Digital filtering
Signal processing

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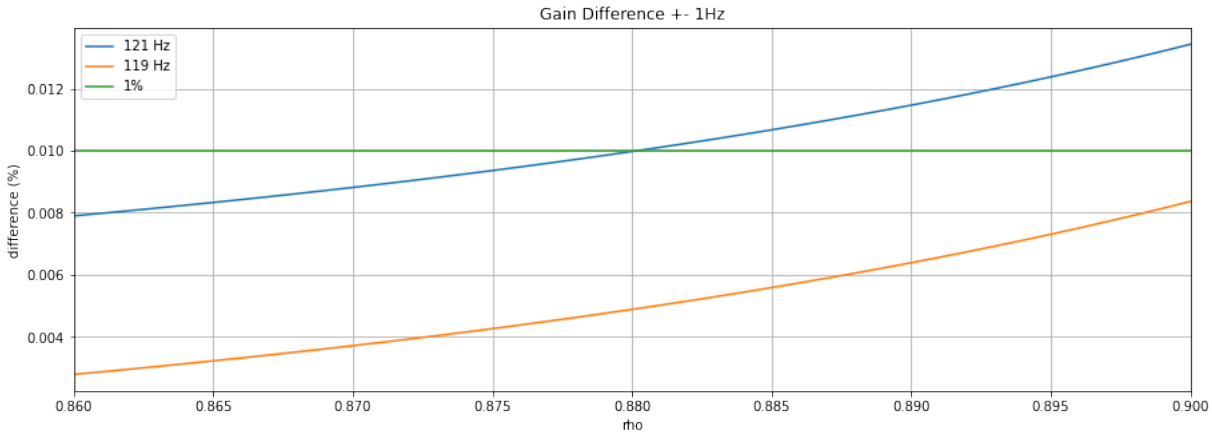
1 Approximation of second order filters

Band-pass filter:

1. Find θ : we compute $\theta = \frac{f}{2\pi} Fe = 1.88$.
2. Find ρ : we set K = 1 and we draw this graph representing the height difference between $\{A_{120\text{Hz}}, A_{121\text{Hz}}\}$ and $\{A_{119\text{Hz}}, A_{120\text{Hz}}\}$.

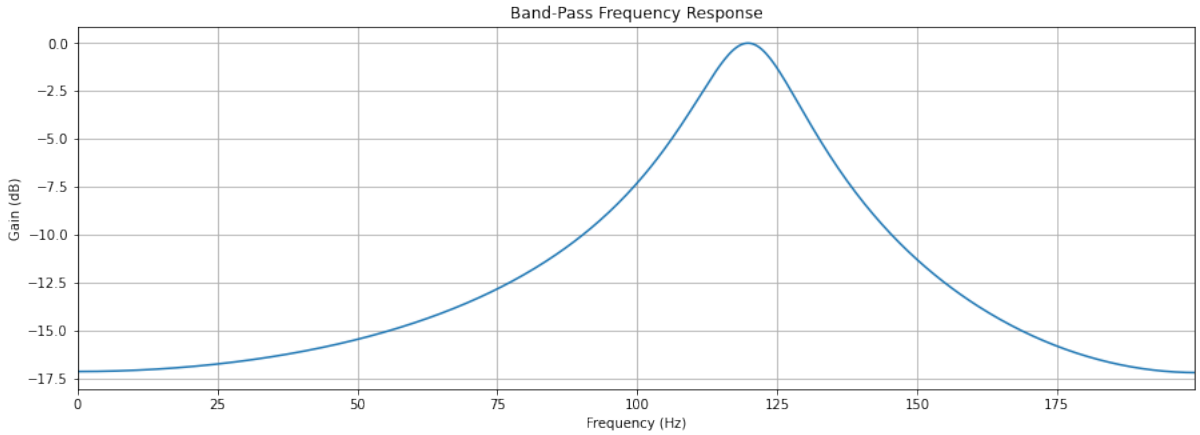


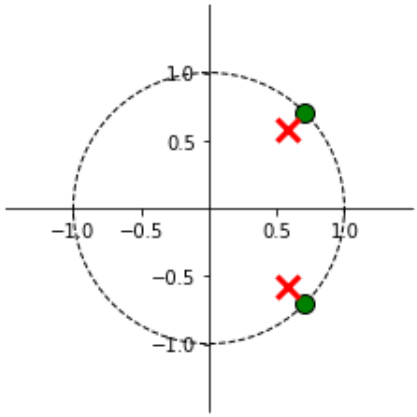
   we set $\rho = 0.87$
3. Find K : we compute $K = \frac{1}{A_{120\text{Hz}}} = 0.2432$.

The transfer function of the band-pass filter is

$$H(Z) = K \frac{1 - \cos(\theta) Z^{-1}}{1 - 2\rho \cos(\theta) Z^{-1} + \rho^2 Z^{-2}}$$

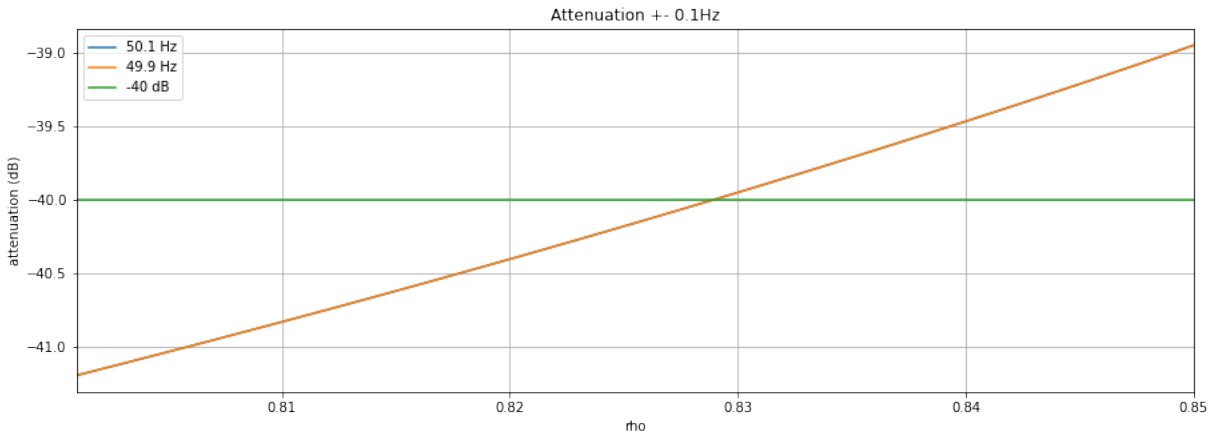
where K = 0.2432, $\rho = 0.87$ and $\theta = 1.88$. The frequency response as well as the poles and zeros position of this filter are the following.





Band-stop filter:

1. Find θ : we compute $\theta = \frac{50}{2\pi} Fe = 0.78$.
2. Find ρ : we set K = 1 and we draw this graph representing the attenuation at 49.9 Hz and 50.1 Hz. We want to attenuate these frequencies at -40dB.



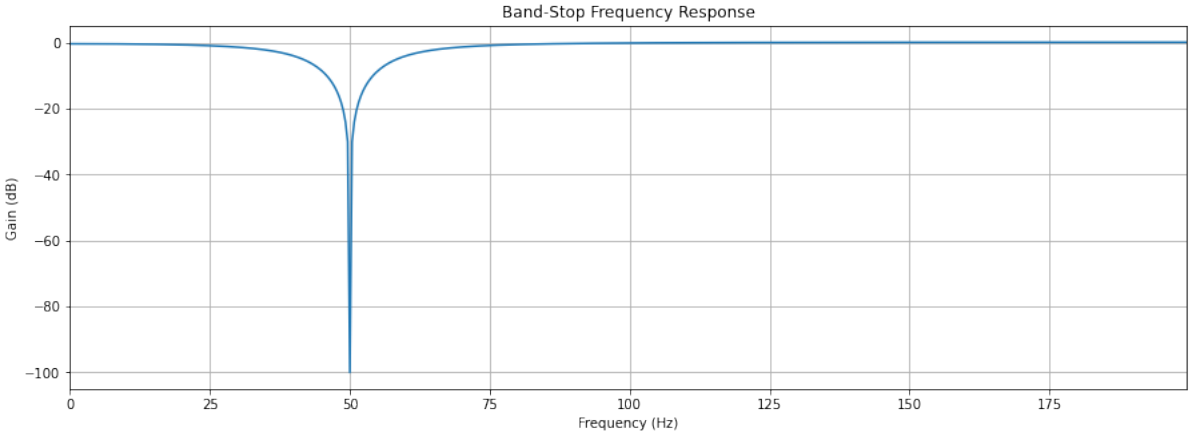
We set $\rho = 0.82$.

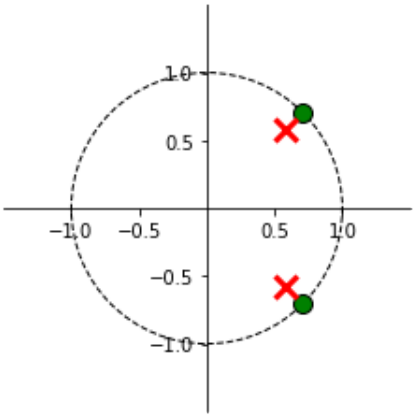
3. Find K : we computed $K = \frac{1}{A_{120\text{Hz}}} = 0.6713$.

The transfer function of the band-stop filter is

$$H(Z) = K\frac{1 - 2\cos(\theta)Z^{-1} + Z^{-2}}{1 - 2\rho\cos(\theta)Z^{-1} + \rho^2 Z^{-2}}$$

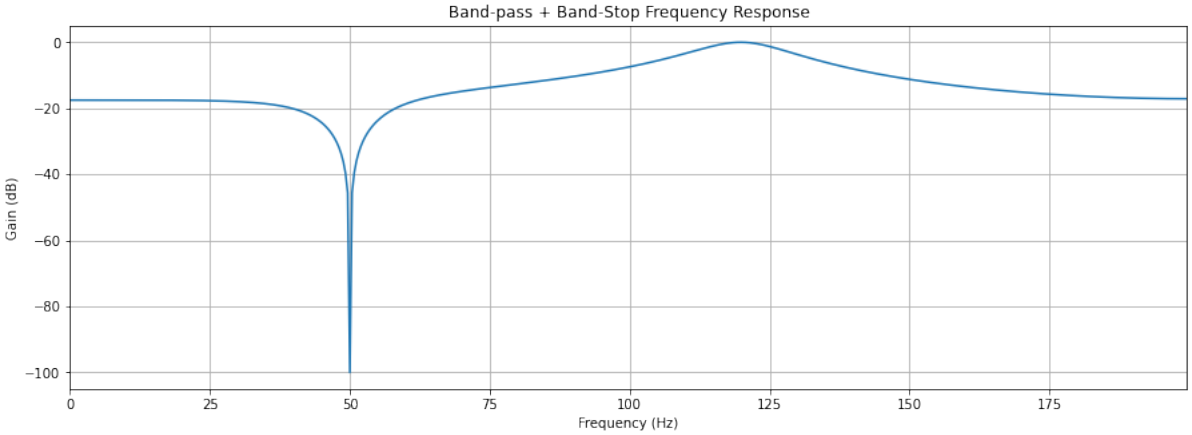
where K = 0.6714, $\rho = 0.82$ and $\theta = 0.78$. The frequency response as well as the poles and zeros position corresponding are the following.

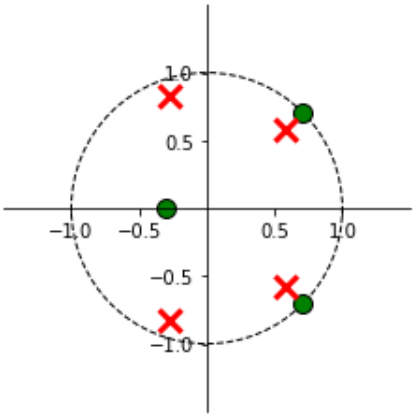




Band-pass + band-stop filter

Now we can add these two filters together and plot their frequency response. The poles and zeros position of this new filter is exactly the same as the two previous ones together.

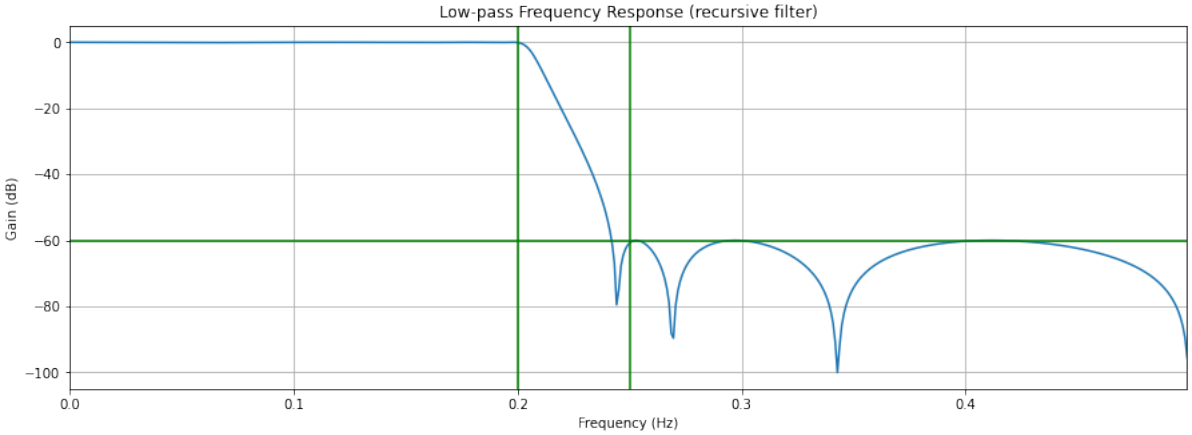


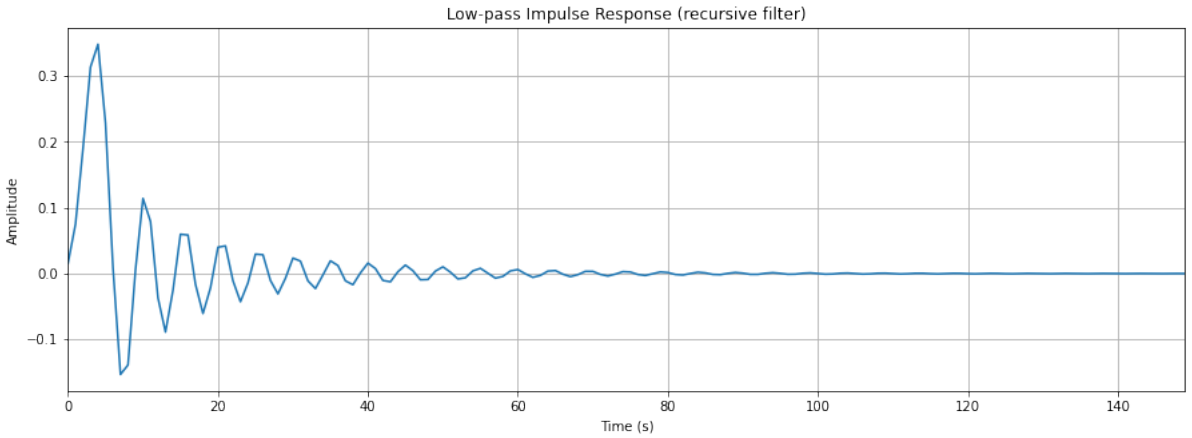


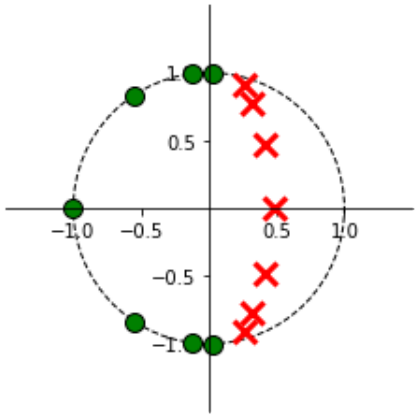
2 Approximation of higher order filters

Recursive filter

We see that the specifications are respected. These specifications are drawn in green on the graph.

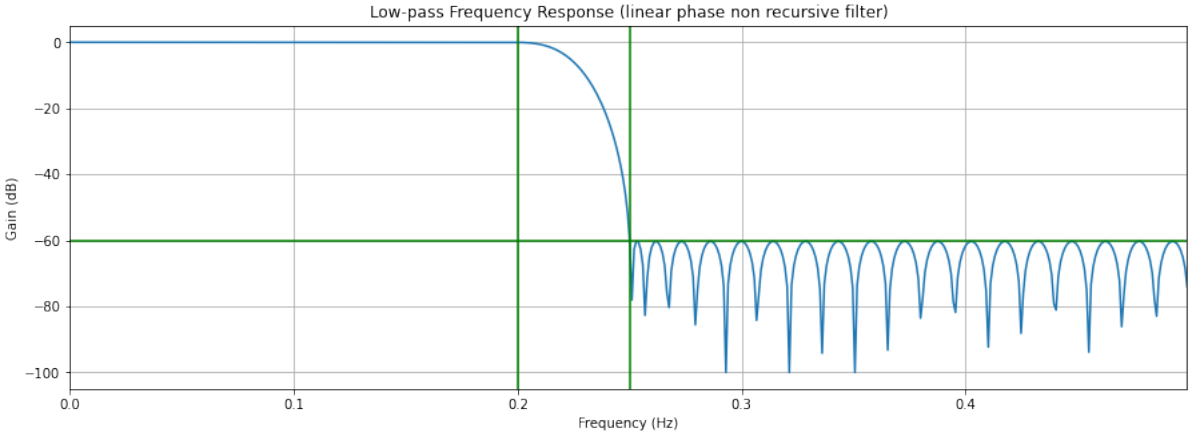


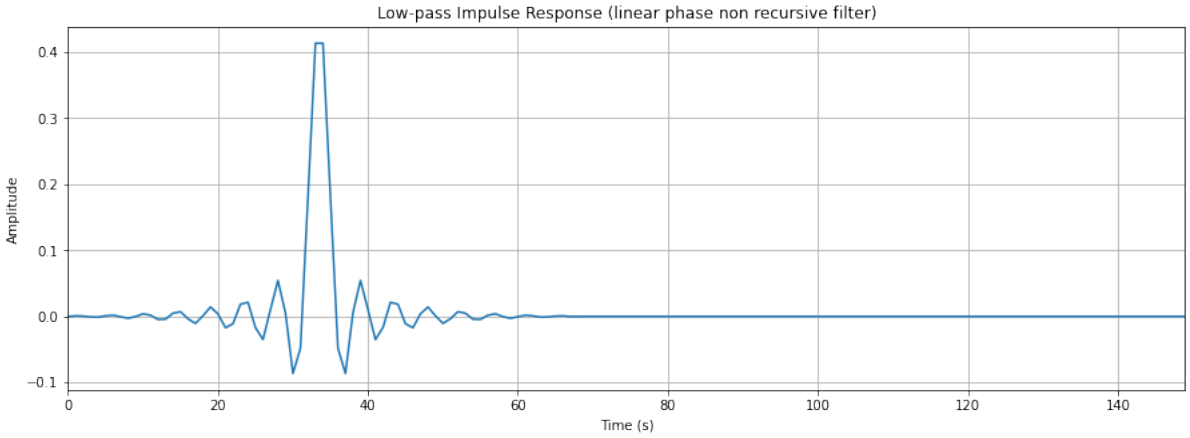


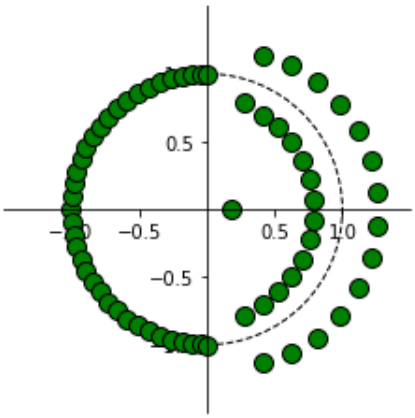


Non-recursive filter

We also see that the specifications are respected. These specifications are drawn in green on the graph.







Comparison between the two filters

For the recursive one, we compute a 7th-order filter and a 68th-order filter for the non-recursive one. We estimate the number of computations to be twice filter's order. So, for the recursive filter we have 14 operations per samples and 136 for the non-recursive one.