

$$\forall x \in V, \quad ||x|| = 0_{\mathbb{K}} \Rightarrow x = 0_V \quad (\text{A.1a})$$

$$\forall x \in V, \forall \lambda \in \mathbb{K}, \quad ||\lambda x|| = |\lambda| ||x|| \quad (\text{A.1b})$$

$$\forall (x, y) \in V^2, \quad ||x + y|| \leq ||x|| + ||y|| \quad (\text{A.1c})$$

$$\forall (x, y, z) \in E^3, \forall (\lambda, \mu) \in \mathbb{K}^2, \quad \langle \lambda x + \mu y ; z \rangle = \lambda \langle x ; z \rangle + \mu \langle y ; z \rangle \quad (\text{A.2a})$$

$$\langle x ; \lambda y + \mu z \rangle = \lambda \langle x ; y \rangle + \mu \langle x ; z \rangle$$

$$\forall (x, y) \in E^2, \quad \langle x ; y \rangle = \langle y ; x \rangle \quad (\text{A.2b})$$

$$\forall x \in E, \quad \langle x ; x \rangle \geqslant 0_{\mathbb{K}} \quad (\text{A.2c})$$

$$\forall x \in E, \quad \langle x ; x \rangle = 0_{\mathbb{K}} \Rightarrow x = 0_E \quad (\text{A.2d})$$

$$\forall x \in E, \quad ||x|| = \sqrt{(x;x)} \quad (A.3)$$

$$\lim_{h \rightarrow 0} \frac{f(u_0 + h) - f(u_0) - Df(u_0) \cdot h}{\|h\|} = 0 \tag{A.4a}$$

$$f(u_0 + h) = f(u_0) + Df(u_0) \cdot h + o(h) \quad , \quad \lim_{h \rightarrow 0} \frac{o(h)}{\|h\|} = 0 \quad (A.4b)$$

$$f'(x_0) = Df(x_0) \quad (A.5)$$

$$\left. \frac{d}{d\lambda} f(u_0 + \lambda h) \right|_{\lambda=0} = \lim_{\lambda \rightarrow 0} \frac{f(u_0 + \lambda h) - f(u_0)}{\lambda} \tag{A.6}$$

$$\forall h \in \mathcal{U}, \quad \lim_{\lambda \rightarrow 0} \frac{f(u_0 + \lambda h) - f(u_0)}{\lambda} = \left. \frac{d}{d\lambda} f(u_0 + \lambda h) \right|_{\lambda=0} = Df(u_0) \cdot h \quad (\text{A.7a})$$

$$\forall h \in \mathcal{U}, \quad f(u + \lambda h) = f(u) + \lambda Df(u) \cdot h + o(\lambda) \quad , \quad \lim_{\lambda \rightarrow 0} \frac{o(\lambda)}{\lambda} = 0 \quad (A.7b)$$

$$f'(x_0) = Df(x_0) \quad (A.8)$$

$$D(f+g)(u) = Df(u) + Dg(u) \quad (\text{A.9})$$

$$D(f \circ h)(u) = Dh(f(u)) \circ Df(u) = Dh(f(u)) \cdot Df(u) \quad (\text{A.10})$$

$$\begin{array}{ccc} f_1 : B_{V_1} & \longrightarrow & B_W \\ u_1 & \longmapsto & f(u_1, u_{02}) \end{array}, \quad \begin{array}{ccc} f_2 : B_{V_2} & \longrightarrow & B_W \\ u_2 & \longmapsto & f(u_{01}, u_2) \end{array} \quad (\text{A.11})$$

$$\boldsymbol{D}_1 f(u) \cdot h_1 = \boldsymbol{D} f(u) \cdot (h_1, 0) \quad (\text{A.12})$$

$$\boldsymbol{D}_2 f(u) \cdot h_2 = \boldsymbol{D} f(u) \cdot (0, h_2) \quad (\text{A.13})$$

$$\boldsymbol{D} f(u) \cdot (h_1, h_2) = \boldsymbol{D}_1 f(u) \cdot h_1 + \boldsymbol{D}_2 f(u) \cdot h_2 \quad (\text{A.14})$$

$$h \in \mathcal{H}, (grad F)(x), h = DF(x) \cdot h \quad (A.15)$$

$$\boldsymbol{F}_{x+h} = \boldsymbol{F}_x + (\textit{grad } F)_x^T H + \boldsymbol{o}(H) \quad , \quad \textit{grad } F_x = \begin{bmatrix} \frac{\partial F}{\partial x_1} \\ \vdots \\ \frac{\partial F}{\partial x_n} \end{bmatrix} \in \mathbb{R}^n \quad (\text{A.16})$$

$$\forall h \in \mathcal{L}^2, DF(x) \cdot h = \langle (grad F)(x), h \rangle = \int (grad F) h \quad (A.17)$$

$$Df(x) = J_x = \frac{df}{dx} = \left[\frac{\partial f}{\partial x_1} \quad \cdots \quad \frac{\partial f}{\partial x_n} \right] = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix} \in \mathcal{M}_{m,n}(\mathbb{R}) \quad (\text{A.18})$$

$$E_{x+h} = E_x + J_x H + o(H) \quad (A.19)$$

$$D F(x) = J_x = \frac{dF}{dx} = \left[\frac{\partial F}{\partial x_1} \quad \cdot \quad \cdot \quad \cdot \quad \frac{\partial F}{\partial x_n} \right] = \nabla F^T \quad (\text{A.20})$$

$$\begin{aligned}
\boldsymbol{D}^2 F(x) = \boldsymbol{H}_x = \frac{d^2 F}{dx}(x) = & \begin{bmatrix} \frac{\partial F_1^2}{\partial x_1^2} & \frac{\partial F_1^2}{\partial x_1 \partial x_2} & \cdots & \frac{\partial F_1^2}{\partial x_1 \partial x_n} \\ \frac{\partial F_1^2}{\partial x_2 \partial x_1} & \frac{\partial F_1^2}{\partial x_2^2} & \cdots & \frac{\partial F_1^2}{\partial x_2 \partial x_n} \\ \vdots & & \ddots & \vdots \\ \frac{\partial F_p^2}{\partial x_n \partial x_1} & \frac{\partial F_p^2}{\partial x_n \partial x_2} & \cdots & \frac{\partial F_p^2}{\partial x_n^2} \end{bmatrix} \in \mathcal{M}_{n,n}(\mathbb{R}) \quad (\text{A.21})
\end{aligned}$$

$$F_{x+n} = F_x + J_x H + \frac{1}{2} H^T H_x H + o(H) \quad (A.22)$$