Modeling of bending-torsion couplings in active-bending structures. Application to the design of elastic gridshell.

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B Parabolic interpolation

B.1 Introduction

In this appendix, we give the required formulas to conduct a parabolic interpolation of a scalar or vector-valued function over an interval.

We look for a polynomial interpolation of order 2 of a continuous scalar or vector-valued function $\boldsymbol{V} : t \mapsto \boldsymbol{V}(t)$ over the interval $[t_0, t_2]$; supposing that the value of the function is known for three distinct parameters $t_0 < t_1 < t_2$:

$$\boldsymbol{V}(t_0) = \boldsymbol{V}_0 \tag{B.1a}$$

$$\boldsymbol{V}(t_1) = \boldsymbol{V}_1 \tag{B.1b}$$

$$\boldsymbol{V}(t_2) = \boldsymbol{V}_2 \tag{B.1c}$$

This interpolation method is employed several times in this thesis, for instance to evaluate the position of a kinetic energy peak during the dynamic relaxation process. It is also employed for evaluating the bending moment and the curvature of a discrete rod at mid-edge, knowing its values at vertices.

Note that this interpolation method is valid if the basis in which $\boldsymbol{V}$ is decomposed does not depend on the parameter t. Otherwise, the classical transportation term should be considered ($\boldsymbol{\omega} \times \boldsymbol{V}$).

B.2 Lagrange interpolating polynomial

The Lagrange interpolation of order two is given by the following polynomial :

$$\boldsymbol{V}(t) = \boldsymbol{V}_0 \frac{(t-t_1)(t-t_2)}{(t_0-t_1)(t_0-t_2)} + \boldsymbol{V}_1 \frac{(t-t_0)(t-t_2)}{(t_1-t_0)(t_1-t_2)} + \boldsymbol{V}_2 \frac{(t-t_0)(t-t_1)}{(t_2-t_0)(t_2-t_1)} \tag{B.2}$$

B.3 Reparametrization

Lets introduce the distances l_0 and l_1 in the parametric space :

$$l_0 = t_1 - t_0 \tag{B.3a}$$
$$l_1 = t_2 - t_1 \tag{B.3b}$$

Lets introduce the change of variable $u = t - t_1$. The polynomial in eq. (B.2) can be rewritten in the form :

$$\boldsymbol{V}(u) = \boldsymbol{V}_0 \frac{u(u - l_1)}{l_0(l_0 + l_1)} - \boldsymbol{V}_1 \frac{(u + l_0)(u - l_1)}{l_0 l_1} + \boldsymbol{V}_2 \frac{u(u + l_0)}{l_1(l_0 + l_1)} \tag{B.4}$$

where :

$$u_0 = -l_0 \tag{B.5a}$$
$$u_1 = 0 \tag{B.5b}$$
$$u_2 = l_1 \tag{B.5c}$$

The derivative of this polynomial is also required to determine the extremum value of $\boldsymbol{V}$. Differentiating eq. (B.4) gives :

$$\boldsymbol{V}'(u) = \boldsymbol{V}_0 \frac{2u - l_1}{l_0(l_0 + l_1)} - \boldsymbol{V}_1 \frac{2u + (l_0 - l_1)}{l_0 l_1} + \boldsymbol{V}_2 \frac{2u + l_0}{l_1(l_0 + l_1)} \tag{B.6}$$

This expression can be factorized to give the more compact form :

$$\boldsymbol{V}'(u) = \left(\frac{\boldsymbol{V}_1 - \boldsymbol{V}_0}{l_0}\right) \frac{l_1 - 2u}{l_0 + l_1} + \left(\frac{\boldsymbol{V}_2 - \boldsymbol{V}_1}{l_1}\right) \frac{l_0 + 2u}{l_0 + l_1} \tag{B.7}$$

B.4 Characteristic values

Using eq. (B.4) the interpolated values of $\boldsymbol{V}$ at mid distance between t_0 and t_1 ($u = -l_0/2$), and at mid distance between t_1 and t_2 ($u = +l_1/2$) are given by :

$$\boldsymbol{V}_{01} = \boldsymbol{V}_0 \frac{l_0 + 2l_1}{4(l_0 + l_1)} + \boldsymbol{V}_1 \frac{l_0 + 2l_1}{4l_1} - \boldsymbol{V}_2 \frac{{l_0}^2}{4l_1(l_0 + l_1)} \tag{B.8a}$$
$$\boldsymbol{V}_{12} = -\boldsymbol{V}_0 \frac{{l_1}^2}{4l_0(l_0 + l_1)} + \boldsymbol{V}_1 \frac{2l_0 + l_1}{4l_0} + \boldsymbol{V}_2 \frac{2l_0 + l_1}{4(l_0 + l_1)} \tag{B.8b}$$

Using eq. (B.7) the interpolated values of $\boldsymbol{V}'$ at mid distance between t_0 and t_1 ($u = -l_0/2$), and at mid distance between t_1 and t_2 ($u = +l_1/2$) are given by :

$$\boldsymbol{V}'_{01} = \frac{\boldsymbol{V}_1 - \boldsymbol{V}_0}{l_0} \tag{B.9a}$$
$$\boldsymbol{V}'_{12} = \frac{\boldsymbol{V}_2 - \boldsymbol{V}_1}{l_1} \tag{B.9b}$$

Remark that this is an interesting result as at these parameters the evaluation of $\boldsymbol{V}'$ boils down to a finite difference scheme.

Using eq. (B.7) and introducing $\alpha = \frac{l_0}{l_0+l_1}$ the interpolated values of $\boldsymbol{V}'$ at t_0, t_1 and t_2 are given by :

$$\boldsymbol{V}_0' = (1+\alpha)\boldsymbol{V}_{01}' - \alpha \boldsymbol{V}_{12}' \tag{B.10a}$$
$$\boldsymbol{V}_1' = (1-\alpha)\boldsymbol{V}_{01}' + \alpha \boldsymbol{V}_{12}' \tag{B.10b}$$
$$\boldsymbol{V}_2' = (\alpha-1)\boldsymbol{V}_{01}' + (2-\alpha) \boldsymbol{V}_{12}' \tag{B.10c}$$

Lets rewrite eq. (B.8a) and (B.8b) with the help of α :

$$\boldsymbol{V}_{01} = \frac{1}{4}\left((2-\alpha)\boldsymbol{V}_0 + \frac{2-\alpha}{1-\alpha}\boldsymbol{V}_1 - \frac{\alpha^2}{1-\alpha}\boldsymbol{V}_2\right) \tag{B.11a}$$
$$\boldsymbol{V}_{01} = \frac{1}{4}\left(-\frac{(1-\alpha)^2}{\alpha}\boldsymbol{V}_0 + \frac{1+\alpha}{\alpha}\boldsymbol{V}_1 + (1+\alpha)\boldsymbol{V}_2\right) \tag{B.11b}$$

B.5 Extremum value

The extremum value of the parabola is obtained for $\boldsymbol{V}'(u^*) = 0$. It's a minimum if $\boldsymbol{V}_{12}' > \boldsymbol{V}_{01}'$ and it's a maximum if $\boldsymbol{V}_{12}' > \boldsymbol{V}_{01}'$:

$$u^* = \frac{l_1 \boldsymbol{V}_{01}' + l_0 \boldsymbol{V}_{12}'}{2(\boldsymbol{V}_{01}' - \boldsymbol{V}_{12}')} \tag{B.12}$$

Remark that if $\boldsymbol{V}_{12}' = \boldsymbol{V}_{01}'$ it does not make sens to compute u^* as in this case the parabola degenerates into a line. The value of the function at this parameter is given by :

$$\boldsymbol{V}(u^*) = \boldsymbol{V}_1 + \frac{(l_1 \boldsymbol{V}_{01}' + l_0 \boldsymbol{V}_{12}')^2}{4(l_0+l_1)(\boldsymbol{V}_{01}' - \boldsymbol{V}_{12}')} \tag{B.13}$$

The parabola in eq. (B.4) now writes :

$$\boldsymbol{V}(u) = -\frac{\boldsymbol{V}_{01}' - \boldsymbol{V}_{12}'}{l_0+l_1}(u-u^*)^2 + \boldsymbol{V}(u^*) \tag{B.14}$$

The extremum is located in $[t_0, t_2]$ if the sign of $\boldsymbol{V}'$ changes on this interval. This condition is satisfied whenever $\boldsymbol{V}_{01}' \cdot \boldsymbol{V}_{12}' < 0$.

Finally, in the special case of a uniform discretization where $l_0 = l_1 = l$, eq. (B.12) and (B.13) become :

$$u^* = \frac{l}{2}\left(\frac{\boldsymbol{V}_0 - \boldsymbol{V}_2}{\boldsymbol{V}_0 - 2\boldsymbol{V}_1 + \boldsymbol{V}_2}\right) \tag{B.15a}$$
$$\boldsymbol{V}(u^*) = \boldsymbol{V}_1 - \frac{u^*}{4l}(\boldsymbol{V}_2 - \boldsymbol{V}_0) \tag{B.15b}$$