Macroeconomics III - Problem Set 4 (Part 2)

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1 Literature Review

In this paper titled "Idiosyncratic Risk in the United States and Sweden: Is There a Role for Government Insurance?" by Martin Flodén and Jesper Lindé investigates the role of government redistribution in addressing individual-specific income risks in the United States and Sweden. Using panel data from the PSID and HINK surveys, the authors estimate wage processes to quantify idiosyncratic risk, decomposing wages into permanent components based on observable characteristics and stochastic components capturing persistent and transitory shocks. They use these findings to parameterize a calibrated general equilibrium model where agents face uninsurable productivity risk, constrained borrowing, and make labor supply and savings decisions. By simulating various tax and transfer policies, the study evaluates the welfare trade-offs between insurance benefits and distortions, concluding that optimal redistribution levels are higher in the U.S. due to greater income risk and lower in Sweden where such risks are more moderate.

In tables VI and VII present the distributional implications for the model economies under benchmark policies while comparing their estimates with actual American and Swedish data. Here we decide to compare the two countries by focusing on the Gini coefficients for wealth, earnings and total income. The first thing we denote is that wealth inequalities are larger than earning inequalities, which are larger than total income inequalities in both countries. These results are in line with the literature arguing wealth is more concentrated than income. Secondly, if the gini on wealth is similar across the US and Sweden, the indexes on earnings and income are significantly lower in the latter. Indeed, the gini on total income (which includes transfers) is 24 points lower in Sweden. This difference can be explained by a lower income risk in the country, mainly due to strong syndicates protecting employment (and labor earnings) as well as a more generous transfer system. On the other hand, because of weak labor protection and lower transfers, Americans would be exposed to greater earnings and income risk, which translates in higher levels of inequalities.

TABLE VI
Distributional Implications—United States

	Gini	Percent of total: Bottom 40%	Top 20%	Top 10%	Top 1%
Wealth					
Actual U.S. data	0.78	1.4	79.5	66.1	29.5
Model, benchmark policy	0.65	2.3	65.2	44.5	8.6
Model, optimal policy	0.66	1.9	66.3	45.5	8.8
Earnings					
Actual U.S. data	0.63	2.8	61.4	43.5	14.8
Model, benchmark policy	0.50	10.1	53.5	34.2	6.7
Model, optimal policy	0.54	7.6	56.7	36.5	7.3
Total income					
Actual U.S. data	0.57	8.8	59.9	45.2	18.6
Model, benchmark policy	0.42	14.6	48.4	30.3	5.8
Model, optimal policy	0.42	14.7	48.5	30.4	5.9

Note. U.S. data adapted from Díaz-Giménez, et al. (1997). $\tau^h = 0.36$ under benchmark policy and $\tau^h = 0.46$ under optimal policy. Earnings are defined as net labor income before taxes. Total income is defined as net factor income plus transfers but before taxes. Note that U.S. data refer to households while the income process in the model is calibrated to match individual wage processes.

TABLE VII
Distributional Implications—Sweden

	Gini	Percent of total: Bottom 40%	Top 20%	Top 10%	Top 1%
Wealth					
Actual Swedish data	0.79	−6	72	49	13
Model, benchmark policy	0.60	4	60	39	6
Model, optimal policy	0.58	5	58	38	6
Earnings					
Actual Swedish data	0.48	8	47	29	5
Model, benchmark policy	0.43	12	46	28	4
Model, optimal policy	0.32	19	39	23	3
Total income					
Actual Swedish data	0.33	19	37	14	5
Model, benchmark policy	0.28	22	38	21	3
Model, optimal policy	0.28	22	37	21	3

Note. Swedish data adapted from Domeij and Klein (1998). $\tau^h = 0.57$ under benchmark policy and $\tau^h = 0.27$ under optimal policy. Earnings are defined as net labor income before taxes. Total income is defined as net factor income plus transfers but before taxes. Note that Swedish data refer to households while the income process in the model is calibrated to match individual wage processes.

MACRO 3

PS5

Exercise: Infinite horizon and small-heterogeneity model

1. Program of an agent i and Bellman equation of each class of agent.

$$\max_{c_t^i, a_{t+1}^i, l_t} \sum_{t=0}^{\infty} \beta^t \left(u(c_t^i) - l_t^i \right)$$

$$\text{s.t.} \quad a_{t+1}^i + c_t^i = (1+r) a_t^i + e_t^i w l_t^i + (1-e_t^i)\delta$$

$$a_t^i \geq \bar{a} \qquad \text{and } \bar{a} > \frac{-\delta}{r}$$

Which gives the following Bellman equations:

Employed: $$V^e(a) = \max_{c, a', l} u(c) - l + \beta(\alpha V^e(a') + (1-\alpha) V^u(a')) + \mu\left(a' + \frac{\delta}{r}\right)$$

$$\text{s.t.} \quad a' + c = (1+r) a + wl$$

$$\left(a \geq \bar{a} \right) \text{ and } \bar{a} > \frac{-\delta}{r})$$

Unemployed: $$V^u(a) = \max_{c, a', l} u(c) + \beta((1-\rho) V^e(a) + \rho V^u(a)) + \mu\left(a' + \frac{\delta}{r}\right)$$

$$\text{s.t.} \quad a' + c = (1+r) a + \delta$$

$$\left(a \geq \bar{a} \quad \text{and } \bar{a} > \frac{-\delta}{r} \right)$$

$$(l = 0)$$

2. Euler condition for both agents:

Employed:

If we integrate the constraint into the Bellman, replacing for c, we get the following FOCs:

$$\frac{\partial V^e(a)}{\partial \ell} = 0 \Leftrightarrow u'(c) = \frac{1}{w}$$

$$\frac{\partial V^e(a)}{\partial a'} = 0 \Leftrightarrow u'(c) = \beta \left(\alpha V_1^{e}(a') + (1-\alpha) V_1^{u}(a')\right) + \mu$$

Envelop: $$\frac{\partial V_1^e(a)}{\partial a} = (1+r)\, u'(c) = \frac{1+r}{w}$$

$$\Rightarrow V_1^e(a') = \frac{1+r}{w}$$

Unemployed:

$$\frac{\partial V^u(a)}{\partial a'} = 0 \Leftrightarrow \beta\left((1-\rho) V_1^{u}(a') + \rho V_1^{u}(a')\right) + \mu$$

Envelop: $$\frac{\partial V^u(a)}{\partial a} = (1+r)\, u'(c)$$

$$\Rightarrow V_1^u(a') = (1+r)\, u'(c')$$

Euler of the employed: $$\frac{1}{w} = \beta(1+r)\left(\frac{\alpha}{w} + (1-\alpha)\, u'(c')\right) + \mu$$

- of the unemployed: $$u'(c) = \beta(1+r)\left(\frac{1-\rho}{w} + \rho\, u'(c')\right) + \mu$$

3. There are three consumption levels in equilibrium, one where the agent is employed, one where they just fell in unemployment and one where they have been unemployed for more than one period:

$$c^e = l w + (1+r)a - a'$$

$$c^{eu} = \delta + (1+r)a^e$$

$$c^{uu} = \delta + (1+r)a^e$$

Note that we derive these levels of consumption from the fact only save when they are employed: $a'^u = 0$.

4. For the employed agent to be borrowing constrained, they would need to be unsatisfied with their level of consumption such that they want to consume more but their incomes are insufficient to finance both the desired level of consumption and minimum savings.
Formally:

$$\frac{1}{w} > \beta(1+r)\left(\frac{\alpha}{w} + (1-\alpha)u'(\delta+(1+r)a^e)\right)$$

5. For the unemployed agent to be credit constrained, they need to hit the lower bound $a' = -\bar{a}$, such that their wealth and home production are insufficient to finance their desired consumption.
Formally:

$$u'(\delta + (1+r)a^e) > \beta(1+r)\left(\frac{1-\rho}{w} + \rho u'(\delta)\right)$$

for the newly unemployed,
and

$$u'(\delta) > \beta(1+r)\left(\frac{1-\rho}{w} + \rho u'(\delta)\right)$$

for the long unemployed.

6. WTS employed agents consume the same amount when they are not borrowing constrained: $\mu = 0$.
From the Euler equation:

$$\frac{1}{w} = \beta(1+r)\left(\frac{\alpha}{w} + (1-\alpha)u'(\delta + (1+r)a^e)\right)$$

Rearranging:

$$\left(\frac{1}{w}\frac{1}{\beta(1+r)} - \frac{\alpha}{w}\right)\frac{1}{1-\alpha} = u'(\delta + (1+r)a^e)$$

$$\delta + (1+r)a^e = u'^{-1}\left[\left(\frac{1}{w\beta(1+r)} - \frac{\alpha}{w}\right)\frac{1}{1-\alpha}\right]$$

$$a^e = \left[u'^{-1}\left[\left(\frac{1}{w\beta(1+r)} - \frac{\alpha}{w}\right)\frac{1}{1-\alpha}\right] - \delta\right]\frac{1}{1+r}$$

a^e is constant, so the employed agent always save the same amount. Moreover, marginal utility of labor is constant and wage is exogenous such that labor income is steady over time.
As a result, the employed agent will always take the same consumption decisions, and they save to self-insure against the risk of becoming unemployed.

7. We start from the transition matrix : $T = \begin{pmatrix} \alpha & 1-\alpha \\ 1-\rho & \rho \end{pmatrix}$

Using the law of large numbers, we compute n^e, the number of employed agents:

$$(n^e, 1-n^e) = (n^e, 1-n^e)\begin{pmatrix} \alpha & 1-\alpha \\ 1-\rho & \rho \end{pmatrix}$$

$$\Rightarrow n^e = \alpha n^e + (1-\rho)(1-n^e)$$

$$n^e = \frac{1-\rho}{2-\alpha-\rho}$$

From there, we deduce the number of newly unemployed:
$n^{eu} = (1-\alpha)n^e$ and of long-term unemployed : $n^{uu} = 1-n^e-n^{eu}$

8. Unemployed agents are credit constrained, so $a^u = 0$.
Thus, the total savings in the economy are done by employed agents : $A_t = n^e_t a^e_t$

$$\Rightarrow A = \frac{1-\rho}{2-\alpha-\rho} \cdot \frac{1}{1+r}\left[u'^{-1}\left[\frac{1}{1-\alpha}\left(\frac{1}{\beta w(1+r)} - \frac{\alpha}{w}\right]\right]-\delta\right]$$

9. $\frac{\partial n^e}{\partial \alpha} > 0$ so the fraction of employed agents in the population increases.
As a result, aggregate savings increase (more people save).

The effect of α on a^e is more ambiguous.
From the formula for a^e we previously recovered, we see savings of the employed might be both increasing and decreasing with α.
This ambiguous effect on individual savings might be the result of :
- a negative substitution effect (agents face less risk of becoming unemployed and decrease precautionary savings).
- a positive income effect (agents see their expected lifetime income increase since they are less likely to fall unemployed, and they might save more as a result).

The effect of α on individual savings will then depends on the strenght of these two opposite forces. However, note that in this case we expect a stronger substitution effect due to the form of a^e!

Thus the effect of α on A will be ambiguous, with a positive income effect due to n^e and a potential substitution effect due to a^e.