Solving for α

I was recently going over some code to generate the Euler-Mascheroni constant, γ, and I realized that the algorithm I had implemented used a hard-coded constant. The constant is called α and has an approximate value of 3.5911. The authors of the paper hinted that α could be calculated, and they gave the following relationship:

$\alpha(\ln(\alpha) - 1) = 1$

Admittedly, I was unsure of how to solve for α, so I did what many would do: I asked a mathematician. Here's what Steve came up with:

$\ln(\alpha)-1 = \alpha^{-1}$

Take $\beta = \alpha^{-1}$; then $\ln(\alpha) = -\ln(\beta)$: $-\beta = \ln(\beta)+1$

Shuffle around: $-1 = \ln(\beta)+\beta$

Exponentiate: $e^{-1} = \beta e^{\beta}$ And now we're at precisely the definition of the Lambert W function:

$\beta = W(e^{-1})$, so $\alpha = 1/W(1/e)$.

$$\alpha = \frac{1}{W\left(\frac{1}{e}\right)}$$

This really sold me on the utility of the Lambert W function (thanks Steve!), which is why I implemented it in the Tungsten library. As a bonus, I no longer need to have one weird hard-coded constant in my Euler-Mascheroni code.