ENGR 4421:Robotics II

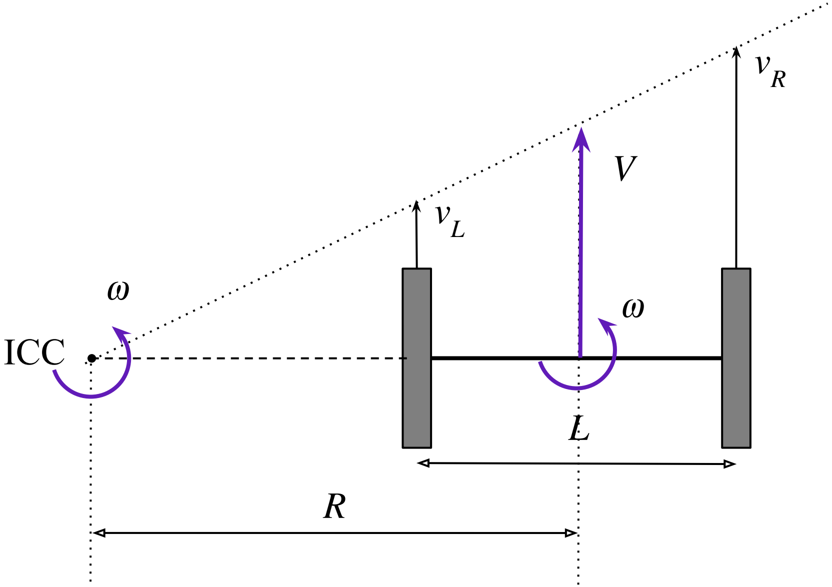
Kinematics of Differential Drive

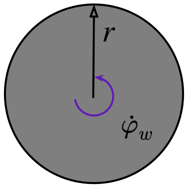
02/01/2022

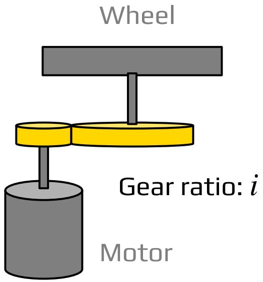
Outline

- Motion: From Motor to Body
- Forward Kinematics (w.r.t. different frames)
- Inverse Kinematics

Motion: From Motor to Body



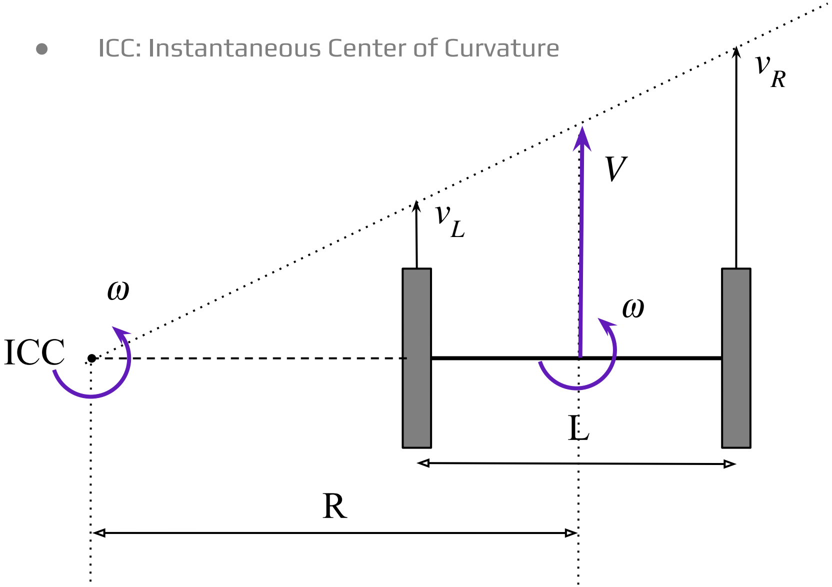


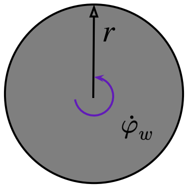


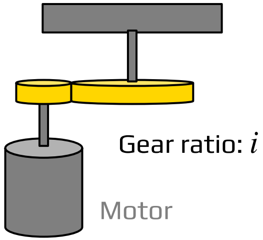
ICC: Instantaneous Center of Curvature
R: radius of curvature
L: wheel separation distance
V: robot linear velocity
ω: robot angular velocity
r: radius of wheel
i: gear ratio
$\dot{\varphi}_w$: angular velocity of wheel
$\dot{\varphi}_m$: angular velocity of motor
v_L: linear velocity of left wheel
v_R: linear velocity of right wheel

Motion: From Motor to Body

- ICC: Instantaneous Center of Curvature







$$\omega\left(R - \frac{L}{2}\right) = v_L$$

$$\omega\left(R + \frac{L}{2}\right) = v_R$$

Rotation about ICC must be same for both wheels.

$$R = \frac{L}{2}\frac{v_L + v_R}{v_L - v_R}$$

Rotation radius.

$$V = \frac{v_L + v_R}{2}$$

Linear speed of robot

$$\omega = \frac{v_R - v_L}{L}$$

Angular speed of robot

$$v_L = V - \frac{\omega L}{2}$$

Linear speed of left wheel

$$v_R = V + \frac{\omega L}{2}$$

Linear speed of right wheel

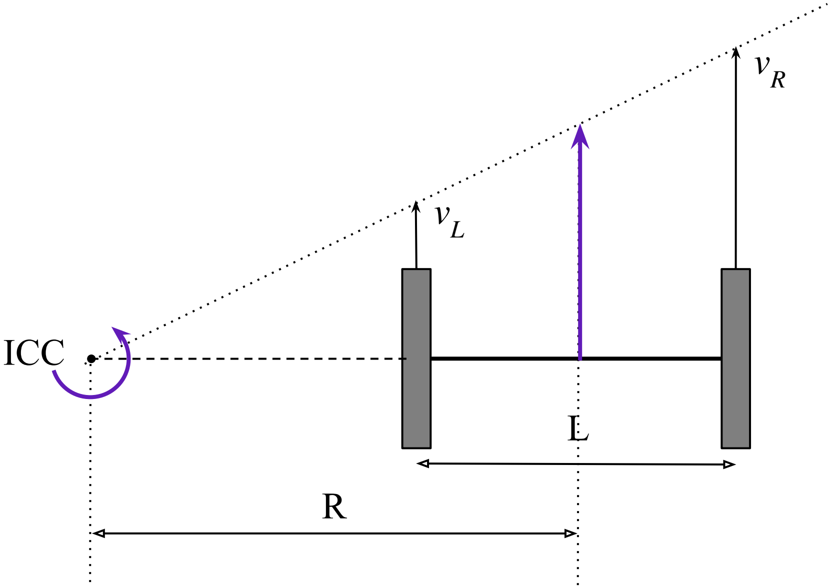
$$v = \dot{\varphi}_w r$$

Angular to linear

$$\dot{\varphi}_w = \frac{\dot{\varphi}_m}{i}$$

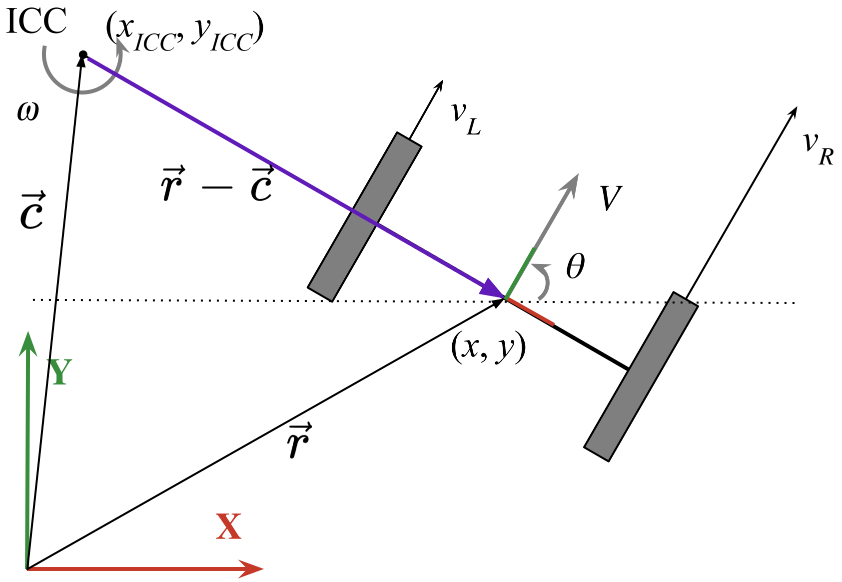
Motor speed to wheel speed

Motion: From Motor to Body



- If $v_L = v_R$, then linear motion in a straight line. R becomes infinite, no rotation ω=0.
- If $v_L = -v_R$, then rotation about the midpoint of the wheel axis, R = 0.
- If $v_L = 0$, then rotation about the left wheel, R = L/2. Rotation about the right wheel if $v_R = 0$.

Forward Kinematics



$$\vec{r} = [x,\ y]^T$$

$$\vec{c} = [x - R\sin(\theta),\ y + R\cos(\theta)]^T$$

$$\vec{r} - \vec{c} = [x - x_{ICC},\ y - y_{ICC}]^T$$

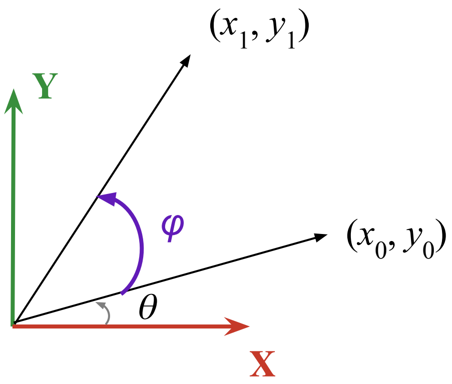
$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - x_{ICC} \\ y - y_{ICC} \\ \theta \end{bmatrix} + \begin{bmatrix} x_{ICC} \\ y_{ICC} \\ \omega\delta t \end{bmatrix}$$

$$x(t) = \int_0^t V(t)\cos(\theta(t))dt$$

$$y(t) = \int_0^t V(t) sin(\theta(t))dt$$

$$\theta(t) = \int_0^t \omega(t)dt$$

Rotation Matrix

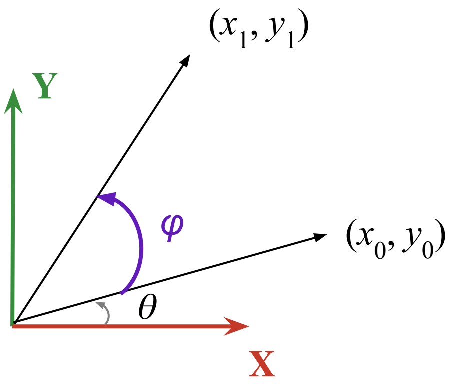


$$r = \sqrt{x_0^2 + y_0^2} = \sqrt{x_1^2 + y_1^2}$$

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} r\cos\theta \\ r\sin\theta \end{bmatrix}$$

$$\begin{aligned}
\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} &= \begin{bmatrix} r\cos(\theta+\varphi) \\ r\sin(\theta+\varphi) \end{bmatrix} \\
&= \begin{bmatrix} r(\cos\theta\cos\varphi - \sin\theta\sin\varphi) \\ r(\sin\theta\cos\varphi + \cos\theta\sin\varphi) \end{bmatrix} \\
&= \begin{bmatrix} x_0\cos\varphi - y_0\sin\varphi \\ y_0\cos\varphi + x_0\sin\varphi \end{bmatrix} \\
&= \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}
\end{aligned}$$

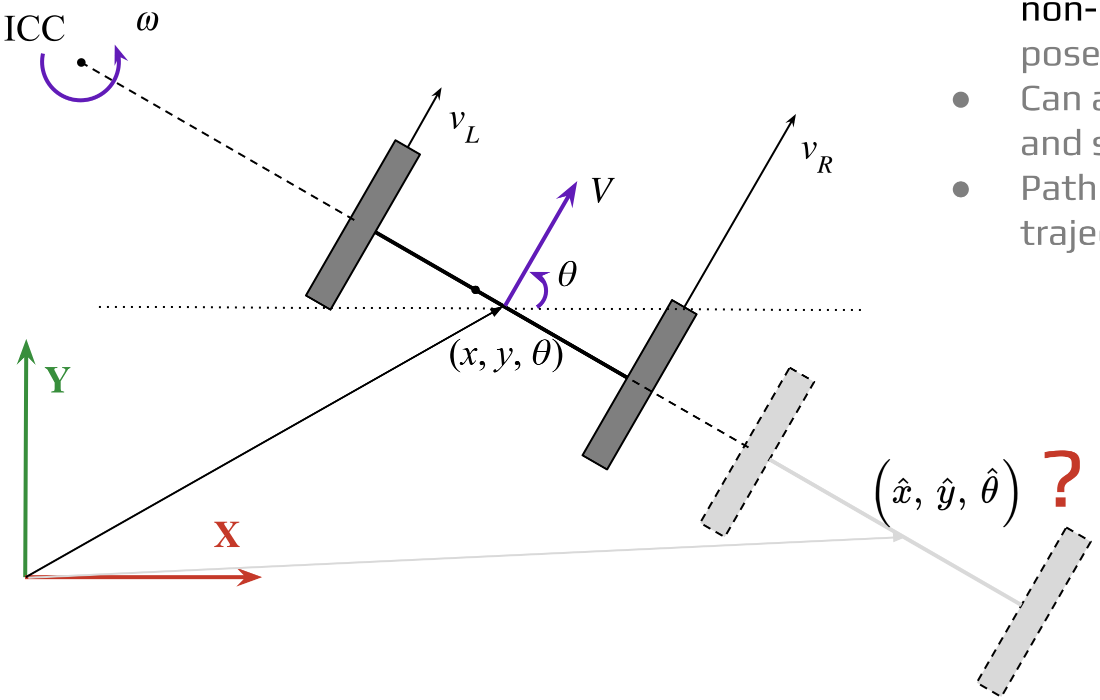
Rotation Matrix



$$R = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}$$

- Rotation matrix preserve length.
- Columns of rotation matrix are orthonormal.
- Transpose of rotation matrix times itself is identity.
- Rotation matrix times its transpose is identity.

Inverse Kinematics



- Given a target $(\hat{x}, \hat{y}, \hat{\theta})$, What is $V(t)$ and $\omega(t)$?
- Two-wheeled differential drive vehicle imposes non-holonomic constraints on establishing its pose (think about lateral translation).
- Can achieve the goal by moving in straight line and spinning in place.
- Path planning algorithms may find smoother trajectories.