Converting between ITRF and NZGD2000

NZGD2000 is aligned with ITRF96 and referenced to epoch 2000.0. However the NZGD2000 coordinates also include the deformation due to earthquakes since 2000, which have been applied as "reverse patches" in the NZGD2000 deformation model.

The procedure for converting coordinates from an ITRF realisation, for example ITRF2008, at a given date to NZGD2000 is to first convert the ITRF2008 coordinate to ITRF96 and then subtract the deformation calculated from the NZGD2000 deformation model at that date.

Conversely to calculate an ITRF coordinate at a specific date from an NZGD2000 coordinate at a specific date the deformation derived from the NZGD2000 deformation model is added to the coordinate to obtain an ITRF96 coordinate, which is then converted to the required ITRF realisation.

These steps are detailed below.

Conversion from other ITRF realisations to ITRF96

The relationship between various ITRF realisations is defined by the International Earth Rotation Service (IERS) and is available at http://itrf.ensg.ign.fr/doc_ITRF/Transfo-ITRF2008_ITRFs.txt. This document defines the relationships between each ITRF realisation and ITRF08 at epoch 2000.0.

The IERS did not determine any significant difference between ITRF96 and ITRF97. However the International GNSS Service (IGS) derived a non-zero transformation between ITRF96 and ITRF97 to IERS (Soler and Snay, 2004). This differs because the IGS used only GPS (rather than including other space techniques) and with GPS only data the difference was statistically significant. Since the NZGD2000 datum is primarily based on GPS observations, the IGS derived relationship is adopted for New Zealand.

The transformation parameters from recent ITRF realisations to ITRF96 are listed below. The values are defined as the value at 2000.0, and the change per year. To convert from ITRF96 to another realisation the values are negated. (Note that Soler and Snay use an opposite sign convention for the rotations R_x, R_y, and R_z to that used by IERS – the table below follows the IERS convention).

ITRF	T_x **(mm)** $\dot{T}_x$ **(mm/yr)**	T_y **(mm)** $\dot{T}_y$ **(mm/yr)**	T_z **(mm)** $\dot{T}_z$**(mm/yr)**	S **(ppb)** $\dot{S}$ **(ppb/yr)**	R_x **(mas)** $\dot{R}_x$**(mas/yr)**	R_y **(mas)** $\dot{R}_y$**(mas/yr)**	R_z **(mas)** $\dot{R}_z$ **(mas/yr)**
ITRF2008 rates	4.8 0.79	2.09 -0.6	-17.67 -1.34	1.40901 -0.10201	-0.16508 -0.01347	0.26897 0.01514	0.11984 0.01973
ITRF2005 rates	6.8 0.49	2.99 -0.6	-12.97 -1.34	0.46901 -0.10201	-0.16508 -0.01347	0.26897 0.01514	0.11984 0.01973
ITRF2000 rates	6.7 0.69	3.79 -0.7	-7.17 0.46	0.06901 -0.18201	-0.16508 -0.01347	0.26897 0.01514	0.11984 0.01973
ITRF97 rates	0 0.69	-0.51 -0.1	15.53 1.86	-1.51099 -0.19201	-0.16508 -0.01347	0.26897 0.01514	0.05984 -0.00027
ITRF96 rates	0 0	0 0	0 0	0 0	0 0	0 0	0 0

The coefficients are scaled by the following factors in the transformation equations below:

c_T = 0.001 (millimetres to metres, applies to T_x, T_y, T_z)

c_S= 1.0×10^{-9} (ppb to ratio, applies to S)

c_R = π/(180×60×60×1000)=4.84814×10^{-9} (mas to radians, applies to R_x, R_y,R_z)

To convert from an ITRF realisation, for example ITRF2008, to ITRF96 the transformation parameters are first determined at the epoch of the transformation. The parameter T_x at time t (in years) is determined as:

$$\begin{aligned}\delta t &= t - 2000 \\ T_{x,t} &= T_x + \delta t.\dot{T}_x\end{aligned} \qquad \textit{Eqn. 1}$$

The other parameters are similarly calculated to the transformation epoch.

The transformation is then applied by converting the ITRF coordinate longitude, latitude, and ellipsoidal height to geocentric Cartesian coordinates, then applying the transformation, and then converting the resulting Cartesian XYZ coordinates back to longitude, latitude, and ellipsoidal height. The conversions between Cartesian and ellipsoidal coordinates are detailed below.

The formula for converting from Cartesian coordinates (X_n, Y_n, Z_n) in realisation n of ITRF to ITRF96 Cartesian coordinates (X_{96}, Y_{96}, Z_{96}) is:

$$\begin{pmatrix} X_{96} \\ Y_{96} \\ Z_{96} \end{pmatrix} = \begin{pmatrix} X_n \\ Y_n \\ Z_n \end{pmatrix} + \begin{pmatrix} c_T.T_{x,t} \\ c_T.T_{y,t} \\ c_T.T_{z,t} \end{pmatrix} + \begin{pmatrix} c_S.S_t & -c_R.R_{z,t} & c_R.R_{y,t} \\ c_R.R_{z,t} & c_S.S_t & -c_R.R_{x,t} \\ -c_R.R_{y,t} & c_R.R_{x,t} & c_S.S_t \end{pmatrix}.\begin{pmatrix} X_n \\ Y_n \\ Z_n \end{pmatrix} \qquad \textit{Eqn. 2}$$

To convert from another ITRF96 to another ITRF realisation the signs of all the parameters are reversed, so the formula is:

$$\begin{pmatrix} X_n \\ Y_n \\ Z_n \end{pmatrix} = \begin{pmatrix} X_{96} \\ Y_{96} \\ Z_{96} \end{pmatrix} + \begin{pmatrix} -c_T.T_{x,t} \\ -c_T.T_{y,t} \\ -c_T.T_{z,t} \end{pmatrix} + \begin{pmatrix} -c_S.S_t & c_R.R_{z,t} & -c_R.R_{y,t} \\ -c_R.R_{z,t} & -c_S.S_t & c_R.R_{x,t} \\ c_R.R_{y,t} & -c_R.R_{x,t} & -c_S.S_t \end{pmatrix}.\begin{pmatrix} X_{96} \\ Y_{96} \\ Z_{96} \end{pmatrix} \qquad \textit{Eqn. 3}$$

Conversion between ellipsoidal and Cartesian coordinates

The conversions between longitude, latitude, and ellipsoidal height (λ, θ, h) and geocentric Cartesian coordinates (X, Y, Z) use the parameters of the GRS80 ellipsoid

Semi-major axis a = 6378137.0 metres

Reciprocal of flattening $1/f$ = 298.257222101

To convert ellipsoidal to geocentric coordinates use the following formulae:

$$b = (1 - f).a \qquad \textit{Eqn. 4}$$

$$\rho = \frac{a^2 \cos\theta}{\sqrt{b^2 \sin^2\theta + a^2 \cos^2\theta}} + h\cos\theta \qquad \textit{Eqn. 5}$$

$$X = \rho \cos \lambda \qquad \text{Eqn. 6}$$

$$Y = \rho \cos \lambda$$

$$Z = \frac{b^2 \sin \theta}{\sqrt{b^2 \sin^2 \theta + a^2 \cos^2 \theta}} + \mathrm{h} \sin \theta$$

There are no simple formulae to convert Cartesian coordinates to ellipsoidal coordinates – this requires an iterative solution to determine the latitude and height. The steps are as follows:

Calculate the longitude and a first approximation to the latitude θ_0

$$\lambda = \tan^{-1}\left(\frac{Y}{X}\right) \qquad \text{Eqn. 7}$$
$$\rho = \sqrt{X^2 + Y^2}$$
$$\theta_0 = \tan^{-1}\left(\frac{a^2 Z}{b^2 \rho}\right)$$

Then the following equation is used to derive successively better approximations to the latitude θ_1, θ_2, … until successive iterations converge to the desired tolerance

$$\theta_{i+1} = \tan^{-1}\left(\left(Z + \frac{(a^2 - b^2) \sin \theta_i}{\sqrt{b^2 \sin^2 \theta_i + a^2 \cos^2 \theta_i}}\right) / \rho\right) \qquad \text{Eqn. 8}$$
$$\text{Repeat until } |\theta_{i+1} - \theta_i| < 10^{-10}$$

The height is determined from the final latitude θ as

$$h = \rho \cos \theta + Z \sin \theta - \sqrt{b^2 \sin^2 \theta + a^2 \cos^2 \theta} \qquad \text{Eqn. 9}$$

Application of deformation model to convert ITRF96 to NZGD2000

To convert ITRF96 coordinates at a specific epoch to NZGD2000 coordinates the deformation component at the observation epoch must be subtracted from the coordinates. The NZGD2000 deformation model is used to calculate this component based on the location (longitude/latitude) of the point and the epoch.

The NZGD2000 deformation model calculates the deformation in terms of east, north, and up displacements $(\delta e_t, \delta n_t, \delta u_t)$ in metres at time t relative to NZGD2000 reference coordinates at the time of observation. The east and north offsets must be converted to degrees $(\delta \lambda_t, \delta \theta_t)$ in order to apply them to the latitude and longitude. (Note that the east/north offsets are defined on the surface of ellipsoid, not at the elevation of the observation point).

$$\delta \lambda_t = \delta e_t \left(\frac{\sqrt{b^2 \sin^2 \theta_i + a^2 \cos^2 \theta_i}}{a^2 \cos \theta}\right)\left(\frac{180}{\pi}\right) \qquad \text{Eqn. 10}$$

$$\delta \theta_t = \delta n_t \left(\frac{(b^2 \sin^2 \theta_i + a^2 \cos^2 \theta_i)^{\frac{3}{2}}}{a^2 \mathrm{b}^2}\right)\left(\frac{180}{\pi}\right) \qquad \text{Eqn. 11}$$

To convert the ITRF96 reference coordinates to NZGD2000 coordinates at time t, these components are subtracted from the coordinates, ie

$$\begin{aligned} \lambda_{\mathrm{NZGD2000}} &= \lambda_{96} - \delta\lambda_t \\ \theta_{\mathrm{NZGD2000}} &= \theta_{96} - \delta\theta_t \\ h_{\mathrm{NZGD2000}} &= h_{96} - \delta u_t \end{aligned} \qquad \textit{Eqn. 12}$$

Example transformation

By way of an example we will transform an ITRF08 coordinate

$\mathrm{LLH}_{\mathrm{ITRF2008}}$ = (174.774752, -41.284944, 48.52)

observed on 27 April 2013 to NZGD2000.

Applying Eqn. 6 the longitude/latitude/ellipsoidal height transforms to a geocentric Cartesian coordinate

$\mathrm{XYZ}_{\mathrm{ITRF2008}}$ = (-4779860.9786, 437125.2533, -4186286.2229)

The date 27 April 2013 is expressed equivalent to year 2013.32. The transformation parameter T_x at this epoch is therefore

$T_{x,2013.32}$ = 4.8+(2013.32-2000.0)×0.79 = 15.3228mm.

The other parameters $T_{y,t}$, $T_{z,t}$, S_t, $R_{x,t}$, $R_{y,t}$ and $R_{z,t}$ are calculated for the epoch in a similar way as 5.902mm, -35.5188mm, 0.0502368ppb, -0.3445004mas, 0.4706348mas, and 0.3826436mas respectively.

Using these parameters in Eqn. 2 the coordinate converts to an ITRF96 coordinate

$\mathrm{XYZ}_{\mathrm{ITRF96}}$ = (-4779860.9739,437125.2316, -4186286.2485)

and applying Eqn. 7 to Eqn. 9 this has latitude, longitude, and height

$\mathrm{LLH}_{\mathrm{ITRF96}}$ = (174.774752252,-41.284944213,48.5319)

The NZGD2000 deformation model must be evaluated at this location and time to convert this to NZGD2000. Using version 20130801 of the deformation model this evaluates to:

$(\delta e, \delta n, \delta u)$ = (-0.2717, 0.4346, 0.0000)

Applying the scaling from Eqn. 10 and Eqn. 11 this converts to

$(\delta\lambda, \delta\theta, \delta h)$ = (-0.00000324, 0.00000391, 0.0)

And subtracting this deformation from the ITRF96 coordinates gives the final NZGD2000 coordinates as

$\mathrm{LLH}_{\mathrm{ITRF96}}$ = (174.77475550, -41.28494813, 48.5319)

References

Soler, T. & R.A. Snay (2004). Transforming positions and velocities between the International Terrestrial Reference Frame of 2000 and North American Datum of 1983, J. Surv. Engrg., ASCE, 130(2), 49-55. Available at http://geodesy.noaa.gov/CORS/Articles/SolerSnayASCE.pdf