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Modeling of bending-torsion couplings in active-bending structures

Application to the design of elastic gridshells

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“Quia nominor leo.”

A Jacques & Christiane, mes grands-parents bien-aimés.

Préface

> Tu ne peux vivre que de cela que tu transformes, et dont un peu chaque jour, puisque tu t'échanges contre, tu meurs.
>
> Antoine de Saint-Exupéry
> *Citadelle*

Si pour une raison quelconque il ne devait subsister qu'une unique page de ce manuscrit, j'aimerais autant que ce soit celle-là. Et qu'alors, seuls vivent les quelques mots de gratitude qui suivent pour les personnes qui m'ont accompagné sur ce chemin de fortune ; chemin initié en 2010 au sortir de l'Ecole Centrale et qui m'a conduit à présenter cette thèse.

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Lyon, le 4 novembre 2017
Lionel du Peloux

Abstract

An *elastic gridshell* is a freeform structure, generally doubly curved, but formed out through the reversible deformation of a regular and initially flat structural grid. Building curved shapes that may seems to offer the best of both worlds : shell structures are amongst the most performant mechanically speaking while planar and orthogonal constructions are much more efficient and economic to produce than curved ones. This ability to “form a form” efficiently is of peculiar importance in the current context where morphology is a predominant component of modern architecture, and envelopes appear to be the neuralgic point for building performances.

The concept was invented by Frei Otto, a German architect and structural engineer who devoted many years of research to gridshells. In 1975 he designed the Multihalle of Mannheim, a $7500\,\mathrm{m}^2$ wooden shell which demonstrated the feasibility of this technology and made it famous to a wide audience. However, despite their potential, very few projects of this kind were built after this major realization. And for good reason, the resources committed at that time cannot guarantee the replicability of this experiment for more standard projects, especially on the economic level. Moreover, the technics and methods developed by Otto’s team in the 1960s have mostly fall into disuse or are based on disciplines that have considerably evolved. New materials, such as composite materials, have recently emerged. They go beyond the limitations of conventional materials such as timber and offer at all levels much better technical performances for this kind of application. Finally, it should be noted that the regulatory framework has also deeply changed, bringing a certain rigidity to the penetration of innovations in the building industry. Therefore, the design of gridshells arises in new terms for current architects and engineers and comes up against the inadequacy of existing tools and methods.

In this thesis, which marks an important step in a personal research adventure initiated in 2010, we try to embrace the issue of the design of elastic gridshells in all its complexity, addressing both theoretical, technical and constructive aspects. In a first part, we deliver a thorough review of this topic and we present in detail one of our main achievements, the ephemeral cathedral of Créteil, built in 2013 and still in service. In a second part, we develop an original discrete beam element with a minimal number of degrees of freedom adapted to the modeling of bending and torsion inside gridshell members with anisotropic cross-section. Enriched with a ghost node, it allows to model more accurately physical phenomena that occur at connections or at supports. Its numerical implementation is presented and validated through several test cases. Although this element has been developed specifically for the study of elastic gridshells, it can advantageously be used in any type of problem where the need for an interactive computation with elastic rods taking into account flexion-torsion couplings is required.

Keywords : gridshell, form-finding, active-bending, free-form, torsion, elastic rod, coupling, fibreglass, composite material.

Résumé

Les structures de type *gridshell élastique* permettent de réaliser des enveloppes courbes par la déformation réversible d'une grille structurelle régulière initialement plane. Cette capacité à "former la forme" de façon efficiente prend tout son sens dans le contexte actuel où, d'une part la forme s'impose comme une composante prédominante de l'architecture moderne, et d'autre part l'enveloppe s'affirme comme le lieu névralgique de la performance des bâtiments.

Fruit des recherches de l'architecte et ingénieur allemand Frei Otto dans les années 1960, elles ont été rendues populaires par la construction de la Multihalle de Mannheim en 1975. Cependant, en dépit de leur potentiel, très peu de projets de ce type ont vu le jour suite à cette réalisation emblématique qui en a pourtant démontré la faisabilité à grande échelle. Et pour cause, les moyens engagés à l'époque ne sauraient assurer la reproductibilité de cette expérience dans un contexte plus classique de projet, notamment sur le plan économique. Par ailleurs, les techniques et les méthodes développées alors sont pour la plus part tombées en désuétude ou reposent sur des disciplines scientifiques qui ont considérablement évoluées. Des matériaux nouveaux, composites, ont vu le jour. Ils repoussent les limitations intrinsèques des matériaux usuels tel que le bois et offrent des performances techniques bien plus intéressantes pour ce type d'application. Enfin, notons que le cadre réglementaire a lui aussi profondément muté, apportant une certaine rigidité vis-à-vis de la pénétration des innovations. Ainsi la conception des gridshells se pose-t-elle en des termes nouveaux aux architectes et ingénieurs actuels et se heurte à l'inadéquation des outils et méthodes existant.

Dans cette thèse, qui marque une étape importante dans une aventure de recherche personnelle initiée en 2010, nous tentons d'embrasser la question de la conception des gridshells élastiques dans toute sa complexité, en abordant aussi bien les aspects théoriques que techniques et constructifs. Dans une première partie, nous livrons une revue approfondie de cette thématique et nous présentons de façon détaillée l'une de nos principales réalisation, la cathédrale éphémère de Créteil, construite en 2013 et toujours en service. Dans une seconde partie, nous développons un élément de poutre discret original avec un nombre minimal de degrés de liberté adapté à la modélisation de la flexion et de la torsion dans les gridshells constitués de poutres de section anisotrope. Enrichi d'un noeud fantôme, il permet de modéliser plus finement les phénomènes physiques au niveau des connexions et des appuis. Son implémentation numérique est présentée et validée sur quelques cas tests. Bien que cet élément ait été développé spécifiquement pour l'étude des gridshells élastiques, il pourra avantageusement être utilisé dans tout type de problème où la nécessité d'un calcul interactif avec des tiges élastiques prenant en compte les couplages flexion-torsion s'avère nécessaire.

Keywords : gridshell, form-finding, active-bending, free-form, torsion, elastic rod, coupling, fibreglass, composite material.

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Introduction

La paternité des structures de type *gridshell élastique* est couramment attribuée à l'architecte et ingénieur allemand Frei Otto, qui les a intensivement étudiées au XX$^{\text{ème}}$ siècle. Fruit de son travail de recherche, il réalise en 1975, en collaboration avec l'ingénieur Edmund Happold du bureau Arup, un projet expérimental de grande ampleur : la Multihalle de Mannheim [1, 2]. Cette réalisation emblématique ancrera durablement les gridshells dans le paysage des typologies structurelles candidates à l'avènement de géométries non-standard, caractérisées par l'absence d'orthogonalité. Cette capacité à *former la forme* de façon efficiente prend tout son sens dans le contexte actuel où, d'une part la forme s'impose comme une composante prédominante de l'architecture moderne (F. Gehry, Z. Hadid, . . .) et d'autre part l'enveloppe s'affirme comme le lieu névralgique de la performance des bâtiments, notamment environnementale.

Littéralement, le terme *grid-shell* désigne une résille à double courbure dont le comportement mécanique s'apparente à celui d'une coque ; c'est à dire que les efforts y transitent principalement de manière membranaire. Ces ouvrages peuvent franchir de grandes portées en utilisant un minimum de matière. Cependant, il semble plus rigoureux et plus fidèle à l'histoire de désigner par *gridshell élastique* la combinaison indissociable d'un principe structurel – le gridshell, une résille qui fonctionne telle une coque – et d'une méthode constructive astucieuse – la déformation réversible d'une grille de poutre initialement plane pour former une surface tridimensionnelle à double courbure. Le projet de Mannheim – dans lequel une grille en bois de trame régulière, initialement plane et sans rigidité de cisaillement est déformée élastiquement jusqu'à la forme désirée via un dispositif d'étaiement, puis contreventée pour mobiliser la raideur d'une coque et finalement couverte d'une toile – pose les bases de ce nouveau concept et le rend populaire auprès d'un large public d'architectes et d'ingénieurs de par le monde.

Cependant, en dépit du potentiel de cette typologie, très peu de projets ont vu le jour suite à la construction de la Multihalle. Il faut en effet attendre 25 ans et le développement des méthodes de calcul numérique pour voir de nouveau éclore quelques réalisations iconiques : Shigeru Ban innove en passant du bois au carton pour la construction du Pavillon de Hanovre en 2000 [3] ; puis viennent les gridshells en bois de Downland en 2002 [4] et de Savill en 2006 [5] qui reprennent fidèlement les principes développés à Mannheim mais emploient des méthodes constructives différentes. Depuis une dizaine d'années le laboratoire Navier a investi ce champ de recherche sous le double aspect de la structure et du matériau, donnant lieu à la réalisation de quelques prototypes (en 2006 et 2007 [6, 7]) et des deux premiers bâtiments de type gridshell élastique en matériau composite construits à ce jour (Solidays 2011 [8] et Créteil 2013 [9]).[1] Plus récemment, on a pu observer un certain engouement pour la construction de pavillons en bois de petite taille, non couverts, réalisés selon des principes similaires à ceux de la Multihalle, essentiellement dans le cadre de workshops pédagogiques ou bien de projets de recherche [10, 11, 12, 13].

Il est naturel de se demander pourquoi cette innovation prometteuse peine ainsi à essaimer ? S'il est vrai que la construction de la Multihalle de Mannheim a permis de prouver la faisabilité économique et technique du concept de gridshell élastique à grande échelle, il faut bien reconnaître que cette prouesse n'a été rendue possible qu'au terme d'un long processus de maturation pour développer et acquérir l'ensemble des compétences scientifiques, techniques, méthodologiques et humaines nécessaires à sa conception et à sa construction.[2]

En vérité, une telle dépense de moyens pour développer et rassembler ces compétences ne saurait assurer la reproductibilité de cette experience sauf en de très rares occasions et pour des projets d'exception. Par ailleurs, les techniques développées à l'époque sont pour partie tombées en désuétude (e.g. la recherche de forme par maquette physique) ou bien ont fortement évoluées voir même mutées (e.g. le calcul numérique). Des matériaux nouveaux, composites, ont vu le jour. Ils repoussent les limitations intrinsèques des matériaux usuels tel que le bois et offrent des performances techniques bien plus intéressantes pour ce type d'application (durabilité, allongement à la rupture, légèreté, résistance mécanique, fiabilité de niveau industrielle, . . .). Enfin, notons que le cadre réglementaire s'est considérablement étoffé apportant aussi son lot de rigidités vis-à-vis de la pénétration des innovations dans le secteur de la construction.

Ainsi la conception des gridshells se pose-t-elle en des termes nouveaux aux architectes et ingénieurs actuels. Elle se heurte aux deux difficultés majeures suivantes :

- La première difficulté est d'ordre technique et concerne la fonctionnalisation de la structure. En effet, bien que le principe du gridshell permette de réaliser des ossatures courbes de manière optimisée, il n'en reste pas moins complexe de constituer à partir de cette résille porteuse une véritable enveloppe de bâtiment capable de répondre à un large panel de critères performantiels (tels que l'étanchéité, l'isolation thermique, l'isolation acoustique, . . .) sur un support qui ne présente aucune rationalité géométrique.[3]

- La seconde difficulté est d'ordre théorique et concerne la mise au point d'outils et de processus de conception adaptés à l'étude de ces structures d'un genre nouveau où Architecture et Ingénierie collaborent de manière indissociable à l'identité formelle de l'ouvrage. L'inadéquation des

[1] Ici, le matériau employé, un composite à base de fibres de verre imprégnées dans une matrice polyester et obtenu par pultrusion, apporte un gain de performance très significatif par rapport au bois et permet de rester sur une conception à simple nape là où le bois aurait nécessité une grille à double nape beaucoup plus complexe à réaliser.

[2] "This is not a case of a building creatively designed, but based on a support system of additive known elements. This design is the result of a symposium of creative thought in the formation, the invention of building elements with the simultaneous integration of the theoretical, scientific contributions from mathematics, geodesy, model measuring, statics as well as control loading and calculation. We are dealing with more than pure 'teamwork', we are dealing with team creation." [Georg Lewenton 1, p. 201]

[3] Pour contourner cette difficulté, une approche prometteuse consiste à identifier des classes de surfaces courbes (comme les maillages isoradiaux) dont certaines propriétés géométriques (e.g. facettes planes, noeuds sans torsion) s'avèrent avantageuses sur le plan constructif [14].

méthodes et des outils de design actuels, orientés davantage vers la justification des ouvrages que vers leur conception, constitue un des principaux freins à la diffusion de cette innovation.

La présent manuscrit s'articule autour de deux grandes parties qui tentent chacune de construire des éléments de réponse aux défis identifiés précédemment. La première partie, composée des chapitres 1 et 2, est destinée à présenter en profondeur le concept de gridshell élastique, son potentiel et les difficultés techniques sous-jacentes (voir part I). La seconde partie, composée des chapitres 3 à 6, est consacrée au développement d'un élément de poutre discret original prenant en compte les sollicitations de flexion et de torsion et applicable à tout type de section dont le centre de torsion est confondu avec le centre de masse, ainsi que certains types de discontinuités liées à la présence de connexions dans les résilles de type gridshell (voir **??**). Cette seconde partie constitue le coeur *académique* de ce travail de thèse.

Dans chapter 1 nous rappelons la genèse de cette invention et nous en donnons une définition précise et actualisée. Puis nous dressons un état des lieux critique des projets réalisés sur ce principe depuis le début des années 1960 à nos jours. Cette brève histoire des gridshells dessine à elle seule le potentiel de ces structures, notamment en terme d'expression formelle et de performance structurelle. Loin de les enfermer dans un style d'architecture particulier, elle en souligne au contraire la formidable variété. Cette revue de projet est complétée par une revue approfondie de la littérature existante sur l'ensemble des domaines connexes à cette thématique (géométrie, structure, matériaux, logiciel).

Dans **??** nous présentons de manière détaillée la conception et la réalisation de la cathédrale éphémère de Créteil, un gridshell élastique en matériau composite construit en 2013 et toujours en service. Cette expérience peu commune a été une source inépuisable pour alimenter ce travail de thèse. Cette relecture expose les méthodes et les outils de conceptions développés pour faire aboutir le projet, les difficultés rencontrées, les pistes d'amélioration. Elle fournit également une analyse économique pour cerner les axes de progrès prioritaires dans l'optique d'une commercialisation future.

Dans **??** nous rappelons les notions fondamentales déjà connues, indispensables à notre étude, pour la caractérisation géométrique de courbes de l'espace et de repères mobiles attachés à des courbes. Ces notions sont présentées pour le cas continu puis pour le cas discret ; ce dernier étant essentiel pour la résolution numérique de notre modèle. Cependant, nous observons que la notion clef de courbure géométrique perd son univocité dans le cas discret. Nous identifions alors plusieurs définitions de la courbure discrète. Puis nous les comparons selon des critères propres à notre application (convergence géométrique, représentativité énergétique, forme d'interpolation). A l'issu de cette analyse, la définition la plus pertinente est retenue pour le développement du nouveau modèle numérique au cours des chapitres suivants.

Dans **??** nous élaborons un premier modèle de poutre à 4-DOFs par une approche variationnelle. Ici nous reprenons et enrichissons un travail initié lors d'une précédente thèse [15] inspirée par des travaux récents sur la simulation des tiges élastiques dans le domaine des *computer graphics* [16], et à laquelle j'ai collaboré [17, 18]. En particulier, notre développement permet d'aboutir à des expressions purement locales des efforts internes et prouve l'équivalence avec le membre statique des équations de Kirchhoff. Sur le plan mathématique, le modèle est développé en continu et son implémentation numérique n'est pas traité.

Dans **??** nous développons une nouvelle approche, plus directe et plus complète, pour construire à partir des équations de Kirchhoff un élément de poutre enrichi par un noeud fantôme et possédant lui aussi un nombre de degré de liberté minimal. L'originalité de cet élément est de pouvoir localiser proprement dans l'espace certains types de discontinuités, notamment des discontinuités de courbures provoquées par des efforts ponctuels ou des sauts de propriétés matérielles. Cela permet une modélisation plus fine des phénomènes physiques au sein de la grille, aussi bien au niveau des connexions que des conditions aux appuis, ce qui était le principal objectif de ce travail de thèse.

Dans **??** nous combinons les résultats des chapitres précédents pour construire un élément de poutre discret tout à fait adapté à la modélisation numérique des gridshells élastiques. Nous présentons la construction de cet élément et la méthode de résolution numérique employée pour trouver l'état d'équilibre statique du système, à savoir le relaxation dynamique. Enfin, nous donnons quelques éléments sur *Marsupilami*, le programme informatique que nous avons mis au point et qui implémente l'élément de poutre discret élaboré au cours de cette thèse. Nous exposons aussi quelques résultats de comparaison avec des logiciels du commerce qui ont permi de valider notre travail. Plus généralement, l'élément développé convient bien pour modéliser des problèmes de couplage flexion-torsion dans des poutres élancées, comme par exemple les phénomènes de repositionnement des câbles et des gaînes accrochées aux bras robots, un matériel industriel qui se démocratise à grande vitesse.

Elastic gridshells Part I

1 Elastic gridshells

1.1 Introduction

This chapter is meant to define and introduce what elastic gridshell structures are. It develops a comprehensive but precise view of the numerous knowledge and know-how that gravitate around this concept.

1.1.1 Overview

We naturally begin this chapter by defining the notion of elastic gridshell and the context in which this technology arose (see §1.2). We briefly highlight the benefits of composite materials for this kind of structure. We then propose two thorough reviews : the first one is dedicated to known built elastic gridshell structures (see §1.3) while the second one is a literature review of the main works related to the topic of elastic gridshells (see §1.4).

1.1.2 Contributions

- We establish a chronological review of known built elastic gridshells, from the very beginning of this technology to the present time. We reveal the richness of this concept by exhibiting the great variety of realised projects. We discuss the specificities brought by each one of these projects.

- We establish an up-to-date review of the existing scientific literature, crossing multiple fields of research (geometry, mechanics, material, . . .).

1.2 Definition

The invention of the *elastic gridshell* concept is commonly attributed to Frei Otto, a German architect who devoted several years to gridshells. In 1975 he achieved the famous *Mannheim Multihalle* [2], a wooden shell of 7500 m^2, in collaboration with the engineer Edmund Happold (Arup). Literally, the word “gridshell” refers to grids behaving like shells : from a mechanical point of view that means stresses acting on the structure are mainly transmitted through compression and tension. These structures can cross large-span with very little material.

However, according to the historic evolution of the concept, to characterise a gridshell as the combination of a structural concept (a grid behaving like a shell, see §1.2.1) and a specific construction process (see §1.2.3) using the bending flexibility of the material (see §1.2.2) seems to be more accurate. The project of Mannheim – in which a wooden regular and planar grid, lacking shear stiffness, is elastically deformed up to a targeted shape with the help of stays, and then braced and covered – is regarded as the starting point of this new concept (see fig. 1.1).

The project of Mannheim is regarded as the starting point of this new concept for which a wooden regular and planar grid, lacking shear stiffness, is elastically deformed up to a targeted shape with the help of stays, and then braced and covered. This type of gridshell, known as elastic gridshell, offers a very elegant manner to materialise freeform shapes from an initially flat and regular grid, which obviously has many practical benefits : planar initial geometry, standard connection nodes, standard profiles and so on.

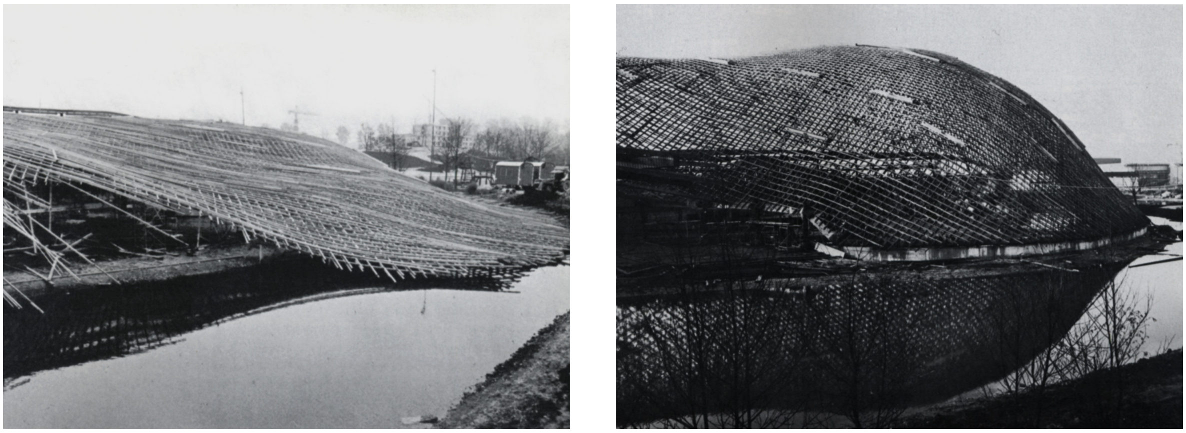
Note that the term *rigid gridshell* is often opposed to the term *elastic gridshell* to indicate reticulated structures that behave like shells but are not formed in an active-bending process.

1.2.1 Structural typology

Their mechanical behaviour is very similar to the one of real shells even if the material is discrete and located in a grid more or less open. Moreover, gridshells benefit from the same advantages as the ones showed by an eggshell : they can cross large span using a low amount of material. Their stiffness is mainly linked to their double-curved shape.

1.2.2 Material flexibility for structural rigidity

In this field of application, composite materials like glass fibre reinforced polymer (GFRP) could favourably replace wood, where both resistance and bending ability of the material is sought [7]. The stiffness of the structure does not derive from the intrinsic material rigidity but principally from its geometric curvature. Ideally, the composite profiles are produced by pultrusion, an economic continuous moulded process. The standardisation of the process guaranties very stable material and mechanical properties. It frees designers from the painful problematic of wood joining and wood durability. The characterisation of this material is presented further in the thesis (see **??**).



(a) Almost flat grid

(b) Deformed grid

Figure 1.1 - Forming process of the timber lattice of Mannheim, Germany.

1.2.3 Erection process

Usually, the grid morphology is not trivial and leads to design numerous costly and complex joints. To overcome this issue, an original and innovative erection process was developed that takes advantage of the flexibility inherent to slender elements. A regular planar grid made of long continuous linear members is built on the ground (see fig. 1.1a). The elements are pinned together so the grid has no in-plane shear stiffness and can accommodate large-scale deformations during erection. Then, the grid is bent elastically to its final shape (see fig. 1.1b). Finally, the grid is frozen in the desired shape with a third layer of bracing members and the structure becomes a shell. This process is illustrated and detailed in the next chapter (see **??**).

1.3 Built elastic gridshells : a review

No thorough historic review is available about executed projects of elastic gridshells although some partial reviews have been done time to time on the occasion of scientific works or construction projects. This review aims at filling this gap by giving an overview of the development of the concept from the very beginning to the very last experiments. Only known built projects have been identified and reported here. The only condition for a project to belong to this review is to comply with the definition of what an elastic gridshell is (see §1.2), independently to any other consideration (material, fabrication, size, cladding, . . .).

The informations collected during this research work are given in table format in appendix (see **??**). A synthetic presentation of these datas is proposed to the reader in fig. 1.2, where projects are ordered by date, span, covered area and material.

The books edited by the *Institut für leichte Flächentragwerke* are of great interest to understand the beginnings. *IL10 Grid Shells* [19] has a precise inventory of the first experiments from 1962 to 1976, while *IL13 Multihalle Mannheim* [1] focuses on the construction of the Multihalle in Mannheim. *Timber gridshells: architecture, structure and craft* [20] is a significant effort but focuses exclusively on medium to large scale projects in timber. A small but general partial review is also available in [21]. An interesting review is also given by Quinn and Gengnagel [22] as part of their research work on new erection methods. A review of bracing and cladding systems is done in [23]. A review of form-finding methods is done in [24]. Finally, various valuable reviews are available in the thesis of Douthe [25], Bouhaya [26], Tayeb [27], and Lafuente Hernández [28].

1.3.1 The beginnings : from the first prototype to the German Pavilion

Frei Otto started his studies in architecture in 1947 in Berlin, Germany, and completed his doctorate on tensile structures in 1953. This first work was published and translated later in the 60's. He then began to work in the field of lightweight structures using physical models such as soap films or hanging nets, and photographic measurements.[1,2] These tools were essentials for his exploration of forms and structures as there were no computers at that time.

Steel Gridshell, Berkeley, USA, 1962

Simultaneously, he became interested by the study of lightweight shells and the way they were form-found. One of his very first elastic gridshell was built in 1962 with students at Berkeley, USA [19, p. 270]. It is funny to remark that this first gridshell was not a timber gridshell but a steel gridshell made out of twin steel rods linked in a grid fashion by bolts with clamping plates (see fig. 1.3a). This first experiment demonstrated at small scale the ability to bend a regular grid with no shear rigidity

[1] In the 19$^{\text{th}}$ and 20$^{\text{th}}$ centuries model testing was at the heart of structural innovation [29]. Analog models were employed successfully by well-known architects and engineers to go beyond the limits of existing knowledge (A. Gaudi, H. Isler, F. Candela, F. Otto, . . .) and are still employed today where numeric models failed to represent accurately some physical phenomenons (for instance in wind analysis for high rise towers and bridges).

[2] "Photography is the medium through which the form and content of a model are communicated. It is one of our most important tools in that it provides the basis for documentation and information, supplements our creative potential [. . .] " [19, p. 56]

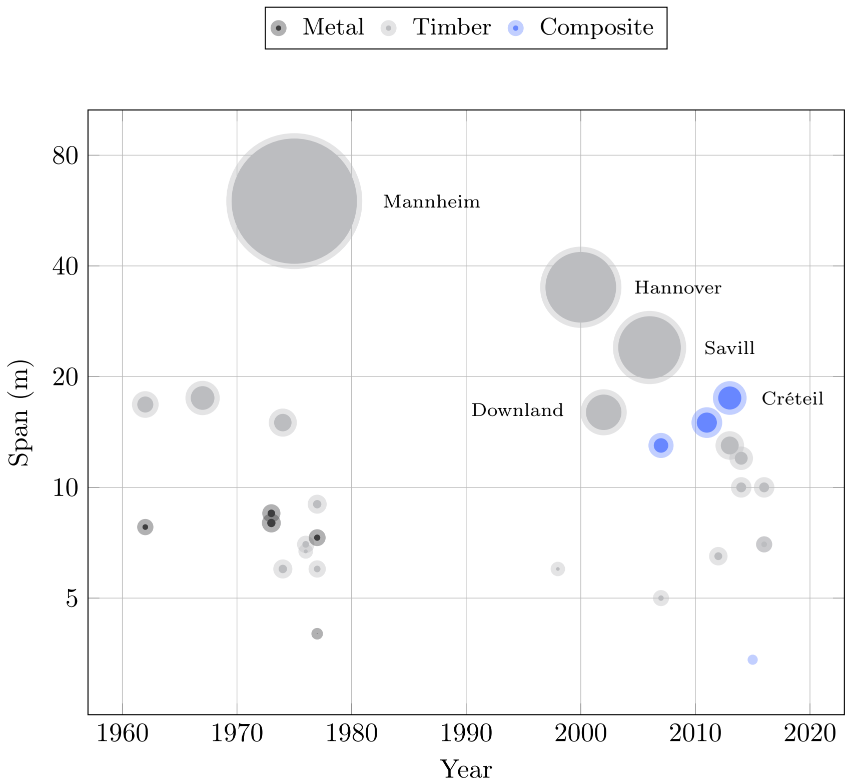


Figure 1.2 - Known elastic gridshells built by the past. The surface of the bubbles is proportional to the covered area. Colour indicates the material employed for the rods.

into a curved shape (see fig. 1.3b). The grid was loosely braced and shell effects were not investigated.

Essen Gridshell, Essen, Germany, 1962

The same year he designed and built a first timber gridshell in Essen, Germany [19, p. 272]. The prototype – a single-layer gridshell spanning $17\,\mathrm{m}$ and covering an area of $198\,\mathrm{m}^2$ – was made with 3-plies laminated timber profiles in hemlock pine (see fig. 1.4a). The cross-section of the profiles was rectangular ($60\,\mathrm{mm}$ x $40\,\mathrm{mm}$) and the elements were assembled in a grid fashion with simple steel bolts. Once erected, nothing was specifically done to improve the in-plane shear stiffness of the grid and activate a shell behaviour. Finally, the structure was covered with a transparent plastic foil nailed directly on the grid's profiles (see fig. 1.4b).

German Pavilion Auditoria, Montreal, Canada, 1967

Five years later, on the occasion of the *1967 International and Universal Exposition* in Montreal, Canada, Frei Otto was appointed to design the German Pavilion : a large cable net tent prefiguring the realisation of the olympic stadium of Munich, Germany, in 1972.[3,4]

The pavilion required two auditoria and these were designed using the principle of elastic gridshell [19, p. 274]. All together, the auditoria covered and area of $365\,\mathrm{m}^2$ and spanned $17.5\,\mathrm{m}$. The construction technique employed in Montreal was quite similar to the one developed in Essen, but this time the grid was fully braced with a layer of nailed plywood boards and offered a proper roofing made out of insulation panels covered with a PVC coated fabric (see figs. 1.5a and 1.5b).

The two gridshells built in Montreal mark a significant step in the maturation process of the technique leading to the major realisation of Mannheim in 1976 : a methodology has emerged to progress "from the inverted form to the gridshell" [19, p. 179] ; main construction details have been validated ; various erection methods have been tested ; mid-scale buildings have been built to host public. However, due to the over complexity of these structures, lots of unknowns remained unsolved at this stage and the behaviour of the structures could not be fully predicted.[5]

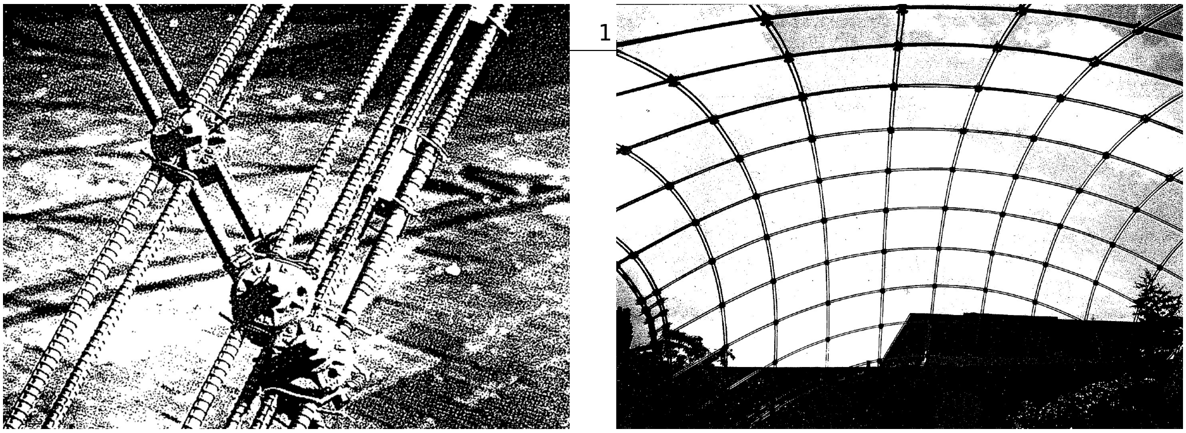
It is worthwhile to mention that several unexecuted large-scale projects were studied by Frei Otto between 1967 and 1973 at the *IL* or at the *Atelier Warmbronn*.[6] These projects are basically documented in [19, pp. 278 - 288] and reveal that he was training his capacity to master large-scale projects with the technique of elastic gridshells for more conventional building projects (wave pool, swimming hall, multi-purpose hall, auditorium, . . .).

[3] Actually, Frei Otto became the director of the newly founded *Institute for Lightweight Structures* (Institut für Leichte Flächentragwerke or IL) at the University of Stuttgart in 1964. It was the IL that was commissioned by the German government to conduct research in connection with the planning of the German pavilion for the exposition in Montreal.

[4] Video of the construction of the German pavilion : https://www.youtube.com/watch?v=Z0mtFMoseUk.

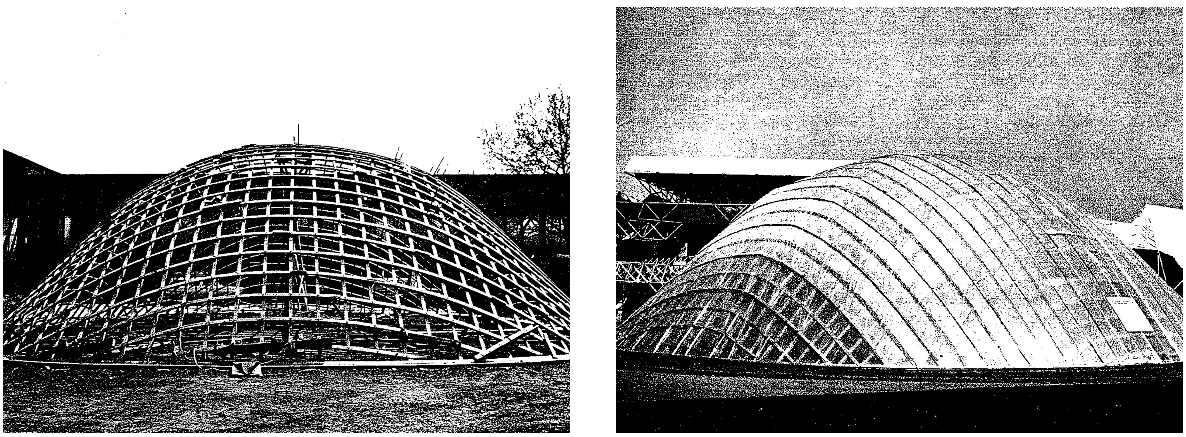
[5] "Snow accumulations in the throat of the common edge beam probably caused one of the two grid shells of project Montreal to buckle in a relatively flat region. The diameter of the buckled area was about 3 meters. Neither grid rod was broken, i.e. the buckling progressed elastically. It might have been possible to press the buckled area back into shape." [19, p. 219]

[6] Atelier Warmbronn is the architectural studio founded by Frei Otto in 1969.



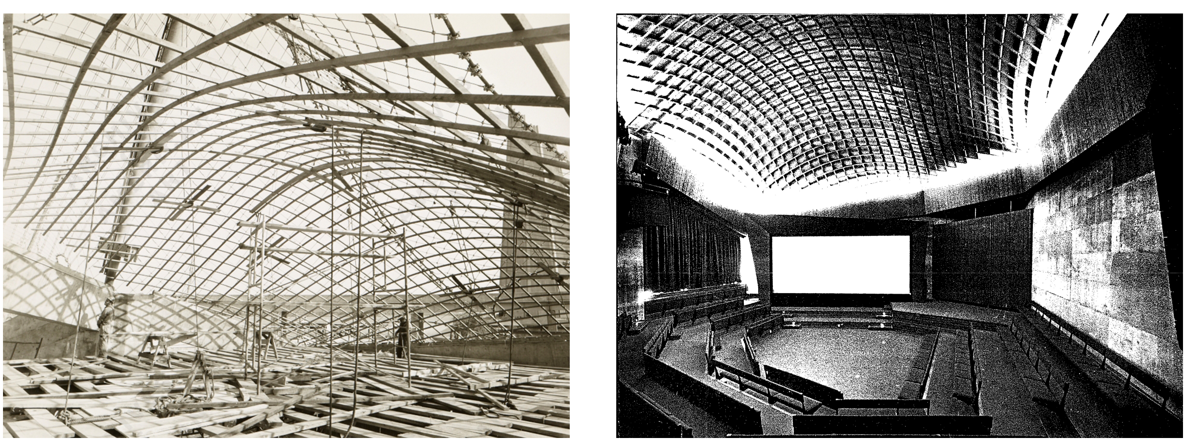
(a) Knot detail **(b)** Steel lattice

Figure 1.3 - Steel gridshell built in 1962 in Berkeley, USA.



(a) Timber lattice **(b)** Plastic foil

Figure 1.4 - Timber gridshell built in 1962 in Essen, Germany.



(a) Grid erection **(b)** Interior view

Figure 1.5 - Timber gridshell built in 1967 in Montreal, Canada.

1.3.2 Mannheim Multihalle : the completion of a decade of research

The project of the Multihalle started in 1970, when the decision was made that Mannheim, Germany, would hold the Bundesgartenschau in 1975.[7] The architects of the project, *Carl Mutschler & Partners*, consulted Frei Otto at *Atelier Warmbronn* as he was starting to get known in the filed of innovative lightweight structures. This is how the idea of the gridshell was introduced in the project [30].

A thorough report on the project is available in [1]. A more condensed but still precise description of the engineering problematics related to this project are available in the excellent papers from Happold and Liddell [2] and Liddell [30].

Multihalle, Mannheim, Germany, 1975

Mannheim is an unprecedented realisation because it is more than twenty times larger than the previously built gridshells in Montreal and is meant to last many years and not only for the duration of a short-term exhibition. The timber lattice, still existing in 2017, covers an area of $7400\,\mathrm{m}^2$ (see fig. 1.6b). It is composed of two interconnected domes, one for the multi-purpose hall (span : $60\,\mathrm{m}$ | height : $20\,\mathrm{m}$) and one for the restaurant (span : $50\,\mathrm{m}$ | height : $18\,\mathrm{m}$).

Although the constructive system deployed in Mannheim clearly inherited from the previous developments, the challenge was such that it had to be revisited. In particular the main additions were the introduction of the double-layer system and the proper bracing of the grid. A major advance was also the use of the very first numeric models to study the structure.

The double-layer system was introduced to tackle two issues : the grid needed some flexibility to be bent into the desired shape, but once erected it should provide sufficient bending stiffness to resist disturbing loads and avoid a buckling collapse.[8] Once erected, the two grids, one sliding on top of the other one, were connected together to form a single grid with much higher ladder profiles (from $50\,\mathrm{mm}$ to $150\,\mathrm{mm}$), increasing their bending stiffness by a factor of about 26 (see fig. 1.6a).

Because the in-plane stiffness of the grid also plays a major role in the resistance to buckling, this question was considered with care. The bracing of the grid was first achieved by preventing the nodes to turn once the grid was erected. This was done by creating some friction in the nodes when tightening the bolts linking the laths, after the grid was erected. Then, additional bracing cables were put in the grid.

Finally, the project of Mannheim was a key project in the development of modern lightweight structures. Great engineers were born in touch with Frei Otto, following his footsteps or collaborating with him. This heritage has irrigated for several decades the engineering of lightweight structures in Europe and gave birth, directly or indirectly, to several studios among which we can cite *Buro Happold* and *Schlaich Bergermann & Partner*.

[7] The Bundesgartenschau is a national horticultural exhibition that takes place every two years in Germany.

[8] Theoretically, self-weight loads would produce only compression in the members because the (funicular) form of the grid resulted from the inversion of a hanging chain model in pure tension.

(a) Interior view

(b) Sky view

Figure 1.6 - Timber gridshell built in 1975 in Mannheim, Germany.

1.3.3 The dry period : 25 years from Mannheim to Hannover

Although the experience of Mannheim proved the feasibility and the potential of gridshell structures for large-scale projects, it also revealed that these projects were subject to an incredible complexity in terms of structural design, geometry, modelling, testing, team work, construction methods ... At that time, very few people could pretend to master all the knowledge and techniques required to design and built timber gridshells and developed in the bosom of the *Institute for Lightweight Structures* in Stuttgart.

This project was obviously well ahead of its time and the engineering cost to design such structures was probably prohibitive considering the tools available at that time. This certainly explains why no elastic gridshells were built during the 25 following years, despite the optimism of the pioneers of the Multihalle.[9]

Note that around 1975 small workshop and experiments lead to the construction of several but small elastic gridshells, as reported in [19]. A non-exhaustive but quite extensive list of known executed gridshell projects is presented in fig. 1.2. The dry period is clearly visible.

1.3.4 The signs of a renewal : Dorset and Doncaster

It is only 20 years later that gridshells started to reappear, in the late 90's mainly in the United Kindom, and for projects that had interest in environmental problematics.

[9] "For many years after its completion, Happold promoted the benefit of the timber gridshell as a construction technique and stated that he could not understand why it had not been adopted more widely. He perceived the benefits to be in the efficiency of the construction method to enable doubly curved (shell) structures to be constructed quickly and cost effectively." [31].

Westminster Lodge, Dorset, England, 1995

In 1995, a small student residence named *Westminster Lodge* was built in Dorset, England. This dwelling was part of a larger project – Hooke Park – aiming at investigating how the local forest resources, in particular immature roundwood thinnings, could be better utilised. The project was lead by ABK, Frei Otto, Buro Happold and Cullinan Studio. Unlike Mannheim, the timber shell was bent and weaved rod by rod on a scaffold platform. But the structural system exhibited a double-layer gridshell pattern very similar to the one employed for the Multihalle (see fig. 1.7a). The rods were made out of splice-jointed roundwood to form long-length poles of diameter $200\,\mathrm{mm}$. The development of this jointing technique, which could be produced directly in the forest, was part of the project's investigations [32]. The grid was braced by a layer of diagonal boards nailed to the roundwood. The structure was finally cladded with a planted turf roof (see fig. 1.7b).

Earth Center, Doncaster, England, 1998

At the same time, a project with a similar spirit arose for the *Earth Center* in Doncaster, England.[10] The project planning started in 1994 and a series of small timber gridshells were designed by Buro Happold and then built in 1998. The landscape structures were single-layer timber gridshells made with oak laths. Once erected with a crane, the grids were braced with crossing diagonal stainless steel cables (see fig. 1.8a). Openings were possibly reinforced with curved timber frames (see fig. 1.8b).

These projects definitely trailed the technique in England and initiated the renewal period (see §1.3.5). Although they remained small-scale projects for which modelling was achieved through physical models, they trained and restored partially the operational ability of Buro Happold to design timber gridshells as pointed by Harris *et al.* [31].

1.3.5 The renewal : Hannover, Downland and Savill

What was missing for elastic gridshells to re-emerge after the major experiment of Mannheim was probably the development of modern numeric tools to ease and speed up the design process.[11] Amongst those tools we should identify two main categories : geometry processing softwares and structural analysis softwares. Recall that in the 70's, geometry processing was done through physical models and photographic measurements [19, pp. 130-135] while structural analysis was conducted through a compound of physical model testing with scaling techniques [1, pp. 130-135], hand calculations and the very first numerical form-finding calculations [19, pp. 184-193] and finite element calculations [19, pp. 210-217]. In the late 90's, the rise in importance of computer methods offered new possibilities.

[10] "The Earth Centre Forest Garden was intended to demonstrate how managed woodland could supply the vast majority of all natural resources needed for human survival."

[11] "The key to the modern use of timber gridshells is the development of computer methods in modelling complex three-dimensional shell structures. For the Mannheim structure, the primary method of form finding was the use of physical models. The Earth Centre structures were small and easily modelled using wire mesh, but when Buro Happold was commissioned to design the Japanese Pavilion for Expo 2000 in Hannover (Architect Shigeru Ban), it was apparent that much more sophisticated computer form finding and analysis would be necessary." [31]

Japan Pavilion, Hannover, Germany, 2000

In 1997, architect Shigeru Ban began to collaborate with Frei Otto and Buro Happold to design the *Japan Pavilion* for *Expo 2000* in Hannover, Germany [3]. This pavilion was a large-scale corrugated gridshell made out of cardboard tubes, about 75 meters long and 25 meters wide. Corrugations bring curvature, and therefore enhance the strength of the shell. The tubes were tied together with a fabric tape, a very low-tech joint (see fig. 1.9a). The structure was covered with a paper membrane specially developed for the project to meet the requirements of the German fire regulations (see fig. 1.9b). For the occasion, a new erection method was set up in which the grid was laid out not at the ground level but at a higher level on a hydraulic scaffold platform. From there, the grid was pushed up into position using the platform's jacks. It was found late that the cardboard tubes were subject to a high level of creep. This required the introduction of new timber arches to reinforce the gridshell and to enlarge the existing timber rafters intended to brace the grid and support the paper membrane (see fig. 1.9a).

Weald and Downland, Singelton, England, 2002

The design of the *Downland* gridshell began right after the completion of the Westminster Lodge (see §1.3.4) where architects from E. Cullinan Studio became acquainted with the engineers from Buro Happold. At Downland, the project team truly revived the technique of large-scale timber gridshells while bringing lots of improvements to the system. The building opened to the public in 2002. Its corrugated shape recalls the one of the Japan Pavilion from which it was inspired (see fig. 1.10b).

The building is 50 meters long and 12.5 to 16 meters wide, covering an area of about $675\,\mathrm{m}^2$ for a height varying from 7 to 9.5 meters [4]. The structure is a double-layer gridshell made of rectangular oak laths of cross-section $50\,\mathrm{mm}$ x $35\,\mathrm{mm}$ (see fig. 1.10a). To produce high grade timber elements, the continuous laths were re-formed from small carefully selected wood pieces, finger-jointed every $60\,\mathrm{cm}$ in $6.0\,\mathrm{m}$ length pieces. These pieces of lath were then scarf-jointed on site every $6\,\mathrm{m}$ to obtained the desired length, up to $50\,\mathrm{m}$.

The grid pitch is $1.0\,\mathrm{m}$ except in weaker areas where it is $0.5\,\mathrm{m}$. There, the grid is twice denser to achieve the required buckling resistance [31]. Rib-lath bracing was preferred to steel cable bracing as ribs were deemed to offer a more convenient support for the cladding and to reduce the complexity of the connection. A new connection system was developed to avoid the cost of drilling thousands of slotted holes that would, in addition, reduce the cross-section area, while maintaining the required scissor behaviour for the deformation of the timber lattice.[12]

The flat lattice was laid out on a scaffold platform. Unlike the Japan Pavilion, the lattice was progressively lowered down into position. This stage took 6 weeks. Once deformed, the shear blocks were introduced in the grid and bracing rib-laths were installed, giving its full strength to the shell. Finally the gridshell was cladded with a mix of polycarbonate plates (to let the light in) and timber boards on top of insulation panels and a rain screen.

It is worthwhile to mention that for the first time the form was not found by inverting some sort of hanging chain model that would produce a pure funicular shape where only compression occurred. Instead, the shape was the result of a numerical computation that took into account the bending be-

[12] This detail was patented by the design team and the client.

haviour of the laths.[13] Harris *et al.* [31] argued that computer models enabled some interactivity in the form-finding process that would not be possible with physical models, leading to a better synergy between architectural and structural requirements. They also argued that physical models contributed invaluably to the development of a creative and efficient design throughout the project.

Lothian Gridshell, Pishwanton, Scotland, 2002

This project deserves some attention because the developed approach was completely different from the projects exposed until now : "Previous projects have portrayed the method as a highly technical use of a low-tech resources. This, however, needs not be the case as we see with this project [. . .]" [33]. The structure was the result of "[. . .] an unusual collaboration between sole practitioner Christopher Day, engineer David Tasker, a crowd of local volunteers and (more unusually) the philosophies of Rudolf Steiner and Johann Wolfgang Goethe" [34].[14]

The single-layer gridshell was made out of local larch. Once erected by hands, the dome-like shape covered about $80\,\mathrm{m}^2$ and spanned 10 meters. The grid was braced with timber boards (see fig. 1.11a) and covered with a planted turf roof (see fig. 1.11b). Some calculations were made but in the end, it had to carry load testing to prove its safety and gain its regulation approval.[15]

Woodland Centre, Filmwell, England, 2003

The gridshell of the Woodland Centre was built 7 years after the project had started (see fig. 1.12a).[16] The building was designed by architect Feilden Clegg and engineers from Atelier One. It was part of a larger research and development project that aimed at developing chestnut – a low grad wood – as a construction material.[17]

The building, still existing, is composed of 5 barrel vaults spanning 12 meters and about 5 meters wide (see fig. 1.12b). It covers about $300\,\mathrm{m}^2$ [35]. Each vault module is a transportable unit composed of two curved arches. A single layer gridshell was then applied to this primary frame and braced with chestnut panels. The grid was made of laths with $75\,\mathrm{mm}$ x $25\,\mathrm{mm}$ rectangular cross-section, assembled with simple bolts. On top of that, insulation materials and a membrane as rainscreen [36].

Savill Garden, Englefield Green, England, 2006

This project saw the light of day thanks to the reputation of the gridshell built in Downland. Again, Buro Happold did the structural design while Green Oak Carpentry realised it. But this time, the architect was Glenn Howells.

The *Savill* gridshell is 90 meters long and 25 meters wide. It covers an area of about $2000\,\mathrm{m}^2$, and

[13] This software was developed under the supervision of Chris Williams of the university of Bath.

[14] From the online paper "The other gridshell" : http://www.bdonline.co.uk/the-other-gridshell/1020435.article

[15] "There were a lot of calculations but no computer-generated models to show they all added up. In fact, the form was previously established with scale models. When it came to gaining Building Regulations approval, the team needed to prove that the building would be strong enough. So Tasker arranged for the unfinished structure to be loaded with about 18 tonnes of sand from a local quarry – equivalent to the maximum predicted snow load, plus a safety factor." [34]

[16] More to be found at : Growing and making Flimwell's chestnut gridshell.

[17] This projet was conducted by the Building Research Establishment.

is therefore almost three times larger than the gridshell in Downland. Once again, the corrugated shape was defined by a parametric equation ($z = f(x, y)$) to enable interactivity between architects and engineers during the form-finding process (see fig. 1.13b). Chris Williams was responsible for this job [5].

In Savill, the forming strategy was quite different than those employed in Mannheim, Hannover or Downland [5]. Firstly, a single layer gridshell – constituted by the bottom two laths jointed with simple bolts – was deformed into the target shape. Secondly, the shear blocks were screwed on these laths. Thirdly, the upper two laths were positioned and screwed on top of the shear blocks to re-form a double-layer gridshell. Finally, the grid was braced with two alternate layers of plywood boards, $12\,\mathrm{mm}$ thick each. Bracing the grid with continuous panels instead of cables or diagonal members was a major architectural choice (see fig. 1.13a). Moreover, it gave a well-defined surface for the cladding composed of $160\,\mathrm{mm}$ of insulation, covered by a waterproof aluminium layer made with standing-seam profiles supporting the oak boards [37].

Another consequence of this forming process was the drastic simplification of the connexion. The system developed for Downland was of no utility in that case and only simple bolts and screws were required. In this project, the pitch of this grid is $1.0\,\mathrm{m}$. The $20\,\mathrm{kms}$ of laths are made from larch and have a $80\,\mathrm{mm}$ x $50\,\mathrm{mm}$ rectangular cross-section. They are spaced from $100\,\mathrm{mm}$ to $150\,\mathrm{mm}$ by the shear blocks.

Of course, the steel perimeter is a major component of the project but is not in the scope of this thesis. For further details the reader is invited to refer to Harris *et al.* [5] and “The Savill building. A visitor centre with a timber gridshell roof gridshell structures” [37].

Chiddingstone Castle Orangery, Kent, England, 2007

The gridshell covering the orangery of Chiddingstone Castle is a very small one. Built in 2007, it is 12 meters long, 5 meters wide and covers about $50\,\mathrm{m}^2$ (see fig. 1.14b). The structural system is derived from the one employed in Downland and is, once again, developed by Buro Happold and the Green Oak Carpentry. But this time the architect is Peter Hulbert.

However this project embed some interesting innovations. Indeed, this time the gridshell is braced with a bidirectional cable network. Twin cables are employed to facilitate clamping on the node connection, which has been adapted from the previous version developed in Downland. This connection is now equipped with an additional threaded hole which can receive the clamping supports for the glazing (see fig. 1.14a). The timber shell is then glazed with triangular panels. Note that the quadrangles of the mesh are not planar any more in the deformed configuration and therefore triangulation of the (flat) glass panels is mandatory.

1.3.6 Gridshell in composite materials : a new perpective

Since 2002, the laboratoire Navier at the Ecole des Ponts ParisTech develops a research program on elastic gridshells that is still ongoing. It focuses on both the use of new materials such as composite materials and the development of modern computer design methods for the generation of complex shapes, their form-finding and their structural analysis.

Douthe *et al.* [7] proved that composite materials in glass fibre reinforced polymers (GFRP) are very suitable for this type of structures where both flexibility and strength of the profiles are required. On the level of mechanical behaviour GFRP surpass wood. They are easy and cheap to produce in long length when they are manufactured by pultrusion, thus avoiding complex jointing issues.[18]

The first gridshells in composte material, Champs-sur-Marne, France, 2006

These developments have been validated by the construction of two prototypes in 2006 (see fig. 1.15a) and in 2007 (see 1.15b) [6]. These structures were left outside for about 7 years. They covered about $150\,\mathrm{m}^2$ each, spanning around 13 meters. The structures were single-layer gridshells made with pultruded GFRP tubes (Ø$41.7\,\mathrm{mm}$ x $3.5\,\mathrm{mm}$) assembled with a standard scaffold swivel connector. The grid was braced with a third layer of tubes and covered with a PVC coated fabric membrane providing full waterproofness.

Here, the performance of composite materials is of real benefit. A single–layer gridshell is enough for this span. The hollow circular cross-section make optimal use of the material. Tubes are provided in 12 meters length and therefore no joints are required for this span. In the end, all these benefits make the constructive system a lot more lighter, simpler and efficient than what a timber gridshell would offer.

Note that the first prototype was manually pushed-up in its deformed shape while the second prototype was assembled member after member on top of an existing blower, similarly to the method employed in Dorset (see fig. 1.7).

Solidays, Champs sur Marne, France, 2011

In 2011, Navier (L. du Peloux, O. Baverel, J-F. Caron, F. Tayeb) used its knowledge to design with a team of students a temporary pavilion for a music festival in Paris, France (see fig. 1.16b).[19] Although the constructive system was similar to the one employed for the two prototypes, the size and the span were more than twice larger [8]. In addition, it was the first gridshell in composite material that hosted some public and therefore had to comply with strict building regulations.

To our knowledge, it is also the first gridshell that was designed using the compass method [19], thus providing an inverse method to design the structure directly from the shape given by the architect. The single-layer gridshell covered about $280\,\mathrm{m}^2$ and was erected by two mobile cranes (see fig. 1.16a).

Ephemeral Cathedral, Créteil, France, 2013

The *Ephemeral Cathedral* of Créteil is the last achievement of this kind [9].[20] It was designed by T/E/S/S (L. du Peloux, B. Vaudeville, T. Gray, S. Aubry) with the assistance of Navier (F. Tayeb, J-F. Caron, O. Baverel, A. Tamaint).[21]. This time, the structure is a real building meant to last a decade and is still in activity since its construction in 2013. A complete review of this project is given in the next chapter of

[18] Video explaining the pultrusion process : https://www.youtube.com/watch?v=4MoHNZB5b_Y

[19] Photos and videos of the construction process at: http://thinkshell.fr/gridshell-solidays-2011/

[20] Photos and videos of the construction process at : http://thinkshell.fr/gridshell-cathedral-2013/

[21] For this project, I was in charge of the project development for T/E/S/S, including structural and technical design, detailing, doors, membrane, drawing production, fabrication and erection with the help of the parishioners, regulations, . . .

this thesis (see **??**).

The single-layer gridshell covers about $350\,\mathrm{m}^2$ and spans 17 meters (see fig. 1.17a). It is covered by a PVC coated fabric membrane (see fig. 1.17b). It was erected by two mobile cranes.

1.3.7 Flourishing timber gridshell pavilions

Since 2010, about 20 timber gridshell pavilions were built around the world, mainly during workshops. Here, we do not review all of these pavilions in detail because they are quite similar although each one has its specificities.

The impetus given by gridshell.it

Around 2010, a research group gathering architectural and engineering skills appeared under the name gridshell.it in Italy. Inspired by the work of Frei Otto, they revisited the structural system developed at Mannheim and adapted it to a range of small-scale timber pavilions.

These pavilions have in common to be double-layer timber gridshells. The structural system is always composed of laths with rectangular cross-section. The laths come in short length from the sawmill (about 3 to 4 meters). They do not try to re-form long-length laths with complex jointing techniques. Instead, they use a simple splice system. Although it is not well architecturally resolved, it is efficient enough for this kind of project. As the laths are short, this detail is repeated frequently in the grid, but the splice system enable a higher level of prefabrication of the grid. Thus, small modules of the size of the laths can be preassembled and connected with the splice system to re-form the full grid. These gridshells are braced either with cables or with individual diagonal members in each cell.

These structures were never meant to provide full waterproofness although some were an occasion to experiment different types of cladding with boards (Lecce 2010, Toledo 2012, Milano 2013) or with textile membranes (Lecce 2009).

One of their first pavilion was built in 2010 in Lecce, Italy (see fig. 1.18a). Their most known project is probably the Toledo pavilion built in 2012 in Naples, Italy. A new pavilion called Toledo 2.0 was built in 2014 in Naples, Italy (see fig. 1.18b). Although it seems that their initial approach focused more on the architectural aspects and the construction process, they rapidly tried to develop dedicated computer design methods [10] and did significant wood testing [12].

Other similar timber pavilions

The ideas of the *gridshell.it* group spread rapidly and similar projects were achieved outside of Italy. Amongst them, we can point out the ZA pavilion built in 2013 in Cluj, Romania [11]; the F^2 pavilion built in 2014 in San Antonio, USA, with an interesting folding skin (see figs. 1.19a and 1.19b) and the pavilion built in 2016 in Trondheim, Norway, which is made of very short length laths spliced every two cells [13, 38].

Specific inputs from the laboratory Navier

In that vein, L. du Peloux from Navier and G. Laurent from Terrell helped two students (S. Hulin and G. Sudres) resp. from the ENSA Grenoble and ENSA Toulouse to design a modular pavilion system for their final year project (2016). These pavilions were designed similarly to the pavilions of gridshell.it but improvements were made. Firstly, a new cable bracing system was developed. It was embedded in the grid and tensioned with spacer plates once the grid was erected (see fig. 1.20a). This system proved its efficiency on site compared to bracing with diagonal members. Secondly, the grid was designed and fabricated so it could be dismantled and reassembled in a different shape. And indeed, a first pavilion was erected in Toulouse the 3rd of June, dismantled, reconfigured, and re-erected in Montpellier the 15th of June. The pavilions shared the same standard grid modules ($2.40\,\mathrm{m}$ x $2.40\,\mathrm{m}$), and dedicated modules were used to adapt the change in shape.[22]

1.3.8 Latest experiments

In 2016, a one-week workshop called Building Freeform 2016 was held at the Ecole des Ponts ParisTech, France. The brief was to explore some innovative methods, including the generation of forms which allow the coverage by flat panels as well as the automation of some production tasks with the use of a robot arm (see fig. 1.21a). The draft studies were conducted upstream of the week, so that students can focus on design issues, implementation and practical achievement.[23]

The second experiment is a hybrid structure (see fig. 1.22). It is part of our reflexion at Navier on how to brace and clad gridshells. Indeed, the bracing of the grid in its final form remains a time consuming step with a lot of manual work. The lack of alternatives to membrane covering is also an important limitation to the development of such technology. The proposed experiment tries to tackle both issues through a novel concept of a hybrid structural skin made of an elastic gridshell braced with a concrete envelope. The idea is to use the gridshell as a formwork for the concrete and to guarantee a mechanical connection between the thin concrete skin and the main grid, so that the concrete ensures the bracing of the grid and that the thickness of the concrete is reduced to a minimum. To demonstrate the feasibility and interest of this structural concept, a $10\,\mathrm{m}^2$ prototype was built at the Ecole des Ponts ParisTech, France (see figs. 1.22a and 1.22b). The main aspects of the design and of the realisation of the prototype are presented by Cuvilliers *et al.* 2017 [23].

[22] For these projects, I did the shape analysis, the meshing with the compass method, the form-finding with my own dynamic relaxation software, wood testing and specification, grid system detailing (nodes, cross-sections, grid pitch, bracing, slotted holes, shear block, . . .) and provided a valuable assistance all along the project.

[23] The co-development of this week was part of my research work. In particular, I provided the form-finding and structural analysis tools and developed the upstream software to generate the fabrication informations required by the milling station. This software was largely parametric so students can truly implement their own design. I was also involved in the planing of the week.

(a) Interior view

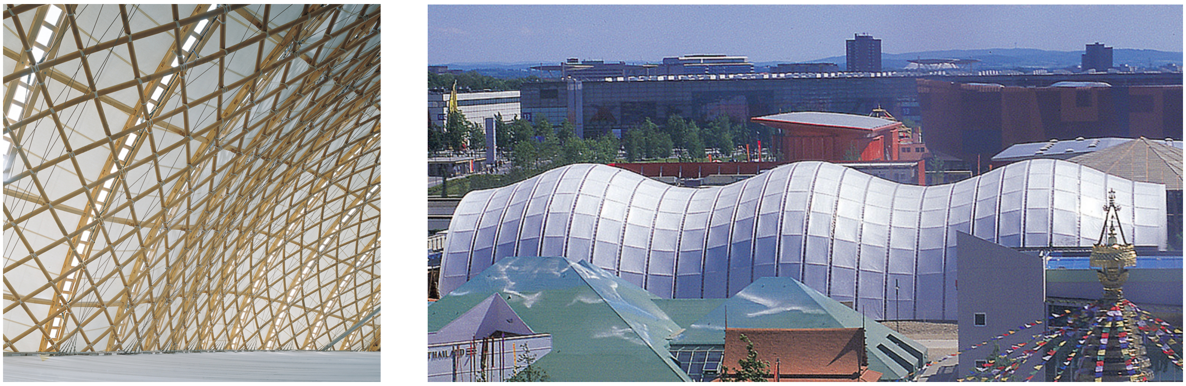
(b) Exterior view

Figure 1.7 - Roundwood gridshell built in 1995 in Dorset, England.

(a) Interior view

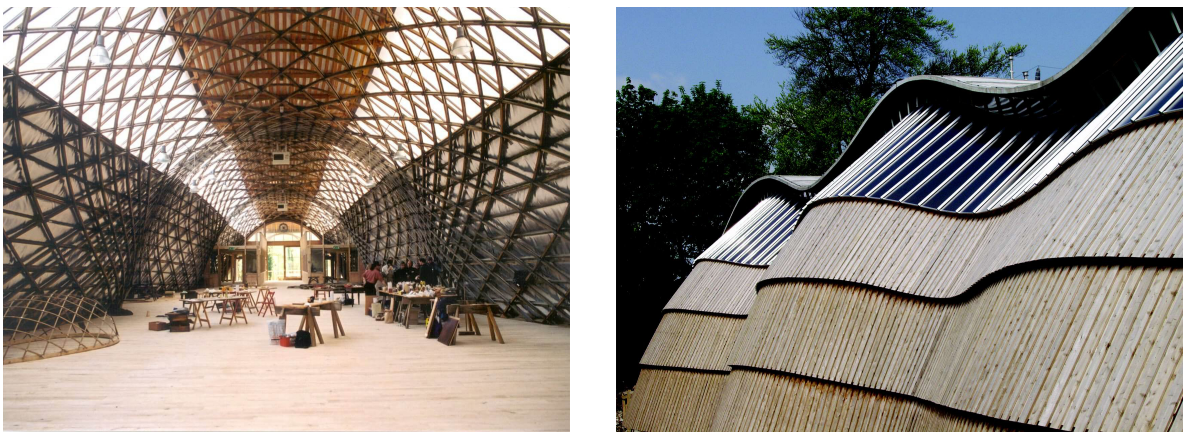
(b) Exterior view

Figure 1.8 - Timber gridshells built in 1998 in Doncaster, England.



(a) Interior view **(b)** Sky view

Figure 1.9 - Cardboard gridshell built in 2000 in Hannover, Germany.



(a) Interior view **(b)** Exterior view

Figure 1.10 - Timber gridshell built in 2002 in Downland, England.

(a) Interior view

(b) Exterior view

Figure 1.11 - Timber gridshell built in 2002 in Pishwanton, England.

(a) Interior view

(b) Exterior view

Figure 1.12 - Timber gridshell built in 2003 in Filmwell, England.

(a) Interior view

(b) Exterior view

(a) Glazing support

(b) Exterior view

Figure 1.14 - Timber gridshell built in 2007 in Kent, England.

(a) First prototype (2006) **(b)** Second prototype (2007)

Figure 1.15 - GFRP gridshell built in 2006 and 2007 in Noisy-Champs, France.

(a) Interior view **(b)** Exterior view

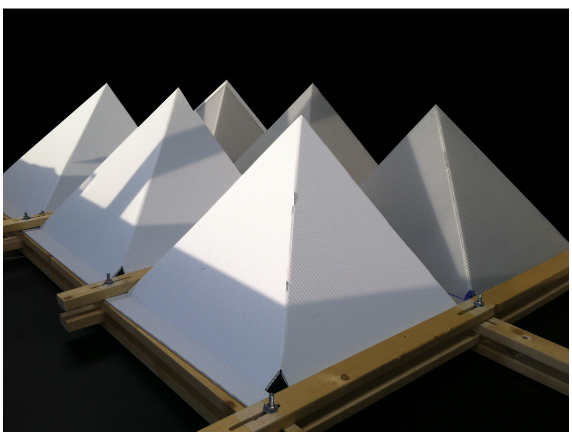
Figure 1.16 - Solidays GFRP gridshell built in 2011 in Paris, France.

(a) Interior view **(b)** Exterior view

(a) Lecce, 2010

(b) Toledo 2.0, 2014

Figure 1.18 - Timber gridshells built by gridshell.it in Italy.



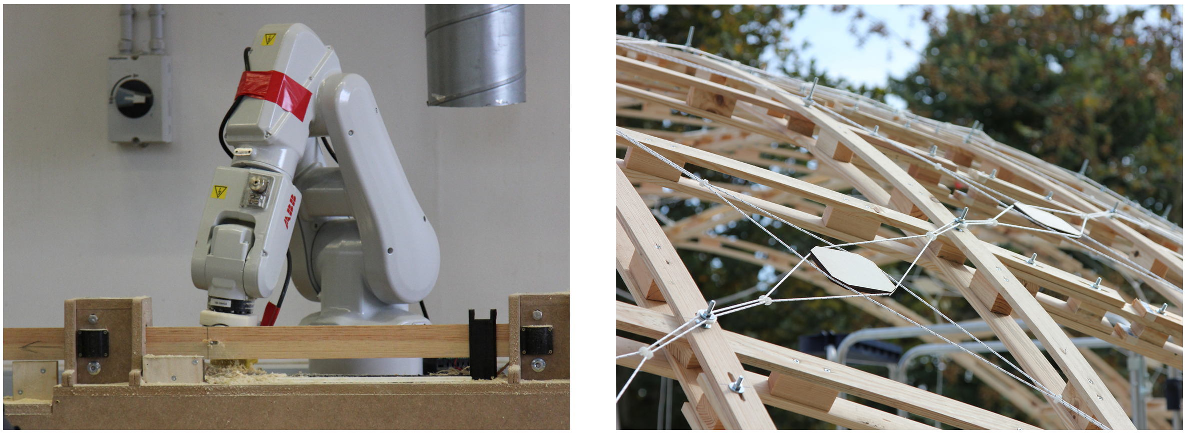
(a) Folding skin

(b) Pavilion

Figure 1.19 - Timber gridshell built in 2013 in San Antonio, USA.

(a) Tensioner

(b) Pavilion



(a) Robotic manufacturing **(b)** Timber lattice

Figure 1.21 - Timber gridshell built in 2016 in Champs-sur-Marne, France.

(a) Interior view **(b)** Exterior view

Figure 1.22 - Hybrid structural skin built in 2016 in Champs-sur-Marne, France.

1.4 Research works on elastic gridshells : a review

In this section we depict the research works that are related to elastic gridshells. Several topics have been identified to organise the review.

1.4.1 Mechanics

Form-finding

Adriaenssens 2000 [39] propose a 6-DOF discrete beam element that integrates in a dynamic relaxation solver. This element is meant for the numerical analysis of bent elements in cable net and gridshell structures.

Adriaenssens *et al.* 1999 [40] present a 3-DOF discrete beam element for the form-finding of elastic rods. This element is valid only for rods that are straight in their rest configuration and that have an isotropic cross-section. Barnes 1999 [41] integrates this element for numerical analysis based on dynamic relaxation. Adriaenssens and Barnes 2001 [42] observe a better stability of this element compare to their previous 6-DOF element.

Barnes *et al.* 2013 [43] try to take account for torsional behaviour in slender rods with anisotropic cross-section. They do not resort to any additional degree of freedom. Instead, they monitor the (geometric) torsion of a discrete space curve by computing the rotation rate between two consecutive osculating planes. This is valid only in rare specific cases where geometric torsion and mechanical torsion agree and is of little practical use.

D'Amico *et al.* 2014 [10] and later Poulsen 2015 [44] implement the 6-DOF beam element developed earlier by Adriaenssens 2000 [39] and use it for the form-finding of gridshells.

du Peloux *et al.* 2015 [17] and Lefevre *et al.* 2017 [18] propose a new 4-DOF element that takes account for both bending and torsion behaviours of slender rods. It relies on the Bishop frame and the notion of parallel transport. It is based on a circular spline interpolation. This element is valid for rods with anisotropic cross-section as well as for rod that are not straight in their rest configuration. They also formulate an elastic joint for the modeling of grids of interconnected beams.

D'Amico *et al.* 2016 [45] propose a similar approach but use a Catmull-Rom spline interpolation. However, dealing with boundary conditions is harder with this interpolation as it requires an additional node.

Kim-Lan Vaulot 2016 [24] revisit the benefits of using scale physical models for the form-finding of elastic gridshells. The grids are made out of Nitinol, a super-elastic material, to make sure the models will always work in the elastic domain of the material.

Bessini *et al.* 2017 [46] propose a beam element based on the Reissner-Simo geometrically exact beam model. Their formulation is compatible with the dynamic relaxation method.

Stability

Bulenda and Knippers 2001 [47] investigate for dome and barrel vault gridshells how imperfections can influence buckling.

Mesnil *et al.* 2015 [48] explore the influence of permanent bending pre-stress on the buckling capacity of strained gridshells. They show that for reasonably sized single-layer elastic gridshells the bending pre-stress does not influence the shape of the buckling modes. They give a simplified formula to estimate the buckling capacity of elastic gridshells under funicular loading.

Mesnil *et al.* 2015 [48] compare the linear buckling of non braced quadrangular gridshells and kagome gridshells.

Lefevre *et al.* [49] explore the buckling of triangulated single-layer elastic gridshells with a dome-like shape. In their analysis they take into account the eccentricity that exists between layers and the anisotropy of the grid. They propose a simplified formula to evaluate the buckling load of such gridshells.

Form-structure interaction

Malek 2012 [50] study how corrugation in shapes affect the mechanics of gridshells.

Jensen *et al.* 2013 [51] propose to interconnect several gridshells to form a stronger structure. Filz and Naicu 2015 [52] also investigate the properties of interconnected gridshells but for the purpose of kinematic effects.

Robustness

Tayeb *et al.* [53] study how the high level of redundancy in a gridshell enhance its resistance to collapse. They show that because of the redundancy, a pseudo ductile behaviour of the structure is still observable when a brittle material is used (such as GFRP).

Implementation

Douthe 2007 [25], Toussaint 2007 [54], Olsson 2012 [55], Poulsen 2015 [44] discuss the implementation details of the dedicated form-finding algorithm they have built.

1.4.2 Geometry

Generation of Chebychev nets

In *IL10 Grid Shells*, Otto 1974 [19] study the uniform mesh net with square cells. They propose a classification for suspended nets (pp. 68-69) and give an inventory of common problems such as overlapping and singularities. They explain how to build valuable physical models for hanging nets (pp. 50-55) and how to measure them with either close-range stereo-photogrammetry, a simple measuring table or the parallel light measurement technique (pp. 130-134). Finally, they propose a geometric method to find

Chebyshev meshes from a given curved shape called the *compass method* (pp. 140-141).

Bouhaya *et al.* 2009 [56] propose an alternative to the compass method for finding gridshell meshes on an imposed surface. This method consists in numerically dropping a grid onto a fixed shape. The simulation is achieved with a dynamic explicit finite element solver. Therefore, the proposed method can take into account the real mechanics of the grid, which is not possible with the compass method.

Bouhaya *et al.* 2014 [57] implement the compass method in a geometry software. For a fixed mesh pitch and starting point they parametrically generate a large number of discrete guidelines on the surface. The generation of a guideline is controlled by a vector of angles controlling the expansion on the surface. The method is then coupled with a genetic algorithm to find meshes where the curvature of the elements is minimised.

Lafuente Hernández *et al.* 2012 [58] propose a variational approach to find grids that minimize the curvature of the elements. This is done by introducing penalty energies. Consequently, the mesh is allowed to move away from the imposed shape and the bars are allowed to dilate from their initial length.

du Peloux *et al.* 2011 [59] implement the compass method in *Grasshopper*. They use it to design two large-scale gridshells in composite material in 2011 [8] and 2013 [9].

Lefevre *et al.* 2015 [49] propose an extended compass method that take into account the eccentricity between the layers of rods. This gap is generally due to the connection system.

Masson and Monasse 2017 [60] prove the existence of a global smooth Chebyshev net on complete, simply connected surfaces when the total absolute curvature is bounded by 2π. In his thesis, Masson 2017 [61] study the conditions of existence of Chebyshev nets with singularities and give methods to construct them.

Pone *et al.* 2016 [62] propose a tool similar to the ones developed by du Peloux *et al.* 2011 [59] and Bouhaya *et al.* 2014 [57].

Morphogenesis

Douthe *et al.* 2016 [63] propose a reverse approach. Instead of trying to fit a mesh on an imposed surface, they construct discrete surfaces that embed the required properties. They show that the dual mesh of an isoradial mesh is a Chebyshev net. They give a method to construct such nets.

Mesnil 2017 [14] propose various methods to generate construction-aware discrete surfaces. Some of them are applicable to gridshells, for instance to produce twist-free grids of grids with planar quadrangular panels.

1.4.3 Material

Douthe *et al.* 2010 [7] look for new materials that could surpass wood when building elastic gridshells. They use Ashby's selection method to show that composite materials in glass fibre reinforced polymers are good candidates. Douthe *et al.* 2006 [6] build the first structure of this kind.

Kotelnikova-Weiler *et al.* 2013 [64] extend the previous approach to draw some recommendations for the selection of materials for actively-bent structures.

Kotelnikova-Weiler 2012 [65] studies the long term behaviour of pultruded GFRP rods subject to permanent combined bending and torsion stresses.

1.4.4 Technology

Erection

In *IL10 Grid Shells*, Otto 1974 [19] propose various methods for erecting elastic gridshells. Quinn and Gengnagel 2014 [22] review several gridshell projects and their erection methods. They question the potential of air-inflated membrane cushions for the erection of strained gridshells. Quinn *et al.* 2016 [66] investigate the benefits of pneumatic falsework to erect strained gridshells.

Liuti *et al.* 2016 [67] present an inflatable membrane technology for the erection of post-formed timber gridshells. They test it on a small-scale structure.

Cladding

Hernández and Gengnagel 2014 [68] try to further improve the efficiency of deployable gridshells by using the cladding membrane to brace the structure. Although this solution is less stiff than the usual ones, it does enhance the deployability and reduce the work spent in the bracing stage.

Cuvilliers *et al.* 2017 [23] develop a concept of a hybrid structural skin, that is an elastic gridshell in composite material braced by a thin fibre reinforced concrete skin. The gridshell serves as a formwork to the concrete skin and the concrete skin is pored directly on the deformed grid. The connection enable a tight collaboration between the structural grid an the concrete, so that the skin is bracing the gridshell.

Optimization

D'Amico *et al.* 2015 [69] describe a procedure to optimise timber gridshell cross-sections. The optimisation is done for a given load case and relatively to the generated stresses. Nevertheless, this optimisation process does not take into account the buckling behaviour of the structure, which usually prevails in such lightweight structures.

Robotisation

Robotisation is investigated in recent timber gridshell projects such as the ZA pavilion [13] and the pavilion built at the ENPC in 2016.[24] Robotic design and manufacturing of timber structures is further explored by Menges *et al.* 2016 [70].

[24] This pavilion has been published on the web : http://thinkshell.fr/freeform-wooden-gridshell-2016/.

1.5 Conclusion

In this chapter, we have tried to immerse ourselves in depth and experience in the complexity of these structures. After a brief description of the concept, we have established two thorough reviews. The first review is dedicated to built elastic gridshell projects from the 1960s to the present day. This brief history draws the potential of these structures, particularly in terms of formal expression and structural performance. Far from confining them to a particular style of architecture, it underlines their great variety and richness. The second review is dedicated to a literature review on all research fields related to this topic (geometry, structure, materials, software).

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Conclusion

Ce travail de thèse s'est intéressé aux modèles de calcul dédiés aux structures précontraintes par flexion. Il s'est inscrit dans un projet de recherche plus large sur les structures de type *gridshell élastique*, développé par le laboratoire Navier. Initié au début des années 2000 par J.-F. Caron et O. Baverel, ce dernier entend revisiter le travail de l'ingénieur et architecte allemand Frei Otto sous le double aspect de la structure et des matériaux composites. J'ai rejoins ce projet en mai 2010 en qualité d'ingénieur de recherche, puis en tant que doctorant à partir d'octobre 2014. Sur ces presque 8 années de collaboration j'ai eu la chance de pouvoir non seulement développer une recherche personnelle sur cette thématique, mais également de pouvoir confronter le fruit de cette recherche à la réalité en concevant et construisant un certain nombre de gridshells en matériau composite ou en bois. Et c'est probablement ce qui caractérise le mieux la spécificité de mon travail : cette confrontation répétée entre théorie et pratique.

Construire courbe se révèle complexe à tous les niveaux et les gridshells n'échappent pas à cette règle. En effet, la définition géométrique de l'ouvrage en constitue la pierre angulaire et, à ce titre, en assure aussi bien l'identité architecturale que la faisabilité sur le plan structurel. Structure et Architecture s'en trouvent ainsi associées de manière symbiotique. Et c'est dans ce lien étroit que se noue leur complexité intrinsèque.

Revue

Dans la première partie de notre travail, nous avons souhaité nous immerger en profondeur et par l'expérience dans la complexité de ces structures. Nous avons commencé notre étude (voir chapter 1) par effectuer une revue critique des projets de gridshell élastique réalisés depuis les années 1960 jusqu'à nos jours. Cette brève histoire dessine à elle seule le potentiel de ces structures, notamment en terme d'expression formelle et de performance structurelle. Loin de les enfermer dans un style d'architecture particulier, elle en souligne au contraire la grande variété. Nous avons complété cette revue de projet par une revue de littérature approfondie sur l'ensemble des domaines de recherche connexes à cette thématique (géométrie, structure, matériaux, logiciel).

Expérimentation et maquette numérique

Nous avons ensuite présenté la plus importante de nos réalisations, la conception et la construction de la cathédrale éphémère de Créteil, premier véritable bâtiment réalisé à ce jour sur le principe du gridshell élastique en matériau composite (voir **??**). Construit en 2013, il est toujours en service. A cette occasion, nous avons mis au point une méthode, des outils et des critères d'évaluation pour permettre à des concepteurs – architectes et ingénieurs – de répondre de façon maîtrisée à un projet de gridshell [9]. Cette méthode s'appuie sur la réalisation d'une maquette numérique interactive qui associe des fonctions de modelage 3D basées sur une représentation NURBS des surfaces, des fonctions de maillage par la méthode du compas, et des fonctions de recherche de forme grâce à un code de calcul non linéaire basé sur la méthode de la relaxation dynamique. Elle a la particularité de recentrer le processus de conception sur la définition d'une forme et redonne ainsi de la place à l'expression de l'intention architecturale, là où la complexité des techniques de recherche de forme (sur modèle physique ou numérique) l'en avait privée. Nous avons montré comment cette liberté « retrouvée » a effectivement servi l'architecture du projet pour créer un espace qui fasse sens vis-à-vis de sa destination (un lieu de culte) et qui ne soit pas le produit de contraintes purement techniques. Ce travail, publié en 2016, s'est récemment vu distingué par l'International Association for Bridge and Structural Engine (IABSE).[25]

Pertinence des outils

Les outils que nous avons mis au point à l'occasion de ce projet ont pallié à l'inadéquation des outils de design existants, qui sont davantage orientés vers la justification des ouvrages que vers leur conception. Ils nous ont permis d'appréhender la problématique de l'interaction forme-maillage-structure avec beaucoup plus d'agilité que si nous avions eu recours aux seuls outils disponibles dans le commerce. Ils ont rendu possible le développement de ce projet de gridshell dans des contraintes de planning et de coût sévères, à l'opposé des moyens engagés pour la multihalle de Mannheim en 1975. Cependant, cette méthode a également montré un certain nombre de limites qui ont restreint notre capacité à développer une représentation riche et fonctionnelle du projet sous la forme d'une maquette numérique.

Limitation des outils

Sur le plan de la fonctionnalité de la représentation, il faut bien reconnaître que la maquette actuelle ne permet ni le niveau d'interactivité ni le niveau de réactivité qu'offrirait une simple maquette physique manipulable *à la main*. Bien que cet aspect n'ai pas constitué l'enjeu principal de notre travail, nous avons porté une grande attention à cette question dans le développement de nos outils, en essayant d'optimiser l'intégration des fonctions et la rapidité du code de calcul pour fournir l'expérience utilisateur la plus fluide et intuitive possible. C'est pour ces mêmes raisons que nous avons choisi d'implémenter nos outils dans le framework *Rhinoceros* & *Grasshopper*.[26] Pour aller plus loin sur les questions d'interactivité on pourrait explorer le champ de la réalité virtuelle et augmentée pour s'affranchir des limitations inhérentes à l'utilisation d'une souris, d'un clavier et d'un écran pour accéder

[25] IABSE Awards 2017, Outstanding Paper Award, Technical Report.

[26] J'ai commencé à développer ces outils sous la forme de scripts python pour *Rhinoceros* à l'occasion du projet Solidays. J'ai progressivement migré ces outils vers C**#** et développé des bibliothèques de composants *Grasshopper*. Aujourd'hui, cette maquette est concrètement contrôlée par un canevas *Grasshopper*.

à la maquette. Pour aller plus loin sur les questions de réactivité on pourrait explorer la piste du calcul parallèle (SIMD, CPU, GPU, ...) pour accélérer les codes de maillage et de recherche de forme ; on pourrait également explorer d'autres méthodes de résolution numériques potentiellement plus rapides que la relaxation dynamique ; ou bien on pourrait encore implémenter des fonctionnalités de raffinement automatique de grille pour travailler avec les modèles les plus légers possibles en terme de degrés de liberté.

Sur le plan de la richesse de la représentation, le code de calcul structurel utilisé reposait sur un élément de poutre discret à seulement trois degrés de liberté [39]. De ce fait, il ne permettait pas la modélisation des phénomènes de torsion et de couplage flexion-torsion dans les éléments structuraux. Bien que ces phénomènes puissent être négligés en première approximation dans le cas de grilles constituées de poutres de section circulaire et rectilignes dans leur configuration naturelle, ces phénomènes peuvent cependant se révéler critiques pour des matériaux fortement anisotropes comme le bois et ou les composites pultrudés, qui en effet résistent mal à des sollicitations de torsion. Par ailleurs, lorsque la section des poutres employées est anisotrope – comme c'est souvent le cas pour les gridshells en bois – ces phénomènes influent fortement sur la forme d'équilibre de la grille et sur le niveau de contrainte observé dans la structure, les poutres pouvant se retrouver soumises à d'importantes courbures selon leur axe fort d'inertie. En outre, l'élément discret à 3 degrés de liberté ne peut représenter la notion de moment que sous la forme d'un couple d'effort. Il reste donc très limité pour modéliser les conditions cinématiques parfois complexes des connexions ou des conditions d'appui, notamment lorsqu'un transfert de moment s'opère (e.g. au niveau d'un encastrement).

Nouveaux modèles de poutre

Dans la seconde partie de notre travail (voir part I) nous avons donc cherché à dépasser les limitations du modèle de calcul employé pour le projet de la cathédrale éphémère de Créteil. L'objectif poursuivi était de renforcer la précision et la complétude des informations mécaniques retournées par la maquette aux concepteurs, sans pour autant sacrifier le niveau d'interactivité et de réactivité précédemment atteint et qui faisait justement la pertinence de cet outil.

Dans une première tentative (voir **??**), à partir de travaux récents sur les tiges élastiques appliqués au champ des computer graphics [16], et dans la continuité d'un précédent travail de thèse auquel nous avons collaboré [27], nous avons, par une approche variationnelle, formulé un élément de poutre discret qui puisse rendre compte des phénomènes de torsion [18]. La description cinématique de l'élément repose ici sur la définition d'une ligne moyenne comprise comme une courbe paramétrique de l'espace ; et d'une section droite positionnée à l'aide d'un repère mobile adapté à cette courbe, lui-même entièrement déterminé, à une constante près, par une unique variable scalaire. Ainsi, cet élément possède un nombre minimal de degrés de liberté, à savoir 4. Cependant, ce nouveau modèle ne répond pas à l'ensemble des limitations identifiées précédemment. En particulier, il ne permet pas de représenter certaines discontinuités qui apparaissent là où les actions mécaniques s'exercent de manière concentrée, comme par exemple au niveau d'un appui, d'une connexion ou bien d'une charge ponctuelle. Cette capacité est pourtant primordiale pour l'étude des détails de la structure, qui sont des points clefs du système constructif comme nous l'avons montré dans notre présentation de la cathédrale éphémère.

Dans une seconde tentative (voir **??**), nous avons donc cherché à combler ces lacunes et à pouvoir rendre compte des discontinuités qui découlent des actions concentrées en sus des phénomènes de torsion. Nous avons commencé par montrer comment, à partir des équations dynamiques de Kirchhoff, nous pouvions formuler de manière relativement directe un élément de poutre à 4 degrés de liberté. Cette approche est apparue plus évidente que la première. Par ailleurs, on a montré qu'elle traitait naturellement la question des actions extérieures et s'insérait parfaitement dans le cadre conceptuel de la relaxation dynamique basé sur le principe fondamental de la dynamique. Puis nous avons développé une réflexion approfondie sur la notion de courbure discrète (voir **??**) qui nous a permis d'identifier les mécanismes géométriques nécessaires à la modélisation des discontinuités de courbure (et donc de moment). En combinant ces résultats nous sommes parvenus à mettre au point un élément de poutre discret à 4 degrés de liberté et 3 noeuds (voir **??**), contre 2 pour les modèles précédents. Les propriétés de section et de matériau sont supposées uniformes sur la longueur de l'élément. Il rend compte du comportement axial, de flexion et de torsion de la poutre, dans le cadre de la théorie de Kirchhoff, pour des sections dont le centre de torsion est confondu avec le centre de masse. Il peut subir des actions concentrées en ses extrémités et des actions distribuées uniformes en partie courante. Les efforts internes sont donc continus sur la longueur de l'élément mais peuvent subir des sauts au niveau des ses extrémités. Nous avons également présenté la démarche à suivre pour implémenter des conditions d'appui de type libre, rigide ou élastique.

Développement d'un code de calcul

Finalement, nous avons présenté succinctement *Marsupilami*, le code de calcul que nous avons mis au point et qui implémente ce nouvel élément. Il se matérialise sous la forme d'une API C**#** libre de toutes dépendances. Cette API a été partiellement implémentée dans une bibliothèque de composants *Grasshopper* pour servir d'interface graphique. De nombreuses pistes ont été explorées concernant l'architecture du code pour le doter de nouvelles possibilités, notamment grâce à l'usage des évènements (raffinement automatique de maillage, force suiveuse, parallélisation des calculs, interaction utilisateur, . . .). Le code essaie de tirer le meilleur parti des abstractions proposées par le langage C**#** pour marier différents types d'éléments, de conditions d'appui et même de noeuds selon leur nombre de degrés de liberté (3, 4 ou 6). Nous avons pu valider la précision de notre nouvel élément en comparant les résultats de *Marsupilami* avec ceux du logiciel *Abaqus* – référence en la matière – sur un certain nombre de cas tests. Réalisés sur des poutres seules, ce travail de validation demande a être poursuivi sur des structures complètes.[27] Cependant, *Marsupilami* n'a pour l'instant rien d'un véritable logiciel que l'on pourrait utiliser dans un contexte de production. Dans son état actuel il s'agit plus d'une *preuve de concept*, qui mériterait un effort de développement conséquent pour établir une première version stable transférable à d'autres utilisateurs.

Perspectives

Les modèles, les outils et les méthodes développés au cours de cette recherche ont rendu possible la conception et la réalisation d'un certain nombre de prototypes à une échelle parfois importante, comme ce fût le cas des gridshells de Solidays en 2011 et de Créteil en 2013. L'expérience acquise sur ces projets a mis en valeur la nécessité de disposer d'outils de conception agiles pour aborder l'interaction forme-maillage-structure. Elle a aussi souligné les éléments qui mériteraient d'être approfondis, parmi

[27] Par exemple un recalcul de la cathédrale de Créteil pourrait peut-être permettre de comprendre certaines des ruptures observées 6 mois après le montage.

lesquels nous retiendrons :

Marsupilami. Le code actuel pourrait devenir une API C**#** fort pratique pour le calcul des gridshells moyennant un effort de développement conséquent. Le travail pourrait consister à consolider et étendre l'API actuelle pour la rendre stable et facilement extensible ; ainsi qu'à développer une interface interactive pour la plateforme *Rhinoceros* & *Grasshopper*. Ce travail devrait garder comme objectif la capacité à générer des maquettes numériques de conception qui soient les plus agiles possible. L'API elle-même pourrait potentiellement faire l'objet d'un développement collaboratif, pourquoi pas en partenariat avec d'autres laboratoires, ce qui permettrait de pourvoir aux compétences nécessaires à un tel projet. En ce sens, une licence de type *Open Source* pourrait permettre une meilleur diffusion du code et donc de toucher de potentiels contributeurs.

Système Constructif. La noix de connexion et le manchon constituent deux détails clefs du système constructif actuel. Ces pièces pourraient faire l'objet de nombreuses améliorations pour en augmenter la légèreté, ou bien pour les rendre plus fonctionnelles afin de faciliter l'assemblage de la grille par les opérateurs. Il en va de même pour le dispositif de contreventement qui impact grandement les coûts de construction comme nous l'avons montré. Dans ce sens nous avons pu tester en 2016, à petite échelle sur trois gridshells en bois d'environ $50\,\mathrm{m}^2$, un système à câble installé sur la grille au sol et activé une fois la déformation terminée. Cela a permis de réduire le temps de travail en hauteur de façon significative et a nécessité des développement nouveaux pour les pièces de connexion du système de triangulation.

Enveloppe. L'enveloppe des ces structures reste un champ difficile à maîtriser du fait de la courbure géométrique. Les membranes employées jusqu'ici ne garantissent aucunes performances acoustiques ou thermiques sérieuses. On pourrait contourner ce problème en identifiant les applications potentielles où ces critères ne sont pas rédhibitoires, comme par exemple pour certaines structures à usage temporaire ou bien pour des couvertures d'espaces industriels qui ne doivent assurer aucune autre fonction que celle de l'étanchéité. Cette question peut être abordée de manière plus globale avec celle de la structure et du contreventement. Nous avons eu l'occasion de développer une réflexion originale sur le sujet, en mettant au point un concept de structure hybride dans lequel l'enveloppe assure à la fois le clos-couvert du bâtiment et le contreventement de la résille en matériau composite [23]. L'idée principale est d'utiliser le gridshell comme cintre pour couler une fine enveloppe en béton fibré par dessus. Une connexion mécanique est assurée entre la résille et le béton pour permettre à l'enveloppe de jouer le rôle de contreventement d'une part ; et minimiser l'épaisseur de béton nécéssaire d'autre part.

Autres. On pourra aussi considérer plus largement les applications potentielles du présent travail, et principalement l'utilisation de l'élément de poutre mis au point dans le domaine de l'*active-bending*. En s'intéressant par exemple au problème de positionnement des gaines sur les bras robotisés, un outil autrefois réservé aux grands industriels et en passe de se démocratiser, qui peuvent venir en contact des outils ou gêner le mobilité du bras et dont les mouvements sont difficiles à prévoir (à cause du couplage flexion torsion). Pour rester dans le secteur de la construction, on pourra également regarder du côté des mécanismes avancés appliqués aux *shading device* [71].

Enfin, bien que ce travail ait permis de mieux comprendre certains aspects des structures de type *gridshell élastique* et d'enrichir la palette des outils d'analyse disponibles pour les concevoir, il reste manifestement beaucoup à faire pour les démocratiser là où elles pourraient apporter une valeur ajoutée significative. Cependant, il nous semble que ce travail n'est désormais plus du ressort de la recherche académique et devrait, pour continuer à vivre, trouver une viabilité économique à moyen terme.

Appendix Part II

A Parabolic interpolation

A.1 Introduction

In this appendix, we give the required formulas to conduct a parabolic interpolation of a scalar or vector-valued function over an interval.

We look for a polynomial interpolation of order 2 of a continuous scalar or vector-valued function $\boldsymbol{V} : t \mapsto \boldsymbol{V}(t)$ over the interval $[t_0, t_2]$; supposing that the value of the function is known for three distinct parameters $t_0 < t_1 < t_2$:

$$\boldsymbol{V}(t_0) = \boldsymbol{V}_0 \tag{A.1a}$$

$$\boldsymbol{V}(t_1) = \boldsymbol{V}_1 \tag{A.1b}$$

$$\boldsymbol{V}(t_2) = \boldsymbol{V}_2 \tag{A.1c}$$

This interpolation method is employed several times in this thesis, for instance to evaluate the position of a kinetic energy peak during the dynamic relaxation process. It is also employed for evaluating the bending moment and the curvature of a discrete rod at mid-edge, knowing its values at vertices.

Note that this interpolation method is valid if the basis in which $\boldsymbol{V}$ is decomposed does not depend on the parameter t. Otherwise, the classical transportation term should be considered ($\boldsymbol{\omega} \times \boldsymbol{V}$).

A.2 Lagrange interpolating polynomial

The Lagrange interpolation of order two is given by the following polynomial :

$$\boldsymbol{V}(t) = \boldsymbol{V}_0 \frac{(t-t_1)(t-t_2)}{(t_0-t_1)(t_0-t_2)} + \boldsymbol{V}_1 \frac{(t-t_0)(t-t_2)}{(t_1-t_0)(t_1-t_2)} + \boldsymbol{V}_2 \frac{(t-t_0)(t-t_1)}{(t_2-t_0)(t_2-t_1)} \tag{A.2}$$

A.3 Reparametrization

Lets introduce the distances l_0 and l_1 in the parametric space :

$$l_0 = t_1 - t_0 \tag{A.3a}$$
$$l_1 = t_2 - t_1 \tag{A.3b}$$

Lets introduce the change of variable $u = t - t_1$. The polynomial in eq. (A.2) can be rewritten in the form :

$$\boldsymbol{V}(u) = \boldsymbol{V}_0 \frac{u(u - l_1)}{l_0(l_0 + l_1)} - \boldsymbol{V}_1 \frac{(u + l_0)(u - l_1)}{l_0 l_1} + \boldsymbol{V}_2 \frac{u(u + l_0)}{l_1(l_0 + l_1)} \tag{A.4}$$

where :

$$u_0 = -l_0 \tag{A.5a}$$
$$u_1 = 0 \tag{A.5b}$$
$$u_2 = l_1 \tag{A.5c}$$

The derivative of this polynomial is also required to determine the extremum value of $\boldsymbol{V}$. Differentiating eq. (A.4) gives :

$$\boldsymbol{V}'(u) = \boldsymbol{V}_0 \frac{2u - l_1}{l_0(l_0 + l_1)} - \boldsymbol{V}_1 \frac{2u + (l_0 - l_1)}{l_0 l_1} + \boldsymbol{V}_2 \frac{2u + l_0}{l_1(l_0 + l_1)} \tag{A.6}$$

This expression can be factorized to give the more compact form :

$$\boldsymbol{V}'(u) = \left(\frac{\boldsymbol{V}_1 - \boldsymbol{V}_0}{l_0} \right) \frac{l_1 - 2u}{l_0 + l_1} + \left(\frac{\boldsymbol{V}_2 - \boldsymbol{V}_1}{l_1} \right) \frac{l_0 + 2u}{l_0 + l_1} \tag{A.7}$$

A.4 Characteristic values

Using eq. (A.4) the interpolated values of $\boldsymbol{V}$ at mid distance between t_0 and t_1 ($u = -l_0/2$), and at mid distance between t_1 and t_2 ($u = +l_1/2$) are given by :

$$\boldsymbol{V}_{01} = \quad \boldsymbol{V}_0 \frac{l_0 + 2l_1}{4(l_0 + l_1)} \quad + \boldsymbol{V}_1 \frac{l_0 + 2l_1}{4l_1} - \boldsymbol{V}_2 \frac{{l_0}^2}{4l_1(l_0 + l_1)} \tag{A.8a}$$
$$\boldsymbol{V}_{12} = -\boldsymbol{V}_0 \frac{{l_1}^2}{4l_0(l_0 + l_1)} + \boldsymbol{V}_1 \frac{2l_0 + l_1}{4l_0} + \boldsymbol{V}_2 \frac{2l_0 + l_1}{4(l_0 + l_1)} \tag{A.8b}$$

Using eq. (A.7) the interpolated values of $\boldsymbol{V}'$ at mid distance between t_0 and t_1 ($u = -l_0/2$), and at mid distance between t_1 and t_2 ($u = +l_1/2$) are given by :

$$\boldsymbol{V}'_{01} = \frac{\boldsymbol{V}_1 - \boldsymbol{V}_0}{l_0} \tag{A.9a}$$
$$\boldsymbol{V}'_{12} = \frac{\boldsymbol{V}_2 - \boldsymbol{V}_1}{l_1} \tag{A.9b}$$

Remark that this is an interesting result as at these parameters the evaluation of $\boldsymbol{V}'$ boils down to a finite difference scheme.

Using eq. (A.7) and introducing $\alpha = \frac{l_0}{l_0+l_1}$ the interpolated values of $\boldsymbol{V}'$ at t_0, t_1 and t_2 are given by :

$$\boldsymbol{V}'_0 = (1+\alpha)\boldsymbol{V}'_{01} - \alpha\boldsymbol{V}'_{12} \tag{A.10a}$$

$$\boldsymbol{V}'_1 = (1-\alpha)\boldsymbol{V}'_{01} + \alpha\boldsymbol{V}'_{12} \tag{A.10b}$$

$$\boldsymbol{V}'_2 = (\alpha-1)\boldsymbol{V}'_{01} + (2-\alpha)\boldsymbol{V}'_{12} \tag{A.10c}$$

Lets rewrite eqs. (A.8a) and (A.8b) with the help of α :

$$\boldsymbol{V}_{01} = \frac{1}{4}\left((2-\alpha)\boldsymbol{V}_0 + \frac{2-\alpha}{1-\alpha}\boldsymbol{V}_1 - \frac{\alpha^2}{1-\alpha}\boldsymbol{V}_2\right) \tag{A.11a}$$

$$\boldsymbol{V}_{01} = \frac{1}{4}\left(-\frac{(1-\alpha)^2}{\alpha}\boldsymbol{V}_0 + \frac{1+\alpha}{\alpha}\boldsymbol{V}_1 + (1+\alpha)\boldsymbol{V}_2\right) \tag{A.11b}$$

A.5 Extremum value

The extremum value of the parabola is obtained for $\boldsymbol{V}'(u^*) = 0$. It's a minimum if $\boldsymbol{V}'_{12} > \boldsymbol{V}'_{01}$ and it's a maximum if $\boldsymbol{V}'_{12} > \boldsymbol{V}'_{01}$:

$$u^* = \frac{l_1\boldsymbol{V}'_{01} + l_0\boldsymbol{V}'_{12}}{2(\boldsymbol{V}'_{01} - \boldsymbol{V}'_{12})} \tag{A.12}$$

Remark that if $\boldsymbol{V}'_{12} = \boldsymbol{V}'_{01}$ it does not make sens to compute u^* as in this case the parabola degenerates into a line. The value of the function at this parameter is given by :

$$\boldsymbol{V}(u^*) = \boldsymbol{V}_1 + \frac{(l_1\boldsymbol{V}'_{01} + l_0\boldsymbol{V}'_{12})^2}{4(l_0+l_1)(\boldsymbol{V}'_{01} - \boldsymbol{V}'_{12})} \tag{A.13}$$

The parabola in eq. (A.4) now writes :

$$\boldsymbol{V}(u) = -\frac{\boldsymbol{V}'_{01} - \boldsymbol{V}'_{12}}{l_0+l_1}(u-u^*)^2 + \boldsymbol{V}(u^*) \tag{A.14}$$

The extremum is located in $[t_0, t_2]$ if the sign of $\boldsymbol{V}'$ changes on this interval. This condition is satisfied whenever $\boldsymbol{V}'_{01} \cdot \boldsymbol{V}'_{12} < 0$.

Finally, in the special case of a uniform discretization where $l_0 = l_1 = l$, eqs. (A.12) and (A.13) become :

$$u^* = \frac{l}{2}\left(\frac{\boldsymbol{V}_0 - \boldsymbol{V}_2}{\boldsymbol{V}_0 - 2\boldsymbol{V}_1 + \boldsymbol{V}_2}\right) \tag{A.15a}$$

$$\boldsymbol{V}(u^*) = \boldsymbol{V}_1 - \frac{u^*}{4l}(\boldsymbol{V}_2 - \boldsymbol{V}_0) \tag{A.15b}$$

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