School based assessment

Additional mathematics

***The use of calculus in the design of a water tank for Frankfield, Jamaica***

r

h

**Name of Candidate: XXXXXXX**

**Candidate Number:**

**Centre Number:XXXXXXX**

**Teacher’s Name:XXXXXXXX**

**Territory: Jamaica**

**Year: XXXX**

Title page

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Acknowledgement

I would first of all like to thank my teacher, Mr. Hendricks, for allowing me to undertake this project for my Additional Mathematics School Based Assessment. Embarking this project has helped to ‘open my eyes’ to the linkage between mathematics and the real world. It has also helped be to better understand the uses of calculus. I would also like to thank my parents who gave me their constant support in my endeavors, as usual. Last but certainly not least, I would like to thank God, who continually strengthens me, allowing me to complete this task as well as many others. I extend my gratitude to all who helped, no matter how great or small.

Aim

1. To satisfy the requirements of the Additional Mathematics School Based Assessment for the CSEC (Caribbean Secondary Education Certificate).
2. To apply mathematical concepts, skills and procedures from a particular topic in order to understand, describe, or explain a real world phenomenon.

*Statement of problem:*

Frankfield, a community in rural Jamaica, requested that a closed, cylindrical water tank, be built for its residents. The tank they desire must be capable of holding 3.6 x106 cm3 of water and is to be manufactured from sheets of stainless metal of negligible thickness. The surface area must be known and also the dimensions of the tank so that the area of the sheet of metal for the project can be determined and hence purchased. It is required to determine the dimensions of a tank which satisfies the capacity specified and requires the minimum amount of sheet material.

*Important Elements to note:*

* The volume and surface area will be used to come up with the mathematical model.
* The minimum surface area will result in minimum cost for constructing the tank.
* Extra material must be added to compensate for any potential errors and flaws.

Method of solving the Problem

It is given that the tank must be cylindrical in shape. Therefore, the design and dimensions of the tank must be done with respect to the radius of the cylinder and the height of the cylinder.

For the ease of notation; let height of the cylinder be , and let the radius of the cylinder be*r*

The volume of a cylinder can be expressed as the product of the area of its circle andits height. Therefore, the volume will be represented by the formula: , where represents pi and will be used as throughout the calculations.

The total surface area of the closed cylinder can be expressed as the area of its 2 circles plus the area of what is essentially a rectangular sheet, making up its height. The width of the rectangle is equal to the circumference of the circle, while the length is equal to the height of the cylinder. Therefore, the total surface area will be represented using the formula:

There are two unknowns in the formula, in the form of the height and the radius. To make solving easier, one of the variables need to be eliminated. Since the volume which the tank needs to hold, was given as 3.6 x106 cm3, the formula for the volume can be rewritten as:

This equation can then be transposed, making the subject:

Makingthe subject is done so that we can substitute the formula for into the formula for the total surface area, thereby eliminating one of the unknowns. Hence:

[The mathematical model]

Calculus must then be used to find the first derivative of the expression as it is assumed that in order to find the minimum surface area, the derivative of the expression must be equated to zero (0). Essentially, will be used to find the minimum surface area required.

Afterwards, calculus will again be used to prove that the mathematical model is truly a minima function. It is given:

To satisfy that a function is a minimum, the second derivative of the function must be greater than 0. Therefore,will prove that the mathematical model is a minima.

Diagram of proposed water tank

Solution of the problem

**Note**

**r - Radius**

**h - Height**

**Volume = 3,600,000cm3**

hcm

r cm

The calculations are done so as to find the value of the minimum total surface area of the tank.

* Let the total surface area of the cylinder =
* Let the height of the cylinder =
* Let the radius of the cylinder =
* Let pi () =

From the formulation given above:

For ease of calculating the derivative of the expression it is broken down into two parts to work with individually.

The derivative of can be found using the product rule:

The derivative of can be found using the quotient rule:

The derivative of the expression is

It is assumed that equating this expression to zero (0) will enable the minimum surface area to be found.

The value of the radius needed to cause the minimum surface area to be achieved, was found to be 83.045 cm.

Having found the value of the last unknown in the equation, it can be used in the equation to find the total surface area of the cylinder.

(Substituting the value of in the equation)

The minimum total surface area of the cylinder is foundto be.

The height of the cylinder can also be found using the known radius:

The height of the cylinder with a radius of 83.045 cm is 166.093 cm.

When the radius is 83.045 cm and the height is 166.093 cm, the total surface area of the cylinder is 130049.235 cm which is assumed to be the minimum.

Proofing:

As stated previously in the method of solving the problem, the mathematical model being a minimum is only true if the second derivative of the function is greater than 0.

Again, for ease of calculation and derivation, the expression will be broken into two parts:

4πr can be derived using the product rule:

The derivative of can be found using the quotient rule:

Therefore,

Substituting the value of r3which we know from previous calculations:

The mathematical model satisfied the requirement of having its second derivative being greater than zero (0), thus proving it is truly a minimum function.

To aidin provingwhether or not the value obtained by the calculations for the minimum surface area is true, an excel spreadsheet was used. The formula for the total surface area was used along with a range of possible radii for the cylinder (including the one obtained by the calculations). The spreadsheet determines the corresponding total surface area that each radius would need in order to satisfy the given volume. The range or radii contained values greater and less than the value obtained in calculations. The results are shown in the table below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Radius (cm) | | Height (cm) | | Volume (cm3) | Total surface Area (cm2) |
| 81.000 | 174.585 | | 3600000 | | 130129.460 |
| 82.000 | 170.353 | | 3600000 | | 130070.021 |
| 83.005 | 166.253 | | 3600000 | | 130049.266 |
| 83.015 | 166.213 | | 3600000 | | 130049.252 |
| 83.025 | 166.173 | | 3600000 | | 130049.243 |
| 83.035 | 166.133 | | 3600000 | | 130049.237 |
| 83.045 | 166.093 | | 3600000 | | 130049.235 |
| 83.055 | 166.053 | | 3600000 | | 130049.236 |
| 83.065 | 166.013 | | 3600000 | | 130049.242 |
| 83.075 | 165.973 | | 3600000 | | 130049.251 |
| 83.085 | 165.933 | | 3600000 | | 130049.264 |
| 83.095 | 165.893 | | 3600000 | | 130049.281 |
| 84.000 | 162.388 | | 3600000 | | 130066.286 |
| 85.000 | 158.540 | | 3600000 | | 130120.168 |

*Table 1: Showing the proof of the minimum total surface area*

As shown in the table, the smallest surface area was generated by the 83.045 cm radius found in the calculations. The table proves that the radius, for which the total surface area is a minimum, converges to the point 83.045 cm. That minimum value for the surface area is 130049.235 cm2.

Final results

Final diagram proposed

**Note**

**r - Radius**

**h - Height**

**Volume = 3,600,000cm3**

**Total Surface Area = 130049.235 cm2**

h = 166.093 cm

r = 83.045 cm

Conclusion

Mathematical concepts, skills and procedures were successfully used in determining the dimensions of a tank as well as the minimum material which was needed to construct it. The mathematical model is practical and the assumptions made in the calculations were proven true through the use of calculus and an Excel spreadsheet. The dimensions of the cylindrical tank were successfully calculated to be 83.045 cm in radius and in height. The minimum total surface area was calculated to be 130049.235 cm2. The minimum value of the total surface area represents the least amount of sheet material which needs to be purchased. It hence also reflects the minimum cost for construction.

The criteria/requirements given were achieved. However, for a community, the capacity of the proposed water tank is small. If the proposed model is approved, then multiple tanks may have to be built to satisfy the needs of the community, depending on the number of residents.