WORCESTER POLYTECHNIC INSTITUTE

PORTFOLIO VALUATION AND RISK MANAGEMENT

Factor Project

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Chapter 1

Introduction

In this project, we first construct six different factor models and compare assets expected return, return variance and covariance matrix across the six models as well as a basic method which simply calculate those parameters by historical data. Then we choose two of these models plus that basic method to construct three different tangency portfolios. By comparing some important factors, we pick a best model among the three. Finally, we make a quantile-based portfolio back-testing.

Chapter 2

Data Collection

Through Yahoo Finance[1], we download 1-years AAL, AAPL, BABA, BIDU, BURL, D, DAL, FB, FDX, FRT, GNC, GOOG, GPRO, L, NKE, O, S, SPWR, T, UPS and S&P 500 historical data from 03/01/16 to 10/31/17 and created a program which extracted all data. We also download factor Momentum from Professor French's website[2]. Python Pandas package helps us to read excel file into data frame. For stock, we extracted the adjust closed prices separately and transferred them into 1-dimensional arrays. They would be accessed in all problems of the project.

[1] https://finance.yahoo.com/

[2] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Chapter 3

Methodology

3.1 Capital Asset Pricing Model

The capital asset pricing model (CAPM) is a standard model for asset returns, it's a theoretical justification of indexing, holding a diversified portfolio in some relative proportions (Ruppert, 2004).

The model based on these assumptions (Marcel, 2017):

- Market prices are in equilibrium for each asset supply equals demand.
- Every one has the same forecasts of expected returns & risk.
- All investors choose portfolio and the risk-free asset.
- The market rewards in investors for assuming unavoidable risk
- No reward for inefficient investing.

3.1.1 Capital Market Line

The capital market line (CML) relates the excess expected return on an efficient portfolio to its risk. "Excess expected return" means the amount by which the expected return of the portfolio exceeds the risk-free rate of return and is also called the risk premium. The CML is

$$\mu_R = \mu_f + \frac{\mu_M - \mu_f}{\sigma_M}\sigma_R, \tag{3.1}$$

where R is the return on a given efficient portfolio (mixture of the market portfolio and the risk-free asset), $\mu_R = E(R)$, μ_f is the rate of return on the risk-free asset, R_M is the return on the market portfolio, $\mu_M = E(R_M)$, σ_M is the standard deviation of the return on the market portfolio. The risk premium of R is $\mu_R - \mu_f$ and the risk premium of the market portfolio is $\mu_M - \mu_f$ (Ruppert, 2004).

3.1.2 Security Market Line

The security market line (SML) is a equation that derived by using CAPM, it can be shown as:

$$\mu_j - \mu_f = \beta_j(\mu_M - \mu_f). \tag{3.2}$$

where β_j is a variable in the linear equation, not the slope; more precisely, μ_j is a linear function of β_j with slope $\mu_M - \mu_f$. In other words, β_j is a slope in one context but is the independent variable in the SML (Ruppert, 2004).

SML can be applied to any asset. Generally, β_j measures how aggressive asset j is. By definition, $\beta_M = 1$ and

- if $\beta_j > 1$ or $\beta_j < -1$, it means it's a aggressive asset;
- if $0 < \beta_j < 1$ or $-1 < \beta_j < 0$, it means it's a less aggressive asset;
- if $\beta_j = 1$ or $\beta_j = -1$, it means it's an average risk asset [Blais note];

Regression Model

The SML is the regression model (Marcel, 2017).

$$R_{jt} = \mu_{ft} + \beta_j(R_{mt} - \mu_{ft}) + \epsilon_{jt} \tag{3.3}$$

where $\epsilon_{jt} \sim \text{WhiteNoise}(0, \sigma^2_{\epsilon j})$, R_{jt} is the j th asset return at time t, R_{mt} is market portfolio, μ_{ft} is the risk free rate at time t. Also, we make a common assumption

$$Cov(\epsilon_{it}, \epsilon_{jt}) = 0, \;\; if i \neq j.$$

Let $\mu_{jt} = E(R_{jt}), \mu_{Mt} = E(R_{Mt})$, if we take expectation of equation (3.3), we have

$$\mu_{jt} = \mu_{ft} + \beta_j(\mu_{Mt} - \mu_{ft}).$$

Also, from equation (3.3), we get

$$Var(R_{jt}) = \sigma_j^2 = \beta_j^2\sigma_M^2 + \sigma^2_{\epsilon j}.$$

where $\beta_j^2\sigma_M^2$ is market/systemic component, and $\sigma^2_{\epsilon j}$ is unique/non-systemic component.

Moreover, we have

$$\begin{aligned}\sigma_{ij} &= Cov(R) = Cov(\mu_{ft} + \beta_i(R_{MT} - \mu_{ft}) + \epsilon_{it}, \mu_{ft} + \beta_j(R_{MT} - \mu_{ft}) + \epsilon_{jt}) \\ &= Cov(\beta_i R_{MT}, \beta_j R_{MT}) \\ &= \beta_i\beta_j\sigma_M^2\end{aligned}$$

$$\begin{aligned}Cov(R_{jt}, R_{MT}) &= Cov(\mu_{ft} + \beta_i(R_{MT} - \mu_{ft}) + \epsilon_{it}, R_{MT}) \\ &= \beta_j\sigma_M^2.\end{aligned}$$

If we let $R^*_{jt} = R_{jt} - \mu_{ft}$, which means excess return for asset j, then equation (3.3) becomes

$$R^*_{jt} = \beta_j R^*_{Mt} + \epsilon_{jt},$$

where $R^*_{Mt} = R_{Mt} - \mu_{ft}$

We can expand this to

$$R^*_{jt} = \alpha_j + \beta_j R^*_{Mt}.$$

It can test for mispricing of CAPM by testing a null hypothesis that $\alpha_j = 0$. If there is a time series for R^*_{jt}, R^*_{Mt}, we regress R^*_{jt} onto R^*_{MT} test $\hat{\alpha_j}$, if $\hat{\alpha_j} > 0$ in a statistically significant way, then we have found meaningful excee return that's not predicted by CAPM (Marcel, 2017).

Risk Reduction by Diversification

Assume there are N assets with returns $R_{1t}, R_{2t}, \cdots, R_{Nt}$ holding period t with weight $\omega_1, \omega_2, \cdots, \omega_N$.

Then the portfolio return is

$$R_{Pt} = \sum_{i=1}^{N} = \omega_i R_{it},$$

where R_{Mt} is market portfolio return.

Combine with equation (3.3), we have

$$\begin{aligned} R_{Pt} &= \sum_{j=1}^{N} \omega_j [\mu_{ft} + \beta_j (R_{Mt} - \mu_{ft}) + \epsilon_{jt}] \\ &= \sum_{j=1}^{N} \omega_j \mu_{ft} + (R_{Mt} - \mu_{ft})(\sum_{j=1}^{N} \omega_j \beta_j) + \sum_{j=1}^{N} \omega_j \epsilon_{jt} \\ &= \mu_{ft} + (\sum_{j=1}^{N} \omega_j \beta_j)(R_{Mt} - \mu_{ft}) + \sum_{j=1}^{N} \omega_j \epsilon_{jt}. \end{aligned}$$

If the ϵ_{jt} are uncorrelated then

$$\sigma_P^2 = Var(R_{Pt}) = (\sum_{j=1}^{N} \omega_j \beta_j)^2 \sigma_M^2 + \sum_{j=1}^{N} \omega_j^2 \sigma_{\epsilon j}^2,$$

where $(\sum_{j=1}^{N} \omega_j \beta_j)^2$ is systemic risk, and $\sum_{j=1}^{N} \omega_j^2 \sigma_{\epsilon j}^2$ is unique/non-systemic risk.

3.2 Robust Regression

Def 1. *A robust regression estimate* $\begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix}$ *is defined as*

$$\begin{bmatrix} \hat{\alpha_i} \\ \hat{\beta_i} \end{bmatrix} = \underset{\alpha_i \beta_i}{argmin} \sum_{t=1}^{n} L(\frac{R_{it} - \alpha_i - \beta_i R_{Mt}}{S}),$$

where S is an estimate of $\sigma_{\epsilon i}$*, L is a symmetric loss, real value function.*

In this project, we introduce 4 different methods to estimate $\begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix}$.

3.2.1 Least Median of Squares

This method lessens the influence of outliers (Marcel, 2017).

$$\begin{bmatrix} \hat{\alpha}_{i,Med} \\ \hat{\beta}_{j,Med} \end{bmatrix} = \underset{\alpha_i \beta_i}{argmin} \underset{t=1,2,\cdots,n}{Median} (\frac{R_{it} - \alpha_i - \beta_i R_{Mt}}{S}).$$

3.2.2 Least Trimmed Squares

We set values smaller than the $\alpha-$quantile to that of the $\alpha-$quantile in left tail. We also set values larger than the $(1-\alpha)-$quantile to that of the $(1-\alpha)-$quantile in the right tail (Marcel, 2017).
Def 2.

$$L(\epsilon) = \begin{cases} q_{\alpha}^2 & \textit{if } \epsilon \leq q_{\alpha} \\ \epsilon \leq q_{\alpha} & \textit{if } q_{\alpha} < \epsilon < q_{1-\alpha} \\ q_{1-\alpha}^2 & \textit{if } \epsilon \geq q_{1-\alpha} \end{cases}$$

$$\begin{bmatrix} \hat{\alpha}_{i,LTS} \\ \hat{\beta}_{j,LTS} \end{bmatrix} = \underset{\alpha_i \beta_i}{argmin} \sum_{t=1}^{n} L(R_{it} - \alpha_i - \beta_i R_{Mt}).$$

3.2.3 Constant beta Shrinkage Estimation

It can be applied for many asserts and few observations, it can exploit information across $\beta' s$ (Marcel, 2017).

Assume there are N assets, for each asset run OLS regression for $\hat{\beta}_{i,OLS}$, we get

$$\hat{\boldsymbol{\beta}} = \begin{bmatrix} \hat{\beta}_{1,OLS} \\ \vdots \\ \hat{\beta}_{N,OLS} \end{bmatrix}.$$

Shrinkage Technique: James-Stein Estimator

$$\hat{\beta}_{i,JS} = \bar{\beta} + \alpha_i(\hat{\beta}_{i,OLS} - \bar{\beta}) = (1-\alpha_i)\bar{\beta} + \alpha_i \hat{\beta}_{i,OLS},$$

where $\bar{\beta}$ is the mean of $\hat{\beta}_{i,OLS}$, $i = 1, 2, \cdots, N$.

In this project, we simply use $\alpha_i = \frac{2}{3}$.

3.2.4 Exponential Weighted Moving Average

Exponential Smoothing

Problems: β may be time varying (Marcel, 2017).

1. Questionable stationarity assumption.
2. Firms may change
    - Mergers & Acquisition

- Change leverage
- Old data points may not be as reliable as recent data points.

We use EWMA to fix these problems (Marcel, 2017).

- Covariance estimate:

$$\begin{aligned}
\sigma_{i,M,t} &= (1-\lambda)\sum_{j=1}^{\infty}\lambda^{j-1}R_{i,t-j}R_{M,t-j} \\
&= (1-\lambda)[R_{i,t-1}R_{M,t-1} + \sum_{j=2}^{\infty}\lambda^{j-1}R_{i,t-j}R_{M,t-j}] \\
&= (1-\lambda)[R_{i,t-1}R_{M,t-1} + \lambda\sum_{j=2}^{\infty}\lambda^{j-2}R_{i,t-j}R_{M,t-j}] \\
&= (1-\lambda)R_{i,t-1}R_{M,t-1} + (1-\lambda)\lambda\sum_{j=1}^{\infty}\lambda^{j-1}R_{i,t-j-1}R_{M,t-j-1} \\
&= (1-\lambda)R_{i,t}R_{M,t} + \lambda\sigma_{i,M,t-1}.
\end{aligned}$$

- Variance estimate:

$$\sigma^2_{M,t} = (1-\lambda)\sum_{j=1}^{\infty}\lambda^{j-1}R^2_{M,t-j} = (1-\lambda)R^2_{M,t} + \lambda\sigma^2_{M,t-1}.$$

Standard value:

- Monthly data: $\lambda = 0.97$.
- Daily data: $\lambda = 0.94$.

3.3 Factor Model

Def 3. *A factor model for excess return is*

$$R_{jt} = \beta_{0j} + \beta_{1j}F_{1t} + \beta_{2j}F_{2t} + \cdots + \beta_{pj}F_{pt} + \epsilon_{jt},$$

where R_{jt} is the return (or excess return) on asset j at time t.

$F_{1t}, \cdots, F_{pt}$ are variables called factors that represent the state of the financial world of time t. $\epsilon_{jt} \sim WN(0, \sigma_{\epsilon j})$ are the uncorrelated unique risks of the individual assets (Ruppert, 2004).

French and Fama model is a special 3 factor model, it's also an extension of CAPM. It has three factors, which are

- Return on Market portfolio $R_{Mt} - \mu_{ft}$,
- Small minus large (SML), which measures difference in returns on a portfolio of small cap stocks & a portfolio of large cap stocks,

- High minus low (HML), which measures difference between returns on a portfolio of high book-to-market value stocks & a low book-to-market values stocks (Marcel, 2017).

Def 4. *Book value is the new worth of a firm according to its accounting (Marcel, 2017).*

Let

$$\boldsymbol{F_t} = \begin{bmatrix} F_{it} \\ \vdots \\ F_{2t} \end{bmatrix}$$

be factors,

$$\Sigma_F$$

be covariance matrix for factors $\boldsymbol{F_t}$. Also let

$$\boldsymbol{\beta_0} = \begin{bmatrix} \beta_{01} \\ \vdots \\ \beta_{0N} \end{bmatrix}$$

be intercept vector,

$$\boldsymbol{\beta} = \begin{bmatrix} \beta_{11} & \cdots & \beta_{1N} \\ \vdots & \ddots & \vdots \\ \beta_{p1} & \cdots & \beta_{pN} \end{bmatrix}$$

be loadings,

$$\boldsymbol{\epsilon_t} = \begin{bmatrix} \epsilon_{1t} \\ \vdots \\ \epsilon_{Nt} \end{bmatrix}$$

be residual vector,

$$\Sigma_\epsilon = \begin{bmatrix} \sigma^2_{\epsilon 1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sigma^2_{\epsilon N} \end{bmatrix}$$

be covariance matrix of residual, and

$$\boldsymbol{R_t} = \begin{bmatrix} R_{1t} \\ \vdots \\ R_{Nt} \end{bmatrix}$$

be return vector (Marcel, 2017).

Hence we have

$$Cov(\boldsymbol{R_t}) = \beta^T \Sigma_F \beta + \Sigma_\epsilon.$$

If we rewrite $\boldsymbol{\beta} = [\beta_1, \cdots, \beta_N]$, then

$$Var(R_{jt}) = \beta_j^T \Sigma_F \beta_j + \sigma^2_{\epsilon j}$$

be the entry jj of $Cov(\boldsymbol{R_t})$.

We also have

$$Cov(R_{it}, R_{jt}) = \beta_i^T \Sigma_F \beta_j.$$

Then we can simply apply multiply regression method to get parameters .

Chapter 4

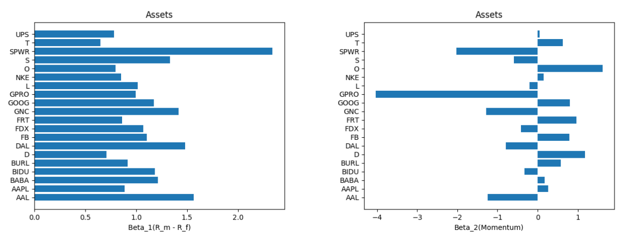
Implementation

We use the data from 3/1/2016 to 12/31/2016 as our training data to train our models, and use the data from 1/1/2017 to 10/31/2017 as our test data to test. We choose 1 year daily treasury yield curve rates with start date 3/1/2016 as risk free rate for constructing models, we choose Bill bond as our risk free asset with 1 year risk free rate 0.002 for testing.

The factors we selected are Momentum and China_US_Exchange. Momentum[1] is the rate of acceleration of a security's price or volume. It is always considered as a factor to analysis the trend line. In finance it can be treated as a measure of how fast an asset's price changing. And it is also availiable on French and Fama website, so we condiered it as one of the suitable factor to choose. Another factor we choose is the China to US exchange rate from Bloomberg[2]. It is also reasonable because we think exchange rate also can reflect the economics and the stock market. So we take the China to US exchange rate to see how it works in our model.

4.1 Construct Factor Models

- An extension of CAPM that includes factor momentum. The parameters β_1 and β_2 are

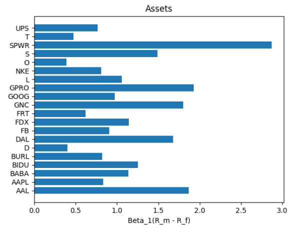


(A) β_1. (B) β_2.

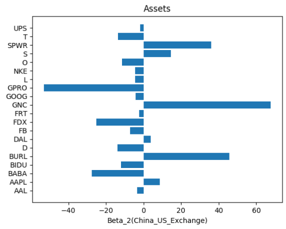
FIGURE 4.1: CAPM plus factor momentum.

- An extension of CAPM that includes factor China_US_Exchange. The parameters β_1 and β_2 are

[1] **https://www.investopedia.com/terms/m/momentum.asp?lgl=rira-layout**.
[2] **Downloan data from Financial Mathematics labotory at Worcester polytechnic Institute** .



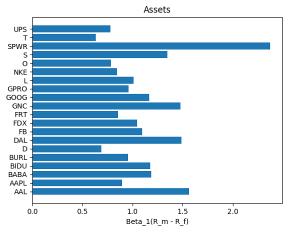
(A) β_1.



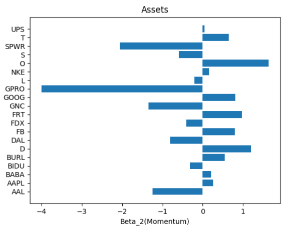
(B) β_2.

FIGURE 4.2: CAPM plus factor China_US_Exchange.

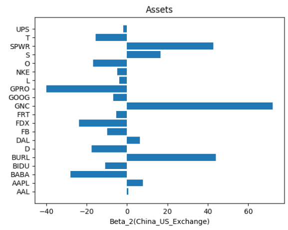
- An extension of CAPM that includes factors momentum and China_US_Exchange. The parameters β_1 , β_2 and β_3 are



(A) β_1.



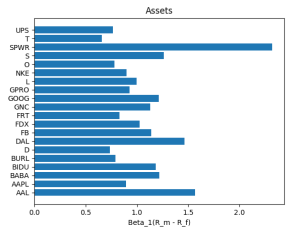
(B) β_2.



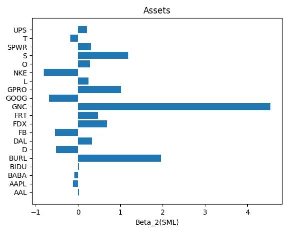
(C) β_3.

FIGURE 4.3: CAPM plus factors momentum and China_US_Exchange.

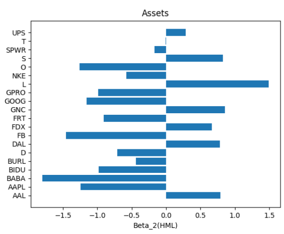
- An extension of French and Fama that includes factor momentum. The parameters β_1 , β_2, β_3 and β_4 are



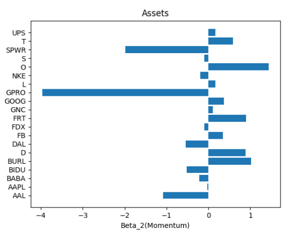
(A) β_1.



(B) β_2.



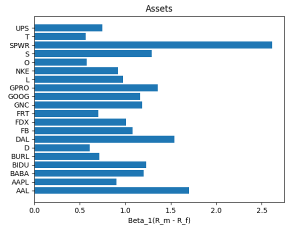
(C) β_3.



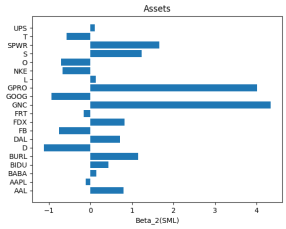
(D) β_4.

FIGURE 4.4: French and Fama plus factors momentum.

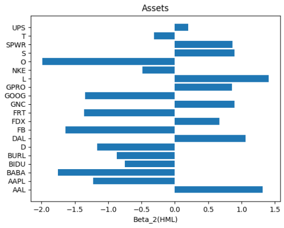
- An extension of French and Fama that includes factor China_US_Exchange. The parameters β_1 , β_2, β_3 and β_4 are



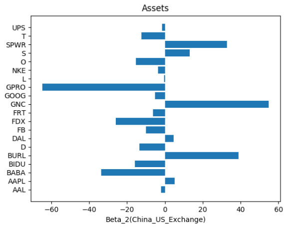
(A) β_1.



(B) β_2.



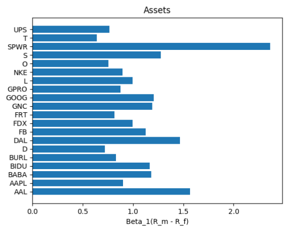
(C) β_3.



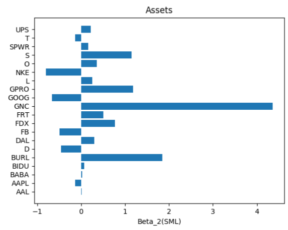
(D) β_4.

FIGURE 4.5: French and Fama plus factors China_US_Exchange.

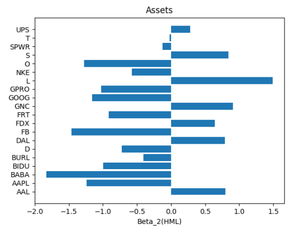
- An extension of French and Fama that includes factors momentum and China_US_Exchange. The parameters β_1 , β_2, β_3, β_4, and β_4 are



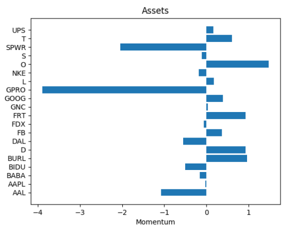
(A) β_1.



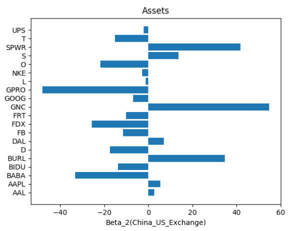
(B) β_2.



(C) β_3.



(D) β_4.



(E) β_5.

FIGURE 4.6: French and Fama plus factors momentum and China_US_Exchange.

Now, we compare the estimates of each asset's expected return and return variance across these six models. The estimates expected return as shown in figure 4.7, we can see all methods yield same results of each asset as comparing with the results simply calculated the mean values of historical data. When we first saw the results, it was surprised us. But, we noticed that it may be a reasonable results. Recall equation (3.3), for each model, if we expectation for both sides, since ϵ_{jt} is White Noise where it's expectation is zero by definition, and we made a common assumption which is

$$Cov(\epsilon_{it}, \epsilon_{jt}) = 0, \text{ if } i \neq j,$$

all models' value should equal to μ_{jt}, which is simply taking mean value of it, more precisely,

$$\mu_{jt} = (\mu_{ft} + \beta_j(\mu_{Mt} - \mu_{ft})) + 0.$$

Asset	CAPM_F_1	CAPM_F_2	CAPM_F_1_F_2	FF_F_1	FF_F_2	FF_F_1_F_2	Historical return
AAL	0.1236	0.1236	0.1236	0.1236	0.1236	0.1236	0.1236
AAPL	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653
BABA	0.2241	0.2241	0.2241	0.2241	0.2241	0.2241	0.2241
BIDU	-0.0958	-0.0958	-0.0958	-0.0958	-0.0958	-0.0958	-0.0958
BURL	0.4181	0.4181	0.4181	0.4181	0.4181	0.4181	0.4181
D	0.1349	0.1349	0.1349	0.1349	0.1349	0.1349	0.1349
DAL	0.0201	0.0201	0.0201	0.0201	0.0201	0.0201	0.0201
FB	0.0486	0.0486	0.0486	0.0486	0.0486	0.0486	0.0486
FDX	0.2968	0.2968	0.2968	0.2968	0.2968	0.2968	0.2968
FRT	-0.0389	-0.0389	-0.0389	-0.0389	-0.0389	-0.0389	-0.0389
GNC	-0.976	-0.976	-0.976	-0.976	-0.976	-0.976	-0.976
GOOG	0.0743	0.0743	0.0743	0.0743	0.0743	0.0743	0.0743
GPRO	-0.3252	-0.3252	-0.3252	-0.3252	-0.3252	-0.3252	-0.3252
L	0.2435	0.2435	0.2435	0.2435	0.2435	0.2435	0.2435
NKE	-0.2105	-0.2105	-0.2105	-0.2105	-0.2105	-0.2105	-0.2105
O	0.0095	0.0095	0.0095	0.0095	0.0095	0.0095	0.0095
S	0.9261	0.9261	0.9261	0.9261	0.9261	0.9261	0.9261
SPWR	-1.3332	-1.3332	-1.3332	-1.3332	-1.3332	-1.3332	-1.3332
T	0.1717	0.1717	0.1717	0.1717	0.1717	0.1717	0.1717
UPS	0.1827	0.1827	0.1827	0.1827	0.1827	0.1827	0.1827

FIGURE 4.7: Estimates expected return table.

From figure 4.8, we can see although there are some differences across different models, the differences are very small. For clearly to see how difference between different models, we plot a "Absolute difference" plot across models as shown in figure 4.9. We can see the maximum absolute difference value between models is less than 1, where the differences are relatively high occur in assets SPWE, S, GPRO, and GNC, others are relatively small.

Asset	CAPM_F_1	CAPM_F_2	CAPM_F_1_F_2	FF_F_1	FF_F_2	FF_F_1_F_2	Historical return
AAL	0.1177	0.1177	0.1181	0.1184	0.1185	0.1188	0.1163
AAPL	0.0354	0.0354	0.0356	0.0357	0.0357	0.0358	0.035
BABA	0.0679	0.0679	0.0682	0.0684	0.0684	0.0686	0.067
BIDU	0.0761	0.0761	0.0764	0.0767	0.0767	0.077	0.0751
BURL	0.0837	0.0837	0.084	0.0844	0.0844	0.0847	0.0825
D	0.0252	0.0252	0.0253	0.0253	0.0254	0.0254	0.0248
DAL	0.0811	0.0811	0.0814	0.0816	0.0816	0.0819	0.0802
FB	0.0378	0.0378	0.0379	0.038	0.038	0.0382	0.0373
FDX	0.0411	0.0411	0.0412	0.0414	0.0413	0.0415	0.0406
FRT	0.026	0.0261	0.0261	0.0262	0.0262	0.0263	0.0257
GNC	0.4372	0.4373	0.4392	0.4409	0.4408	0.4428	0.4311
GOOG	0.0258	0.0258	0.0259	0.0259	0.0259	0.026	0.0255
GPRO	0.3303	0.3308	0.3315	0.3327	0.3334	0.3339	0.3263
L	0.0232	0.0232	0.0233	0.0233	0.0233	0.0233	0.023
NKE	0.0368	0.0368	0.037	0.0371	0.0371	0.0373	0.0363
O	0.0487	0.0488	0.0489	0.0491	0.0492	0.0493	0.0481
S	0.2484	0.2484	0.2496	0.2507	0.2507	0.2518	0.2449
SPWR	0.4321	0.4323	0.4338	0.4355	0.4357	0.4372	0.4266
T	0.0181	0.0181	0.0182	0.0182	0.0183	0.0183	0.0179
UPS	0.0117	0.0117	0.0117	0.0117	0.0117	0.0118	0.0116

FIGURE 4.8: Estimates return variance table.

Then, we use matrix norm 2 to compare the covariance matrix across models, the table results is shown in figure 4.10. We can see, model containing factor China_US_Exchange usually has large covariance matrix difference when comparing with other model

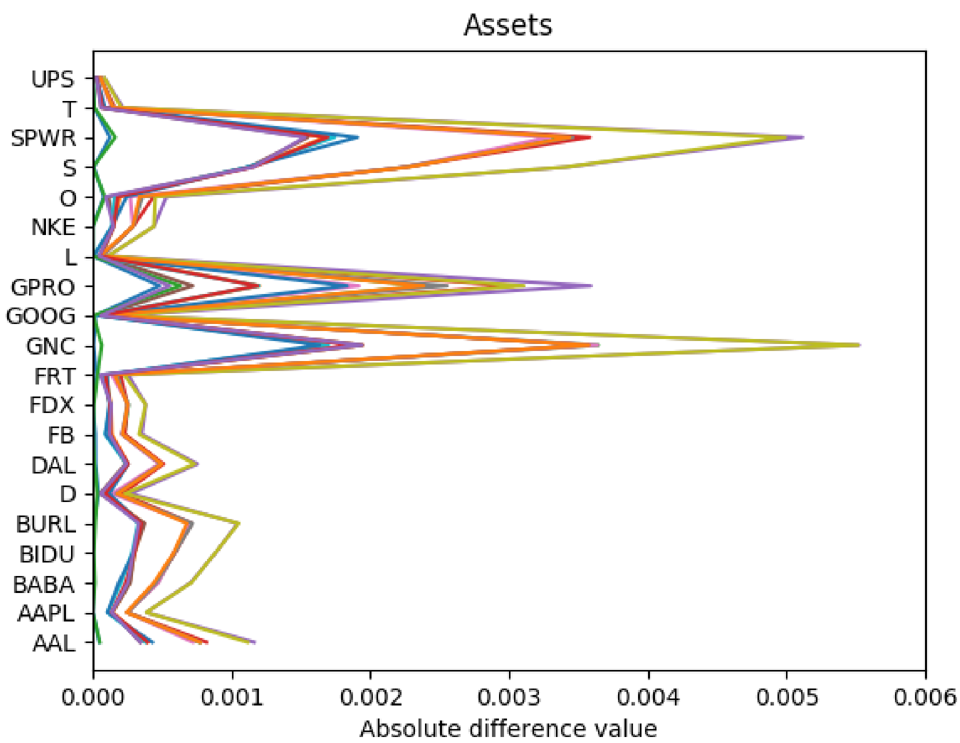


FIGURE 4.9: Difference across models.

not containing this factor. Also, looking at the last column, we can see that the covariance matrix difference between the model by simply using historical data to calculate those variance and other models are around 0.88, especially when comparing with models contain factor China_US_Exchange, the differences increase a little to 0.99 or 0.97.

Values	CAPM_F_1	CAPM_F_2	CAPM_F_1_F_2	FF_F_1	FF_F_2	FF_F_1_F_2	Historical return
CAPM_F_1	0.0	0.076	0.0107	0.0212	0.0412	0.024	0.0902
CAPM_F_2	0.076	0.0	0.0765	0.0818	0.0565	0.0823	0.0995
CAPM_F_1_F_2	0.0107	0.0765	0.0	0.0213	0.0411	0.0217	0.0884
FF_F_1	0.0212	0.0818	0.0213	0.0	0.0414	0.0096	0.0884
FF_F_2	0.0412	0.0565	0.0411	0.0414	0.0	0.0411	0.0973
FF_F_1_F_2	0.024	0.0823	0.0217	0.0096	0.0411	0.0	0.0882
Historical return	0.0902	0.0995	0.0884	0.0884	0.0973	0.0882	0.0

FIGURE 4.10: Covariance matrix comparison across models by using matrix norm 2.

4.2 Portfolio Optimization

We use these expected return and covariance estimates from two of the above models to form optimal tangency portfolio as of January 1,2017 with a 10 month holding period. Also, we use only historical asset returns to estimate parameters to form tangency portfolio. In the hold period, we won't re-balance our portfolio. We choose CAPM containing factor momentum model and French and Fama containing factors momentum and Chian_US_Exchange model to form optimal tangency portfolio.

4.2.1 Estimate return, variance and covariance comparing with actual return, variance and covariance.

For the 20 assets, we compute the actual return and return variances and covariances for the holding period of January 1, 2017 to October 31, 2017.

Return comparison

From figure 4.11 we can see the difference between estimate expected return and actual expected return of each aseets are a little bit large. The most two large difference assets are SPWR and S, which are corresponding with high variances in the figure 4.8.

Assets	Model return	Actual return
AAL	0.1236	0.0098
AAPL	0.1653	0.4184
BABA	0.2241	0.7978
BIDU	-0.0958	0.4227
BURL	0.4181	0.1097
D	0.1349	0.0925
DAL	0.0201	0.0337
FB	0.0486	0.4799
FDX	0.2968	0.2139
FRT	-0.0389	-0.1512
GNC	-0.976	-0.5129
GOOG	0.0743	0.2952
GPRO	-0.3252	0.1931
L	0.2435	0.0639
NKE	-0.2105	0.0949
O	0.0095	-0.0343
S	0.9261	-0.2707
SPWR	-1.3332	0.0796
T	0.1717	-0.1986
UPS	0.1827	0.0513

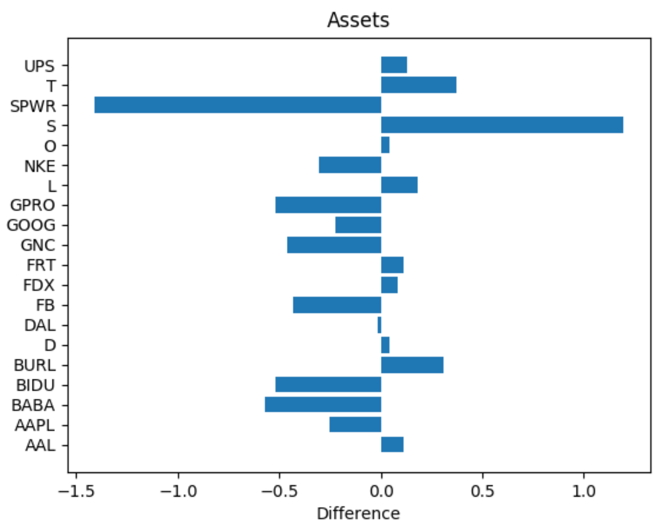


FIGURE 4.11: Return comparison.

Variance comparison

From the variance table in figure 4.13, we can see the difference among models of each assets are very small comparing the scale of actual return variance. So we simply use one model data to plot the return variance between asset return variance generated by models and actual return variance as shown on the bottom in the figure 4.13. Some asset's variance between two methods are very small like GNC and

NKE, but some are very high like SPWR and GPRO. So, it indicates that by using training data to estimate assets variances is not totally reliable when facing different assets.

Covariance comparison

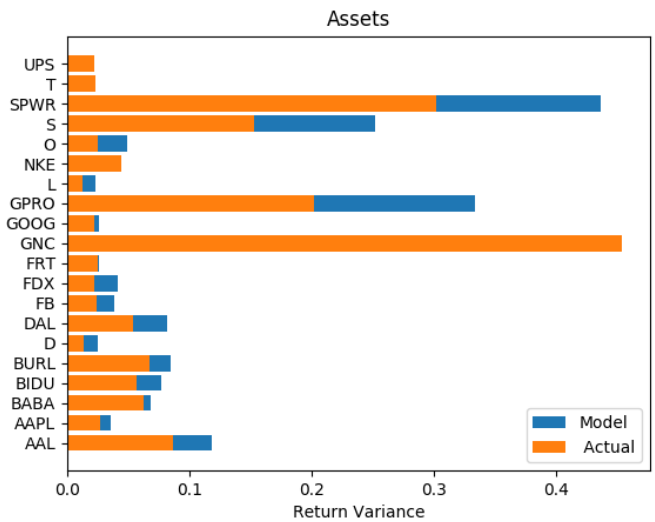
From figure 4.12 and figure 4.10, we can see the three covariance matrix generated by three models are different from actual testing data's covariance matrix.

Models	CAPM	FF	Historical return variance
Actual	0.2296	0.2324	0.2664

FIGURE 4.12: Covariance comparison by using matrix norm 2.

Values	CAPM return variance	FF return variance	Historical return variance	Actual return variance
AAL	0.1177	0.1188	0.1163	0.0863
AAPL	0.0354	0.0358	0.035	0.0274
BABA	0.0679	0.0686	0.067	0.063
BIDU	0.0761	0.077	0.0751	0.0571
BURL	0.0837	0.0847	0.0825	0.0676
D	0.0252	0.0254	0.0248	0.0138
DAL	0.0811	0.0819	0.0802	0.0534
FB	0.0378	0.0382	0.0373	0.0236
FDX	0.0411	0.0415	0.0406	0.0226
FRT	0.026	0.0263	0.0257	0.0248
GNC	0.4372	0.4428	0.4311	0.4546
GOOG	0.0258	0.026	0.0255	0.0223
GPRO	0.3303	0.3339	0.3263	0.2025
L	0.0232	0.0233	0.023	0.0122
NKE	0.0368	0.0373	0.0363	0.0445
O	0.0487	0.0493	0.0481	0.0249
S	0.2484	0.2518	0.2449	0.1526
SPWR	0.4321	0.4372	0.4266	0.302
T	0.0181	0.0183	0.0179	0.0234
UPS	0.0117	0.0118	0.0116	0.0221

(A) Table



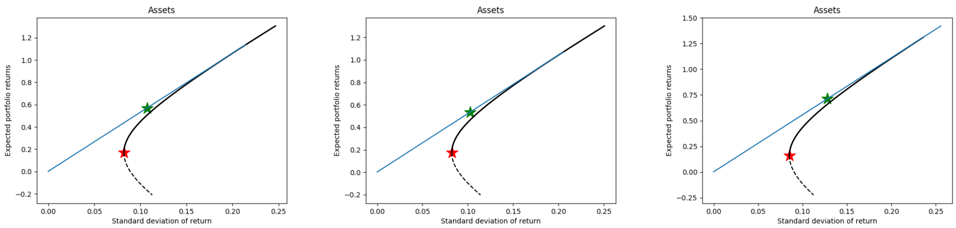
(B) Plot

FIGURE 4.13: Variance comparison.

4.2.2 Portfolio Performance

We assume our initial cash value is $ 1000000, when construct tangency portfolio, we but $ 500000 risk free assets, and other money to construct risk portfolio.

First, we plot three efficient frontiers as well as tangency portfolios. The results as shown in figure 4.14. We can see all expected return of three models, which indicated by green star in plots, are above 0.5 which is a big number. It's may because our 20 assets historical expected returns exist some very high values as shown in figure 4.15.



(A) Efficient frontier with CAPM model. (B) Efficient frontier with French and Fama model. (C) Efficient frontier with historical data.

FIGURE 4.14: Three efficient frontiers and tangency portfolios.

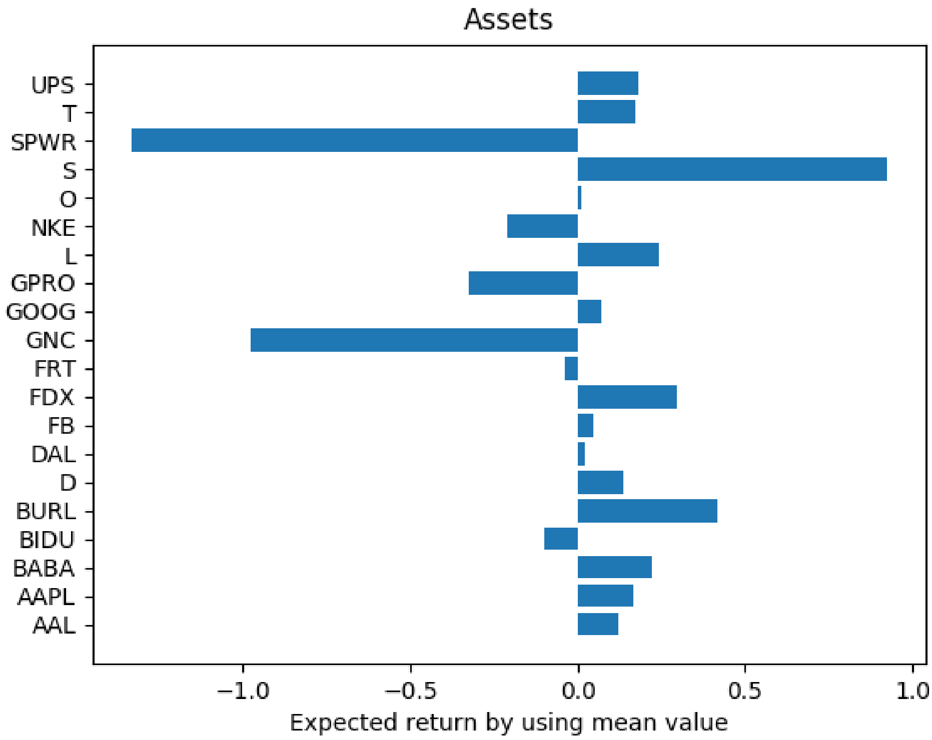
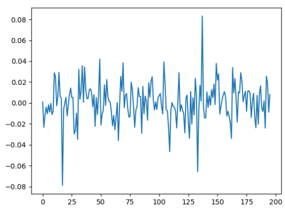


FIGURE 4.15: Expected return of all assets.

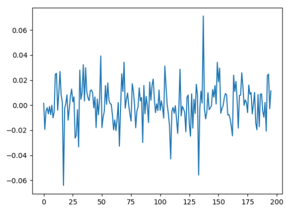
Then we compare the actual return with three methods, we can see from figure 4.16 that historical method gives us a better return. Furthermore, we plot the portfolio daily return of each model and also the variances of each model in figure 4.17 and figure 4.18. Those show the variances are small of each portfolio generated by three methods, and the pattern are same across models.

Models	CAPM_F_1_F_2	FF_F_1_F_2	Historical
Actual return	0.1086	0.2336	0.5302
Estimate return	0.5671	0.5339	0.7094

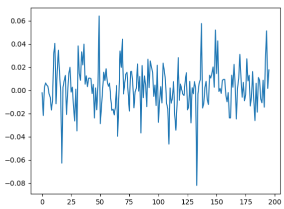
FIGURE 4.16: Actual return and estimate return.



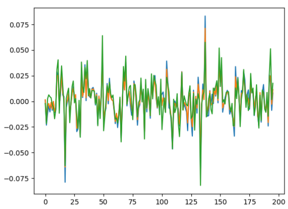
(A) Portfolio daily return by CAPM.



(B) Portfolio daily return by French and Fama model.



(C) Portfolio daily return by historical method.



(D) Portfolio daily return by three methods.

FIGURE 4.17: Portfolio daily return.

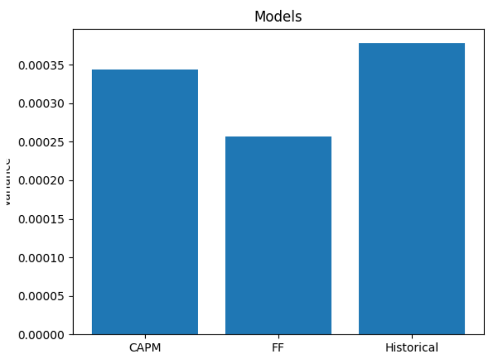
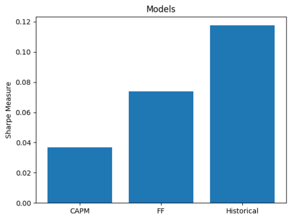


FIGURE 4.18: Portfolio return variance.

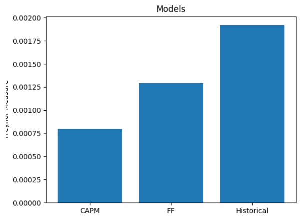
Moreover, we use 4 methods to evaluate our portfolio performance. From figure 4.19, we can see the historical method give us best performance, if exclude historical method, then French and Fama with factor momentum and China_US_Exchange give a better performance than CAPM with factor momentum.

Models	CAPM_F_1	FF_F_1_F_2	Historical
Sharpe Measure	0.0366	0.0737	0.1175
Treynor Measure	0.0008	0.0013	0.0019
Information Ratio	-0.0014	0.0308	0.0843
Sortino Ratio	0.0524	0.1085	0.1773

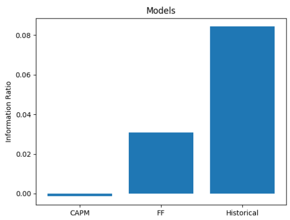
(A) All methods.



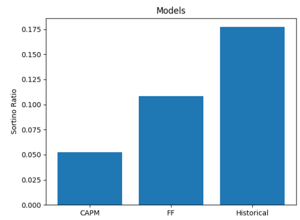
(B) Sharpe Measure.



(C) Treynor Measure.



(D) Information Ratio.



(E) Sortino Ratio.

FIGURE 4.19: Evaluate performance.

4.2.3 Model conclusion

Bese on the results as shown in above, we can conclude that historical method is better than other two models, whatever comparing by real portfolio return, portfolio return variance or four evaluation methods. But, the conclusion maybe vary when we change our assets selection, choose another models from the six models we constructed before, change our factors rather than momentum and China_US_Exchange, or even just simply change data size.

If just according the analysis as above, we won't change our mind to just use historical method to forming portfolio in the prior project that uses Interactive Brokers.

4.3 Quantile-Based Portfolio Back-testing

We use Sharpe ratio to rank 20 assets from best to worst.

4.3.1 CAPM with factor momentum

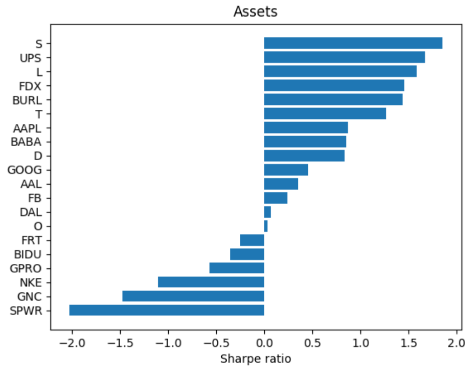


FIGURE 4.20: Sharpe Ratio of 20 assets.

We noticed that the first three assets are S, UPS and L. The last three assets are SPWR, GNC and NKE. we will use the six assets to construct our portfolio.

We form a portfolio with a 10 month holding period of January 1, 2016 to October 31, 2016 that consist of

- Equally weighted ω_t long positions in the S, UPS and L.
- Equally weighted ω_b short positions in the SPWR, GNC and NKE.

Then we have

$$3 * \omega_t + 3 * \omega_b = 1, \quad \omega_b <= 0, \omega_t >= 0$$

$\Leftrightarrow$

$$\omega_t + \omega_b = \frac{1}{3}, \quad \omega_b <= 0, \omega_t >= 0,$$

$\Leftrightarrow$

$$\omega_t \geq \frac{1}{3}, \omega_b \leq 0.$$

where ω_b and ω_t cannot be zero synchronously.

Assume $\Delta X_1, \Delta X_2, \Delta X_3$ be the asset price differences of assets S, UPS and L between time January 1, 2016 and October 31, 2016, more precisely, they are the asset price at time October 31, 2016 minus the asset price at time January 1, 2016. Similarly, assume $\Delta Y_1, \Delta Y_2, \Delta Y_3$ be the asset price differences of assets SPWR, GNC and NKE between time January 1, 2016 and October 31, 2016. Then our portfolio return R_p is

$$R_p = 3 * \omega_t * (\Delta X_1 + \Delta X_2 + \Delta X_3) + 3 * \omega_b * (\Delta Y_1 + \Delta Y_2 + \Delta Y_3)$$

$\Leftrightarrow$

$$R_p = 1 + 3\omega_t(\Delta X_1 + \Delta X_2 + \Delta X_3 - \Delta Y_1 - \Delta Y_2 - \Delta Y_3)$$

Since $k = \Delta X_1 + \Delta X_2 + \Delta X_3 - \Delta Y_1 - \Delta Y_2 - \Delta Y_3$ is a constant, R_p is just a proportional function. If $K >= 0$, then the range of R_p is

$$[1 + k, +\infty),$$

if $K <= 0$, then the maximum value of R_p is

$$(-\infty, 1 + k].$$

So each model with same variance and covariance which are same as the testing data parameters. The portfolio daily return is

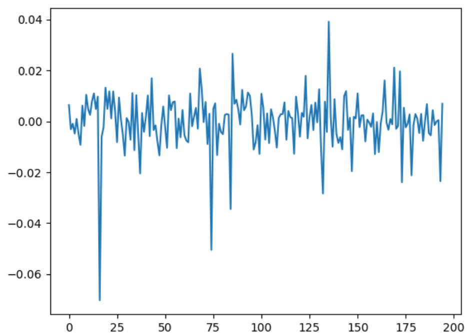


FIGURE 4.21: Portfolio Daily return.

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