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par

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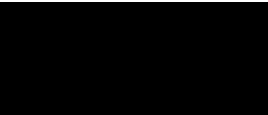
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Torsion Part I

Connection Part II

6 Bench for HPC

6.1 Introduction

In this section aims at providing basic but reliable guidelines to produce fast and mannagable code for our algorithms

[AMR02]

6.2 Languages

- Csharp - Julia - C++ - Intel MKL - OpenBLAS

6.3 From syntax to processor

A short story about how a code is translated to get machin instructions

6.4 Benchmark

Bibliography

[AMR02] Ralph Abraham, Jerrold E. Marsde, and Tudor Ratiu. *Manifolds, Tensor Analysis, and Applications (Ralph Abraham, Jerrold E. Marsden and Tudor Ratiu)*. 2002.

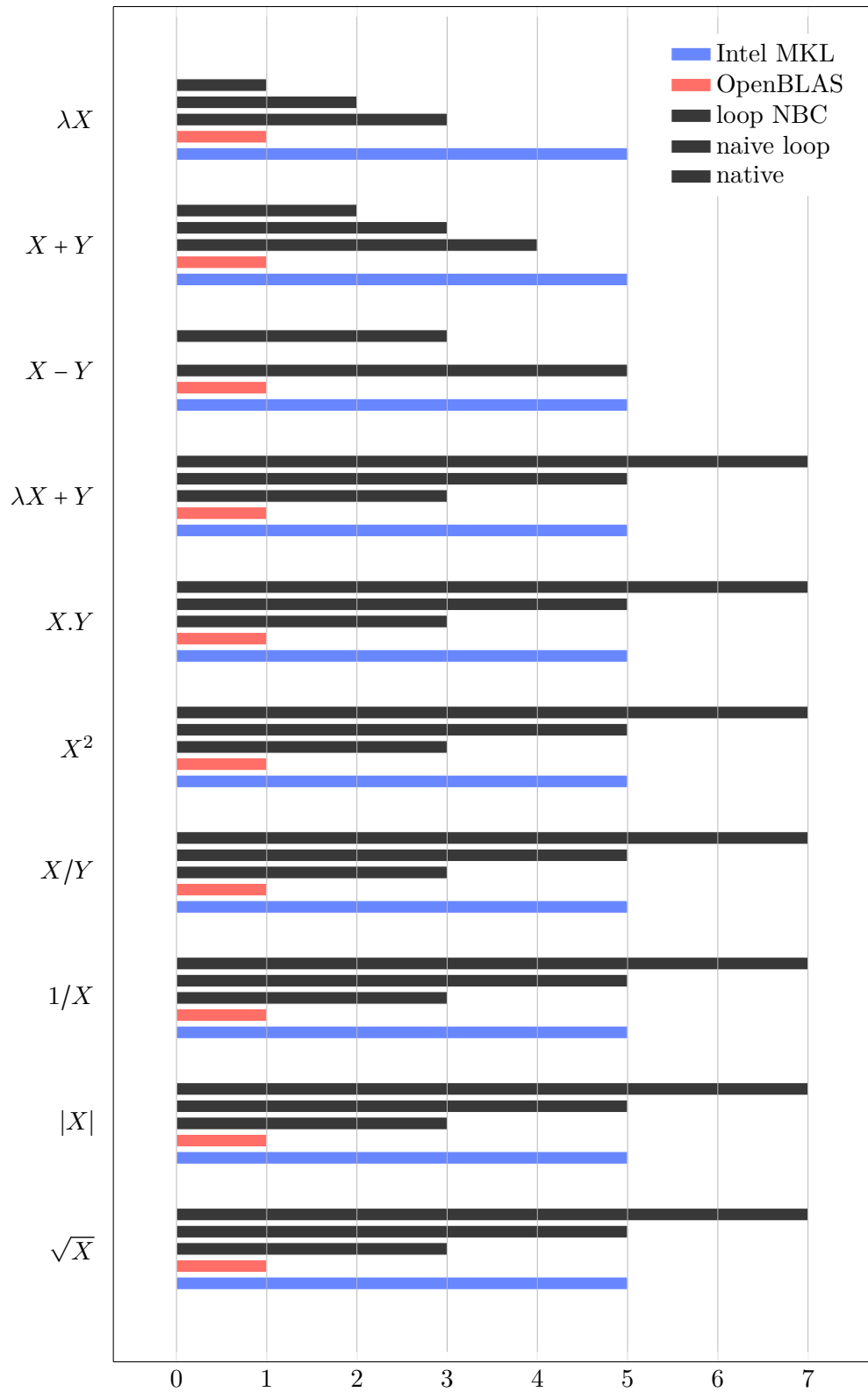


Figure 6.1 – Each operator is evaluated on a vector of Float64 of size $n = 10^6$ for about 10s. Results are given relatively to MKL performance (MKL = 1).