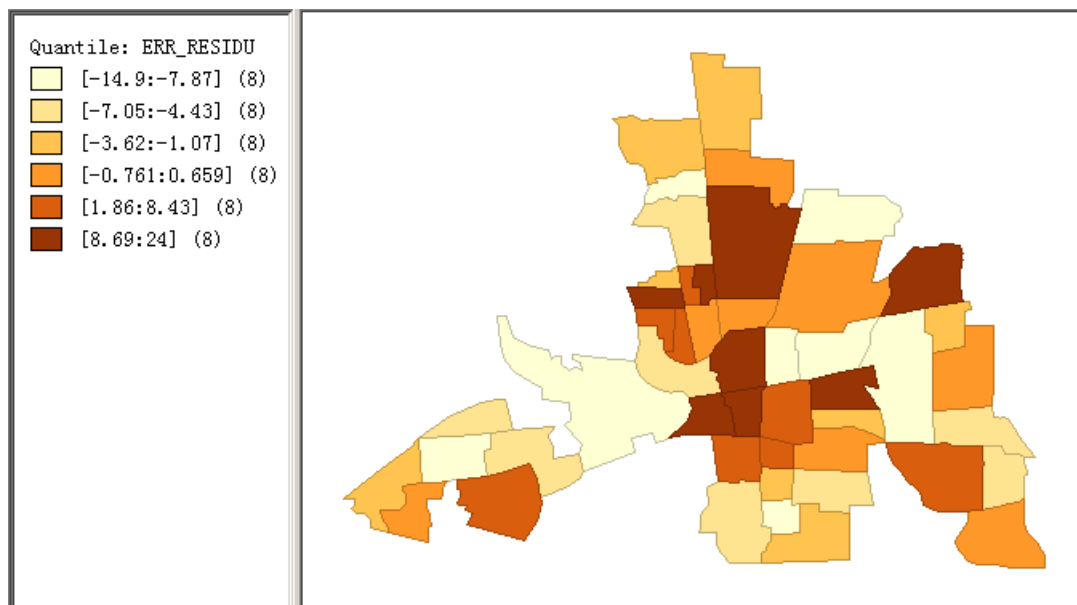


Figure 21 Choropleth map for the residuals of spatial error model

The spatial autocorrelation test for residuals of the spatial error model is quite similar to that of the spatial lag model. Moran's I of the residuals for spatial error model is 0.015, smaller than that of the spatial lag model. The permutation test show a p-value of 0.326. It can be concluded that there is no spatial autocorrelation for the residuals after the spatial error model. The result of spatial error model is slightly better than that of spatial lag model for this case.