

# Statistical Assessment of Optimal Drug Combination

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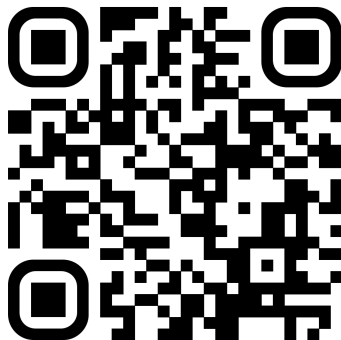


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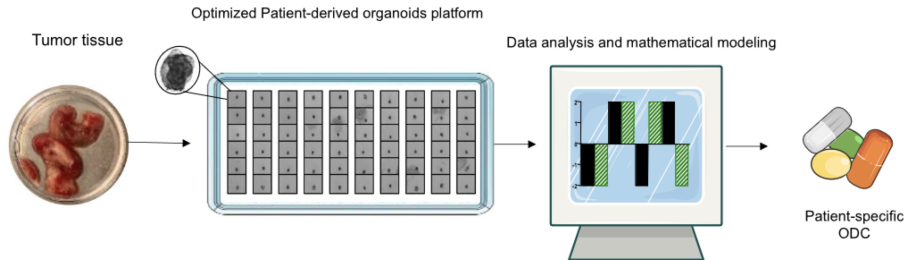


Access the material:



[https://github.com/lionelvoirol/stat\\_modelling\\_drug\\_combination](https://github.com/lionelvoirol/stat_modelling_drug_combination)

# Context



Adapted from Ramzy et al., Journal of Experimental & Clinical Cancer Research, 2023

# Drug combination

- Optimized Drug Combination (ODC) for disease treatment.
- The identification of ODC: target disease/malignant cells while causing a minimal harm to healthy cells.
- This is a complicated task from a statistical perspective. Currently **no standard statistical procedures** exist.
- Example of existing methods: **Therapeutically Guided Multidrug Optimization**<sup>1,2,3</sup> mainly developed by Prof. Nowak-Sliwinska's group.
- Experiments allow to test a measure of interest (quantifying how well a drug works) for different drug combinations recorded over multiple replicates.

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<sup>1</sup>Weiss et al., Cancers, 2019; <sup>2</sup>Zoetemelk et al., Molecular Oncology, 2023; <sup>3</sup>Ramzy et al., Journal of Experimental & Clinical Cancer Research, 2023

# Example

- Goal: Investigate the marginal treatment effects of three drugs (i.e., drug  $A$ ,  $B$  and  $C$ ).
- Record a measure of interest for multiple replicates which have been exposed to one of the treatments.
- As an example, consider 3 replicates per treatment, with only one drug applied in each treatment  $\implies 3 \text{ replicates} \times 3 \text{ drugs} = 9 \text{ experiments}$ .

ID	Measure	Treatment	Replicate	Drug $A$	Drug $B$	Drug $C$
1	$Y_1$	$A$	1	1	0	0
2	$Y_2$	$A$	2	1	0	0
3	$Y_3$	$A$	3	1	0	0
4	$Y_4$	$B$	1	0	1	0
5	$Y_5$	$B$	2	0	1	0
6	$Y_6$	$B$	3	0	1	0
7	$Y_7$	$C$	1	0	0	1
8	$Y_8$	$C$	2	0	0	1
9	$Y_9$	$C$	3	0	0	1

# Estimating marginal treatment effects

- Intuitive approach: Consider a suitable location estimator (e.g., the sample mean) of the measure **for each treatment**.
- Estimated effect of drugs:

$$\bar{Y}_A = \frac{Y_1 + Y_2 + Y_3}{3}, \quad \bar{Y}_B = \frac{Y_4 + Y_5 + Y_6}{3}, \quad \bar{Y}_C = \frac{Y_7 + Y_8 + Y_9}{3}.$$

# Estimating marginal treatment effects

Alternative approach: linear regression

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ \vdots & \vdots & \vdots \\ x_{n1} & x_{n2} & x_{n3} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$
$$\mathbf{Y} = \mathbf{X} \boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

- $n$ : number of experiments.
- $\mathbf{Y}$ : vector of measurements.
- $\mathbf{X}$ : design matrix, where  $x_{ij} = 1$  if drug  $j$  is applied in experiment  $i$ , and  $x_{ij} = 0$  otherwise.
- $\boldsymbol{\beta}$ : vector of unknown parameters.
- $\boldsymbol{\varepsilon}$ : zero mean random vector which, for example, follows a  $\mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}_n)$  where  $\sigma^2$  is an unknown *nuisance* parameter.

# Estimating marginal treatment effects

In this example, the model can be written as:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_A \\ \beta_B \\ \beta_C \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_9 \end{bmatrix}$$
$$Y = X\beta + \varepsilon$$



# Estimating marginal treatment effects

Define  $\hat{\beta}_{OLS}$  as the Ordinary Least Square (OLS) estimator of  $\beta$ :

$$\hat{\beta}_{OLS} = \underset{\beta}{\operatorname{argmin}} \|\mathbf{Y} - \mathbf{X}\beta\|_2^2 = \underset{\beta}{\operatorname{argmin}} \sum_{i=1}^n (Y_i - \mathbf{x}_i^T \beta)^2 = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y}.$$

Is this approach equivalent to the sample mean?

# Estimating marginal treatment effects

$$\hat{\beta}_{\text{OLS}} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y} = \left( \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \right)^{-1}$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \\ Y_6 \\ Y_7 \\ Y_8 \\ Y_9 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} \sum_{i=1}^3 Y_i \\ \sum_{i=4}^6 Y_i \\ \sum_{i=7}^9 Y_i \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} \sum_{i=1}^3 Y_i \\ \sum_{i=4}^6 Y_i \\ \sum_{i=7}^9 Y_i \end{bmatrix} = \begin{bmatrix} \bar{Y}_A \\ \bar{Y}_B \\ \bar{Y}_C \end{bmatrix}$$

# A more realistic design

- In practice: typically more than 3 drugs and consider different dosages.
- **Objective:** find a combination of  $k$  drugs among  $h$  drugs. Each drug can have  $m$  dosages. (We assume same number of dosages for all drugs for simplicity.)
- **Example:** Suppose we want to investigate the effect of all treatments composed of  $k = 2$  drugs among a total of  $h = 3$  drugs (i.e., drug  $A, B, C$ ), where each drug can be administrated at  $m = 2$  dosages (e.g., low dosage encoded as 1 or high dosage encoded as 2).
- How many possible combinations will we consider?

# A more realistic design

- 3 possible combinations of drugs (i.e.,  $AB$ ,  $AC$  and  $BC$ ).
- 4 possible combinations of dosages (i.e.,  $\{1, 1\}$ ,  $\{1, 2\}$ ,  $\{2, 1\}$ ,  $\{2, 2\}$ ).
- Therefore, we have 12 combinations in total (i.e.,  $A1B1$ ,  $A1B2$ ,  $A2B1$ ,  $A2B2$ ,  $A1C1$ ,  $A1C2$ ,  $A2C1$ ,  $A2C2$ ,  $B1C1$ ,  $B1C2$ ,  $B2C1$ ,  $B2C2$ ).
- Consider 3 replications for each combination, we obtain the following data structure.

ID	Measure	Combination	Replicate	A1	A2	B1	B2	C1	C2
1	$Y_1$	$A1B1$	1	1	0	1	0	0	0
2	$Y_2$	$A1B1$	2	1	0	1	0	0	0
3	$Y_3$	$A1B1$	3	1	0	1	0	0	0
4	$Y_4$	$A1B2$	1	1	0	0	1	0	0
5	$Y_5$	$A1B2$	2	1	0	0	1	0	0
6	$Y_6$	$A1B2$	3	1	0	0	1	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
33	$Y_{33}$	$B2C1$	3	0	0	0	1	1	0
34	$Y_{34}$	$B2C2$	1	0	0	0	1	0	1
35	$Y_{35}$	$B2C2$	2	0	0	0	1	0	1
36	$Y_{36}$	$B2C2$	3	0	0	0	1	0	1

# A more realistic design

- In general, how many combinations of  $k$  drugs based on  $h$  drugs at  $m$  dosages can we consider?

$$\text{Number of combinations} = \binom{h}{k} m^k.$$

- Suppose  $k = 4$ ,  $h = 11$  and  $m = 2$ , we have

$$\text{number of combinations} = \binom{11}{4} 2^4 = 5,280,$$

with 3 replications this would correspond to  $3 \times 5,280 = 15,840$  experiments...

- Hence, it quickly becomes impossible to experimentally test all treatments.

# Experimental design

- **Experimental design** is an area of statistics that involves the random assignment of subjects or treatments to different groups.
- Objective: construct a set of treatments that effectively sample the entire spectrum of possible treatments, optimizing the information extracted from the data.
- Given the number of potential treatments to test and the number of experiments to be performed, an experimental design protocol constructs a design matrix  $\mathbf{X}$  that explores “at best” the space of possible treatments.
- The theory of experimental design is rather involved and beyond the scope of this presentation. For simplicity, we use the results of Xu, Jaynes and Ding, 2014<sup>4</sup> to construct  $\mathbf{X}$ .

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<sup>4</sup>Xu et al., Statistica Sinica, 2014

# Parameter estimation

- In our example: we have 5,280 treatments to test.
- Is it possible to estimate the treatment effects of **all combinations**?
- Consider a linear regression model with 5,280 parameters:

$$Y_i = \nu_1 A_{1i} B_{1i} C_{1i} D_{1i} + \nu_2 A_{1i} B_{1i} C_{1i} D_{2i} + \dots \\ + \nu_{5279} H_{2i} I_{2i} J_{2i} K_{1i} + \nu_{5280} H_{2i} I_{2i} J_{2i} K_{2i} + \varepsilon_i$$

where  $\nu_1$  the true efficacy of combination  $A_1 B_1 C_1 D_1$  and

$$A_{1i} = \begin{cases} 1 & \text{if drug } A \text{ at dosage 1 is considered in experiment } i \\ 0 & \text{otherwise} \end{cases}$$

- What is the meaning of  $H_{2i} I_{2i} J_{2i} K_{2i}$ ?
- Is this a good approach?

# Overparameterized model

- Alternative: consider an **overparameterized model** allowing to “separate” the effects of each drug and their interactions.
- In our example, we have  $k = 4$ ,  $h = 11$ , and  $m = 2$ , so we consider

$$\begin{aligned} Y_i = & \underbrace{\beta_1 A_{1i} + \beta_2 A_{2i} + \beta_3 B_{1i} + \dots + \beta_{21} K_{1i} + \beta_{22} K_{2i}}_{\text{marginal effects}} \\ & + \underbrace{\gamma_1 A_{1i} B_{1i} + \gamma_2 A_{1i} B_{2i} + \dots + \gamma_{219} J_{2i} K_{1i} + \gamma_{220} J_{2i} K_{2i}}_{\text{first order interactions}} \\ & + \underbrace{\eta_1 A_{1i} B_{1i} C_{1i} + \eta_2 A_{1i} B_{1i} C_{2i} + \dots + \eta_{1319} I_{2i} J_{2i} K_{1i} + \eta_{1320} I_{2i} J_{2i} K_{2i}}_{\text{second order interactions}} \\ & + \underbrace{\nu_1 A_{1i} B_{1i} C_{1i} D_{1i} + \dots + \nu_{5279} H_{2i} I_{2i} J_{2i} K_{1i} + \nu_{5280} H_{2i} I_{2i} J_{2i} K_{2i}}_{\text{third order interactions}} \\ & + \varepsilon_i \end{aligned}$$

- We define  $\theta = [\beta_1, \dots, \beta_{22}, \gamma_1, \dots, \gamma_{220}, \eta_1, \dots, \eta_{1320}, \nu_1, \dots, \nu_{5280}]$  as the vector of unknown parameters.
- Now we have 1,562 MORE parameters. Why is this a good idea?



# How many interactions?

- We should only consider interactions up to  $(k - 1)$ -th order. Why?

$$\text{Total number of interactions} = \sum_{i=2}^k \binom{h}{i} m^i$$

- In our example, we consider treatments composed of  $k = 4$  drugs, where each drug can be expressed at  $m = 2$  dosages, with a total of  $h = 11$  possible drugs. Therefore, there are  $\sum_{i=2}^4 \binom{11}{i} 2^i = 6,820$  interactions (see coding exercise [example\\_3.R](#)).
- For the full model with all marginal effects plus all interactions:

$$\text{Total number of parameters} = \sum_{i=1}^k \binom{h}{i} m^i$$

# Spare estimation

- Generally all drugs expressed at a specific dosage are assumed to have a non-zero treatment effect. However, **it is reasonable to think that most interactions have zero effect**. But which ones?
- This can be viewed as a **model selection** problem, where the variables to be chosen are the ones encoding significant interactions and the variables encoding the **marginal treatment effects are always included**.
- There are  $2^p$  possible models when considering  $p$  variables.
- For example, with 3 variables ( $X_1$ ,  $X_2$  and  $X_3$ ), we have  $2^3 = 8$  models which can be expressed as:

- 1  $Y_i = \beta_0 + \varepsilon_i$  (The empty model)
- 2  $Y_i = \beta_0 + \beta_1 X_{1i} + \varepsilon_i$
- 3  $Y_i = \beta_0 + \beta_1 X_{2i} + \varepsilon_i$
- 4  $Y_i = \beta_0 + \beta_1 X_{3i} + \varepsilon_i$
- 5  $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \varepsilon_i$
- 6  $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{3i} + \varepsilon_i$
- 7  $Y_i = \beta_0 + \beta_1 X_{2i} + \beta_2 X_{3i} + \varepsilon_i$
- 8  $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \varepsilon_i$

# Best subset selection

- When the number of interactions is relatively small, one approach is to consider **all possible subsets**.
- Procedure:
  - Identify all possible regression models derived from all possible combinations of the interactions.
  - For each model, compute the model performance evaluation metric such as the Akaike information criterion (AIC).
  - Select the *best* model(s) according to the considered criteria.
  - See [example\\_2.R](#) for more details.
- In our example: 6,820 interactions  $\implies 2^{6820}$  possible models.
- As a comparison:  $60 \times 60 \times 24 \times 365 \times 2023 = 63797328000 \simeq 2^{36}$ .
- NP-hard problem: conventional **exhaustive search is impossible**. Some hope with mixed integer optimization algorithms (near-optimal solutions)<sup>5</sup>.

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<sup>5</sup> Bertsimas et al., Annals of Statistics, 2016

# Penalized regression

- We are considering **high-dimensional settings** (in the sense that there are much more parameters  $p$  to estimate than observations  $n$ , or  $p \gg n$ ).
- Very hot (and very hard) topic in statistics.
- A major aspect of statistical and machine learning methods is to reduce complexity (avoid **overfitting**) in order to improve prediction accuracy and interpretability.
- A common strategy to control the risk of overfitting and improve predictive accuracy is to use **regularization** methods. For example, in the linear regression model, we consider:

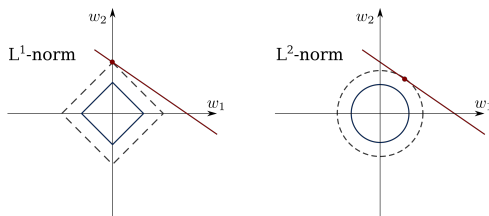
$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \quad \hat{\boldsymbol{\beta}}_{\text{Pen}} = \underset{\boldsymbol{\beta}}{\operatorname{argmin}} \|\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}\|_2^2 + \text{penalty}(\boldsymbol{\beta})$$

- If  $\text{penalty}(\boldsymbol{\beta}) = 0 \implies$  OLS estimator.
- If  $\text{penalty}(\boldsymbol{\beta}) = \lambda \|\boldsymbol{\beta}\|_1 \implies$  Lasso.

# Geometric interpretation

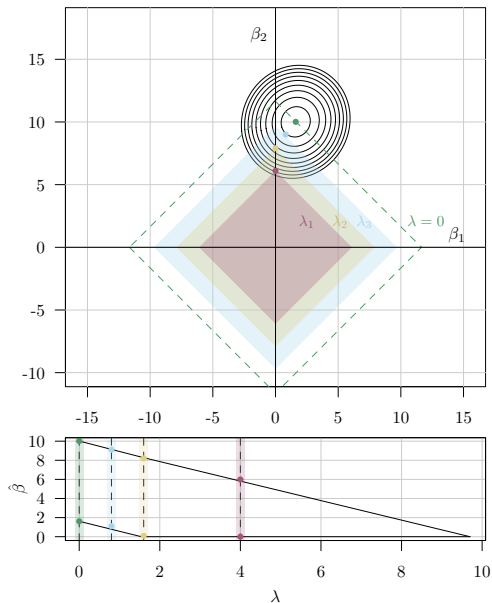
- Lasso (or least absolute shrinkage and selection operator)<sup>6</sup> is a penalization method developed to perform feature selection.
- Lasso considers the  $\ell_1$  norm multiplied by an hyperparameter  $\lambda$  as the **penalty term** added to the usual linear regression cost function.
- Procedure shrinks the coefficients toward zero value, hence performing **feature selection** (often improving predictive performance and interpretability).

$$\hat{\beta}_{\text{Lasso}} = \underset{\beta}{\operatorname{argmin}} \|Y - X\beta\|_2^2 + \lambda \|\beta\|_1$$



<sup>6</sup>Tibshirani, Journal of the Royal Statistical Society Series B, 1996

# Geometric interpretation



# Incorporating domain knowledge: Example in finance

Is it possible to “help” the Lasso by incorporating domain knowledge?

- In finance the **No-Arbitrage** assumption informally means that there cannot be free investments with a positive payoff.
- This assumption is the core of financial models.
- It can be translated into constraints on the admissible models.
- Penalties  $\text{penalty}(\beta)$  encoding these constraints can be constructed<sup>7</sup>.
- Example: given 11 variables (or  $2^{11} = 2048$  models), the penalties result in only 32 models, hence leading to considerable improvements.

$M_1$	✓	×	×	×	×	×	✓	×	×	×	×	×	×	×
$M_2$	✓	✓	×	×	×	×	✓	×	×	×	×	×	×	×
$M_3$	✓	×	×	×	×	×	✓	×	×	×	×	×	×	×
$M_4$	✓	×	×	×	×	×	×	×	×	×	×	×	×	×
$M_5$	✓	✓	×	×	×	×	✓	✓	✓	×	×	×	×	×
$M_6$	✓	✓	✓	×	×	×	✓	×	×	×	×	×	×	×
$M_7$	✓	×	×	×	×	×	✓	×	×	×	×	×	×	×
$M_8$	✓	✓	×	×	×	×	✓	✓	✓	×	×	×	×	×
$M_9$	✓	×	×	×	×	×	×	×	×	×	×	×	×	×
$M_{10}$	✓	×	×	×	×	×	✓	×	×	×	×	×	×	×
$M_{11}$	✓	×	×	×	×	×	×	×	×	×	×	×	×	×
$M_{12}$	✓	✓	×	×	×	×	✓	×	×	×	×	×	×	×
$M_{13}$	✓	✓	×	×	×	×	✓	×	×	×	×	×	×	×
$M_{14}$	✓	×	×	×	×	×	×	×	×	×	×	×	×	×
$M_{15}$	✓	×	×	×	×	×	✓	×	×	×	×	×	×	×
$M_{16}$	✓	×	×	×	×	×	✓	×	×	×	×	×	×	×
$M_{17}$	✓	×	×	×	×	×	✓	×	×	×	×	×	×	×
$M_{18}$	✓	✓	×	×	×	×	✓	×	×	×	×	×	×	×
$M_{19}$	✓	✓	×	×	×	×	✓	×	×	×	×	×	×	×
$M_{20}$	✓	✓	×	×	×	×	✓	×	×	×	×	×	×	×
$M_{21}$	✓	×	×	×	×	×	×	×	×	×	×	×	×	×
$M_{22}$	✓	×	×	×	×	×	×	×	×	×	×	×	×	×
$M_{23}$	✓	×	×	×	×	×	×	×	×	×	×	×	×	×
$M_{24}$	✓	✓	×	×	×	×	×	×	×	×	×	×	×	×
$M_{25}$	✓	✓	×	×	×	×	×	×	×	×	×	×	×	×
$M_{26}$	✓	✓	×	×	×	×	×	×	×	×	×	×	×	×
$M_{27}$	✓	×	×	×	×	×	×	×	×	×	×	×	×	×
$M_{28}$	✓	×	×	×	×	×	×	×	×	×	×	×	×	×
$M_{29}$	✓	×	×	×	×	×	×	×	×	×	×	×	×	×
$M_{30}$	✓	✓	×	×	×	×	×	×	×	×	×	×	×	×
$M_{31}$	✓	✓	×	×	×	×	×	×	×	×	×	×	×	×
$M_{32}$	✓	✓	×	×	×	×	×	×	×	×	×	×	×	×

<sup>7</sup>Bakalli et al., Journal of Econometrics, 2023

# Incorporating domain knowledge

In the context of drug combinations, it is generally believed that:

- All **marginal treatment effects** of drugs are significant.
- Interactions up to  $(k - 1)$ -th order should be included in the model.  
Moreover, the proportion of significant covariates associated with interactions **decreases as we consider higher order interactions**.

To incorporate this information, we consider the adaptive Lasso<sup>8</sup> instead of the Lasso:

$$\hat{\beta}_{\text{AdaLasso}} = \underset{\beta}{\operatorname{argmin}} \|\mathbf{Y} - \mathbf{X}\beta\|_2^2 + \lambda \sum_{j=1}^p w_j |\beta_j|,$$

where  $w_j$  are predetermined (or estimated) weights.

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<sup>8</sup>Zou, Journal of American Statistical Association, 2006



# Incorporating domain knowledge

- The model we consider can be expressed as:

$$\begin{aligned} Y_i = & \underbrace{\beta_1 A_{1i} + \beta_2 A_{2i} + \beta_3 B_{1i} + \dots + \beta_{21} K_{1i} + \beta_{22} K_{2i}}_{\text{marginal effects}} \\ & + \underbrace{\gamma_1 A_{1i} B_{1i} + \gamma_2 A_{1i} B_{2i} + \dots + \gamma_{219} J_{2i} K_{1i} + \gamma_{220} J_{2i} K_{2i}}_{\text{first order interactions}} \\ & + \underbrace{\eta_1 A_{1i} B_{1i} C_{1i} + \eta_2 A_{1i} B_{1i} C_{2i} + \dots + \eta_{1319} I_{2i} J_{2i} K_{1i} + \eta_{1320} I_{2i} J_{2i} K_{2i}}_{\text{second order interactions}} \\ & + \underbrace{\nu_1 A_{1i} B_{1i} C_{1i} D_{1i} + \dots + \nu_{5279} H_{2i} I_{2i} J_{2i} K_{1i} + \nu_{5280} H_{2i} I_{2i} J_{2i} K_{2i}}_{\text{third order interactions}} \\ & + \varepsilon_i \end{aligned}$$

- We can express the penalty term as follows:

$$\lambda \sum_{j=1}^p w_j |\theta_j| = \frac{\lambda}{W_1} \|\beta\|_1 + \frac{\lambda}{W_2} \|\gamma\|_1 + \frac{\lambda}{W_3} \|\eta\|_1 + \frac{\lambda}{W_4} \|\nu\|_1$$

- It is not straightforward to determine  $W_i$ , with  $i = 1, \dots, 4$ .
- A possible (and intuitive) strategy is to use

$$W_i = W_i^{(1)} W_i^{(2)}$$

- $W_i^{(1)}$ : Empirical proportion of non-zero effect in the  $(i - 1)$ -th order.
- $W_i^{(2)}$ : Average effect of the non-zero effects in the  $(i - 1)$ -th order.

# How to assess the efficacy of a drug combination?

- In our example, we have 5,280 possible treatments (i.e., combinations of drugs at specific dosage).
- The effect of each treatment can be estimated by a **linear combination of the estimated parameters**. For example, for the treatment composed of drugs  $A$ ,  $B$ ,  $C$  and  $D$  at dosage 1 (i.e., treatment  $A1B1C1D1$ ):

$$\hat{Y}_{A1B1C1D1} = \hat{\beta}_1 + \hat{\beta}_3 + \hat{\beta}_5 + \hat{\beta}_7 + \hat{\gamma}_1 + \dots + \hat{\eta}_1 + \dots + \hat{\nu}_1 = \mathbf{z}_1^T \hat{\boldsymbol{\theta}}$$

- Here the  $i$ -th element of  $\mathbf{z}_1$  is either 0 or 1, depending on whether the  $i$ -th parameter of  $\boldsymbol{\theta}$  is associated to treatment 1 (i.e., treatment  $A1B1C1D1$ ).
- We define  $\mathbf{z}_j$  where  $j = 1, \dots, 5280$  for all combinations.
- See [example\\_5.R](#).

# Best combination or best combinations?

- We define the optimal treatment as:

$$\hat{j} = \underset{j \in \{1, \dots, 5280\}}{\operatorname{argmin}} \quad \mathbf{z}_j^T \hat{\boldsymbol{\theta}}$$

- Are there other treatments that are non-distinguishable given the data at hand?  
Probably...
- Since the variance of  $\hat{\boldsymbol{\theta}}$ , say  $\boldsymbol{\Sigma}$ , can be estimated by  $\hat{\boldsymbol{\Sigma}}$ , we have:

$$\operatorname{var}(\mathbf{z}_j^T \hat{\boldsymbol{\theta}}) = \mathbf{z}_j^T \boldsymbol{\Sigma} \mathbf{z}_j \approx \mathbf{z}_j^T \hat{\boldsymbol{\Sigma}} \mathbf{z}_j$$

- Hence, it is possible to compute an upper-tail (asymptotic) confidence interval at the  $(1 - \alpha)$  confidence level:

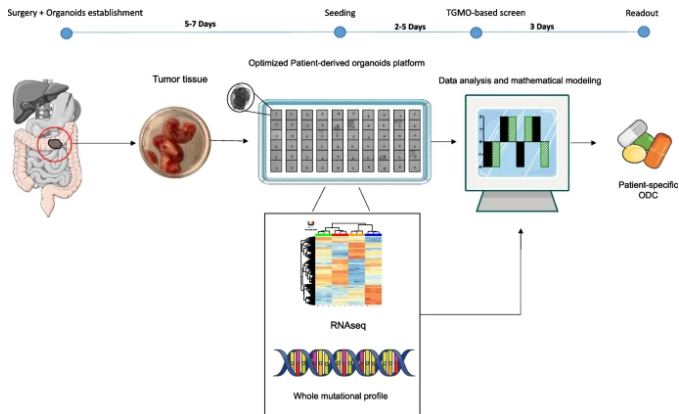
$$\mathbf{z}_j^T \hat{\boldsymbol{\theta}} + z_{1-\alpha} \sqrt{\mathbf{z}_j^T \hat{\boldsymbol{\Sigma}} \mathbf{z}_j}$$

where  $z_{1-\alpha}$  denotes the  $(1 - \alpha)$ -th quantile of the standard normal distribution.

- Consider all treatments satisfying:

$$\mathbf{z}_j^T \hat{\boldsymbol{\theta}} < \mathbf{z}_{\hat{j}}^T \hat{\boldsymbol{\theta}} + z_{1-\alpha} \sqrt{\mathbf{z}_j^T \hat{\boldsymbol{\Sigma}} \mathbf{z}_j}$$

# Example: Colorectal cancer

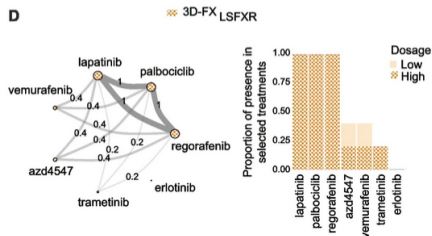
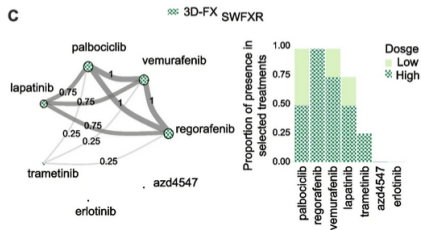
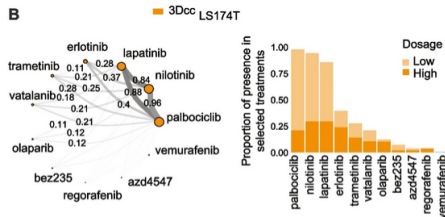
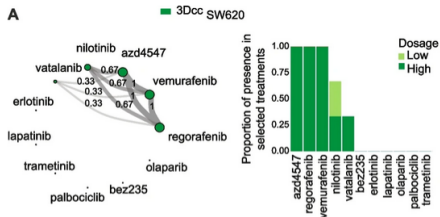


Adapted from Ramzy et al., Platform combining statistical modeling and patient-derived organoids to facilitate personalized treatment of colorectal carcinoma, *Journal of Experimental & Clinical Cancer Research*, 2023

# Example: Colorectal cancer

- Colorectal cancer is the third most diagnosed cancer worldwide with newly diagnosed cases exceeding one million per year.
- The effect of a given treatment (i.e., a combination of drugs) is measured by its **therapeutic window** defined as the difference between healthy cell viability and cancer cell viability.
- We aim to identify treatments with the largest therapeutic windows, i.e., the ones that are highly aggressive toward cancerous cells while having the minimum impact on healthy cells.
- To identify these treatments  $\implies$  **adaptive Lasso approach with weights incorporating domain knowledge.**

# Example: Colorectal cancer

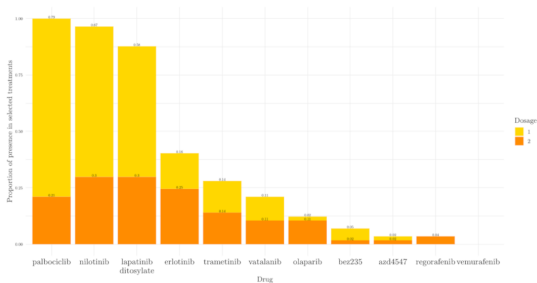


# Example: Colorectal cancer





# Example: Colorectal cancer



# If you want to know more...



## Data Analytics

for Pharmaceutical Sciences

**Chapter 1: A Brief Introduction to R and RMarkdown**

**Chapter 2: Introduction to Statistical Inference**

**Chapter 3: Analysis of Variance**

<https://intro-data-analytics.netlify.app>



## Introduction to Data Science

with R

**Chapter 1: Data Structures**

Learn to organize data in order to use it more efficiently.

**Chapter 2: Control Structures**

Understand how control structures dictate the flow of execution of a sequence of operations.

**Chapter 3: Functions**


Write your own functions.

<https://intro-to-ds.netlify.app/>

# AI helper to simplify coding

In the future, many aspects of data analytics will become **more streamlined due to the advances in AI** (e.g., natural language processing tools). Building on these new development, we created a web application which allows to generate R code:

## Data Analysis AI Helper



### Data Analytics

for Pharmaceutical Sciences

Dataset: diet. To switch, click Reset.

[Reset](#)

**Example requests:**

Boxplot, weight loss for diet B

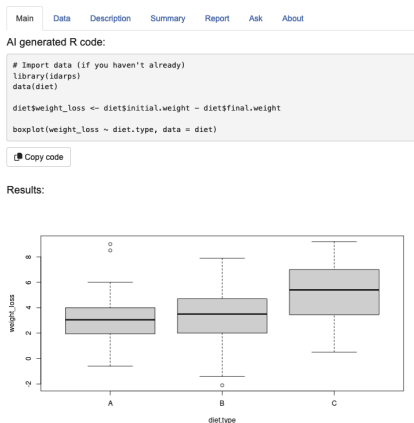
Construct new variable called weight\_loss as initial.weight - final.weight

Make a boxplot for this new variable and compare diet.type

[Re-submit](#) [Copy text](#)

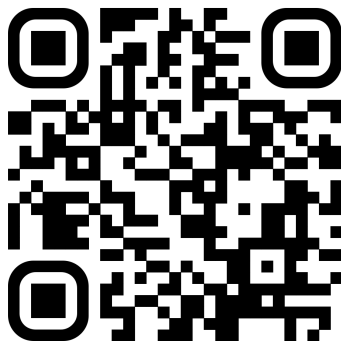
**All code chunks**

2 ☐ Continue from this chunk



<https://data-analytics-lab.shinyapps.io/ai-helper-pharma>

# Thank you very much for your attention!



[https://github.com/lionelvoirol/stat\\_modelling\\_drug\\_combination](https://github.com/lionelvoirol/stat_modelling_drug_combination)



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