as a product of primes.

=> according to well-ordering principle - the smallest element in S, $S_0 = \min(S)$ then S_0 is not a product of primes meaning S_0 is not a prime but a composite number i.e. $S_0 = m \times n$ where $m, n > 1$
=> $m, n < S_0$ so ~~m,n~~ $m, n < \min(S)$

=> $m, n \notin (S)$ but m and n ~~< S0 = m,n~~ ~~are~~ are product of primes since they do not belong to S, but $S_0 = m \times n$ however $S_0 \in S$ thus a contradiction.

Question 10

10) Induction : Base case $n = 2$

$\Rightarrow$ n here has factors 1, 2 and n is a prime number. Suppose for all $k \leq n$ is either prime or can be expressed as a product of a collection of prime factors

Suppose there are $2 \leq c, d \leq n$ in a way that $cd = n+1$

$\Rightarrow$ In the case that $n+1$ is not prime, cd c, d must either be prime or are the product of a set of prime factors, cd is the product of a prime factors

$\Rightarrow$ if c, d does not exists then $n+1$ is prime

$\therefore$ $n+1$ is either prime or can be expressed a the product of prime factors.

Proof by Contradiction :

Suppose that every number $n \in \mathbb{N}$ can be expected as a product of prime numbers
: $S = \{ s \in \mathbb{N} : S$ cannot be expressed

Question 3

De Morgans Law

① $\overline{S_1 \cup S_2} = \overline{S_1} \cap \overline{S_2}$

Say there exist some $t \in \overline{S_1 \cup S_2}$

if $t \in \overline{S_1 \cup S_2}$
$\Rightarrow t \notin S_1 \cup S_2$
$\Rightarrow t \notin S_1$ and $t \notin S_2$
$\Rightarrow t \in \overline{S_1}$ and $t \in \overline{S_2}$
$\Rightarrow t \in \overline{S_1} \cap \overline{S_2}$

② $\overline{S_1 \cap S_2} = \overline{S_1} \cup \overline{S_2}$
Suppose there exist some $s \in \overline{S_1 \cap S_2}$

$\Rightarrow s \in \overline{S_1}$ and $s \in \overline{S_2}$
$\Rightarrow s \notin S_1$ and $s \notin S_2$
$\Rightarrow s \notin S_1$ or $s \notin S_2$
$\Rightarrow s \notin S_1 \cup S_2$
$\Rightarrow s \in \overline{S_1 \cup S_2}$

Question 9

$\Rightarrow 2 - \sqrt{2} = m/n$

$\Rightarrow (2-\sqrt{2})(2-\sqrt{2}) = m^2/n^2$

$\Rightarrow 4 - 2\sqrt{2} - 2\sqrt{2} + 2 = m^2/n^2$

$\Rightarrow 6 - 2\sqrt{2} - 2\sqrt{2} = m^2/n^2$

$\Rightarrow 6 - 4\sqrt{2} = m^2/n^2$

$\Rightarrow -4\sqrt{2} = m^2/n^2 - 6$

$\Rightarrow 6 - m^2/n^2 = 4\sqrt{2}$

Noting that $\sqrt{2}$ is irrational we see that $4 \times \sqrt{2}$ is also irrational. this is a contradiction to a fact that $2-\sqrt{2}$ is rational thus we conclude that $2-\sqrt{2}$ is also irrational

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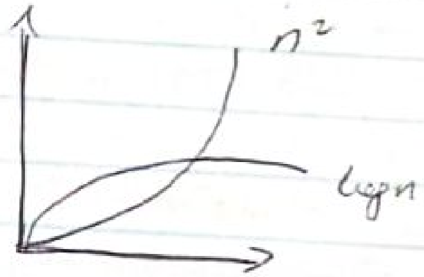
Question 5

a) Given $n^2 + 5 \log n = O(n^2)$
Time Complexity :

It is clear from the given expression above Comparison function we notice that as $n \to \infty$ n^2 will grow faster in an exponential growth than ~~log~~ $\log n$



~~Th~~Therefore n^2 is dominating the Complexity of the function as n grows larger

Question 7

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Using Induction:

$$i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{1(1+1)(2(1)+1)}{6}$$

$$= \frac{6}{6}$$

$$= 1$$

Assuming that this is true for $n = k$, let's show it is true for $k+1$ also.

$$\sum_{i=1}^{k+1} i^2 = \sum_{i=1}^{k} i^2 + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)+(k+1)^2}{6}$$

$$= \frac{(k+1)[(k+2)(2k+3)]}{6}$$

Note:

$k+2$ can be written as $(k+1)+1$

$$\sum_{i=1}^{k+1} i^2 = \frac{(k+1)([((k+1)+1)(2((k+1)+1)])}{6}$$

By induction this is true for $(k+1)$, and $(k+2)$.

Question 1

Show that for a finite Set S $|2^S| = 2^{|S|}$ where 2^S is the powerset of the set S. Suppose that Set S has n elements Such that $|S| = n$.

Assume there exist T such that is associated with a n-tuple i.e $T = \{t_1, t_2, \dots\}$ where for all $p = 1, 2, 3, 4, \dots n$

$$p = \begin{cases} 1 & \text{if } x \in T \\ 0 & \text{otherwise} \end{cases}$$

Therefore $|2^S|$ = number of n tuples in which $p \in \{0, 1\}$
= power of 2, n times
$= 2^n$

$|2^S| = 2^{|S|}$

Question 2

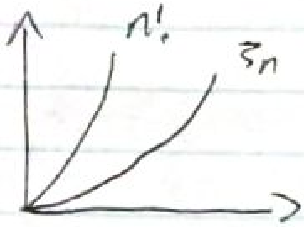
Suppose you're given two disjoint sets A and B. Such that $A \cap B = \emptyset$ Note $|A \cap B| = 0$ and we know

$A \cup B = A + B - A \cap B$
$= |A| + |B| - |A \cap B|$
$= |A| + |B| - 0$
$= |A| + |B|$

Question 5

b) $3^n = O(n!)$

Lets find some constant c and:



$3n$ $3^n = n(n-1)!$

Suppose $n = 1$: $3^1 = n(n-1) = 1(1-1)!$

$= 1$

for $n = 2$

$3^2 = 2(2-1)!$

$9 = 2$

$n = 100$

$100^2 = 100(99)!$

$= 100 \leq 99!$

thus $100 < 99!$

$\therefore 3^n - O(n!)$

Given most higher values the statement c true.

$\therefore 3^n = O(n!)$

$n! = O(n^n)$

Question 8

$\sqrt{3} = \frac{s}{t}$

$3 = s^2/t^2$

$3t^2 = s^2$

$\Rightarrow s^2$ is a multiple of 3 thus is divisible by 3, now let $s = 3p$ for $p \in \mathbb{Z}$

$\therefore \quad 3t^2 = (3p)^2$

$3t^2 = 9p^2$

$t^2 = 3p^2$

$\Rightarrow t^2$ is a multiple of 3 thus is also divisible by 3. Since both s and t are both divisible by 3 we can say this is contradiction meaning they have a common denominator 3.

$\therefore \sqrt{3}$ is irrational.