$$r = X_{\alpha} d_{\alpha} + u_k d_k$$