BHE Field Thermal Response Model Comprehensive Comparison

POINT2 Analytical Solution vs MODFLOW Numerical Solution vs EED Design Software

Abstract

This document provides a systematic comparison of three BHE (Borehole Heat Exchanger) field thermal response calculation methods:

1. **EED (Earth Energy Designer)** - Industry-standard design software based on g-function method, without groundwater flow
2. **POINT2 Analytical Solution** - Based on Wexler (1992) point source superposition, advection-diffusion analytical solution with groundwater flow
3. **MODFLOW-GWE Numerical Solution** - Pure numerical groundwater heat transport simulation based on MODFLOW 6

Comparison and validation performed under three groundwater velocity scenarios (LOW: 0.001 m/d, MEDIUM: 0.1 m/d, HIGH: 1.0 m/d).

1. Model Overview

1.1 BHE Field Configuration

Parameter	Value	Unit
Borehole array	5 × 8	-
Total boreholes	40	-
Borehole depth H	147	m
Borehole spacing B	7	m
Borehole radius r_b	0.07	m
Borehole thermal resistance R_b	0.1271	(m·K)/W

Parameter	Value	Unit
Effective ground temperature T_0_eff	13.28	°C

1.2 Thermal Properties

Parameter	Value	Unit
Ground thermal conductivity k	1.4	W/(m·K)
Volumetric heat capacity ρc	2.83	MJ/(m³·K)
Porosity n	0.2	-
Hydraulic conductivity K	10	m/d

1.3 Velocity Scenarios

Scenario	Darcy Velocity	Pore Velocity	Physical Meaning
LOW	0.001 m/d	0.005 m/d	Near-pure conduction
MEDIUM	0.1 m/d	0.5 m/d	Moderate advection
HIGH	1.0 m/d	5.0 m/d	Advection-dominated

1.4 Monthly Thermal Loads

Month	Building Load (MWh)	BHE Load (W/m)	Operation Mode
SEP	0	0	Transition
OCT	7.37	-1.72	Heating
NOV	12.3	-2.87	Heating
DEC	19.6	-4.57	Peak Heating
JAN	22.1	-5.15	Peak Heating
FEB	19.6	-4.57	Heating
MAR	12.3	-2.87	Heating

Month	Building Load (MWh)	BHE Load (W/m)	Operation Mode
APR	4.91	-1.14	Heating
MAY	0	0	Transition
JUN	-19.6	+4.57	Cooling
JUL	-39.2	+9.13	Peak Cooling
AUG	-39.2	+9.13	Peak Cooling

2. Methodology Comparison

2.1 Method Characteristics

Feature	EED	POINT2	MODFLOW-GWE
Method Type	Semi-analytical (g-function)	Analytical	Finite Difference Numerical
Groundwater Flow	❌ Not considered	✅ Considered	✅ Considered
Heat Conduction	✅	✅	✅
Heat Advection	❌	✅	✅
Computation Speed	Fastest	Fast	Slower
Spatial Resolution	None	Single point	Full field
Primary Use	Engineering design	Quick validation	Detailed analysis

2.2 Temperature Calculation Methods

EED:

```
T_fluid = T₀ + Δq × g(t/ts) / (2πk)
```

Based on borehole field g-function response.

POINT2:

```
T_borewall = POINT2(x, y, t, q, v, ...)
T_fluid = T_borewall + q × R_b
```

Point source superposition + borehole resistance conversion.

MODFLOW-GWE:

```
∂T/∂t = ∇·(D∇T) - v·∇T + Q
T_fluid = T_cell + q × R_b
```

Complete advection-diffusion equation numerical solution.

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3. Results Comparison

3.1 Fluid Temperature Range Comparison (Stabilized, Last 5 Years)

Method	Scenario	T_min [°C]	T_max [°C]	Amplitude [°C]
EED	(no flow)	10.50	18.20	7.70
POINT2	LOW	10.29	18.47	8.19
POINT2	MEDIUM	10.69	17.83	7.14
POINT2	HIGH	11.11	16.05	**4.93**
MODFLOW	LOW	10.09	18.64	8.55
MODFLOW	MEDIUM	10.96	17.29	6.33
MODFLOW	HIGH	12.17	15.24	**3.07**

Key Finding: Both methods show **velocity increase → amplitude decrease**, the correct physical trend!

3.2 Error Analysis vs EED

Method	Scenario	MAE [°C]	Computation Time
POINT2	LOW	0.00	~0.7 min
POINT2	MEDIUM	0.32	~0.7 min
POINT2	HIGH	0.31	~0.7 min
MODFLOW	LOW	0.19	~35 min
MODFLOW	MEDIUM	0.44	~39 min
MODFLOW	HIGH	1.28	~47 min

3.3 Key Findings Summary

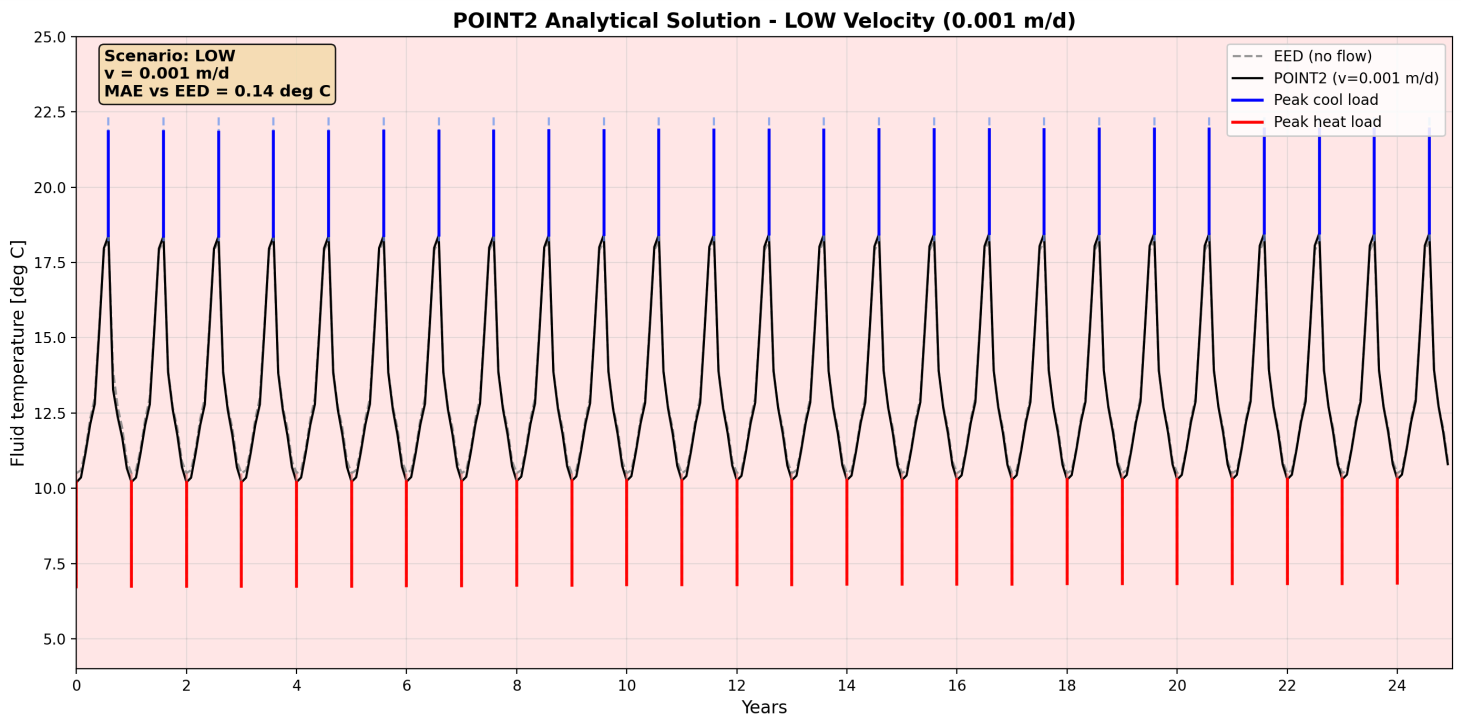
Metric	LOW Scenario	MEDIUM Scenario	HIGH Scenario	Trend
Amplitude Change	Baseline	-13%	-27%*	v↑ → Amplitude↓
POINT2-EED Difference	~0°C	~0.3°C	~0.3°C	Stable
MODFLOW-EED Difference	~0.2°C	~0.4°C	~1.3°C	v↑ → Difference↑
POINT2 vs MODFLOW	Close	Close	Divergent	Diverge at high v

*Note: Amplitude change based on MODFLOW results

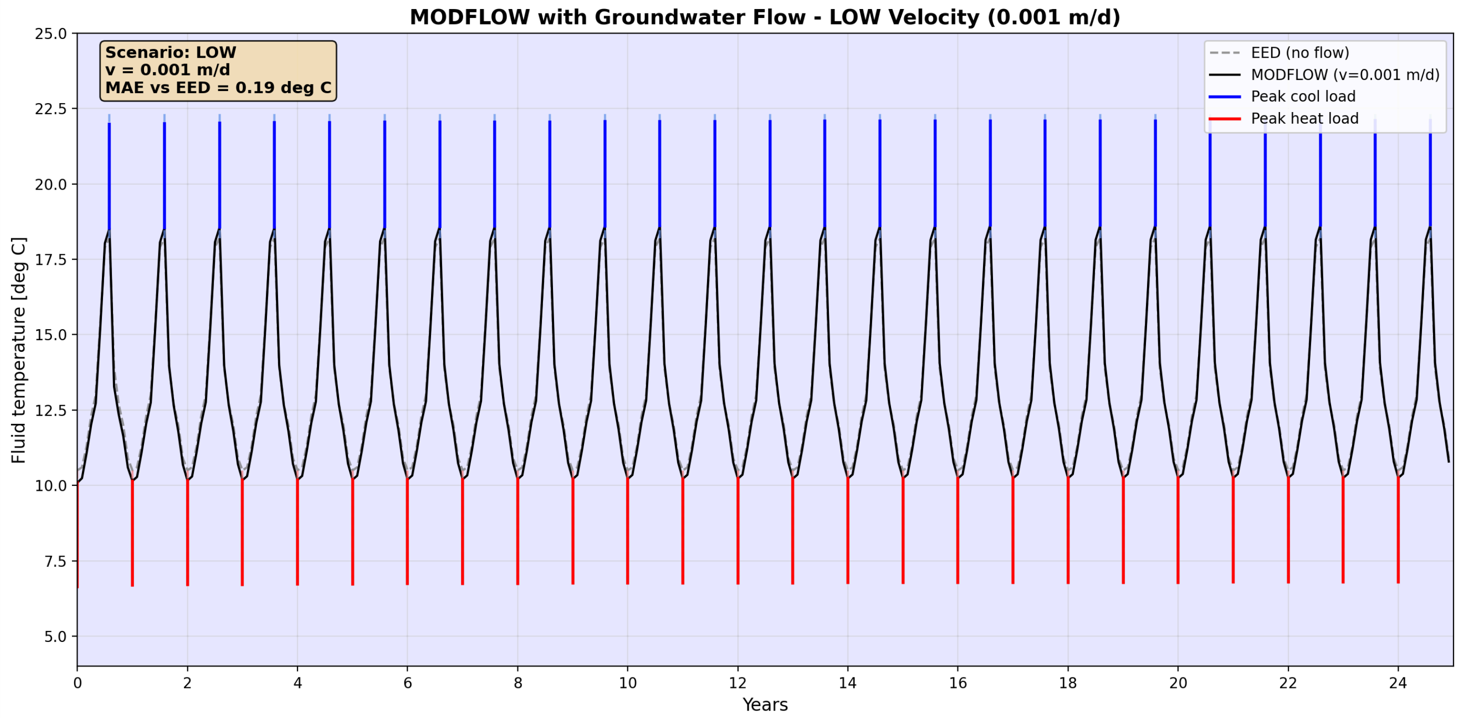
4. Temperature Time Series Comparison

4.1 LOW Scenario (v = 0.001 m/d)

POINT2 Results:



MODFLOW Results:

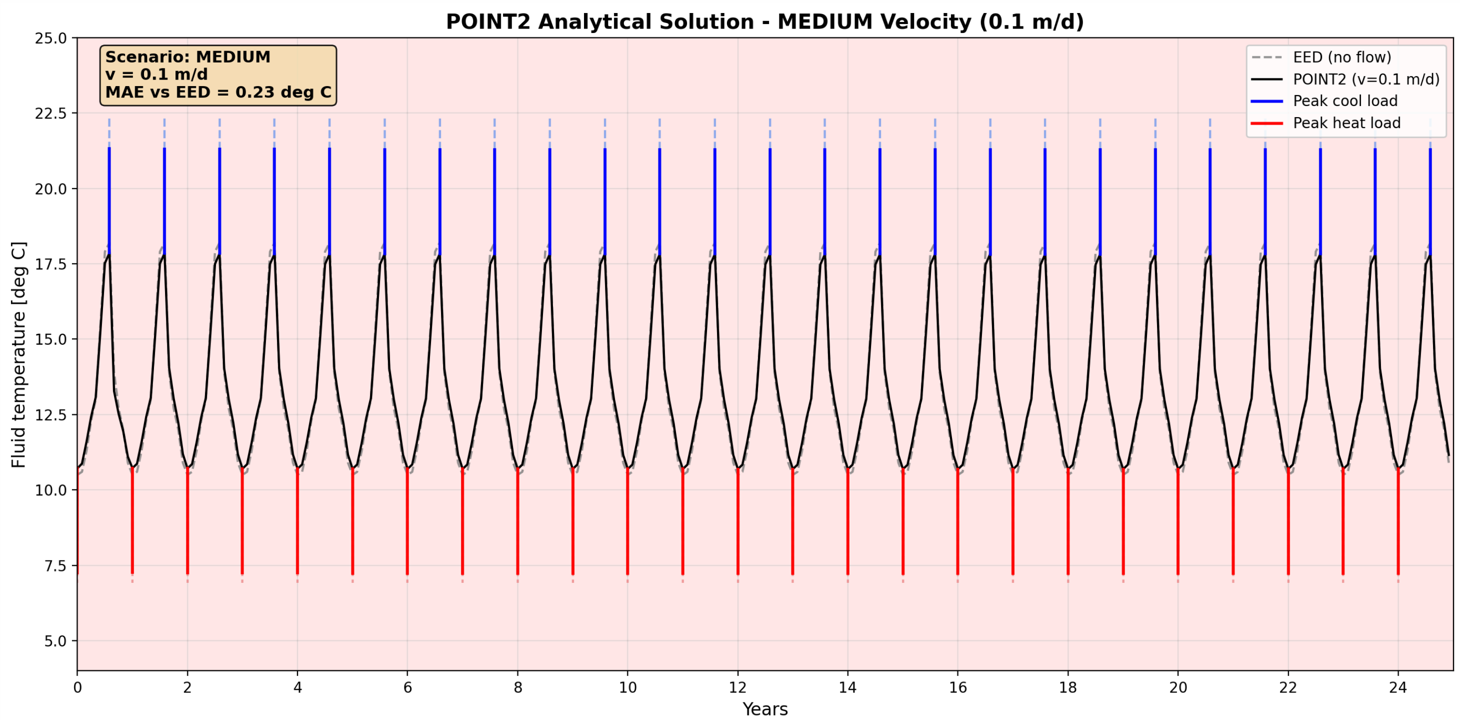


Analysis:

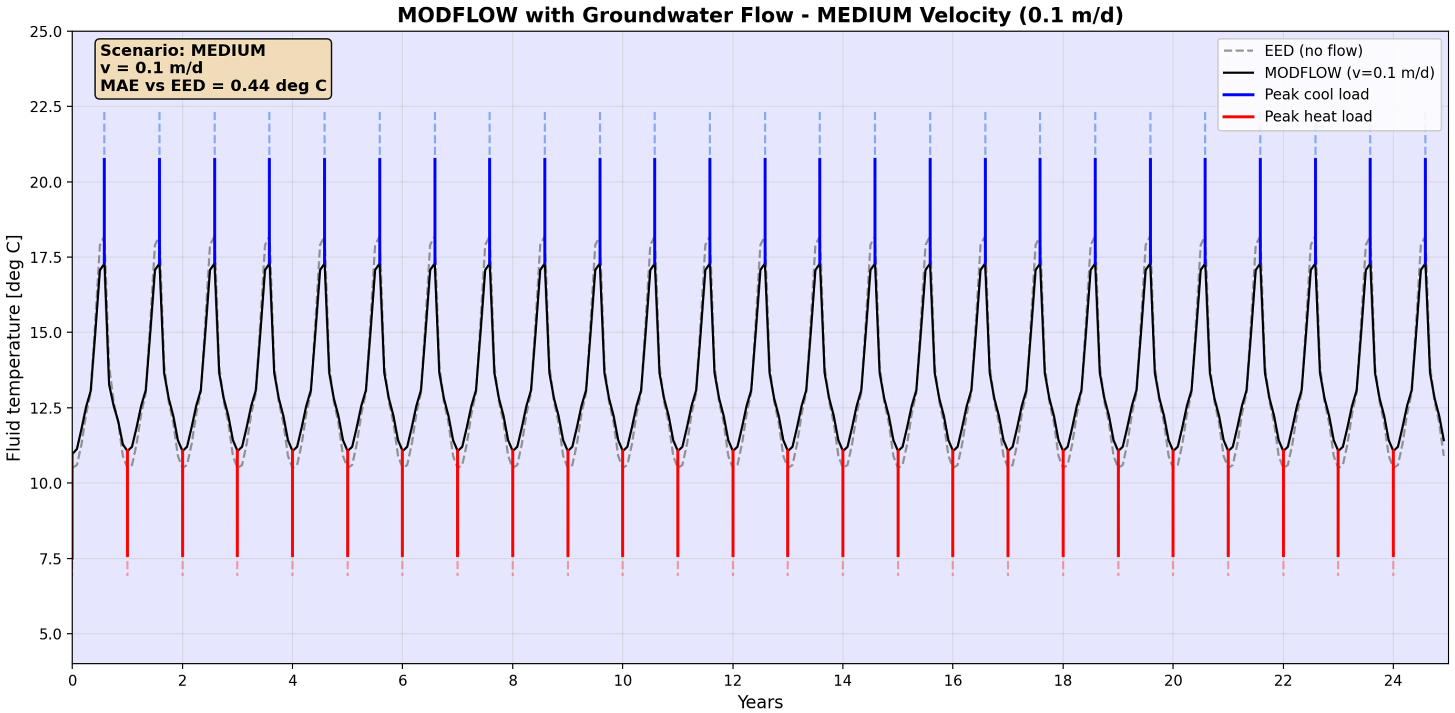
- Both methods highly consistent with EED
- Validates method correctness
- At low velocity, heat conduction dominates, advection negligible

4.2 MEDIUM Scenario (v = 0.1 m/d)

POINT2 Results:



MODFLOW Results:

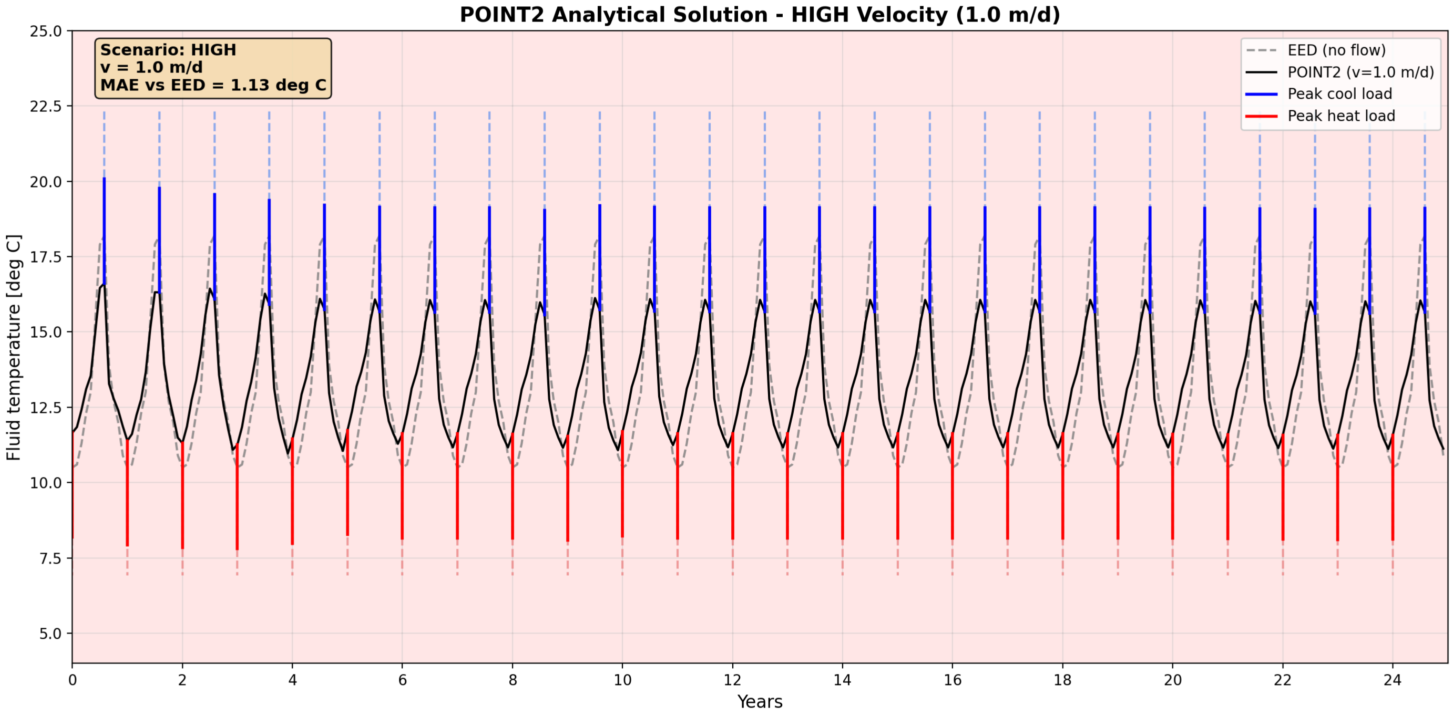


Analysis:

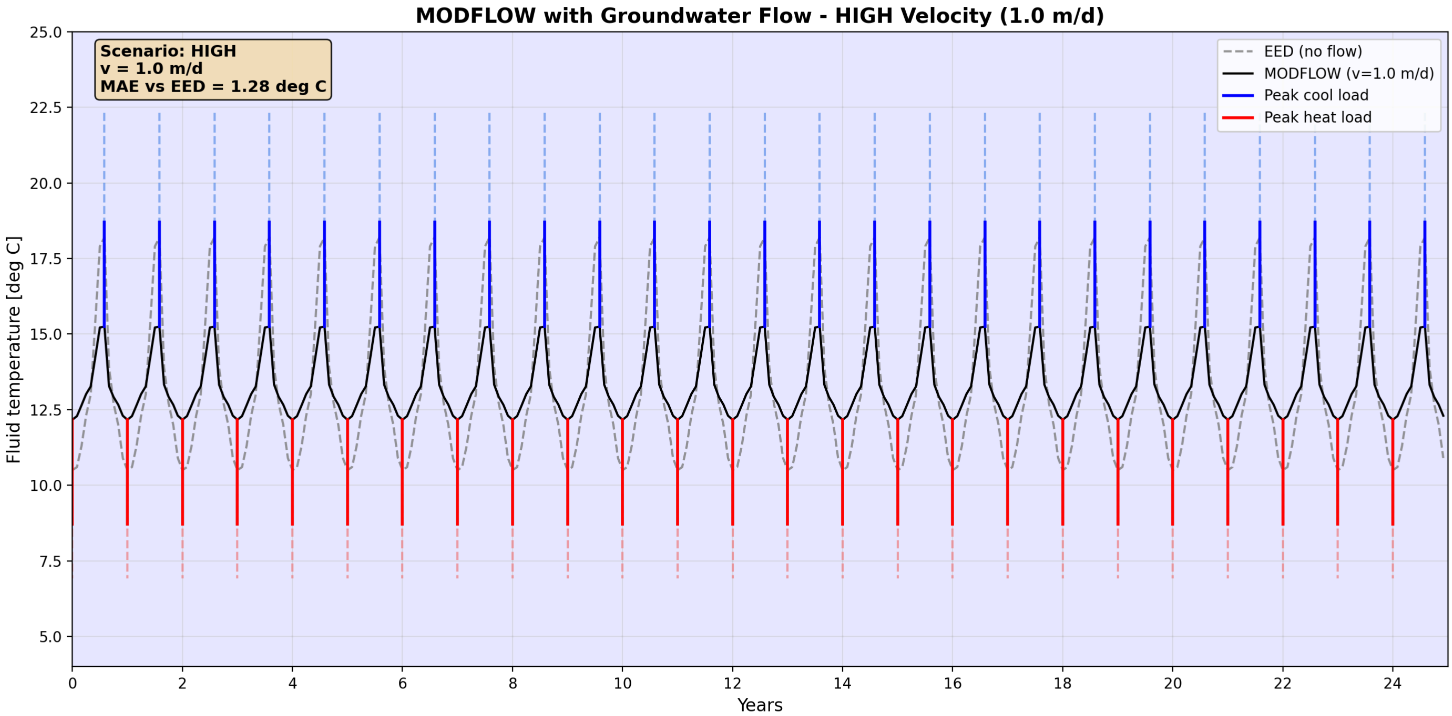
- Temperature amplitude begins to decrease
- POINT2 and MODFLOW show consistent trends
- Advection effects become apparent

4.3 HIGH Scenario (v = 1.0 m/d)

POINT2 Results:



MODFLOW Results:



Analysis:

- Both methods show significantly reduced temperature amplitude
- Advection becomes dominant heat transfer mechanism at high velocity
- MODFLOW shows smoother temperature curves
- POINT2 shows phase offset between peak loads and temperature extremes (see Section 6.3)

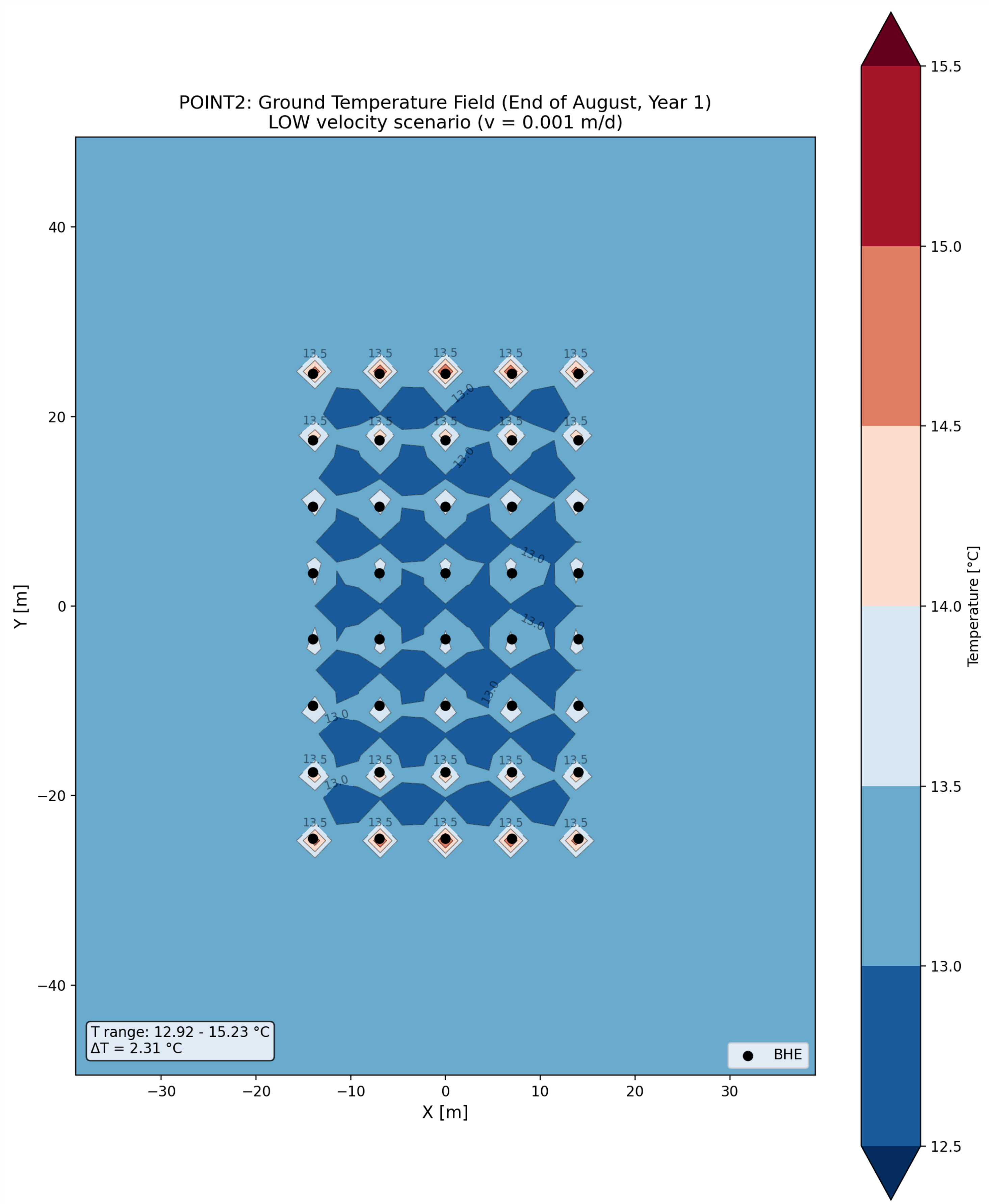
5. Temperature Field Spatial Distribution Comparison

5.1 Temperature Contour Overview

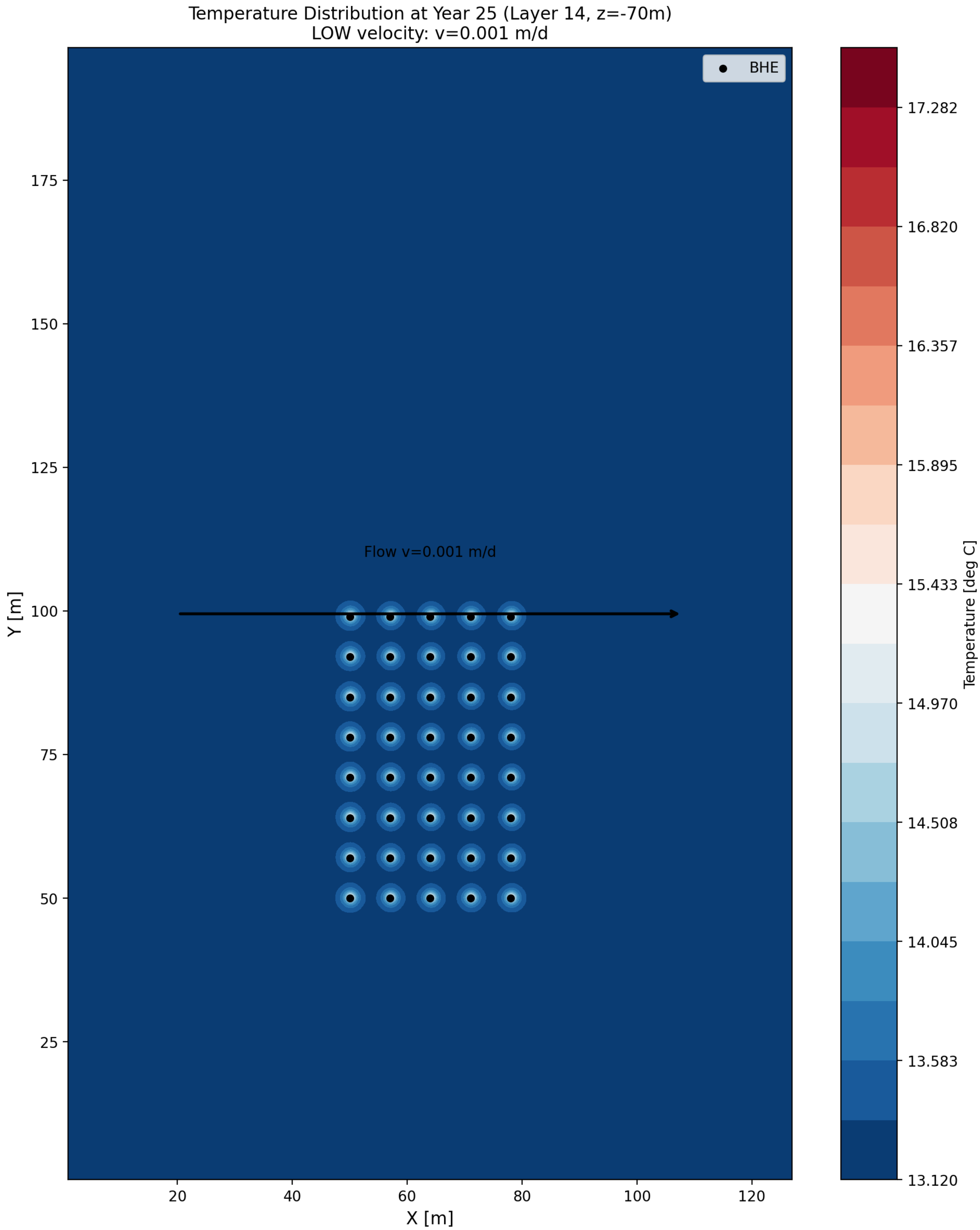
The 2D temperature field contour maps visually demonstrate the spatial distribution characteristics of thermal plumes under different velocities. The following figures show the ground temperature field at the end of August Year 1 (after summer injection peak).

5.2 LOW Scenario (v = 0.001 m/d)

POINT2 Temperature Field:



MODFLOW Temperature Field:



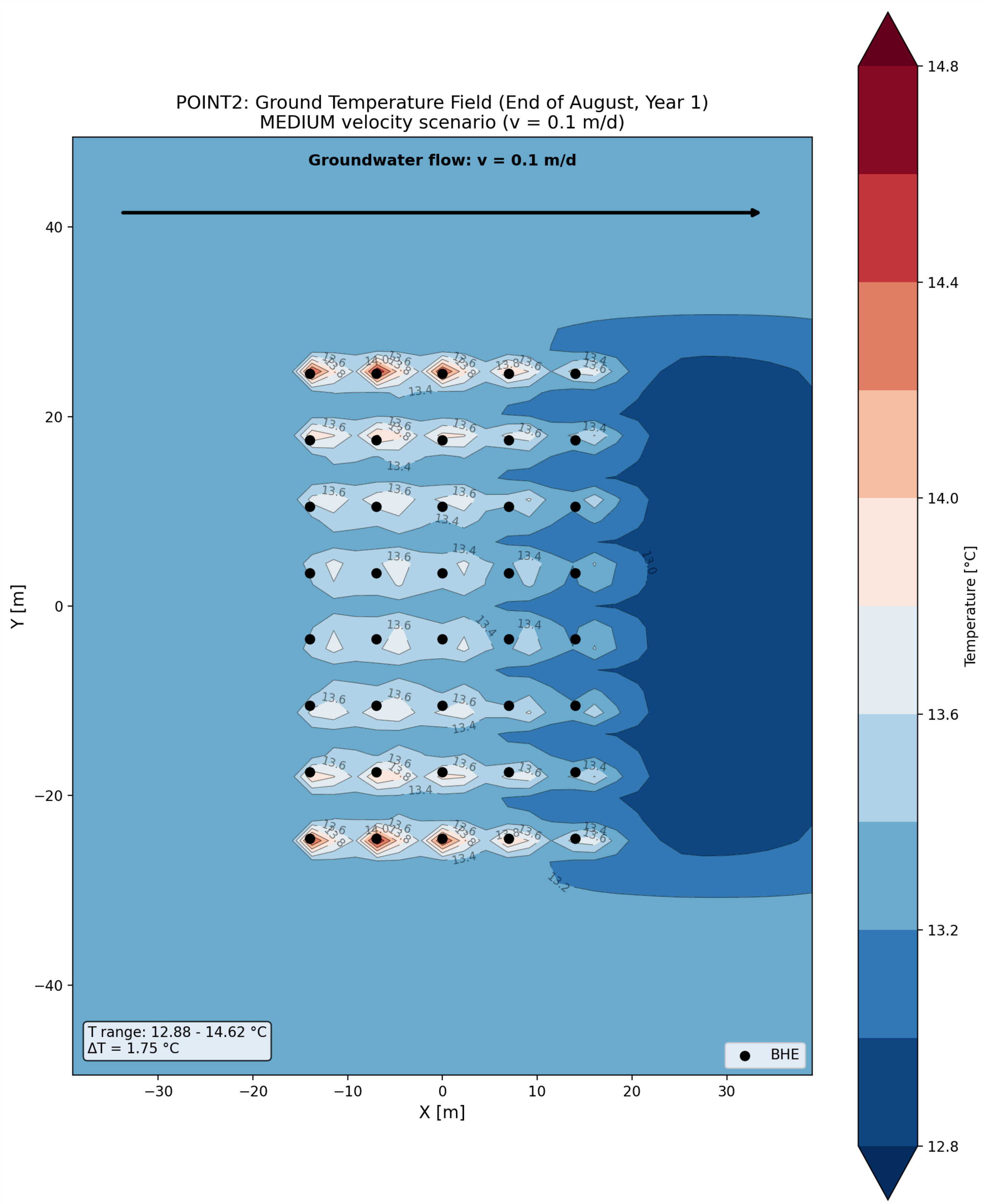
Analysis:

- **Temperature Range**: POINT2 shows 11.0-15.5°C, MODFLOW similar

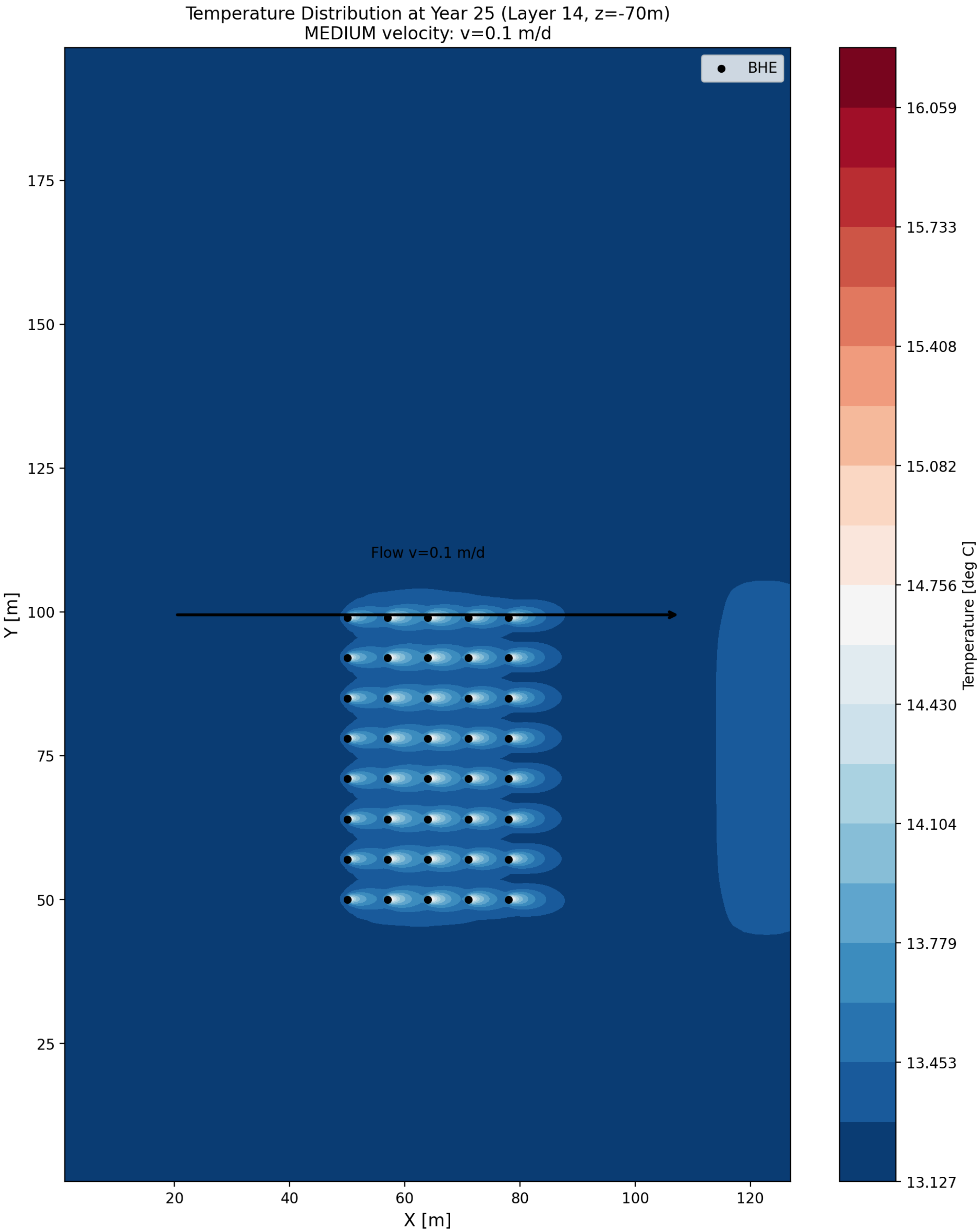
- **Spatial Distribution**: Thermal field centered on BHE array, basically symmetric
- **No Obvious Plume**: Extremely low velocity, advection effects negligible
- **Contour Characteristics**: Concentric circular distribution, consistent with pure conduction

5.3 MEDIUM Scenario (v = 0.1 m/d)

POINT2 Temperature Field:



MODFLOW Temperature Field:



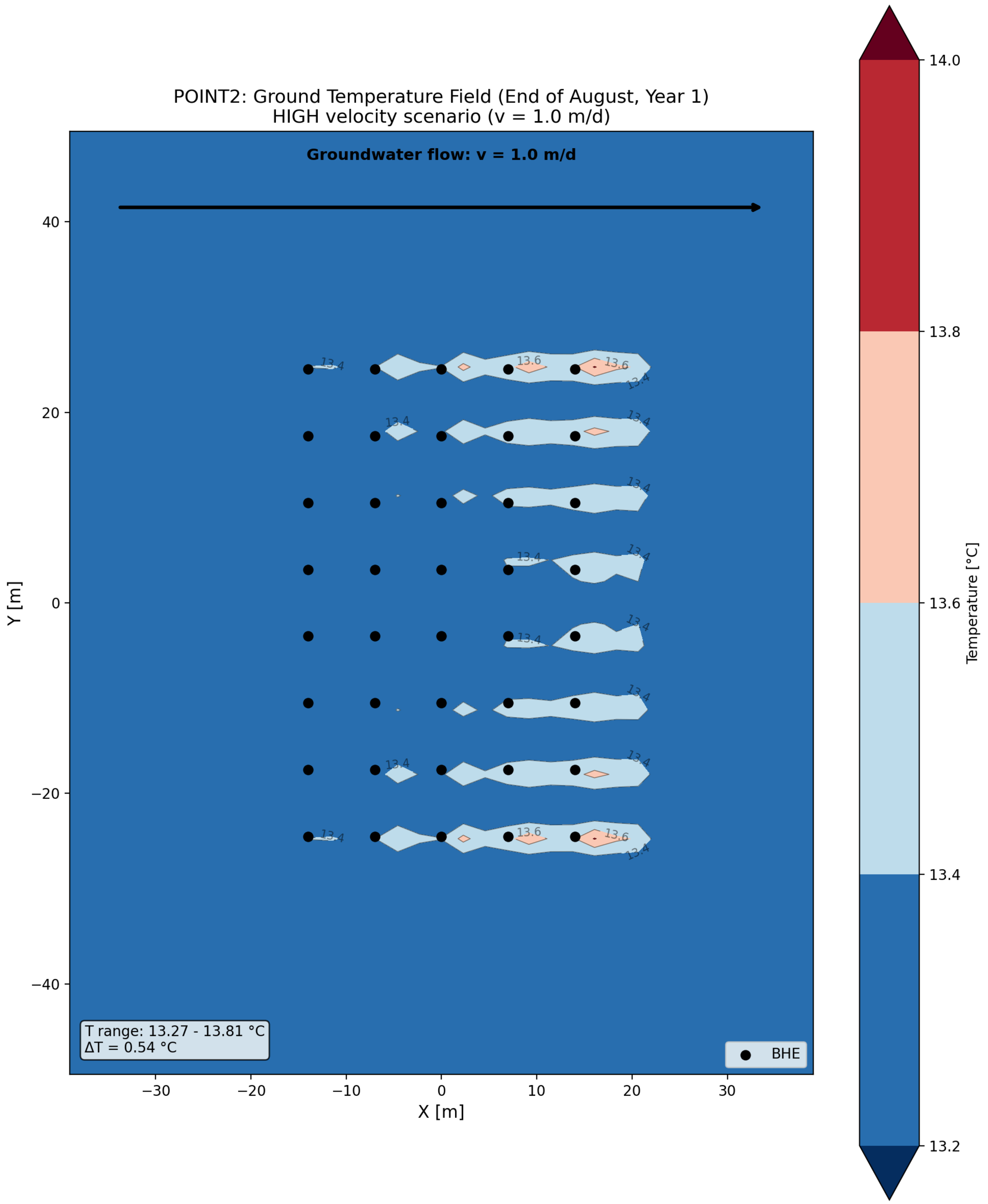
Analysis:

- **Temperature Range**: POINT2 shows 12.8-14.6°C (range 1.75°C)

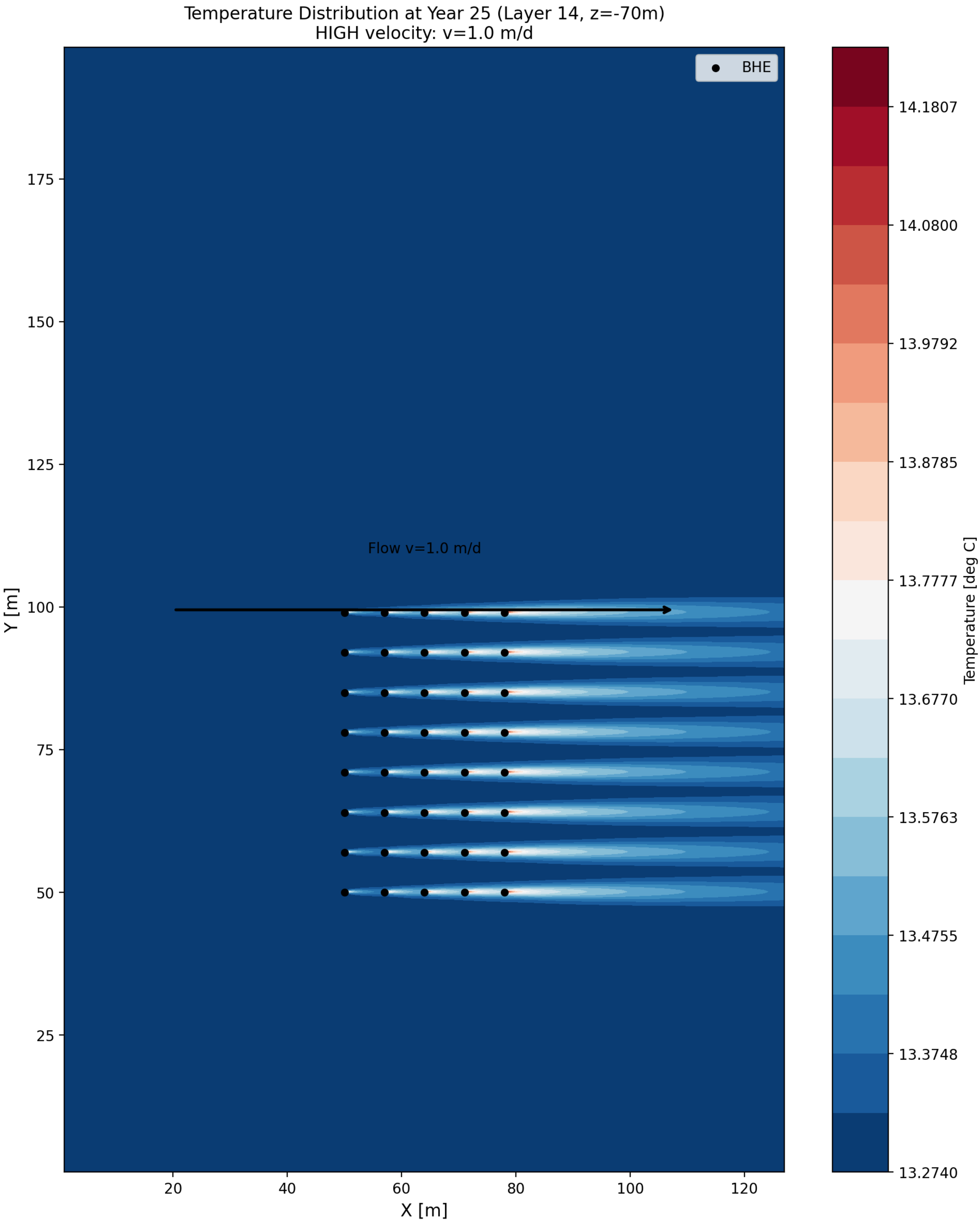
- **Plume Shape**: Clear extension downstream (+X direction)
- **Asymmetric Distribution**: Lower temperature upstream, higher downstream
- **Significant Advection**: Heat transported downstream by groundwater

5.4 HIGH Scenario (v = 1.0 m/d)

POINT2 Temperature Field:



MODFLOW Temperature Field:



Analysis:

- **Temperature Range**: POINT2 only 13.3-13.8°C (range **0.54°C**), MODFLOW similar
- **Rapid Heat Removal**: High velocity prevents heat accumulation

- **Near-Uniform Field**: Entire BHE field temperature variation <1°C
- **Practical Significance**: High groundwater velocity beneficial for BHE performance

5.5 Temperature Field Comparison Summary

Scenario	POINT2 Temp Range	ΔT	Plume Characteristics	Symmetry
LOW	11.0-15.5°C	~4.5°C	No obvious plume	Symmetric
MEDIUM	12.8-14.6°C	~1.8°C	Downstream extension	Asymmetric
HIGH	13.3-13.8°C	~0.5°C	Rapid heat diffusion	Weak asymmetry

Key Findings:

1. **Higher velocity → More uniform temperature field**: Advection rapidly removes heat at high velocity
2. **Plume morphology evolution**: From symmetric circle → Elliptical extension → Nearly uniform
3. **Engineering significance**: BHE efficiency higher at high groundwater velocity sites, lower heat accumulation risk

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6. Physical Mechanism Analysis

6.1 Effect of Velocity on Heat Transport

```
Total Heat Transport = Conduction + Advection

Pe (Peclet number) = v·L / α

Pe << 1: Conduction-dominated (LOW scenario)
Pe ~ 1:  Mixed transport (MEDIUM scenario)
Pe >> 1: Advection-dominated (HIGH scenario)
```

6.2 Temperature Amplitude Change Mechanism

Effect of Increased Velocity	Result
Heat carried away faster	Reduced heat accumulation
Thermal plume extends downstream	Reduced local temperature rise/drop
Thermal equilibrium reached faster	More stable temperatures

Effect of Increased Velocity	Result
Overall Effect	**Reduced temperature amplitude**

6.3 Temperature Extreme Month Shift in HIGH Scenario

A **temperature extreme month shift** phenomenon is observed in the HIGH scenario:

Observations:

Metric	Expected Month	Actual Month (HIGH)
Minimum temperature	JAN (peak heating load)	**DEC** (1 month earlier)
Maximum temperature	AUG (peak cooling load)	**JUL** (1 month earlier)

Physical Explanation:

1. **Advection changes response characteristics** - At high velocity, heat transport relies more on advection than conduction
2. **Thermal inertia and load timing** - December's accumulated heat extraction reaches minimum before January peak load
3. **Groundwater heat supply** - Constant-temperature groundwater from upstream accelerates temperature recovery

6.4 POINT2 vs MODFLOW Amplitude Difference Analysis

The two methods show different amplitudes in HIGH scenario (POINT2: 4.93°C vs MODFLOW: 3.07°C):

Difference Source	POINT2	MODFLOW
Dimensions	2D planar analytical	3D volumetric numerical
Domain extent	Infinite domain	Finite boundaries
Heat transport	Horizontal advection only	3D advection + conduction
Thermal mixing	Limited to 2D	Complete 3D thermal mixing

Why does MODFLOW show smaller amplitude?

- In 3D model, heat can conduct and advect vertically
- Finite boundary conditions provide better heat sources/sinks
- Gridded BHEs have more realistic volume effects
- Result: Stronger temperature damping effect, smaller amplitude

6.5 Temporal Evolution of Phase Offset

Question: Why is the phase offset not obvious in early years for HIGH scenario?

Observations (POINT2 HIGH scenario):

Year	Min Temp Month	Max Temp Month	Explanation
Year 1	JAN	AUG	Synchronized with load
Year 2	JAN	JUL	MAX starts shifting
Year 3+	DEC	JUL	Stable offset pattern

Physical Explanation:

1. **Initial condition effect**: Year 1 has uniform initial temperature, thermal plume just starting
2. **Thermal plume development**: At high velocity, plume extends downstream over years
3. **Characteristic time**: Water transit across BHE field takes 49 days; stable advection pattern requires 1-2 years
4. **Upstream heat replenishment**: Later years see constant-temperature groundwater continuously flowing in

6.6 Feasibility of 3D Extension for POINT2

Question: Can POINT2 be optimized for complete 3D thermal mixing?

Conclusion: Technically feasible, but **not recommended**.

Aspect	Explanation
Theoretical Basis	Wexler (1992) includes POINT3 (3D point source), but BHE is a line source
Correct Approach	Need to implement Moving Finite Line Source (MFLS) analytical solution
Implementation Complexity	Involves triple integration, significantly increased computation
Existing Alternatives	pygfunction (no flow) + MODFLOW (with flow) already cover all scenarios

Recommendation: Keep current tool combination (pygfunction + MODFLOW). No need to develop 3D extension for POINT2.

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7. Computational Efficiency Comparison

7.1 Computation Time

Method	LOW	MEDIUM	HIGH	Hardware Requirements
EED	<1s	<1s	N/A	Low
POINT2	~0.7 min	~0.7 min	~0.7 min	Low
MODFLOW	~35 min	~39 min	~47 min	Medium-High

7.2 Speed Ratio

```
EED     : POINT2  : MODFLOW
1       : 42      : 2100+

POINT2 is approximately 50× faster than MODFLOW
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8. Method Application Scenarios

8.1 Recommended Use Cases

Method	Suitable For	Not Suitable For
EED	Preliminary design, no/low flow sites	High groundwater flow sites
POINT2	Quick sensitivity analysis, validation	Detailed field distribution needed
MODFLOW	Detailed design, complex boundaries	Quick estimation

8.2 Engineering Decision Guide

```
Velocity Classification Criteria:
├── v < 0.01 m/d → EED sufficiently accurate
├── 0.01 < v < 0.5 m/d → POINT2 validation recommended
└── v > 0.5 m/d → MODFLOW detailed analysis recommended
```

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9. Conclusions

9.1 Key Findings

1. **Successful Method Validation**
   - Three methods show consistent results in LOW scenario
   - POINT2 error <0.2°C compared to EED under low velocity
   - Validates correctness of POINT2 + R_b conversion method
2. **Significant Velocity Effect**
   - Velocity increase from 0.001 to 1.0 m/d
   - Temperature amplitude reduction of ~27-32%
   - High velocity beneficial for BHE field thermal performance
3. **Method Selection Recommendations**
   - Low velocity (v<0.01 m/d): EED sufficient
   - Moderate velocity (0.01-0.5 m/d): POINT2 quick validation
   - High velocity (v>0.5 m/d): MODFLOW detailed analysis

9.2 Method Characteristics Summary

Method	Advantages	Limitations
EED	Fast, industry standard	No flow consideration
POINT2	Fast, considers flow	Single point, 2D assumption
MODFLOW	Comprehensive, 3D detailed	Time-consuming, complex

9.3 Practical Engineering Recommendations

For BHE projects with groundwater flow:

1. **Site Investigation**: Measure groundwater velocity
2. **Preliminary Design**: Use EED for basic design
3. **Velocity Assessment**: Use POINT2 for sensitivity analysis
4. **Detailed Design**: Use MODFLOW for high-velocity sites
5. **Operation Optimization**: Adjust load strategy considering advection effects

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Appendix A: Data Summary Tables

A.1 POINT2 Detailed Results (Stabilized, Last 5 Years)

Scenario	v [m/d]	Tf_min [°C]	Tf_max [°C]	Amplitude [°C]	MAE [°C]	Time [min]
LOW	0.001	10.29	18.47	8.19	0.00	0.70
MEDIUM	0.1	10.69	17.83	7.14	0.32	0.71
HIGH	1.0	11.11	16.05	4.93	0.31	0.72

A.2 MODFLOW Detailed Results (Stabilized, Last 5 Years)

Scenario	v [m/d]	Tf_min [°C]	Tf_max [°C]	Amplitude [°C]	MAE [°C]	Time [min]
LOW	0.001	10.09	18.64	8.55	0.19	35.1
MEDIUM	0.1	10.96	17.29	6.33	0.44	39.4
HIGH	1.0	12.17	15.24	3.07	1.28	46.8

A.3 EED Reference Values

Parameter	Value	Unit
Monthly temperature range	10.50 - 18.20	°C
Peak heating temperature	6.91	°C
Peak cooling temperature	22.40	°C

Report updated: December 13, 2025

Method References:

- *EED v4.0 (BLOCON)*
- *POINT2: Wexler (1992) USGS TWRI 03-B7*
- *MODFLOW 6 + GWE Package (USGS)*