# ENGR 4421: Robotics II

## Kinematics of Differential Drive

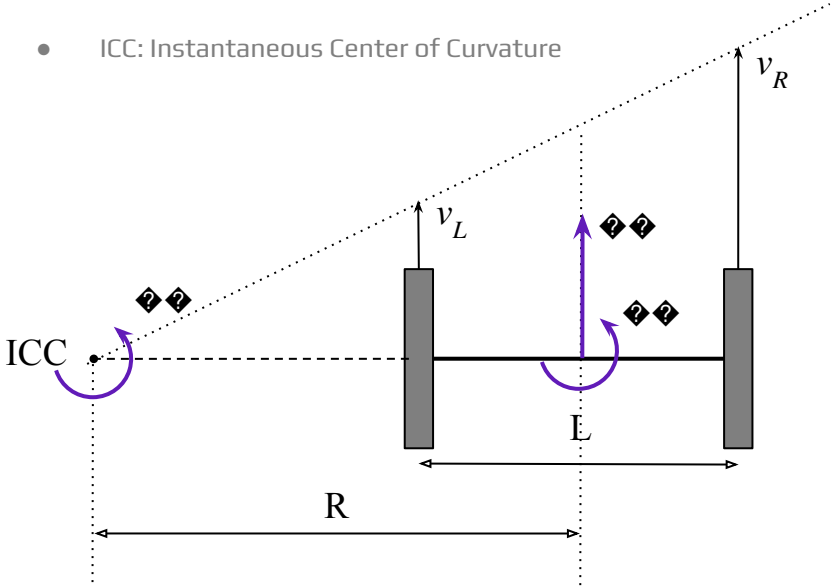
01/27/2022

# Outline

- Motion: From Motor to Body
- Forward Kinematics (w.r.t. different frames)
- Inverse Kinematics

# Motion: From Motor to Body

- ICC: Instantaneous Center of Curvature




$$\omega\left(R-\frac{L}{2}\right)=v_L$$
$$\omega\left(R+\frac{L}{2}\right)=v_R$$

Rotation about ICC must be same for both wheels.

$$R=\frac{L}{2}\frac{v_L+v_R}{v_L-v_R}$$

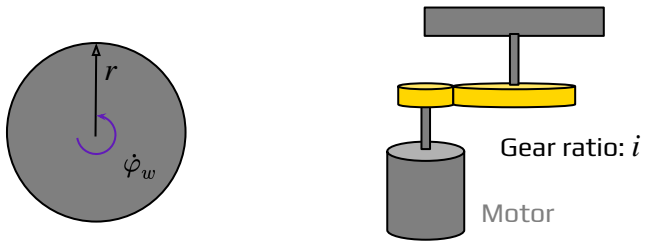
Rotation radius.

$$V=\frac{v_L+v_R}{2}$$

Linear x

$$\omega=\frac{V_R-V_L}{L}$$

Angular z



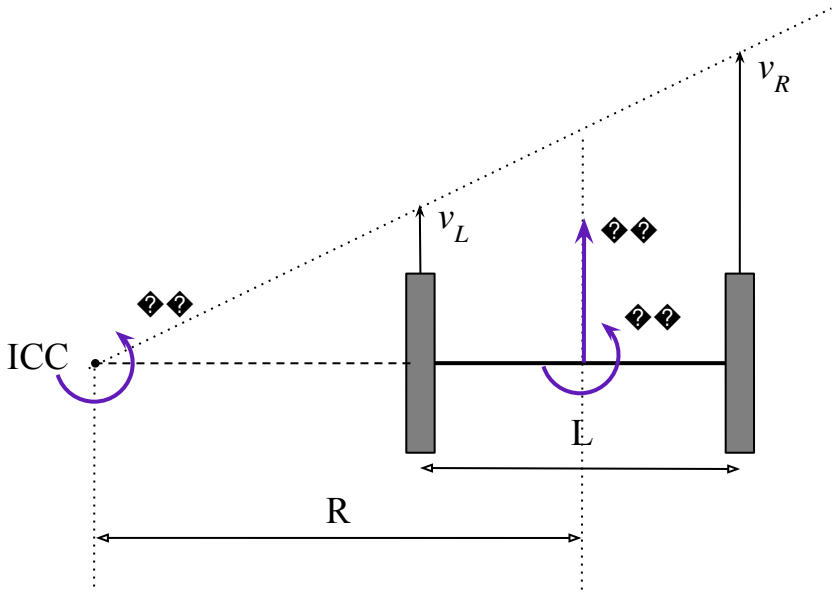

$$v=\dot{\varphi}_w r$$

Angular to linear

$$\dot{\varphi}_w=\frac{\dot{\varphi}_m}{i}$$

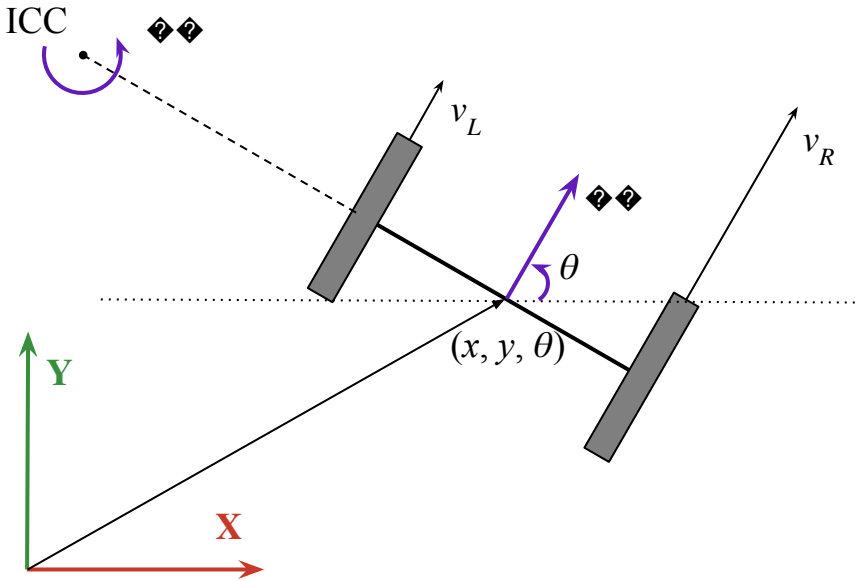
Motor speed to wheel speed

# Motion: From Motor to Body




- If $v_L = v_R$, then linear motion in a straight line. R becomes infinite, no rotation $\omega$=0.
- If $v_L = -v_R$, then rotation about the midpoint of the wheel axis, R = 0.
- If $v_L = 0$, then rotation about the left wheel, R = L/2. Rotation about the right wheel if $v_R = 0$.

# Forward Kinematics




$$ICC = [x - R\sin(\theta),\ y + R\cos(\theta)]$$

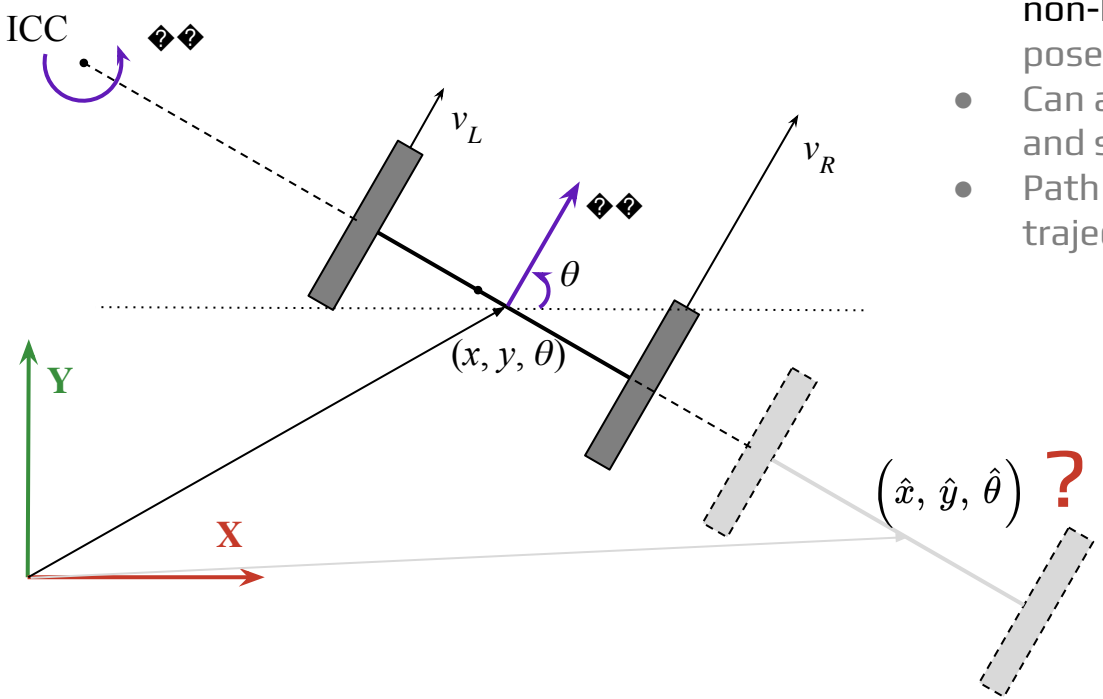
$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - ICC_x \\ y - ICC_y \\ \theta \end{bmatrix} + \begin{bmatrix} ICC_x \\ ICC_y \\ \omega\delta t \end{bmatrix}$$

$$x(t) = \int_0^t V(t)\cos(\theta(t))dt$$

$$y(t) = \int_0^t V(t)sin(\theta(t))dt$$

$$\theta(t) = \int_0^t \omega(t)dt$$

# Inverse Kinematics




- Given a target $(\hat{x}, \hat{y}, \hat{\theta})$, What is $V(t)$ and $\omega(t)$?
- Two-wheeled differential drive vehicle imposes non-holonomic constraints on establishing its pose (think about lateral translation).
- Can achieve the goal by moving in straight line and spinning in place.
- Path planning algorithms may find smoother trajectories.