

Modeling of bending-torsion couplings in active-bending structures. Application to the design of elastic gridshell.



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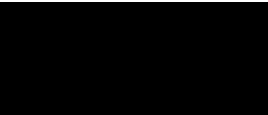


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B Parabolic interpolation

B.1 Introduction

In this appendix, we give the required formulas to conduct a parabolic interpolation of a scalar or vector-valued function over an interval.

We look for a polynomial interpolation of order 2 of a continuous scalar or vector-valued function $\mathbf{V} : t \mapsto \mathbf{V}(t)$ over the interval $[t_0, t_2]$; supposing that the value of the function is known for three distinct parameters $t_0 < t_1 < t_2$:

$$\mathbf{V}(t_0) = \mathbf{V}_0 \tag{B.1a}$$

$$\mathbf{V}(t_1) = \mathbf{V}_1 \tag{B.1b}$$

$$\mathbf{V}(t_2) = \mathbf{V}_2 \tag{B.1c}$$

This interpolation method is employed several times in this thesis, for instance to evaluate the position of a kinetic energy peak during the dynamic relaxation process. It is also employed for evaluating the bending moment and the curvature of a discrete rod at mid-edge, knowing its values at vertices.

Note that this interpolation method is valid if the basis in which \mathbf{V} is decomposed does not depend on the parameter t . Otherwise, the classical transportation term should be considered ($\boldsymbol{\omega} \times \mathbf{V}$).

B.2 Lagrange interpolating polynomial

The Lagrange interpolation of order two is given by the following polynomial :

$$\mathbf{V}(t) = \mathbf{V}_0 \frac{(t - t_1)(t - t_2)}{(t_0 - t_1)(t_0 - t_2)} + \mathbf{V}_1 \frac{(t - t_0)(t - t_2)}{(t_1 - t_0)(t_1 - t_2)} + \mathbf{V}_2 \frac{(t - t_0)(t - t_1)}{(t_2 - t_0)(t_2 - t_1)} \tag{B.2}$$

B.3 Reparametrization

Lets introduce the distances l_0 and l_1 in the parametric space :

$$l_0 = t_1 - t_0 \quad (\text{B.3a})$$

$$l_1 = t_2 - t_1 \quad (\text{B.3b})$$

Lets introduce the change of variable $u = t - t_1$. The polynomial in [eq. \(B.2\)](#) can be rewritten in the form :

$$\mathbf{V}(u) = \mathbf{V}_0 \frac{u(u - l_1)}{l_0(l_0 + l_1)} - \mathbf{V}_1 \frac{(u + l_0)(u - l_1)}{l_0 l_1} + \mathbf{V}_2 \frac{u(u + l_0)}{l_1(l_0 + l_1)} \quad (\text{B.4})$$

where :

$$u_0 = -l_0 \quad (\text{B.5a})$$

$$u_1 = 0 \quad (\text{B.5b})$$

$$u_2 = l_1 \quad (\text{B.5c})$$

The derivative of this polynomial is also required to determine the extremum value of \mathbf{V} . Differentiating [eq. \(B.4\)](#) gives :

$$\mathbf{V}'(u) = \mathbf{V}_0 \frac{2u - l_1}{l_0(l_0 + l_1)} - \mathbf{V}_1 \frac{2u + (l_0 - l_1)}{l_0 l_1} + \mathbf{V}_2 \frac{2u + l_0}{l_1(l_0 + l_1)} \quad (\text{B.6})$$

This expression can be factorized to give the more compact form :

$$\mathbf{V}'(u) = \left(\frac{\mathbf{V}_1 - \mathbf{V}_0}{l_0} \right) \frac{l_1 - 2u}{l_0 + l_1} + \left(\frac{\mathbf{V}_2 - \mathbf{V}_1}{l_1} \right) \frac{l_0 + 2u}{l_0 + l_1} \quad (\text{B.7})$$

B.4 Characteristic values

Using [eq. \(B.4\)](#) the interpolated values of \mathbf{V} at mid distance between t_0 and t_1 ($u = -l_0/2$), and at mid distance between t_1 and t_2 ($u = +l_1/2$) are given by :

$$\mathbf{V}_{01} = \mathbf{V}_0 \frac{l_0 + 2l_1}{4(l_0 + l_1)} + \mathbf{V}_1 \frac{l_0 + 2l_1}{4l_1} - \mathbf{V}_2 \frac{l_0^2}{4l_1(l_0 + l_1)} \quad (\text{B.8a})$$

$$\mathbf{V}_{12} = -\mathbf{V}_0 \frac{l_1^2}{4l_0(l_0 + l_1)} + \mathbf{V}_1 \frac{2l_0 + l_1}{4l_0} + \mathbf{V}_2 \frac{2l_0 + l_1}{4(l_0 + l_1)} \quad (\text{B.8b})$$

Using [eq. \(B.7\)](#) the interpolated values of \mathbf{V}' at mid distance between t_0 and t_1 ($u = -l_0/2$), and at mid distance between t_1 and t_2 ($u = +l_1/2$) are given by :

$$\mathbf{V}'_{01} = \frac{\mathbf{V}_1 - \mathbf{V}_0}{l_0} \quad (\text{B.9a})$$

$$\mathbf{V}'_{12} = \frac{\mathbf{V}_2 - \mathbf{V}_1}{l_1} \quad (\text{B.9b})$$

Remark that this is an interesting result as at these parameters the evaluation of \mathbf{V}' boils down to a finite difference scheme.

Using [eq. \(B.7\)](#) and introducing $\alpha = \frac{l_0}{l_0+l_1}$ the interpolated values of \mathbf{V}' at t_0 , t_1 and t_2 are given by :

$$\mathbf{V}'_0 = (1 + \alpha)\mathbf{V}'_{01} - \alpha\mathbf{V}'_{12} \quad (\text{B.10a})$$

$$\mathbf{V}'_1 = (1 - \alpha)\mathbf{V}'_{01} + \alpha\mathbf{V}'_{12} \quad (\text{B.10b})$$

$$\mathbf{V}'_2 = (\alpha - 1)\mathbf{V}'_{01} + (2 - \alpha)\mathbf{V}'_{12} \quad (\text{B.10c})$$

Lets rewrite [eq. \(B.8a\)](#) and [\(B.8b\)](#) with the help of α :

$$\mathbf{V}_{01} = \frac{1}{4} \left((2 - \alpha)\mathbf{V}_0 + \frac{2 - \alpha}{1 - \alpha}\mathbf{V}_1 - \frac{\alpha^2}{1 - \alpha}\mathbf{V}_2 \right) \quad (\text{B.11a})$$

$$\mathbf{V}_{01} = \frac{1}{4} \left(-\frac{(1 - \alpha)^2}{\alpha}\mathbf{V}_0 + \frac{1 + \alpha}{\alpha}\mathbf{V}_1 + (1 + \alpha)\mathbf{V}_2 \right) \quad (\text{B.11b})$$

B.5 Extremum value

The extremum value of the parabola is obtained for $\mathbf{V}'(u^*) = 0$. It's a minimum if $\mathbf{V}'_{12} > \mathbf{V}'_{01}$ and it's a maximum if $\mathbf{V}'_{12} < \mathbf{V}'_{01}$:

$$u^* = \frac{l_1\mathbf{V}'_{01} + l_0\mathbf{V}'_{12}}{2(\mathbf{V}'_{01} - \mathbf{V}'_{12})} \quad (\text{B.12})$$

Remark that if $\mathbf{V}'_{12} = \mathbf{V}'_{01}$ it does not make sens to compute u^* as in this case the parabola degenerates into a line. The value of the function at this parameter is given by :

$$\mathbf{V}(u^*) = \mathbf{V}_1 + \frac{(l_1\mathbf{V}'_{01} + l_0\mathbf{V}'_{12})^2}{4(l_0 + l_1)(\mathbf{V}'_{01} - \mathbf{V}'_{12})} \quad (\text{B.13})$$

The parabola in [eq. \(B.4\)](#) now writes :

$$\mathbf{V}(u) = -\frac{\mathbf{V}'_{01} - \mathbf{V}'_{12}}{l_0 + l_1}(u - u^*)^2 + \mathbf{V}(u^*) \quad (\text{B.14})$$

The extremum is located in $[t_0, t_2]$ if the sign of \mathbf{V}' changes on this interval. This condition is satisfied whenever $\mathbf{V}'_{01} \cdot \mathbf{V}'_{12} < 0$.

Finally, in the special case of a uniform discretization where $l_0 = l_1 = l$, [eq. \(B.12\)](#) and [\(B.13\)](#) become :

$$u^* = \frac{l}{2} \left(\frac{\mathbf{V}_0 - \mathbf{V}_2}{\mathbf{V}_0 - 2\mathbf{V}_1 + \mathbf{V}_2} \right) \quad (\text{B.15a})$$

$$\mathbf{V}(u^*) = \mathbf{V}_1 - \frac{u^*}{4l}(\mathbf{V}_2 - \mathbf{V}_0) \quad (\text{B.15b})$$