Coursera Game Theory from Stanford
Week 1 Quiz
Solution of @Linzhuo

1.

1 \ 2	x	y	z
a	1,2	2,2	5,1
b	4,1	3,5	3,3
c	5,2	4,4	7,0
d	2,3	0,4	3,0

Find the strictly dominant strategy:

c is the strictly dominant strategy (of player 1). Note that strategy y is not strictly dominant for player 2, because when player 1 is at strategy a, u2(a,x) = u2(a,y).

2.

1 \ 2	x	y	z
a	1,2	2,2	5,1
b	4,1	3,5	3,3
c	5,2	4,4	7,0
d	2,3	0,4	3,0

Find a very weakly dominant strategy that is not strictly dominant.

See the analysis of last question. The answer is y.

3.

1 \ 2	x	y	z
a	1,2	2,2	5,1
b	4,1	3,5	3,3
c	5,2	4,4	7,0
d	2,3	0,4	3,0

When player 1 plays d, what is player 2's best response:

u2(d,x) = 3
u2(d,y) = 4
u2(d,z) = 0
The best response is y.

4.

1 \ 2	x	y	z
a	1,2	2,2	5,1
b	4,1	3,5	3,3
c	5,2	4,4	7,0
d	2,3	0,4	3,0

Find all strategy profiles that form pure strategy Nash equilibria (there may be more than one, or none):

For a, the BR of player 2 is x,y
For b -> y
For c -> y
For d -> y

For x, the BR of player 1 is c
For y -> c
For z -> c

The strategy profile s = <s1,s2> is a “pure strategy ” Nash Equilibrium if and only if: for all i, si is the BR of s-i. That is to say, for playler 1, s1 is the BR of s2 and for player 2, s2 is also the BR of player1.

(c,y) is NE. c->y, y->c. The others are not.

5. There are 2 players who have to decide how to split one dollar. The bargaining process works as follows. Players simultaneously announce the share they would like to receive s_1 and s_2, with $0 \leq s_1, s_2 \leq 1$. If $s_1 + s_2 \leq 1$, then the players receive the shares they named and if $s_1 + s_2 > 1$, then both players fail to achieve an agreement and receive zero. This game is known as `Nash Bargaining'.

Which of the following is a strictly dominant strategy?

- a)1;
- b) 0.5;
- c) 0;
- d) None of the above.

The def of strictly dominant: s_i strictly dominates s_i’ if for all s_-i in S_{-i} that u_i(s_i, S_{-i}) > u_i(s_i’, S_{-i}).

S1,s2	0	0.5	1
0	0,0	0,0.5	0,1
0.5	0.5,0	0.5,0.5	0,0
1	1,0	0,0	0,0

If player 1 selects 0.5, his utiliy is less than when he selects 1 and player 2 selects 0;
If player 1 selects 1, his utility is less than when he select 0.5 and player 2 selects 0.5;

If player1 selects 0, his utility is less than when he selects 0.5 and player 2 selects 0 or 0.5. The same case also apply to player 2. Therefore, select "None of the above".

6. There are 2 players who have to decide how to split one dollar. The bargaining process works as follows. Players simultaneously announce the share they would like to receive s_1 and s_2, with $0 \leq s_1, s_2 \leq 1$. If $s_1 + s_2 \leq 1$, then the players receive the shares they named and if $s_1 + s_2 > 1$, then both players fail to achieve an agreement and receive zero.

Which of the following strategy profiles is a pure strategy Nash equilibrium?

- a) (0.3, 0.7);
- b) (0.5, 0.5);
- c) (1.0, 1.0);
- d) All of the above

S1,s2	0	0.3	0.5	0.7	1
0	0,0	0,0.3	0,0.5	0,0.7	0,1
0.3	0.3,0	0.3,0.3	0.3,0.5	0.3,0.7	0,0
0.5	0.5,0	0.5,0.3	0.5,0.5	0,0	0,0
0.7	0.7,0	0.7,0.3	0,0	0,0	0,0
1	1,0	0,0	0,0	0,0	0,0

BR_s2(s1=0.5) = 0.5, BR_s1(s2=0.5) = 0.5 -> (0.5,0.5) is a pure strategy N.E.
BR_s2(s1=0.3) = 0.7, BR_s1(s2=0.7) = 0.3 ->(0.3,0.7) is a pure strategy N.E.
BR_s2(s1=1.0) = all, BR_s2(s2=x) = 1.0, solving -> x can be 1.0 -> (1.0, 1.0) is a pure strategy N.E.
Select "All of the above".

7. Two firms produce identical goods, with a production cost of $c > 0$ per unit.

Each firm sets a nonnegative price (p_1 and p_2).

All consumers buy from the firm with the lower price, if $p_i \neq p_j$. Half of the consumers buy from each firm if $p_i = p_j$.

D is the total demand.

Profit of firm i is:

- 0 if $p_i > p_j$ (no one buys from firm i);
- $D\frac{p_i - c}{2}$ if $p_i = p_j$(Half of customers buy from firm i);
- $D(p_i - c)$ if $p_i < p_j$ (All customers buy from firm i)

Find the pure strategy Nash equilibrium:

- a) Both firms set $p = 0$.
- b) Firm 1 sets $p = 0$, and firm 2 sets $p = c$.
- c) Both firms set $p = c$.
- d) No pure strategy Nash equilibrium exists.

The utility matrix:

P1, p2	0	C
0	-DC/2, -DC/2	-DC, 0
C	0, -DC	0, 0

(C,C) is the pure strategy N.E.
Choose c).

8.

- Three voters vote over two candidates (A and B), and each voter has two pure strategies: vote for A and vote for B.
- When A wins, voter 1 gets a payoff of 1, and 2 and 3 get payoffs of 0; when B wins, 1 gets 0 and 2 and 3 get 1. Thus, 1 prefers A, and 2 and 3 prefer B.
- The candidate getting 2 or more votes is the winner (majority rule).

Find all ***very weakly*** **dominant** strategies (click all that apply: there may be more than one, or none).

- [ ] a) Voter 1 voting for A.
- [ ] b) Voter 1 voting for B.
- [ ] c) Voter 2 (or 3) voting for A.
- [ ] d) Voter 2 (or 3) voting for B.

Analysis:

Utility:	case
u(A,A,A) = (1,0,0)	1
u(A,A,B) = (1,0,0)	2
u(A,B,A) = (1,0,0)	3
u(A,B,B) = (0,1,1)	4
u(B,A,A) = (1,0,0)	5
u(B,A,B) = (0,1,1)	6
u(B,B,A) = (0,1,1)	7
u(B,B,B) = (0,1,1)	8

For Voter 1:

Voting for A are from case 1 to 4. Comparing voting for B, which are case 5 to 8, u1(A,x,y) >= u1(B,x,y). (Note that x,y pair must be the identical for each case, so we are comparing between case1 and case5, case2 and case6, case3 and case7, case4 and case8). Thus voting for A very weakly dominates voting for B, on Voter 1's side. On asides, voting for B does not dominates voting for A.

For Voter 2 and 3:

Voter 2 votes for B (case 3,4,7,8): u2(x,B,y) >= u2(x,A,y), by comparing between case3 and case1, case4 and case2, case7 and case5, case8 and case6.

Voter 3 votes for B(case 2,4,6,8): u3(x,y,B) >= u3(x,y,A) by comparing between case2 and case1, case4 and case3, case6 and case5, case8 and case7.

Conclusion: Voter 2 or 3 voting for B very weakly dominates voting for A.

The answer is: a)d).

9.

- Three voters vote over two candidates (A and B), and each voter has two pure strategies: vote for A and vote for B.
- When A wins, voter 1 gets a payoff of 1, and 2 and 3 get payoffs of 0; when B wins, 1 gets 0 and 2 and 3 get 1. Thus, 1 prefers A, and 2 and 3 prefer B.
- The candidate getting 2 or more votes is the winner (majority rule).

Find **all** pure strategy Nash equilibria (click all that apply)? Hint: there are three.

- [ ] a) 1 voting for A, and 2 and 3 voting for B.
- [ ] b) All voting for A.
- [ ] c) All voting for B.
- [ ] d) 1 and 2 voting for A, and 3 voting for B.

For a):

The best response of player 1 when player 2 and 3 vote for B is voting for A and/or voting for B. Because: u1(A,B,B) = 0 and u1(B,B,B) = 0, the player1 has same utility. When player1 votes A and player2 votes B, u3(A,B,A) = 0, u3(A,B,B) = 1, so player3 has best response of voting B. When player1 votes A and player3 votes B, u2(A,A,B) = 0, u2(A,B,B) = 1, so player2 has best response of voting B. In sum, (A,B,B) is a pure strategy N.E.

For b):

When player1 and player2 both vote for A, u3(A,A,A) = 0, u3(A,A,B) = 0, both voting for A and B are best response.

When player1 and player3 both vote for A, u2(A,A,A) = 0, u2(A,B,A) = 0, both voting for A and B are best response.

When player2 and player3 both vote for A, u1(A,A,A) = 1, u1(B,A,A) = 1, both voting for A and B are best response.

In sum, (A,A,A) is a pure strategy N.E.

For c):

When player1 and player2 both vote for B, u3(B,B,A) = 1, u3(B,B,B) = 1, both voting for A and B are best response.

When player1 and player3 both vote for B, u2(B,A,B) = 1, u2(B,B,B) = 1, both voting for A and B are best response.

When player2 and player3 both vote for B, u1(A,B,B) = 0, u1(A,B,B) = 0, both voting for A and B are best response.

In sum, (B,B,B) is a pure strategy N.E.

For d):

When player 1 and 2 both vote for A, u3(A,A,A) = 0, u3(A,A,B) = 0, both voting for A and B are best response.

When player1 votes for A and player3 votes for B, u2(A,A,B) = 0, u2(A,B,B) = 1, thus voting for A is not best response for player 2. (A,A,B) is not a pure strategy N.E.

Answer: a)b)c).