

# ENGR 3421: Robotics I

Kinematics of Differential Drive

10/14/2025



# Outline

- Motion: From Motor to Robot
- Forward Kinematics

# Encoder to Motor Velocity (Before Gearbox)

1. Encoder Counts Changing Rate = Encoder Counts Difference / Time Difference (s)

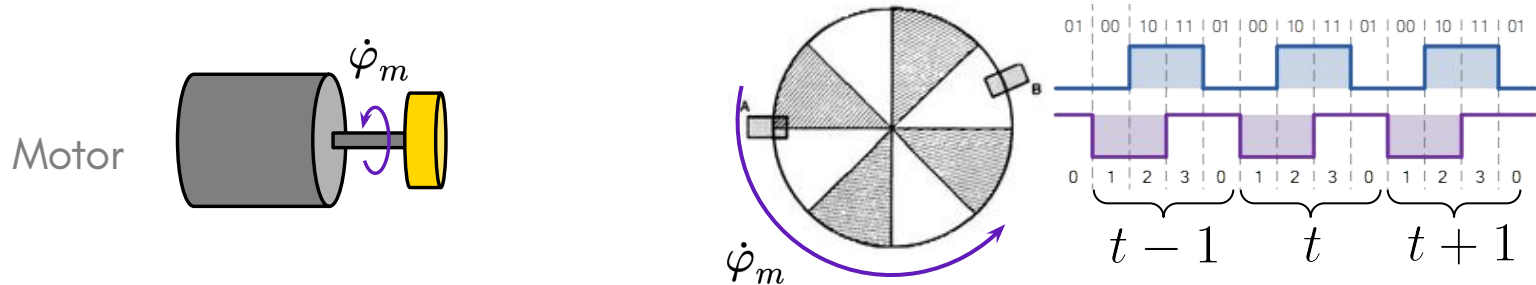
$$\dot{C} = \frac{\Delta C}{\Delta t} = \frac{C_t - C_{t-1}}{\Delta t}, \text{ counts/s}$$

2. Angular Velocity = Encoder Counts Changing Rate / Counts Per Revolution

$$\dot{\varphi}_m = \frac{\dot{C}}{CPR}, \text{ revs/s}$$

3. Angular Velocity (rads/s) = 2 \* pi (radians) \* Angular Velocity (revs/s)

$$\dot{\varphi}_m = 2\pi\dot{\varphi}_m, \text{ rads/s}$$



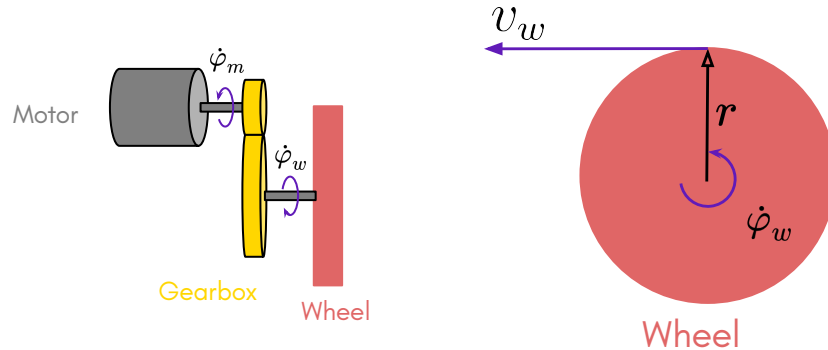
# Motor to Wheel Velocity (After Gearbox)

1. Wheel's Angular Velocity (rads/s) = Motor's Angular Velocity / Gear Ratio

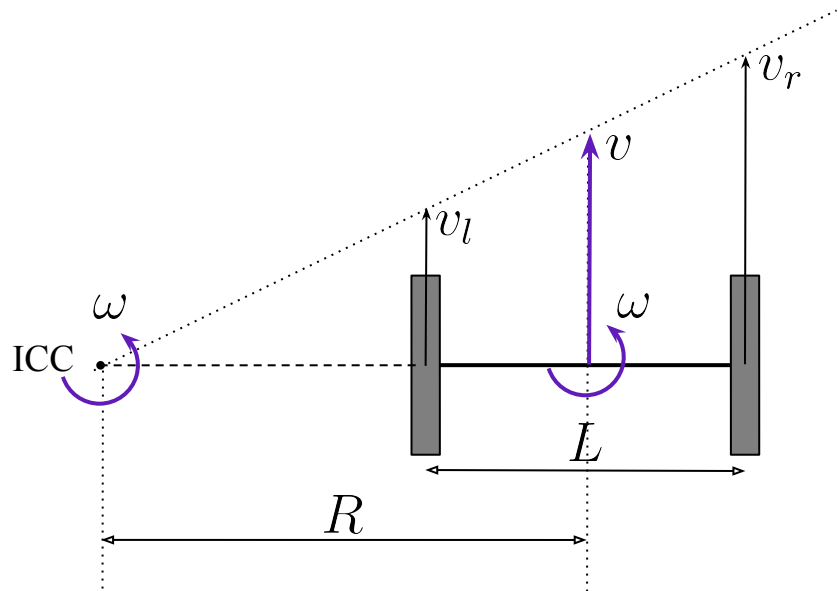
$$\dot{\varphi}_w = \frac{\dot{\varphi}_m}{i}, \text{ rads/s}$$

2. Wheel's Linear Velocity (m/s) = Wheel's Angular Velocity \* Wheel's Radius (m)

$$v_w = \dot{\varphi}_m r, \text{ m/s}$$



# Motion Notations



$ICC$ : Instantaneous Center of Curvature

$R$ : radius of curvature

$L$ : wheel separation distance

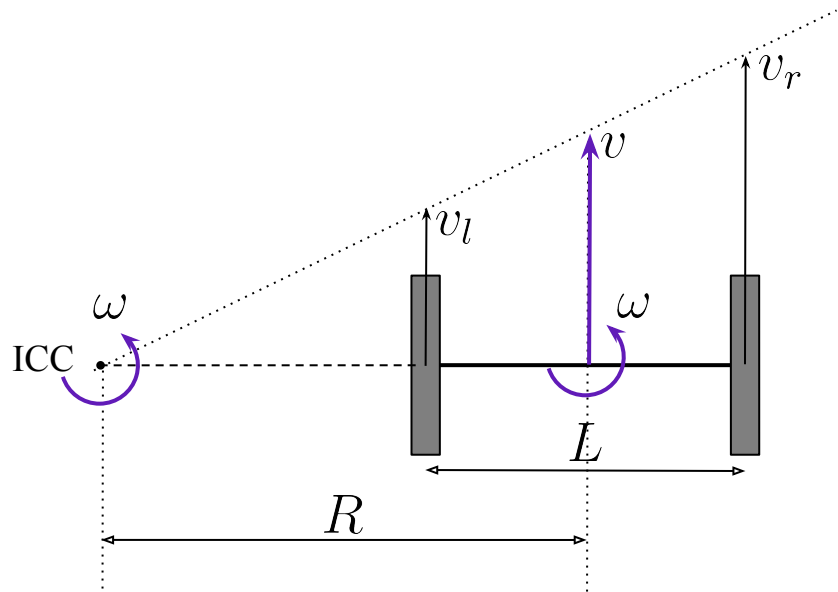
$v$ : robot linear velocity

$\omega$ : robot angular velocity

$v_l$ : linear velocity of left wheel

$v_r$ : linear velocity of right wheel

# Motion: From Wheel to Robot



$$\omega\left(R - \frac{L}{2}\right) = v_l$$
$$\omega\left(R + \frac{L}{2}\right) = v_r$$

Rotation about ICC must be same for both wheels.

$$v_l = v - \frac{\omega L}{2}$$

Linear velocity of left wheel

$$v_r = v + \frac{\omega L}{2}$$

Linear velocity of right wheel

$$R = \frac{L}{2} \frac{v_l + v_r}{v_l - v_r}$$

Rotation radius.

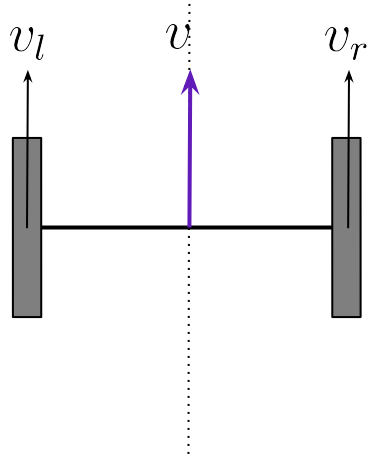
$$v = \frac{v_l + v_r}{2}$$

Robot's linear velocity

$$\omega = \frac{v_r - v_l}{L}$$

Robot's angular velocity

**Special Case:**  $v_l = v_r$

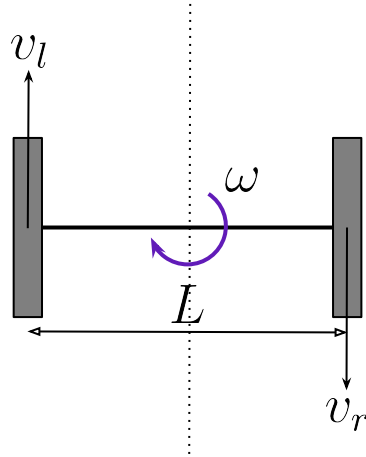


$$v = v_l = v_r$$

$$\omega = 0$$

ICC disappears.

**Special Case:**  $v_l = -v_r$



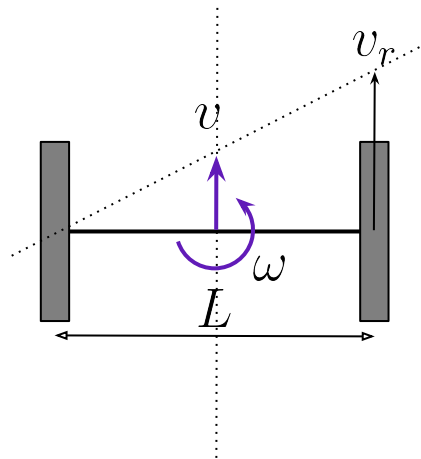
$$v = 0$$

$$\omega = \frac{2v_r}{L}$$

ICC at axle center.



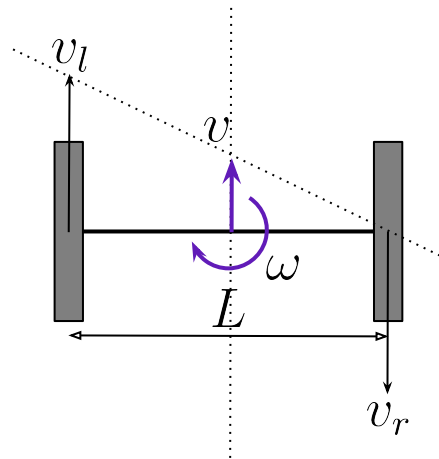
**Special Case:**  $v_l = 0$  **or**  $v_r = 0$



$$v = \frac{v_r}{2}$$

$$\omega = \frac{v_r}{L}$$

ICC at left wheel center.

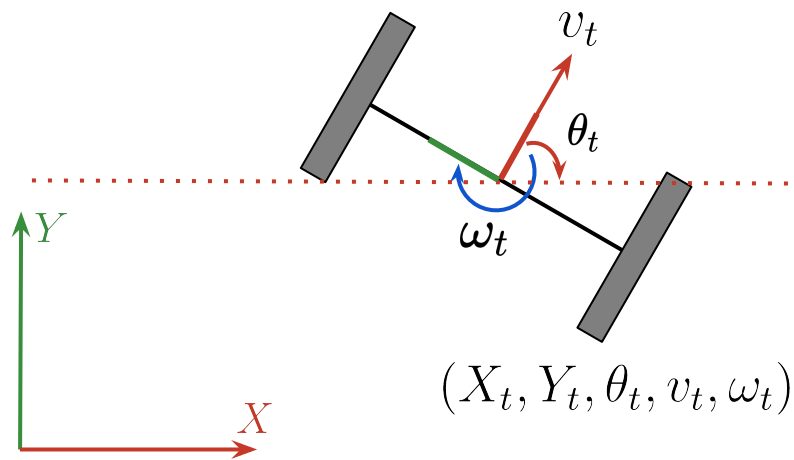


$$v = \frac{v_l}{2}$$

$$\omega = \frac{-v_l}{L}$$

ICC at right wheel center.

# Forward Kinematics (Discrete)

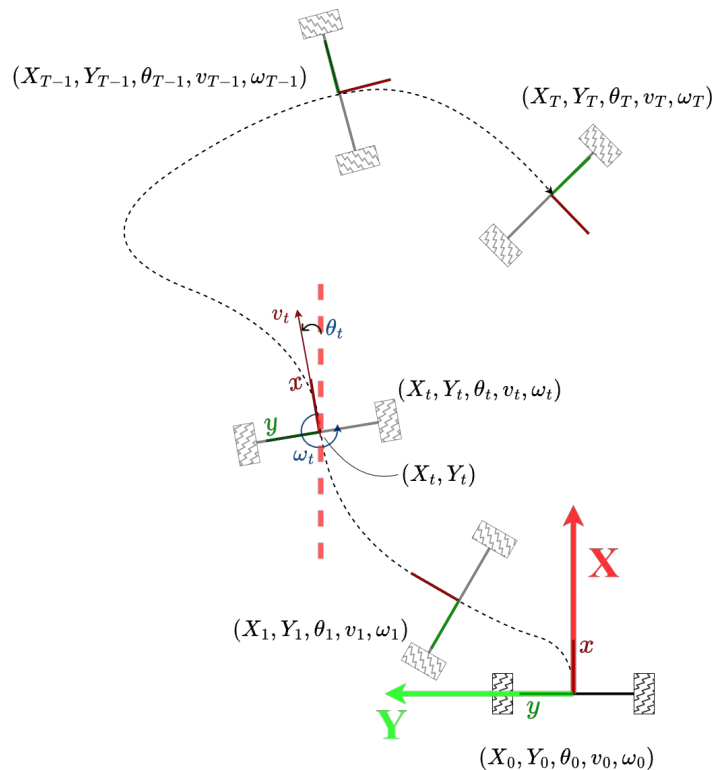


$$\begin{aligned} X_{t+1} &= X_t + \Delta X \\ &= X_t + v_t \cos \theta_t \Delta t \end{aligned}$$

$$\begin{aligned} Y_{t+1} &= Y_t + \Delta Y \\ &= Y_t + v_t \sin \theta_t \Delta t \end{aligned}$$

$$\begin{aligned} \theta_{t+1} &= \theta_t + \Delta \theta \\ &= \theta_t + \omega_t \Delta t \end{aligned}$$

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