$$\forall x \in V, \qquad \|x\| = 0_{\mathbb{K}} \Rightarrow x = 0_V \tag{A.1a}$$

$$\forall x \in V, \forall \lambda \in \mathbb{K}, \qquad \|\lambda x\| = |\lambda| \, \|x\| \tag{A.1b}$$

$$\forall (x, y) \in V^2, \qquad \|x + y\| \leqslant \|x\| + \|y\| \tag{A.1c}$$

$$\forall (x,y,z) \in E^3, \forall (\lambda, \mu) \in \mathbb{K}^2, \qquad \langle \lambda x + \mu y \,;\, z \rangle = \lambda \langle x \,;\, z \rangle + \mu \langle y \,;\, z \rangle \tag{A.2a}$$

$$\langle x \,;\, \lambda y + \mu z \rangle = \lambda \langle x \,;\, y \rangle + \mu \langle x \,;\, z \rangle$$

$$\forall (x,y) \in E^2, \qquad \langle x \,;\, y \rangle = \langle y \,;\, x \rangle \tag{A.2b}$$

$$\forall x \in E, \qquad \langle x \,;\, x \rangle \geqslant 0_{\mathbb{K}} \tag{A.2c}$$

$$\forall x \in E, \qquad \langle x \,;\, x \rangle = 0_{\mathbb{K}} \Rightarrow x = 0_E \tag{A.2d}$$

$$\forall x \in E, \quad \|x\| = \sqrt{\langle x\,;x\rangle} \tag{A.3}$$

$$\lim_{h\to 0} \frac{f(u_0+h) - f(u_0) - \boldsymbol{D}f(u_0)\cdot h}{\|h\|} = 0 \tag{A.4a}$$

$$f(u_0+h) = f(u_0) + \boldsymbol{D}f(u_0)\cdot h + o(h) \quad , \quad \lim_{h\to 0}\frac{o(h)}{\|h\|} = 0 \tag{A.4b}$$

$$f'(u_0) = \boldsymbol{D} f(u_0) \tag{A.5}$$

$$\frac{d}{d\lambda} f(u_0 + \lambda h)\bigg|_{\lambda=0} = \lim_{\lambda \to 0} \frac{f(u_0 + \lambda h) - f(u_0)}{\lambda} \tag{A.6}$$

$$\forall h \in \mathcal{U}, \quad \lim_{\lambda \to 0} \frac{f(u_0 + \lambda h) - f(u_0)}{\lambda} = \frac{d}{d\lambda} f(u_0 + \lambda h)\bigg|_{\lambda=0} = \boldsymbol{D} f(u_0) \cdot h \tag{A.7a}$$

$$\forall h \in \mathcal{U}, \quad f(u+\lambda h) = f(u) + \lambda \boldsymbol{D} f(u_0) \cdot h + o(\lambda) \quad , \quad \lim_{\lambda \to 0} \frac{o(\lambda)}{\lambda} = 0 \tag{A.7b}$$

$$f'(u_0) = \boldsymbol{D} f(u_0) \tag{A.8}$$

$$\boldsymbol{D}(f+g)(u) = \boldsymbol{D}f(u) + \boldsymbol{D}g(u) \tag{A.9}$$

$$\boldsymbol{D}(f \circ h)(u) = \boldsymbol{D}h(f(u)) \circ \boldsymbol{D}f(u) = \boldsymbol{D}h(f(u)) \cdot \boldsymbol{D}f(u) \tag{A.10}$$

$$\begin{array}{rccc} f_1: & \mathcal{B}_{V_1} & \longrightarrow & \mathcal{B}_W \\ & u_1 & \longmapsto & f(u_1, u_{02}) \end{array} \quad , \quad \begin{array}{rccc} f_2: & \mathcal{B}_{V_2} & \longrightarrow & \mathcal{B}_W \\ & u_2 & \longmapsto & f(u_{01}, u_2) \end{array} \tag{A.11}$$

$$\boldsymbol{D}_1 f(u) \cdot h_1 = \boldsymbol{D} f(u) \cdot (h_1, 0) \tag{A.12}$$

$$\boldsymbol{D}_2 f(u) \cdot h_2 = \boldsymbol{D} f(u) \cdot (0, h_2) \tag{A.13}$$

$$\boldsymbol{D} f(u) \cdot (h_1, h_2) = \boldsymbol{D}_1 f(u) \cdot h_1 + \boldsymbol{D}_2 f(u) \cdot h_2 \tag{A.14}$$

$$\forall h \in \mathcal{H}, \quad \langle (grad\ F)(x)\, , h \rangle = \boldsymbol{D}F(x) \cdot h \tag{A.15}$$

$$\boldsymbol{F}_{x+h} = \boldsymbol{F}_x + (grad\ F)_x^T H + \boldsymbol{o}(H) \quad , \quad grad\ F_x = \begin{bmatrix} \frac{\partial F}{\partial x_1} \\ \vdots \\ \frac{\partial F}{\partial x_n} \end{bmatrix} \in \mathbb{R}^n \tag{A.16}$$

$$\forall h \in \mathcal{L}^2, \quad \boldsymbol{D}F(x) \cdot h = \langle (grad\ F)(x)\,, h \rangle = \int (grad\ F) h \tag{A.17}$$

$$\boldsymbol{D}f(x) = \boldsymbol{J}_x = \frac{df}{dx} = \begin{bmatrix} \frac{\partial f}{\partial x_1} & \cdots & \frac{\partial f}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \cdots & \dfrac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial f_m}{\partial x_1} & \cdots & \dfrac{\partial f_m}{\partial x_n} \end{bmatrix} \in \mathcal{M}_{m,n}(\mathbb{R}) \tag{A.18}$$

$$\boldsymbol{F}_{x+h} = \boldsymbol{F}_x + \boldsymbol{J}_x H + \boldsymbol{o}(H) \tag{A.19}$$

$$\boldsymbol{D}F(x) = \boldsymbol{J}_x = \frac{dF}{dx} = \begin{bmatrix} \frac{\partial F}{\partial x_1} & \cdots & \frac{\partial F}{\partial x_n} \end{bmatrix} = \nabla F^T \tag{A.20}$$

$$\boldsymbol{D}^2 F(x) = \boldsymbol{H}_x = \frac{d^2 F}{dx}(x) = \begin{bmatrix} \frac{\partial F_1^2}{\partial x_1^2} & \frac{\partial F_1^2}{\partial x_1 \partial x_2} & \cdots & \frac{\partial F_1^2}{\partial x_1 \partial x_n} \\ \frac{\partial F_1^2}{\partial x_2 \partial x_1} & \frac{\partial F_1^2}{\partial x_2^2} & \cdots & \frac{\partial F_1^2}{\partial x_2 \partial x_n} \\ \vdots & & \ddots & \vdots \\ \frac{\partial F_p^2}{\partial x_n \partial x_1} & \frac{\partial F_p^2}{\partial x_n \partial x_2} & \cdots & \frac{\partial F_p^2}{\partial x_n^2} \end{bmatrix} \in \mathcal{M}_{n,n}(\mathbb{R}) \tag{A.21}$$

$$\boldsymbol{F}_{x+h} = \boldsymbol{F}_x + \boldsymbol{J}_x H + \frac{1}{2} H^T \boldsymbol{H}_x H + \boldsymbol{o}(H) \tag{A.22}$$