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Thèse n. 1234 2011
présenté le 12 Mars 2011
à la Faculté des Sciences de Base
laboratoire SuperScience
programme doctoral en SuperScience
École Polytechnique Fédérale de Lausanne

pour l'obtention du grade de Docteur ès Sciences par

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Lausanne, EPFL, 2011

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## Torsion Part I

## Connection Part II

### 6 Bench for HPC

### 6.1 Introduction

In this section aims at providing basic but reliable guidlines to produce fast and mannagable code for our algorithms

[AMR02]

### 6.2 Languages

- Csharp - Julia - C++ - Intel MKL - OpenBLAS

### 6.3 From syntax to processor

A short story about how a code is translated to get machin instructions

#### 6.4 Benchmark

### **Bibliography**

[AMR02] Ralph Abraham, Jerrold E. Marsde, and Tudor Ratiu. Manifolds, Tensor Analysis, and Applications (Ralph Abraham, Jerrold E. Marsden and Tudor Ratiu). 2002.

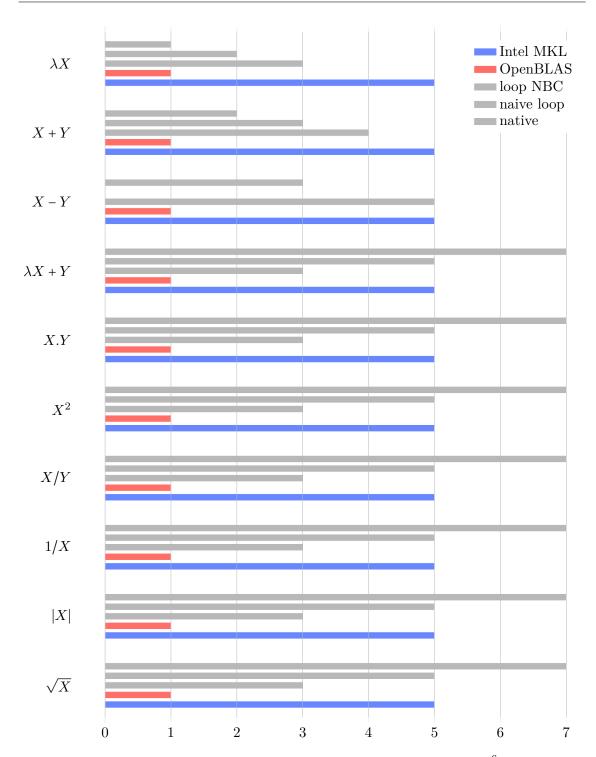


Figure 6.1 – Each operator is evaluated on a vector of Float64 of size  $n = 10^6$  for about 10s. Results are given relatively to MKL performance (MKL = 1).