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par

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# 6 Bench for HPC

## 6.1 Introduction

In this section aims at providing basic but reliable guidlines to produce fast and mannagable code for our algorithms

Most compilers with which you are probably familiar are standalone programs which take as input some source code text and compile it into machine code (or some other target representation).

[AMR02]

## 6.2 Languages

- Csharp - Julia - C++ - Intel MKL - OpenBLAS

## 6.3 From syntax to processor

A short story about how a code is translated to get machin instructions

## 6.4 Benchmark

## Bibliography

[AMR02] Ralph Abraham, Jerrold E. Marsde, and Tudor Ratiu. *Manifolds, Tensor Analysis, and Applications (Ralph Abraham, Jerrold E. Marsden and Tudor Ratiu)*. 2002.

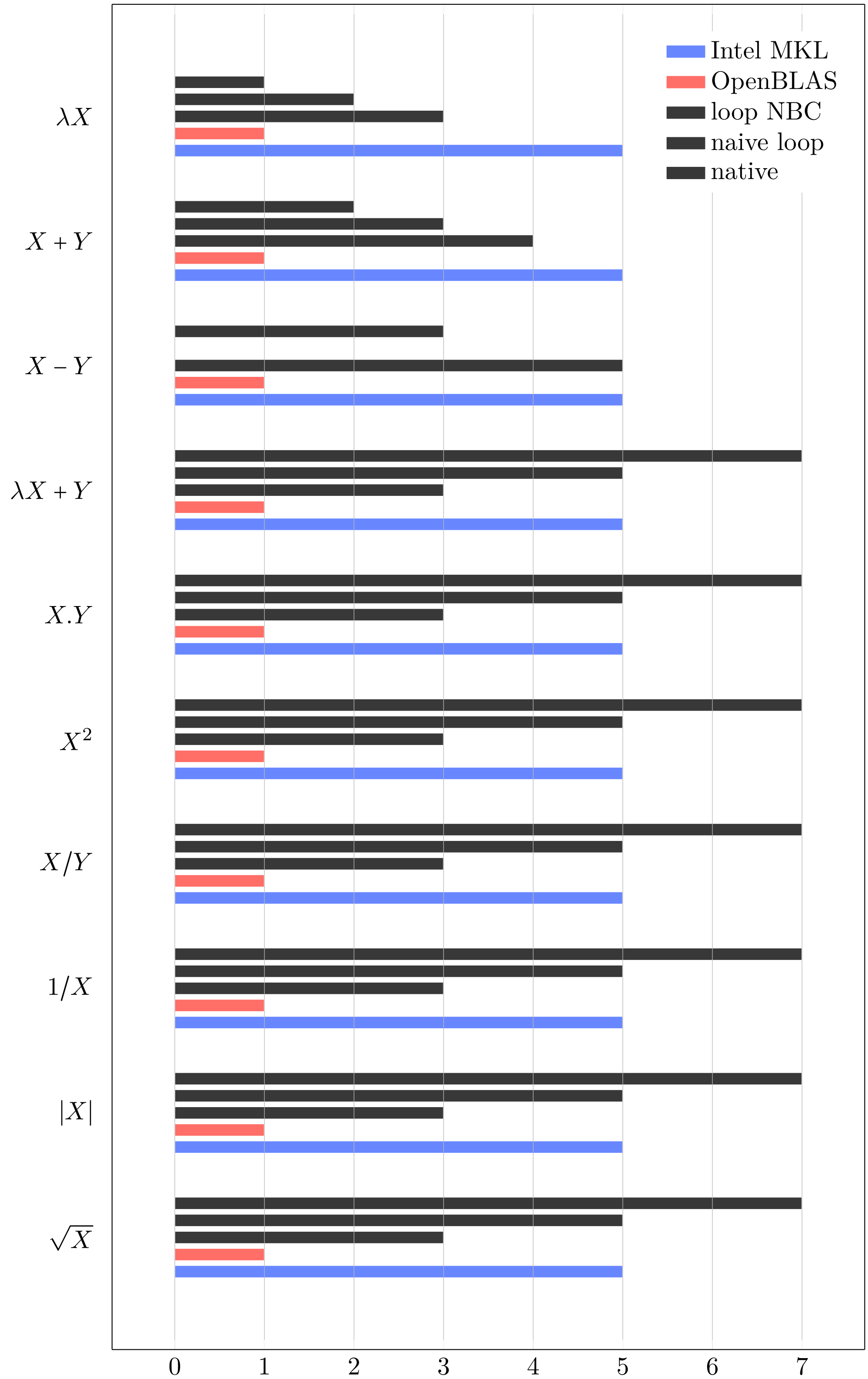


Figure 6.1 – Each operator is evaluated on a vector of Float64 of size $n = 10^6$ for about 10s. Results are given relatively to MKL performance (MKL = 1).