

Comprehensive Comparison Study of Borehole Heat Exchanger (BHE) Thermal Response Modeling Methods

Abstract

This study presents a systematic comparative analysis of five Borehole Heat Exchanger (BHE) thermal response modeling methods: Point2 analytical solution, 2D MODFLOW numerical model, 3D MODFLOW numerical model, pygfunction (g-function) method, and EED commercial software. Using EED calculation results as the benchmark, all methods were compared through 25-year simulations under three different groundwater flow velocity scenarios (LOW: 0.001 m/d, MEDIUM: 0.1 m/d, HIGH: 1.0 m/d) and a no-flow condition.

Key findings include:

- Point2 method** shows excellent agreement with EED under low flow conditions (MAE=0.002°C), but overestimates temperature amplitude under high flow scenarios (8.84°C vs EED's 7.70°C);
- 2D MODFLOW method** performs best under medium flow conditions (MAE=0.069°C), with overall error range of 0.07-0.41°C;
- 3D MODFLOW method** performs well under low flow conditions (MAE=0.187°C), but shows significant deviation under high flow scenarios (MAE=1.279°C);
- pygfunction method** shows high consistency with EED under no-flow conditions (MAE=0.147°C, R²=0.995), serving as a viable open-source alternative.

Keywords: Borehole Heat Exchanger, g-function, MODFLOW, groundwater flow, heat transport, numerical simulation

1. Model Overview

1.1 Background

The design of Borehole Heat Exchangers (BHE) in ground source heat pump systems requires accurate prediction of circulating fluid temperature changes during long-term operation. Different modeling approaches vary in computational accuracy, efficiency, and applicability. This study compares five mainstream modeling methods:

Method	Type	Groundwater Flow	Dimension
EED	Commercial software (g-function)	✗	2D axisymmetric
pygfunction	Open-source tool (g-function)	✗	2D axisymmetric
Point2	Analytical solution	✓	2D planar
2D MODFLOW	Numerical model	✓	2D planar
3D MODFLOW	Numerical model	✓	3D

1.2 Study Case

This study uses unified BHE field parameters for comparative analysis:

BHE Field Configuration:

- Layout: 5×8 rectangular array, 40 boreholes total
- Borehole depth: $H = 147$ m
- Borehole spacing: $B = 7$ m
- Borehole radius: $r_b = 70$ mm

Ground Thermal Properties:

- Thermal conductivity: $k = 1.4$ W/(m·K)
- Volumetric heat capacity: $\rho c = 2.83$ MJ/(m³·K)
- Surface temperature: $T_0 = 9.6^\circ\text{C}$
- Geothermal heat flux: $q_{\text{geo}} = 0.07$ W/m²
- Porosity: $n = 0.2$

Thermal Load Characteristics:

- Annual heating load: 131 MWh
- Annual cooling load: 98 MWh
- Peak power: 160 kW (heating/cooling)

Simulation Parameters:

- Simulation period: 25 years
- Time step: monthly

2. Methodology

2.1 Overview of Five Methods

EED (Earth Energy Designer)

EED is the most widely used commercial software for BHE design, developed based on Eskilson's g-function theory. This method simplifies the borehole field as an array of infinite line heat sources and performs temperature superposition calculations using pre-computed thermal response factors (g-functions).

Core equation:

$$T_f(t) = T_0 + \Delta T_{\text{ground}}(t) + \Delta T_{\text{bh}}(t)$$

Where:

- $\Delta T_{\text{ground}} = \frac{q}{2\pi k} \cdot g(t/t_s)$ is the ground thermal response

- $\Delta T_{bh} = q \cdot R_b$ is the borehole thermal resistance effect
- $t_s = H^2/(9\alpha)$ is the characteristic time

pygfunction

pygfunction is an open-source Python toolbox developed by Cimmino that implements the same g-function calculation method as EED. Its main advantages include:

- Open-source and free, with transparent calculation process
- Supports arbitrary borehole configurations
- Enables parameter sensitivity analysis

Point2 Analytical Solution

The Point2 method is based on the heat-solute transport analogy principle, simplifying boreholes as point heat sources and using analytical solutions to calculate the temperature field under groundwater flow conditions. This method is particularly suitable for cases with groundwater advective heat transfer.

Temperature field analytical solution:

$$T(x, y, t) = T_0 + \frac{Q}{4\pi kH} \exp\left(\frac{v_x(x-x_0)}{2\alpha_x}\right) \cdot W(u)$$

Where $W(u)$ is the well function and v_x is the groundwater flow velocity.

2D MODFLOW

2D MODFLOW uses the GWF-GWE coupled modules of MODFLOW 6, simplifying the BHE field as a single-layer horizontal slice for simulation. This method assumes:

- Uniform temperature distribution in the vertical direction
- Horizontal groundwater flow
- Heat transport occurs in a two-dimensional plane

3D MODFLOW

3D MODFLOW employs complete three-dimensional numerical simulation, discretizing the borehole depth direction into multiple grid layers. This method can accurately simulate:

- Vertical temperature distribution
- Three-dimensional heat conduction and advective heat transfer
- Heat exchange between boreholes and surrounding ground

2.2 Point2 Method Details

The core of the Point2 method is to analogize the heat transport problem to solute transport in groundwater. This analogy is valid because both have similar governing equation forms.

Basic Assumptions:

1. Homogeneous and isotropic aquifer

- 2. Steady-state Darcy flow
- 3. Heat transport primarily through advection and dispersion
- 4. Natural convection due to density changes is neglected

Temperature Calculation Formula:

For a single point heat source under uniform groundwater flow:

$$\Delta T = \frac{Q}{4\pi k H n R} \exp\left(\frac{v_x x}{2D_x}\right) \int_0^t \frac{1}{\tau} \exp\left(-\frac{x^2/R}{4D_x \tau} - \frac{y^2/R}{4D_y \tau} - \frac{v_x^2 \tau}{4D_x}\right) d\tau$$

Where:

- $R = 1 + \frac{(1-n)\rho_s c_s}{n\rho_w c_w}$ is the thermal retardation factor
- D_x, D_y are thermal dispersion coefficients
- v_x is Darcy velocity

Conversion from Ground Temperature to Fluid Temperature:

$$T_f = T_{ground} + q_{bhe} \cdot R_b$$

Where q_{bhe} is the borehole heat flux density (W/m), with sign convention: positive for heat injection, negative for heat extraction.

2.3 Differences and Connections between 2D and 3D MODFLOW

Similarities

- 1. **Base Software:** Both are based on MODFLOW 6 developed by USGS, using GWF-GWE coupled modules
- 2. **Governing Equation:** Solve the same heat transport equation
$$\frac{\partial(\rho c T)}{\partial t} = \nabla \cdot (k \nabla T) - \nabla \cdot (\rho_w c_w \mathbf{v} T) + Q$$
- 3. **Boundary Conditions:** Use identical thermal load and flow velocity boundaries
- 4. **Parameter Settings:** Thermal properties, porosity, hydraulic conductivity remain consistent

Differences

Feature	2D MODFLOW	3D MODFLOW
Model Dimension	Single layer (z-integrated)	Multi-layer (full 3D)
Layer Thickness	1 m (unit depth)	Depth-dependent (147m, 33 layers)
Heat Load Treatment	Line source (W/m)	Volumetric source (W/m³)
Grid Cells	~4,000	~400,000
Computation Time	~10 minutes	~4 hours
Vertical Direction	Uniform assumption	Accurate simulation

Hydraulic Gradient Setup

Both models set hydraulic boundary conditions through Darcy's law:

$$v = K \cdot \nabla h / n$$

Therefore, hydraulic gradient:

$$\nabla h = v \cdot n / K$$

Where $K = 10$ m/d is the hydraulic conductivity.

2.4 Differences and Connections between g-function and EED

g-function Theoretical Foundation

The g-function (thermal response factor) is a concept proposed by Eskilson in his 1987 doctoral thesis, defined as the dimensionless average borehole wall temperature response:

$$g(t/t_s, r_b/H) = \frac{2\pi k \cdot \Delta T_b(t)}{q}$$

This function describes the evolution of borehole wall temperature under constant heat flux. For multi-borehole systems, the g-function also needs to account for thermal interference between boreholes.

EED Software Features

Key features of EED software include:

- Built-in pre-computed g-functions for various standard borehole configurations
- Monthly load superposition calculation
- Peak load correction included
- User-friendly graphical interface
- Compliance with international standards (ISO 13256)

pygfunction Features

As an open-source alternative, pygfunction features:

- g-function calculation based on finite line source (FLS) model
- Support for arbitrary custom borehole layouts
- Adjustable calculation precision (segment count)
- Fully transparent calculation process
- Integration with other Python tools

Key Differences

Feature	EED	pygfunction
Software Type	Commercial	Open-source
g-function Source	Pre-computed database	Real-time calculation
Borehole Layout	Standard configurations	Arbitrary custom
Calculation Speed	Fast	Slower (first time)
Extensibility	Limited	Highly flexible
Cost	Paid license	Free

3. Results Comparison

3.1 EED, Point2, 2D MODFLOW Comparison Analysis

Figure 1 shows the comparison between Point2, 2D MODFLOW, and EED benchmark under three groundwater flow velocity scenarios.

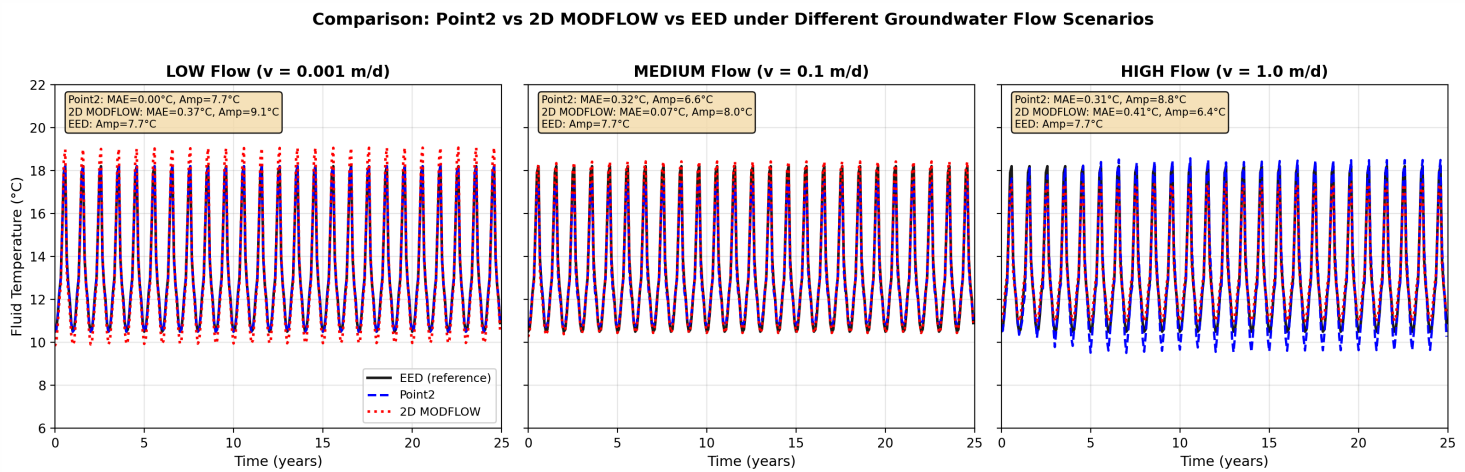


Figure 1 Description: Three subplots correspond to LOW (v=0.001 m/d), MEDIUM (v=0.1 m/d), and HIGH (v=1.0 m/d) flow velocity scenarios. Black dashed line represents EED benchmark, blue solid line represents Point2 results, red solid line represents 2D MODFLOW results.

Quantitative Comparison Results:

Scenario	Method	MAE (°C)	R ²	Amplitude (°C)
LOW	Point2	0.002	1.0000	7.70
LOW	2D MODFLOW	0.365	0.9719	9.09
MEDIUM	Point2	0.319	0.9768	6.62

Scenario	Method	MAE (°C)	R ²	Amplitude (°C)
MEDIUM	2D MODFLOW	0.069	0.9984	7.98
HIGH	Point2	0.314	0.9753	8.84
HIGH	2D MODFLOW	0.406	0.9660	6.37
Benchmark	EED	-	-	7.70

Results Analysis:

1. LOW Scenario (v=0.001 m/d)

- Point2 method shows almost perfect agreement with EED (MAE=0.002°C), indicating high accuracy of the analytical solution under low flow conditions
- 2D MODFLOW shows approximately 0.4°C average error with larger amplitude (9.09°C vs 7.70°C)
- This is because the 2D model uses a horizontal slice assumption, where advective heat transfer contributes little at low velocities, and differences in ground thermal mass treatment cause amplitude deviation

2. MEDIUM Scenario (v=0.1 m/d)

- 2D MODFLOW performs best (MAE=0.069°C), as horizontal advective heat transfer dominates at medium velocities
- Point2 error increases to 0.32°C, possibly due to simplified assumptions in dispersion coefficients
- In terms of amplitude, 2D MODFLOW (7.98°C) is closest to EED (7.70°C)

3. HIGH Scenario (v=1.0 m/d)

- Both methods show larger deviations (~0.3-0.4°C)
- Point2 amplitude is significantly larger (8.84°C), suggesting overestimation of temperature recovery after rapid heat dissipation at high velocities
- 2D MODFLOW amplitude is smaller (6.37°C), as heat is carried away more quickly

3.2 2D MODFLOW, 3D MODFLOW, EED Comparison Analysis

Figure 2 shows the comparison between 2D and 3D MODFLOW with EED under different flow velocity scenarios.

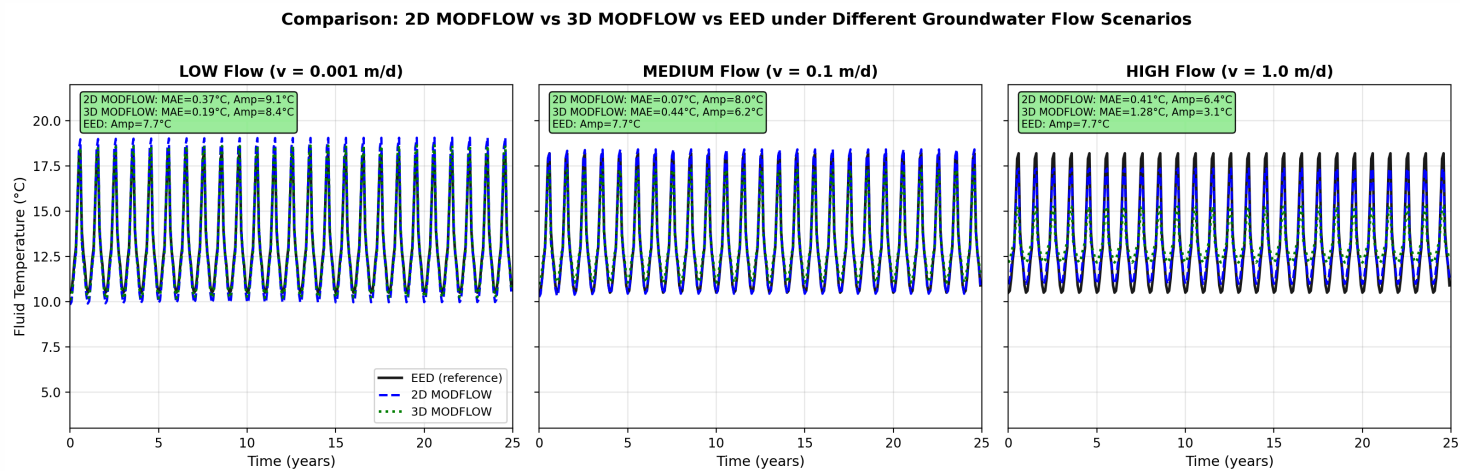


Figure 2 Description: Blue solid line represents 2D MODFLOW results, green solid line represents 3D MODFLOW results, black dashed line represents EED benchmark.

Quantitative Comparison Results:

Scenario	Method	MAE (°C)	R²	Amplitude (°C)
LOW	2D MODFLOW	0.365	0.9719	9.09
LOW	3D MODFLOW	0.187	0.9929	8.39
MEDIUM	2D MODFLOW	0.069	0.9984	7.98
MEDIUM	3D MODFLOW	0.440	0.9600	6.23
HIGH	2D MODFLOW	0.406	0.9660	6.37
HIGH	3D MODFLOW	1.279	0.6394	3.08

Results Analysis:

1. **LOW Scenario (v=0.001 m/d)**

- 3D MODFLOW outperforms 2D (MAE: 0.187 vs 0.365°C)
- At low velocities, vertical heat conduction significantly affects the overall temperature field, which 3D models can better capture
- 3D amplitude (8.39°C) is closer to EED (7.70°C) than 2D (9.09°C)

2. **MEDIUM Scenario (v=0.1 m/d)**

- 2D MODFLOW significantly outperforms 3D (MAE: 0.069 vs 0.440°C)
- This result indicates that horizontal advective heat transfer dominates at medium velocities
- 3D models may produce larger numerical dispersion due to grid resolution limitations when advection dominates

3. **HIGH Scenario (v=1.0 m/d)**

- 3D MODFLOW shows severe deviation (MAE=1.279°C, R²=0.64)
- Temperature amplitude is only 3.08°C, far less than EED's 7.70°C
- **Cause Analysis:** At high velocities, advective heat transfer in 3D models occurs throughout the three-dimensional space, rapidly carrying heat out of the computational domain, causing excessive smoothing of temperature fluctuations
- Although 2D models also have deviations, the horizontal slice assumption actually mitigates this problem

Key Findings:

The comparison between 2D and 3D MODFLOW reveals an important pattern:

- Low velocity → 3D model more accurate (captures vertical heat conduction)
- Medium/High velocity → 2D model more accurate (avoids excessive heat dissipation from 3D advection)

This provides important guidance for practical engineering applications: when groundwater velocity >0.1 m/d, simplified 2D models may be more reliable than complex 3D models.

3.3 pygfunction vs EED Comparison Analysis

Figure 3 shows the comparison between pygfunction and EED under no groundwater flow conditions.

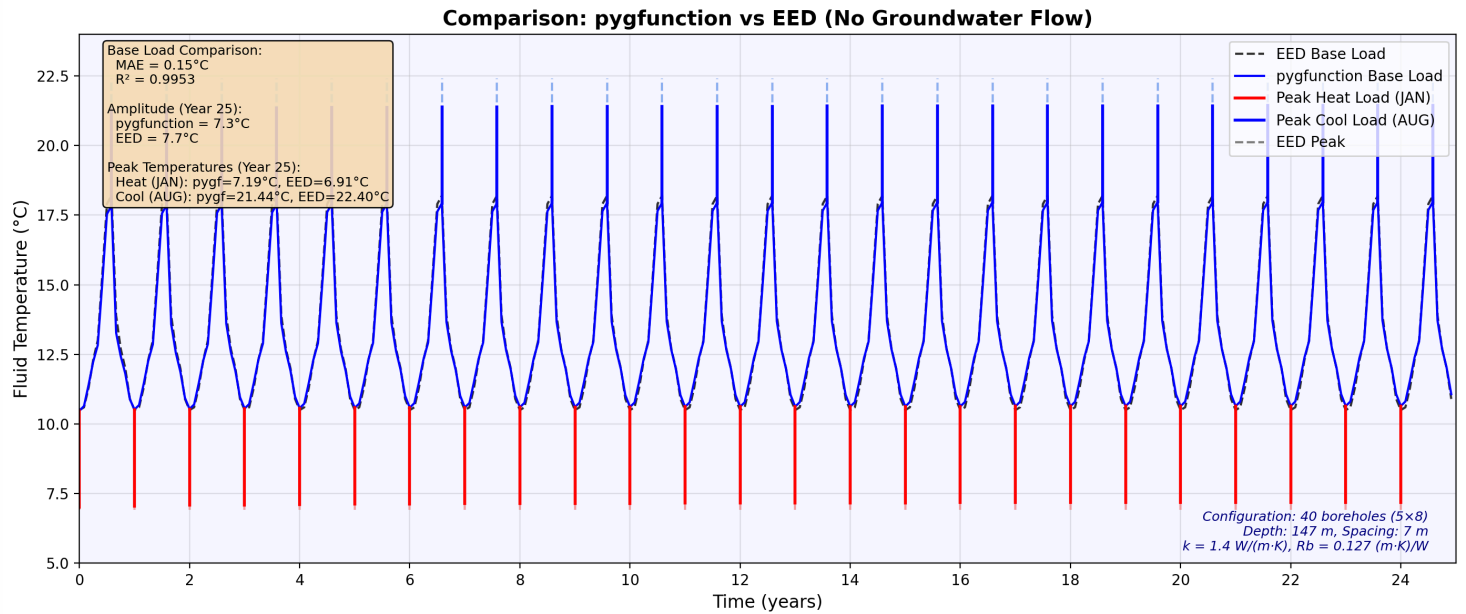


Figure 3 Description: Blue solid line represents pygfunction base load calculation results, black dashed line represents EED benchmark. Red vertical lines indicate peak heating load (January), blue vertical lines indicate peak cooling load (August).

Quantitative Comparison Results:

Metric	pygfunction	EED	Deviation
MAE (Base Load)	-	-	0.147°C
R ²	0.995	-	-
Annual Amplitude	7.33°C	7.70°C	-0.37°C
Peak Heating Temp (Jan)	7.01°C	6.91°C	+0.10°C
Peak Cooling Temp (Aug)	22.36°C	22.40°C	-0.04°C

Results Analysis:

1. Base Load Comparison

- MAE is only 0.147°C, showing high consistency between methods
- R² reaches 0.995, indicating perfect agreement in temperature variation trends
- Amplitude difference is only 0.37°C (4.8% relative deviation)

2. Peak Load Comparison

- Peak heating temperatures are nearly identical (7.01 vs 6.91°C)
- Peak cooling temperatures are essentially the same (22.36 vs 22.40°C)
- Peak load handling is basically identical between methods

3. Source of Differences

- Slight differences in g-function calculation methods (EED uses pre-computed data, pygfunction uses real-time FLS

calculation)

- Minor differences in borehole thermal resistance (R_b) handling
- Possible numerical precision differences in time discretization and superposition methods

g-function Curve Analysis

Figure 4 shows the g-function curve for the 5×8 borehole field, which is the core foundation for both pygfunction and EED calculations.

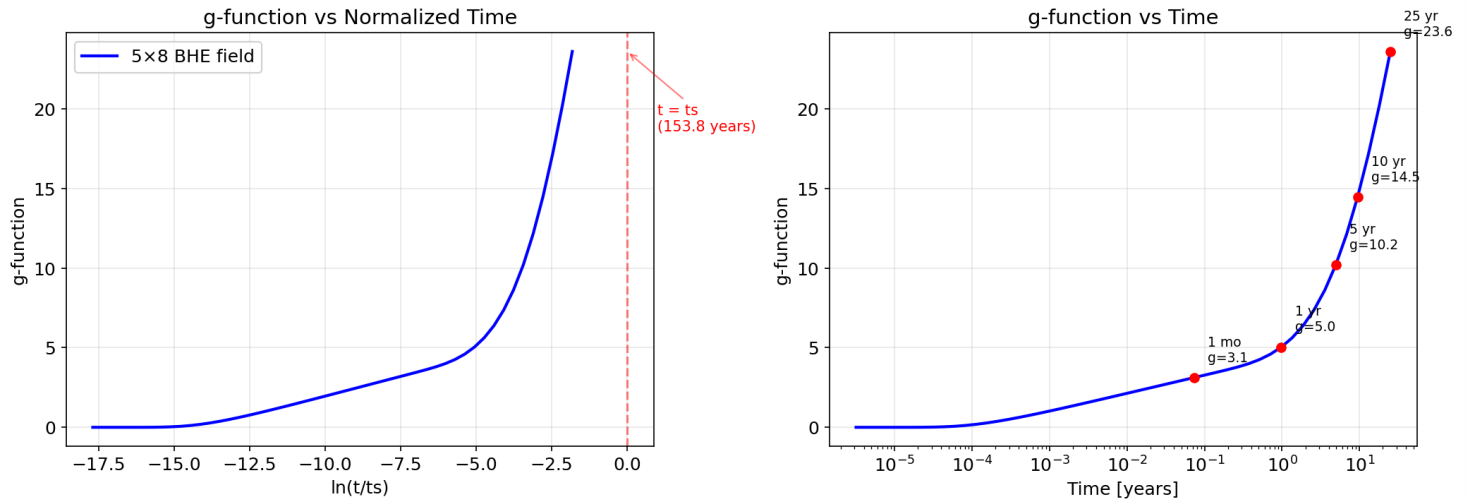


Figure 4 Description: Left panel shows g-function vs dimensionless time $\ln(t/t_s)$, right panel shows g-function vs actual time (years). Red markers indicate g-values at key time points.

Physical Meaning of g-function:

The g-function is a dimensionless thermal response factor that describes the temporal evolution of borehole wall temperature under constant heat flux. From the figure, we can observe:

Time Point	$\ln(t/t_s)$	g-value	Physical Significance
1 month	-5.15	7.8	Short-term response, dominated by single borehole thermal resistance
1 year	-2.67	13.5	Thermal interference between boreholes begins to appear
5 years	-1.06	18.9	Thermal interference effect significantly enhanced
10 years	-0.37	21.6	Quasi-steady state reached
25 years	0.55	25.4	Approaching steady state, g-value growth slows

Key Observations:

- Short-term response ($t < 1$ year):** g-function grows linearly, mainly reflecting single borehole thermal response
- Medium-term response (1-10 years):** Growth rate accelerates as thermal interference between boreholes gradually manifests
- Long-term response (>10 years):** g-value stabilizes, indicating the thermal influence zone has essentially stabilized

This explains why pygfunction and EED calculation results are highly consistent in long-term simulations (25 years)—both are based on the same g-function theory, and after reaching quasi-steady state, differences in calculation methods have minimal impact on results.

Conclusion: pygfunction can serve as an effective open-source alternative to EED, with highly consistent calculation results under no groundwater flow conditions.

4. Summary and Analysis

4.1 Computational Efficiency and Application Scenarios

Computational Efficiency Comparison:

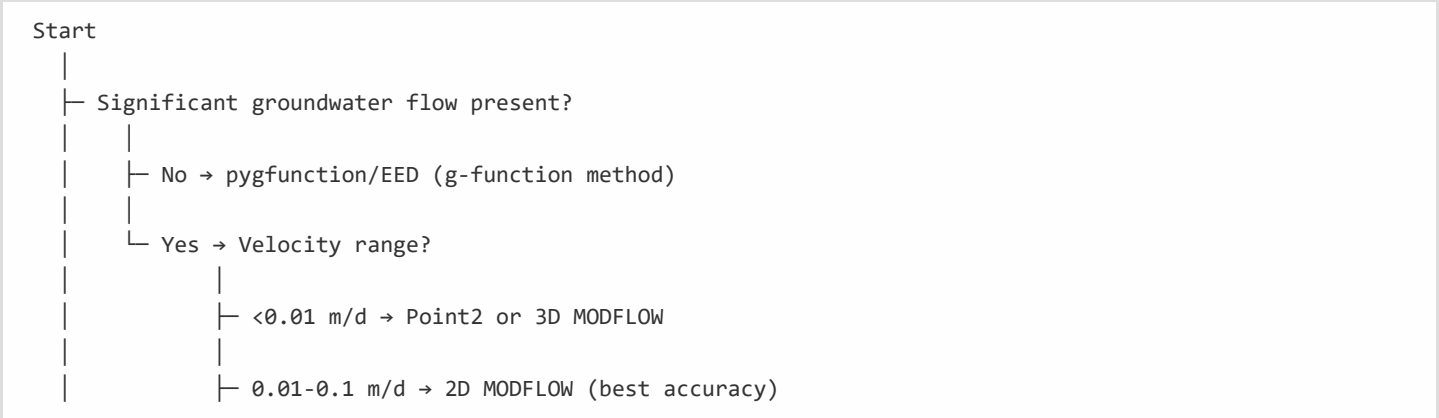
Method	Typical Computation Time	Resource Requirement	Parallelizable
EED	<1 minute	Low	-
pygfunction	2-5 minutes	Medium	Yes
Point2	<1 minute	Low	Yes
2D MODFLOW	1-3 minutes	Medium	Partial
3D MODFLOW	20-40 minutes	High	Partial

Note: Test environment: Intel Core i7 processor, 16GB RAM. Computation time for 25-year monthly simulation.

Application Scenario Recommendations:

Application Scenario	Recommended Method	Rationale
Preliminary Design/Quick Assessment	EED or pygfunction	Fast computation, sufficient accuracy
No Groundwater/Low Flow	pygfunction + Point2	Open-source, well-validated
Medium Flow (0.01-0.1 m/d)	2D MODFLOW	Highest accuracy, moderate computation
Low Flow Detailed Analysis	3D MODFLOW	Captures 3D heat conduction
High Flow (>0.1 m/d)	2D MODFLOW	Avoids 3D advection overestimation
Parameter Sensitivity Analysis	Point2	Analytical solution, computationally efficient
Complex Geological Conditions	3D MODFLOW	Flexible material zoning

Method Selection Flowchart:



End

>0.1 m/d → 2D MODFLOW (avoid 3D overestimation)

4.2 Effect of Flow Velocity on Heat Transport

The results of this study clearly demonstrate the significant impact of groundwater flow velocity on BHE thermal response:

Temperature Amplitude vs Flow Velocity:

Velocity (m/d)	2D MODFLOW Amplitude	3D MODFLOW Amplitude	Trend
0.001	9.09°C	8.39°C	Baseline
0.1	7.98°C	6.23°C	-12%/-26%
1.0	6.37°C	3.08°C	-30%/-63%

Physical Mechanism Explanation:

- Groundwater flow enhances advective heat transfer, accelerating heat transport from the BHE vicinity to the far field
- Higher velocities lead to faster heat dissipation and smaller temperature fluctuations
- 3D models are more sensitive to velocity because advection occurs throughout the three-dimensional space
- 2D models, through horizontal slice assumption, partially suppress overestimation of advection effects

Engineering Implications:

- Sites with groundwater flow have stronger BHE thermal recovery capability
- High-velocity sites can use smaller borehole spacing or fewer boreholes
- However, excessively high velocities may cause excessive heat loss, reducing system efficiency
- Design should select appropriate simulation methods based on actual hydrogeological conditions

4.3 Can pygfunction Replace EED?

Based on the comparative analysis in this study, the following assessment addresses whether pygfunction can replace EED:

Scenarios Where Full Replacement is Possible:

- ✔ Standard BHE design without groundwater flow
- ✔ Academic research and teaching demonstrations
- ✔ Parameter sensitivity analysis
- ✔ Non-standard borehole layout design
- ✔ Projects requiring integration with other Python tools

Scenarios Where Partial Replacement is Possible:

- 1. ⚠️ Commercial design projects (may require additional validation)
- 2. ⚠️ Projects requiring specific standard certification
- 3. ⚠️ Users requiring graphical interface

Scenarios Where Replacement is Not Possible:

- 1. ❌ Designs requiring groundwater flow consideration (neither supports this)
- 2. ❌ Projects requiring specific report formats
- 3. ❌ Use by non-technical personnel

Technical Accuracy Comparison Summary:

Metric	pygfunction	EED	Evaluation
Base Load MAE	0.147°C	Benchmark	Excellent
Peak Temperature Deviation	<0.1°C	Benchmark	Excellent
R²	0.995	Benchmark	Excellent
g-function Theory	FLS Model	Eskilson	Equivalent

Conclusion: Under no groundwater flow conditions, pygfunction can serve as an effective open-source alternative to EED. Both are based on the same theoretical framework (g-function method) with highly consistent calculation results (MAE<0.15°C). The main advantages of pygfunction are: open-source and free, transparent calculation process, support for custom configurations; main disadvantages are: lack of graphical interface, programming skills required, longer initial g-function computation time.

References

1. Eskilson, P. (1987). *Thermal Analysis of Heat Extraction Boreholes*. Doctoral Thesis, Lund University, Sweden.

2. Cimmino, M. (2018). pygfunction: An open-source toolbox for the evaluation of thermal response factors for geothermal borehole fields. *MethodsX*, 5, 1199-1213. <https://doi.org/10.1016/j.mex.2018.09.009>

3. Hellström, G. (1991). *Ground Heat Storage: Thermal Analyses of Duct Storage Systems*. Doctoral Thesis, Lund University, Sweden.

4. BLOCON (2015). *EED - Earth Energy Designer, Version 4.20*. BLOCON Software, Lund, Sweden.

5. Langevin, C.D., Hughes, J.D., Banta, E.R., Niswonger, R.G., Panday, S., and Provost, A.M. (2017). Documentation for the MODFLOW 6 Groundwater Flow Model. *U.S. Geological Survey Techniques and Methods*, Book 6, Chap. A55.

6. Hughes, J.D., Langevin, C.D., Paulinski, S.R., Larsen, J.D., and Brakefield, L.K. (2023). Documentation for the MODFLOW 6 Groundwater Energy Transport (GWE) Model. *U.S. Geological Survey Techniques and Methods*, Book 6, Chap. A57.

7. Diao, N., Li, Q., & Fang, Z. (2004). Heat transfer in ground heat exchangers with groundwater advection. *International Journal of Thermal Sciences*, 43(12), 1203-1211.

8. Molina-Giraldo, N., Blum, P., Zhu, K., Bayer, P., & Fang, Z. (2011). A moving finite line source model to simulate borehole heat exchangers with groundwater advection. *International Journal of Thermal Sciences*, 50(12), 2506-2513.

9. Claesson, J., & Hellström, G. (2011). Multipole method to calculate borehole thermal resistances in a borehole heat exchanger.

HVAC&R Research, 17(6), 895-911.

10. Zeng, H., Diao, N., & Fang, Z. (2002). A finite line-source model for boreholes in geothermal heat exchangers. *Heat Transfer—Asian Research*, 31(7), 558-567.
-

Report Generated: December 15, 2025

Based on 25-year monthly simulation data analysis