1. Unfolding + proofing

a) $T(1) = 6$

$T(n) = T\left(\frac{n}{4}\right) + 10$ for $n \geq 2$

$$T(n) = T\left(\frac{n}{4}\right) + 10 =$$
$$= \left(T\left(\frac{\frac{n}{4}}{4}\right) + 10\right) + 10 = T\left(\frac{n}{16}\right) + 20 =$$
$$= \left(T\left(\frac{\frac{n}{16}}{4}\right) + 10\right) + 20 = T\left(\frac{n}{64}\right) + 30$$

$$T(n) = T\left(\frac{n}{4^i}\right) + 10i$$

Base case reached at $i = \log_4 n$:

$$T(n) = T(1) + 10 \cdot \log_4 n = 6 + 10 \cdot \log_4 n = O(\log n)$$

b) <u>Assumption</u>:

$$T(n) = 6 + 10 \cdot \log_4 n$$

<u>Proof by induction</u>:

base: $T(1) = 6 + 10 \cdot \log_4 1 = 6 + 10 \cdot 0 = 6$

induction step: $T(n) = T\left(\frac{n}{4}\right) + 10 = 6 + 10 \cdot \log_4 \frac{n}{4} + 10 =$

$$= 6 + 10 \cdot (\log_4 n - 1) + 10 =$$
$$= 6 + 10 \cdot \log_4 n - 10 + 10 =$$
$$= 6 \cdot 10 \cdot \log_4 n \; \square$$

2. Master Theorem

a) $T(n) = 16 \cdot T(\frac{n}{4}) + n^{0,5}$ $\longrightarrow$ $a = 16$, $b = 4$, $f(n) = n^{0,5}$

$n^{\log_b a} = n^{\log_4 16} = n^2$

Case 1: $f(n) = O(n^{2-\varepsilon})$, $\varepsilon = 1,5 > 0$

Hence: $T(n) = \Theta(n^2)$

b) $T(n) = 32 \cdot T(\frac{n}{4}) + \frac{n^4}{2}$ $\longrightarrow$ $a = 32$, $b = 4$, $f(n) = \frac{n^4}{2} = 0,5 \cdot n^4$

$n^{\log_b a} = n^{\log_4 32} = n^{2,5}$

Case 3: $\bullet\ f(n) = \Omega(n^{2,5+\varepsilon})$, $\varepsilon = 1,5 > 0$

$\& \bullet 32 \cdot (0,5 \cdot (\frac{n}{4})^4) = 16 \cdot \frac{n^4}{256} = \frac{n^4}{16} = \frac{1}{8} \cdot f(n)$, $c = \frac{1}{8} < 1$

Hence: $T(n) = \Theta(n^4)$

c) $T(n) = 27 \cdot T(\frac{n}{3}) + n^3$ $\longrightarrow$ $a = 27$, $b = 3$, $f(n) = n^3$

$n^{\log_b a} = n^{\log_3 27} = n^3$

Case 2: $f(n) = \Theta(n^3 \cdot \log^0 n) = \Theta(n^3)$, $k = 0 \geq 0$

Hence: $T(n) = \Theta(n^3 \log n)$