# ENGR 4350: Applied Deep Learning

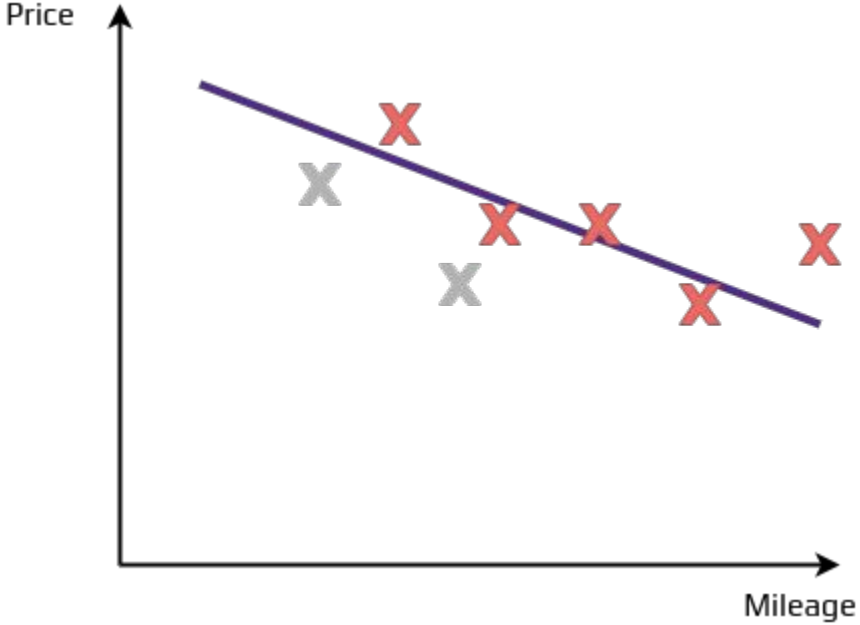
## Logistic Regression: Part 1

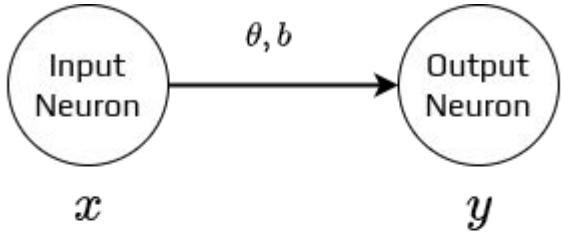
08/31/2022

# Outline

- An example of neural network
- Logistic Regression
  - Forward Pass
  - Loss Function
  - Gradient Descent

# A Neural Network Example

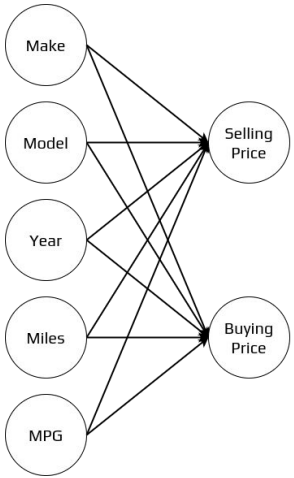



$$y = \sigma(\theta x + b)$$

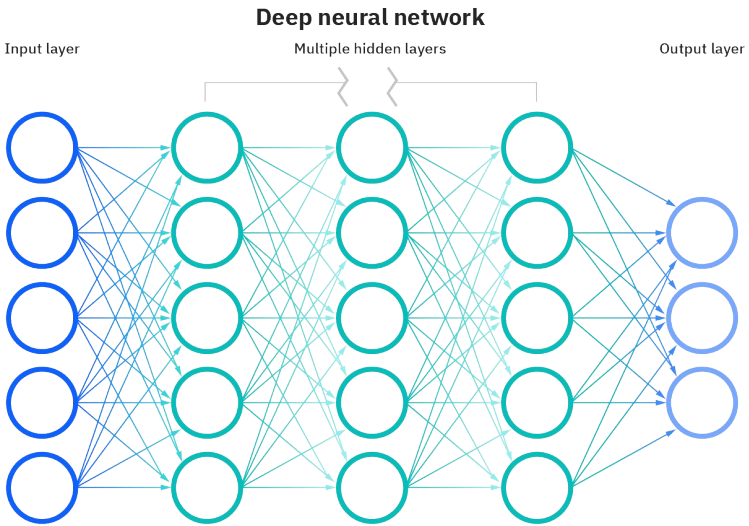
# A Neural Network Example

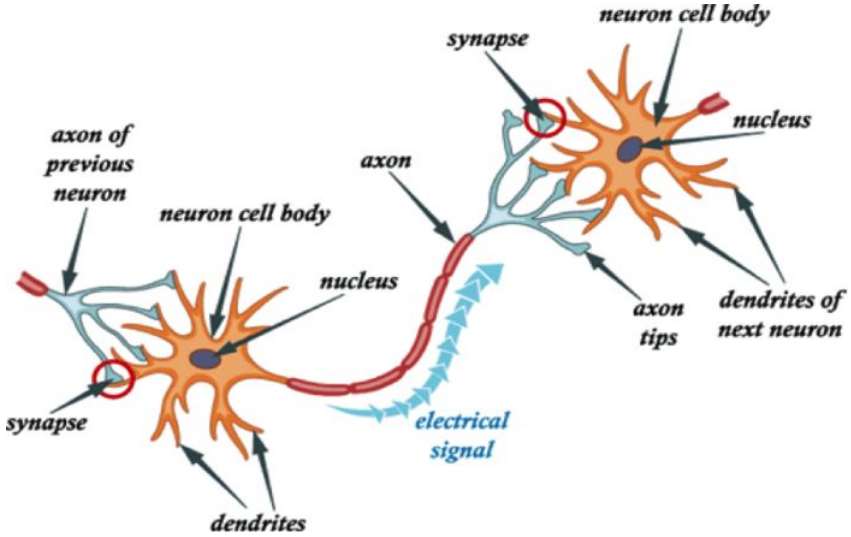
| Make | Model | Year | Mileage | MPG | **Buying Price** | **Selling Price** |
|---|---|---|---|---|---|---|
| Ford | Edge | 2018 | 50,000 | 23 | **$19,000** | **$10,000** |
| Toyota | Land Cruiser | 2020 | 10,000 | 15 | **$80,000** | **$50,000** |
| VW | Golf | 2010 | 150,000 | 36 | **$7,000** | **$2,000** |




$$\vec{y} = \sigma(\vec{\theta}\vec{x} + \vec{b})$$

# A Neural Network Example



# Binary Classifcation

- Complex decision makings can be simplified to classification problems.
  - Vehicle control
  - Robotic arm control
  - Gaming
  - ...
- Binary classification works most of the time.
  - Visitor identification
  - Animal protection
  - Farming
  - ...

# Logistic Regression

Logistic regression estimates the probability of an event occurring, $P(\mathbf{y} = \mathbf{1} \mid \mathbf{x})$ such as voted or didn't vote, based on a given dataset of independent variables. Since the outcome is a probability, the dependent variable is bounded between 0 and 1.

- Classification
- Prediction
- Rating

...

E.g. Given the "make, model, year, mileage, MPG" of vehicles, estimate probabilities of prices of these vehicles under $20,000.

# Problem Settings

Dataset: $\left\{\left(\mathbf{x}^{(1)}, y^{(1)}\right), \left(\mathbf{x}^{(2)}, y^{(2)}\right), \ldots, \left(\mathbf{x}^{(m)}, y^{(m)}\right)\right\}$

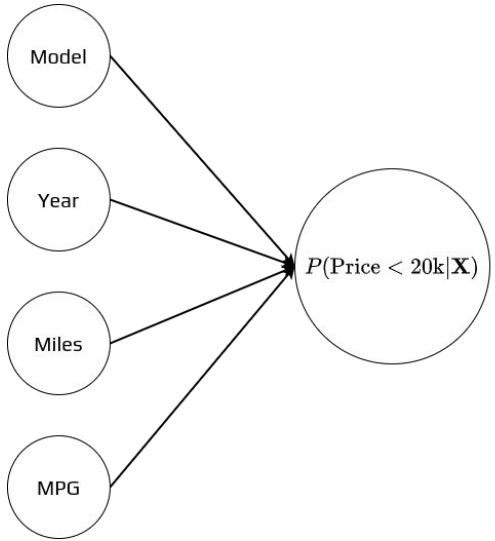
Features: $\mathbf{X} = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & . & . & . & x_n^{(1)} \\ x_1^{(2)} & x_2^{(2)} & . & . & . & x_n^{(2)} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ x_1^{(m)} & x_2^{(m)} & . & . & . & x_n^{(m)} \end{bmatrix}$

Labels: $\mathbf{y} = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ . \\ . \\ . \\ y^{(m)} \end{bmatrix}$

$m$ examples, $n$ independent variables

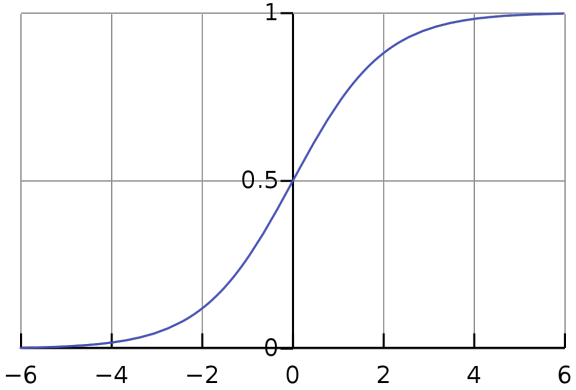
# Forward Pass / Prediction

| Id (Model) | Year | Mileage | MPG | **Buying Price** |
|---|---|---|---|---|
| 5 (Ford Edge) | 2018 | 50,000 | 23 | **1 ($19,000)** |
| 105 (Toyota Landcruiser) | 2020 | 10,000 | 15 | **0 ($80,000)** |
| 233 (VW Golf) | 2010 | 150,000 | 36 | **1 ($7,000)** |



# Forward Pass / Prediction

Sigmoid function: $\sigma(z) = \frac{1}{1+e^{-z}}$




Input: $\mathbf{X}$

Weights: $\mathbf{w} \in \mathbb{R}^{n_\mathbf{x}}$, bias: $b \in \mathbb{R}$

$$\mathbf{w} = \begin{bmatrix} w_1 & w_2 & . & . & . & w_n \end{bmatrix}$$

Output: $\hat{\mathbf{y}} = \sigma\left(\mathbf{X}\mathbf{w}^\mathbf{T} + b\right)$

$$\begin{bmatrix} x_1^{(1)} & x_2^{(1)} & . & . & . & x_n^{(1)} \\ x_1^{(2)} & x_2^{(2)} & . & . & . & x_n^{(2)} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ x_1^{(m)} & x_2^{(m)} & . & . & . & x_n^{(m)} \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \\ . \\ . \\ . \\ w_n \end{bmatrix} = \begin{bmatrix} \hat{y}^{(1)} \\ \hat{y}^{(2)} \\ . \\ . \\ . \\ \hat{y}^{(m)} \end{bmatrix}$$

# Loss Function

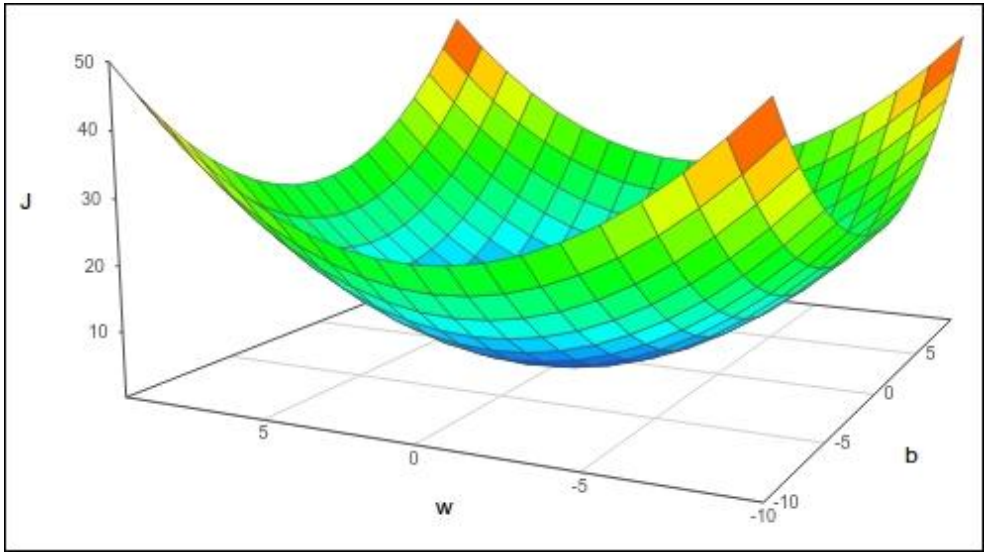
Dataset: $\left\{\left(\mathbf{x}^{(1)}, y^{(1)}\right), \left(\mathbf{x}^{(2)}, y^{(2)}\right), \ldots, \left(\mathbf{x}^{(m)}, y^{(m)}\right)\right\}$

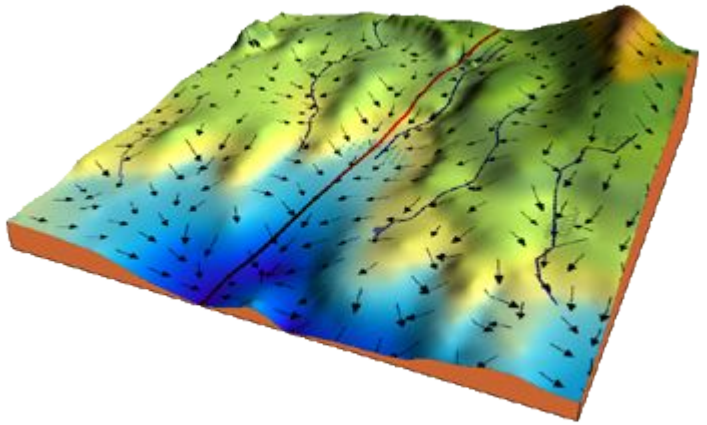
Mean squared error (MSE) loss function: $\mathcal{L}(\hat{\mathbf{y}}, \mathbf{y}) = \frac{1}{2}(\hat{\mathbf{y}} - \mathbf{y})^2$

Cross entropy loss function: $\mathcal{L}(\hat{\mathbf{y}}, \mathbf{y}) = -(\mathbf{y} \log \hat{\mathbf{y}} + (1 - \mathbf{y}) \log\ (1 - \hat{\mathbf{y}}))$

Cost function: $J(\mathbf{w}, b) = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{\mathbf{y}}, \mathbf{y})$

# Gradient Descent



Find $\mathbf{w}$ and $b$ that minimize $J(\mathbf{w}, b)$

# Gradient Descent

repeat until converge {

$$\mathbf{w} := \mathbf{w} - \alpha \frac{\partial J}{\partial \mathbf{w}}$$

$$b := b - \alpha \frac{\partial J}{\partial b}$$

}

$\alpha$ is the learning rate