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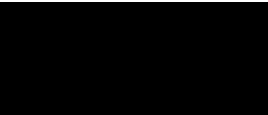
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Torsion Part I

Connection Part II

6 Bench for HPC

6.1 Introduction

In this section aims at providing basic but reliable guidelines to produce fast and mannagable code for our algorithms

Most compilers with which you are probably familiar are standalone programs which take as input some source code text and compile it into machine code (or some other target representation).

[AMR02]

6.2 Languages

- Csharp - Julia - C++ - Intel MKL - OpenBLAS

6.3 From syntax to processor

A short story about how a code is translated to get machin instructions

6.4 Benchmark

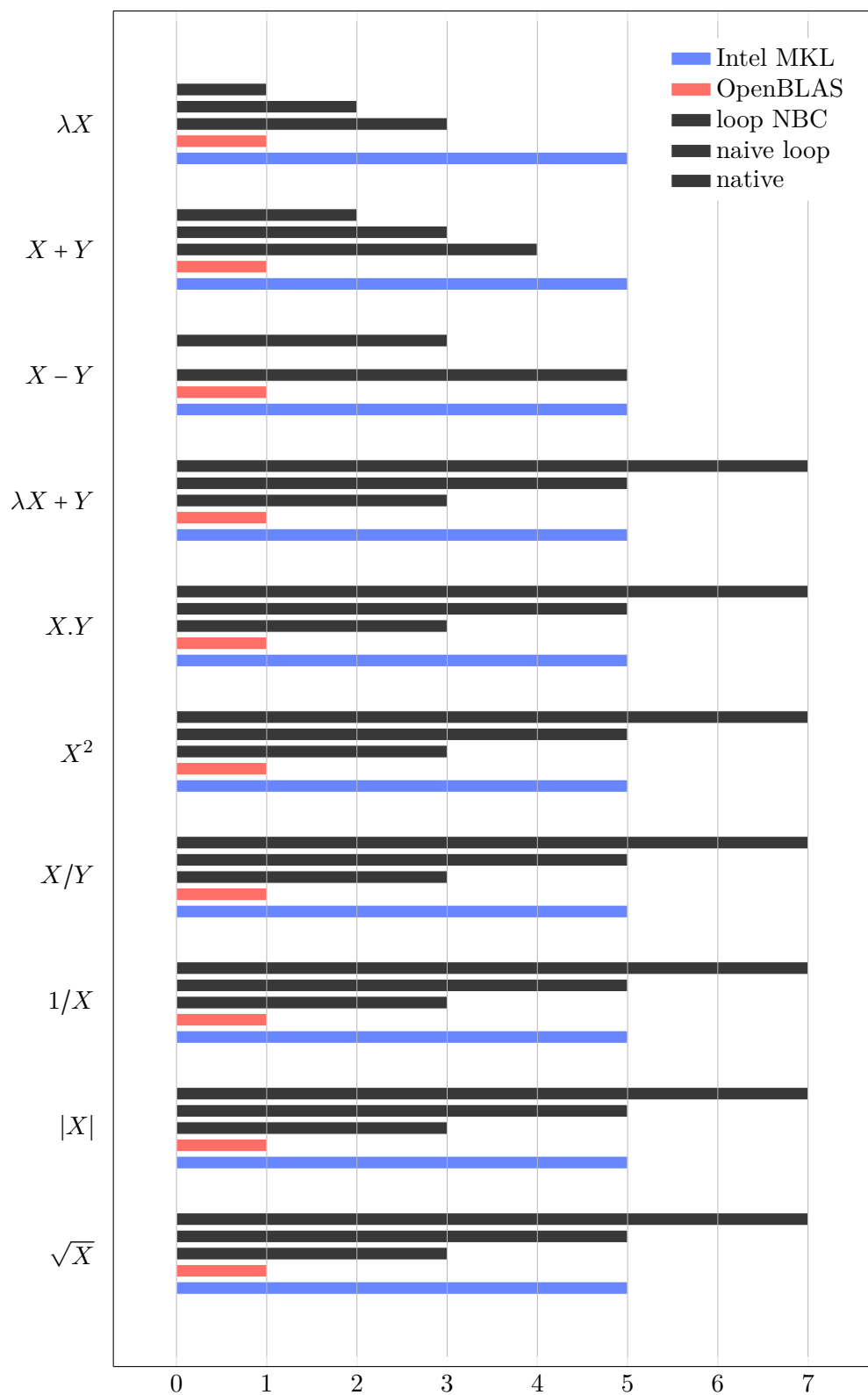


Figure 6.1 – Each operator is evaluated on a vector of Float64 of size $n = 10^6$ for about 10s. Results are given relatively to MKL performance (MKL = 1).

6.5 Memory allocation and garbage collection

```
1 using VML
2
3 # wrap functions to avoid global scoping while testing
4 function sqrt_jvectorized{T<:Number}(a::Vector{T})
5     sqrt(a)
6 end
7 function sqrt_jbroadcast{T<:Number}(a::Vector{T})
8     broadcast(sqrt,a)
9 end
10 function sqrt_jbroadcast!{T<:Number}(dest::Vector{T}, a::Vector{T})
11     broadcast!(sqrt,dest,a)
12 end
13 function sqrt_jmap{T<:Number}(a::Vector{T})
14     map!(sqrt,a)
15 end
16 function sqrt_jmap!{T<:Number}(dest::Vector{T}, a::Vector{T})
17     map!(sqrt,dest,a)
18 end
19 function sqrt_jloop{T<:Number}(dest::Vector{T},a::Vector{T})
20     @inbounds for i in eachindex(a) dest[i]=sqrt(a[i]) end
21 end
22
23 function sqrt_bench()
24     # define vector size and floating precision
25     n = 1_000
26     T = Float64
27     # allocate vectors
28     dest = ones(T,n)
29     @time a = rand(T,n)
30     # bench
31     gc()
32     gc_enable(false)
33     @time sqrt_jvectorized(a)
34     @time sqrt_jbroadcast(a)
35     @time sqrt_jbroadcast!(dest,a)
36     @time sqrt_jmap(a)
37     @time sqrt_jmap!(dest,a)
38     @time VML.sqrt(a)
39     @time VML.sqrt!(dest,a)
40     gc_enable(true)
41 end
42
43 sqrt_bench()
```

sqrt Float64	CPU (ns/el)	Allocation (Bytes)
Allocation	4	8,112
Julia vectorized	9	8,080
Julia broadcast	13	8,352
Julia broadcast!	7	16
Julia map	100	48,000
Julia map!	92	48,000
VML (allocation)	6	8,080
VML (in-place)	4	0

Table 6.1 – Memory allocations for various methods computing $\text{sqrt}(a)$ for $n = 10^4$

Toutes les syntaxes ne sont pas égales en termes de gestion de la mémoire. Le problème, c'est le passage de la GC qui est couteux en temps. Donc il faut essayer de minimiser l'utilisation mémoire. Idéalement le problème peu rester dans le cache du processeur (mémoire d'accès bcp plus rapide que la RAM). Donc la meilleur stratégie consiste à pré-allouer les tableaux et à faire des opération "in-place" au maximum, c'est à dire d'écraser les donner au fur et à mesure du calcul.

On remarque que l'allocation mémoire est très différente d'une fonction à l'autre. Il est important de privilégier des opération "in-place" pour contenir l'allocation mémoire, sinon on risque de déclencher la GC qui est couteuse.

Les temps CPU sont indicatifs car le bench est fait sur une durée caractéristique trop courte

```

1 using DataFrames, VML
2
3 function sqrt_bench()
4
5     # vector size
6     N = [1,2,3,4,5,6,7,8,9,
7          10,20,30,40,50,60,70,80,90,100,200,300,400,500,750,
8          1_000,2_500,5_000,7_500,10_000,100_000,1_000_000]
9
10    # dataframe for results
11    df = DataFrame(N=[], ALLOC=Float64[], JULIA=Float64[], MKL=Float64[])
12
13    @inbounds for i in 1:length(N)
14        T = Float64
15        n = N[i]
16        a = rand(T,n)
17        dest = zeros(T,n)
18
19        # evaluate sqrt and allocation
20        # for small n @elapsed applies to a bunch of evaluations
21        nrep = 1000
22        ncycle = 10_000 ÷ n + 1
23
24        # trigger garbage collection
25        gc()
26        talloc = 0.0 ; talloc = 0.0 ; tsqrt = 0.0
27        for j in 1:nrep
28            talloc += @elapsed for k in 1:ncycle Vector{T}(n) end
29            tsqrt += @elapsed for k in 1:ncycle sqrt(a) end
30        end
31
32        # scale results (ns/element)
33        talloc = talloc / nrep / ncycle / n * 1e9
34        tsqrt = tcu / nrep / ncycle / n * 1e9
35
36        # write results
37        push!(df,[n,talloc, tsqrt])
38    end
39    df
40 end
41
42 sqrt_bench()

```

Ici on met en évidence la non linéarité du coût d'allocation par élément d'un tableau de taille n . On remarque que la différence entre le coût de sqrt et le coût de l'allocation est

constante : c'est le coût de sqrt hors allocation. Attention, cette notion est "language dependent" car les allocations sont gérées par la GC.

Remarque, on trouverait sans doute la même chose pour MKL, à cause du marshalling : le coût d'appel à une fonction C est supérieur à celui d'une fonction managée (cf HPC .Net)

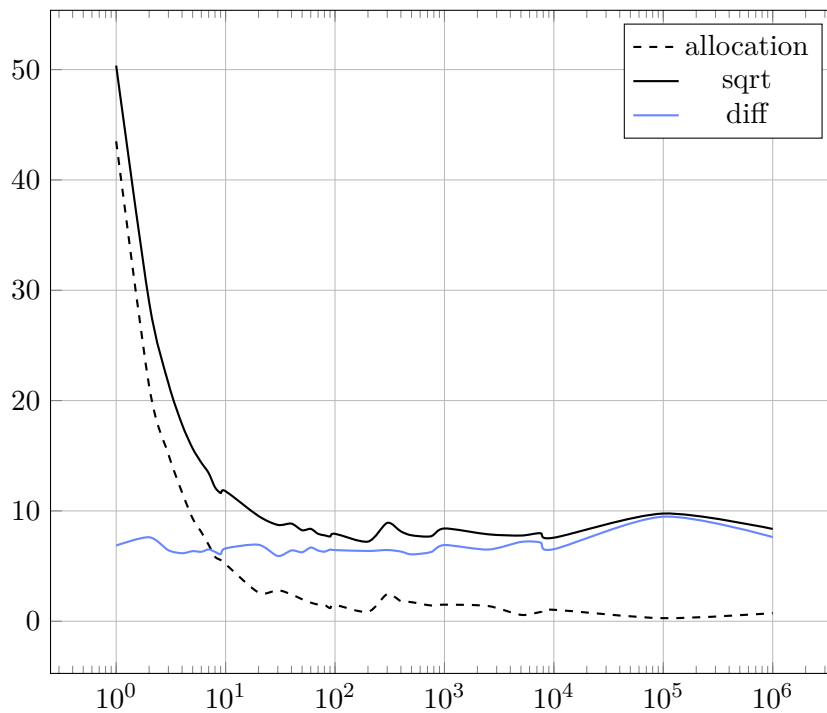


Figure 6.2 – Nonlinear cost of CPU time in ns/el of memory allocation for arrays (Float64).

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