

Modeling of bending-torsion couplings in active-bending structures. Application to the design of elastic gridshell.



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1 Elastic gridshells

1.1 Building free-forms

1.1.1 Non-standard forms

1.1.2 Importance of free-forms in modern architecture

1.1.3 Canonical approaches to build free-forms

1.1.4 Main challenges

1.1.5 Elastic gridshell : a definition

The invention of the gridshell concept is commonly attributed to Frei Otto, a German architect who devoted several years to gridshells. In 1975 he achieved the famous *Mannheim Multihalle* [HL75], a wooden shell of 7500 m², in collaboration with the engineer Edmund Happold (Arup). Literally, the word “gridshell” refers to grids behaving like shells : from a mechanical point of view that means stresses acting on the structure are mainly transmitted through compression and traction. These structures can cross large-span with very little material.

However, according to the historic evolution of the concept, characterizing a gridshell as the combination of a structural concept – a grid behaving like a shell – and a specific construction process – using the bending flexibility of the material – seems to be more accurate. The Mannheim project (in which a wooden regular and planar grid, lacking shear stiffness, is elastically deformed up to a targeted shape with the help of stays, and then braced and covered) is regarded as the starting point of this new concept.

The Mannheim project is regarded as the starting point of this new concept for which a wooden regular and planar grid, lacking shear stiffness, is elastically deformed up to a targeted shape with the help of stays, and then braced and covered. This type of gridshell,

known as elastic gridshell, offers a very elegant manner to materialize freeform shapes from an initially flat and regular grid, which obviously has many practical benefits: planar geometry, standard connection nodes, standard profiles and so on.

At present, the architectural significance of grid shells can hardly be fully envisioned. Too little experience has been gained as yet. However, the first signs of a grid shell architecture are already apparent. The projects planned and executed to date and the results of this research work indicate possibilities that open up a wide field of applications for grid shells. Two things stand out above all. The first is the nearly unlimited variety of forms that can be realized with grid shells, the second is the fact that there is no closed shell surface, but rather a simple, spatially curved grid composed of rods. Both features are of fundamental significance. They fully characterize the architectural essence of the grid shell support structure. [BHS74, p. 250]

“From the inverted form to the gridshell” [BHS74, p. 179]

1.2 Elastic gridshells : a thorough project review

No thorough historic review is available about executed projects of elastic gridshells although some partial reviews have been done time to time on the occasion of scientific works or construction projects. This review aims at filling this gap by giving an overview of the development of the concept from the very beginning to the very last experiments. Only known built projects have been identified and reported here. The only condition for a project to belong to this review is to comply with the definition of what an elastic gridshell is (see ??), independently to any other consideration (material, fabrication, size, cladding, ...).

The informations collected during this research work are given in table format in appendix (see ??). A synthetic presentation of these datas is proposed to the reader in fig. 1.1, where projects are ordered by date, span, covered area and material.

The books edited by the *Institut für leichte Flächentragwerke* are of great interest to understand the beginnings. *IL10 Grid Shells* [BHS74] has a precise inventory of the first experiments from 1962 to 1976, while *IL13 Multihalle Mannheim* [Bur+78] focuses on the construction of the Multihalle in Mannheim. *Timber Gridshells: Architecture, structure and craft* [CT17] is a significant effort but focuses exclusively on medium to large scale projects in timber. A small but general partial review is also available in [CC16]. An interesting review is also given by Quinn and Gengnagel [QG14] as part of his research work on new erection methods. A review of bracing and cladding systems is done in [Cuv+17]. A review of formfinding methods is done in [Vaulot2016]. Finally, various valuable reviews are available in the thesis by Douthé [Dou07], Bouhaya [Bou10], Tayeb [Tay15], and Lafuente Hernández [Laf15].

1.2.1 The beginnings : from the first prototype to the German Pavilion

Frei Otto started his studies in architecture in 1947 in Berlin, Germany, and completed his doctorate on tensile structures in 1953. This first work was published and translated later in the 60's. He then began to work in the field of lightweight structures using physical models such as soap films or hanging nets, and photographic measurements.^{1,2} These tools were essentials for his exploration of forms and structures as there were no computers at that time.

¹In the 19th and 20th centuries model testing was at the heart of structural innovation [Add13]. Analog models were employed successfully by well-known architects and engineers to go beyond the limits of existing knowledge (A. Gaudi, H. Isler, F. Candela, F. Otto, ...) and are still employed today where numeric models failed to represent accurately some physical phenomena (for instance in wind analysis for high rise towers and bridges).

²“Photography is the medium through which the form and content of a model are communicated. It is one of our most important tools in that it provides the basis for documentation and information, supplements our creative potential [...]” [BHS74, p. 56]

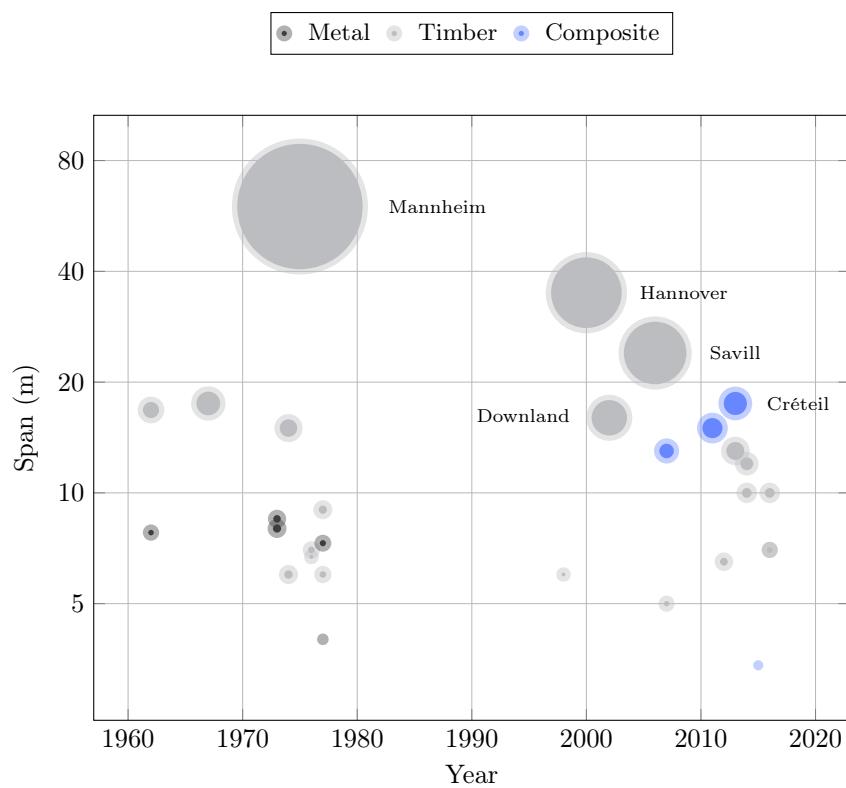
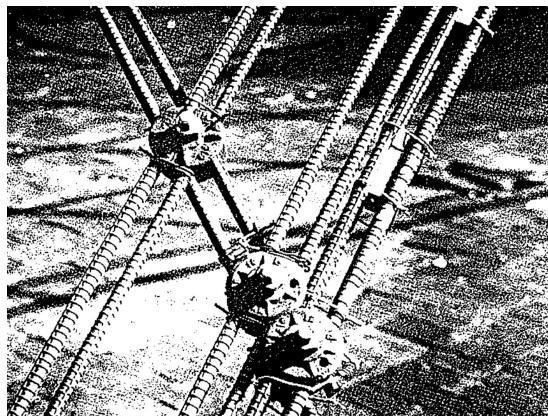


Figure 1.1 – Known elastic gridshells built by the past. The surface of the bubbles is proportional to the covered area. Color indicates the material employed for the rods.



(a) Knot detail



(b) Steel lattice

Figure 1.2 – Steel gridshell built in 1962 in Berkeley, USA.

Steel Gridshell, Berkeley, USA, 1962

Simultaneously, he became interested by the study of lightweight shells and the way they were form-found. One of his very first elastic gridshell was built in 1962 with students at Berkeley, USA [BHS74, p. 270]. It is funny to remark that this first gridshell was not a timber gridshell but a steel gridshell made out of twin steel rods linked in a grid fashion by bolts with clamping plates (see fig. 1.2a). This first experiment demonstrated at small scale the ability to bend a regular grid with no shear rigidity into a curved shape (see fig. 1.2b). The grid was loosely braced and shell effects were not investigated.

Essen Gridshell, Essen, Germany, 1962

The same year he designed and built a first timber gridshell at Essen, Germany [BHS74, p. 272]. The prototype – a single-layer gridshell spanning 17 m and covering an area of 198 m² – was made with 3-ply laminated timber profiles in hemlock pine (see fig. 1.3a). The cross-section of the profiles was rectangular (60 mm x 40 mm) and the elements were assembled in a grid fashion with simple steel bolts. Once erected, nothing was specifically done to improve the in-plane shear stiffness of the grid and activate a shell behavior. Finally, the structure was covered with a transparent plastic foil nailed directly on the grid's profiles (see fig. 1.3b).

German Pavilion Auditoria, Montreal, Canada, 1967

Fiver years later, on the occasion of the *1967 International and Universal Exposition* in Montreal, Canada, Frei Otto was appointed to design the German Pavilion : a large cable net tent prefiguring the realization of the olympic stadium of Munich, Germany, in

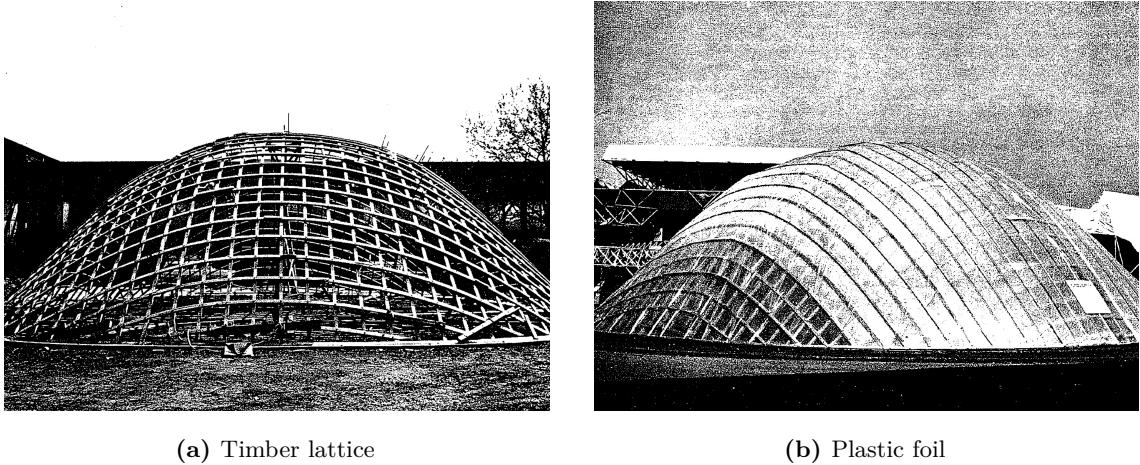


Figure 1.3 – Timber gridshell built in 1962 in Essen, Germany.

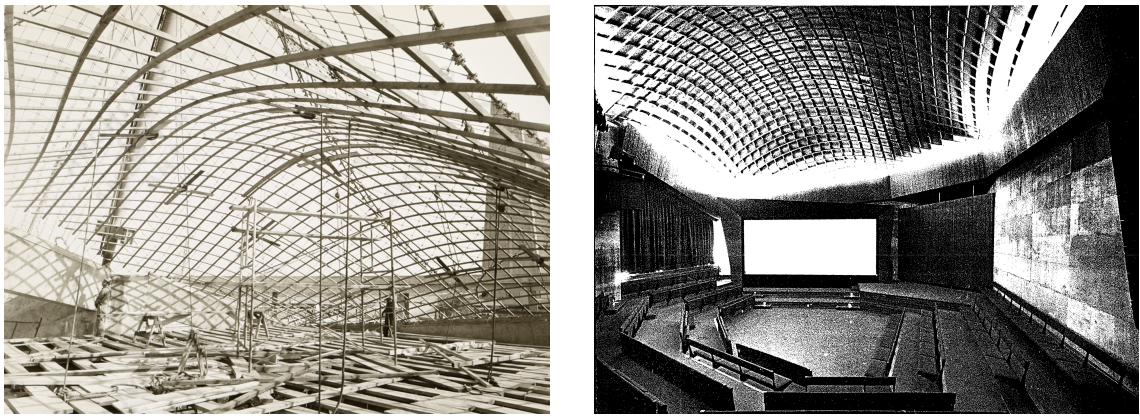


Figure 1.4 – Timber gridshell built in 1967 in Montreal, Canada.

1972.³ The pavilion required two auditoria and these were designed using the principle of elastic gridshell [BHS74, p. 274]. All together, the auditoria covered an area of 365 m² and spanned 17.5 m. The construction technique employed in Montreal was quite similar to the one developed in Essen, but this time the grid was fully braced with a layer of nailed plywood boards and offered a proper roofing made out of insulation panels covered with a PVC coated fabric (see figures 1.4a and 1.4b).

The two gridshells built in Montreal mark a significant step in the maturation process of the technique leading to the major realization of Mannheim in 1976 : a methodology has emerged to progress “from the inverted form to the gridshell” [BHS74, p. 179] ; main

³Actually, Frei Otto became the director of the newly founded *Institute for Lightweight Structures* (Institut für Leichte Flächentragwerke or IL) at the University of Stuttgart in 1964. It was the IL that was commissioned by the German government to conduct research in connection with the planning of the German pavilion for the exposition in Montreal.

construction details have been validated ; various erection methods have been tested ; mid-scale buildings have been built to host public. However, due to the over complexity of these structures, lots of unknowns remained unsolved at this stage and the behavior of the structures could not be fully predicted.⁴

It is worth to mention that several unexecuted large-scale projects were studied by Frei Otto between 1967 and 1973 at the *IL* or at the *Atelier Warmbronn*.⁵ These projects are basically documented in [BHS74, pp. 278 - 288] and reveal that he was training his capacity to master large-scale projects with the technique of elastic gridshells for more conventional building projects (wave pool, swimming hall, multi-purpose hall, auditorium, . . .).

1.2.2 Mannheim Multihalle : the completion of a decade of research

The project of the Multihalle started in 1970, when the decision was made that Mannheim, Germany, would hold the Bundesgartenschau in 1975.⁶ The architects of the project, *Carl Mutschler & Partners*, consulted Frei Otto at *Atelier Warmbronn* as he was starting to get known in the field of innovative lightweight structures. This is how the idea of the gridshell was introduced in the project [Lid15].

A thorough report on the project is available in [Bur+78]. A more condensed but still precise description of the engineering problematics related to this projects are available in the excellent papers from Happold and Liddell [HL75] and Liddell [Lid15].

Mannheim Multihalle, Mannheim, Germany, 1975

Mannheim is an unprecedented realization because it is more than twenty times larger than the previously built gridshells in Montreal and is meant to last many years and not only for the duration of a short-term exhibition. The timber lattice, still existing in 2017, covers an area of 7400 m² (see fig. 1.5b). It is composed of two interconnected domes, one for the multi-purpose hall (span : 60 m | height : 20 m) and one for the restaurant (span : 50 m | height : 18 m).

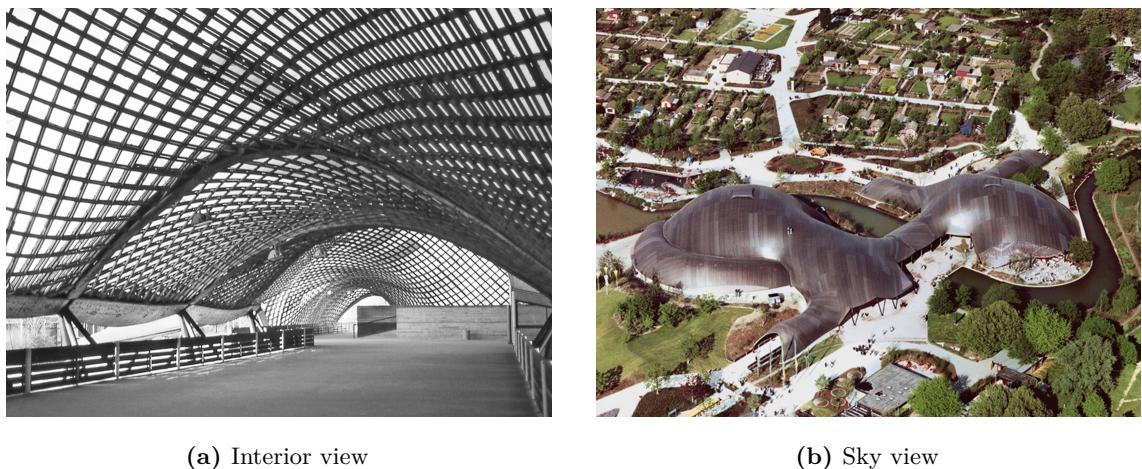
Although the constructif system deployed at Mannheim clearly inherited from the previous developments, the challenge was such that it had to be revisited. In particular the main additions were the introduction of the double-layer system and the proper bracing of the grid. A major advance was also the use of the very first numeric models to study the structure.

The double-layer system was introduced to tackle two issues : the grid needed some

⁴“Snow accumulations in the throat of the common edge beam probably caused one of the two grid shells of project Montreal to buckle in a relatively flat region. The diameter of the buckled area was about 3 meters. Neither grid rod was broken, i.e. the buckling progressed elastically. It might have been possible to press the buckled area back into shape.” [BHS74, p. 219]

⁵Atelier Warmbronn is the architectural studio founded by Frei Otto in 1969.

⁶The Bundesgartenschau is a national horticultural exhibition that takes place every two years in Germany.



(a) Interior view

(b) Sky view

Figure 1.5 – Timber gridshell built in 1975 in Mannheim, Germany.

flexibility to be bent into the desired shape, but once erected it should provide sufficient bending stiffness to resist disturbing loads and avoid a buckling collapse.⁷ One erected, the two grids, one sliding on top of the other one, were connected together to form a single grid with much higher ladder profiles (from 50 mm to 150 mm), increasing their bending stiffness by 26 (see fig. 1.5a).

Because the in-plane stiffness of the grid also plays a major role in the resistance to buckling, this question was considered with care. The bracing of the grid was first achieved by preventing the nodes to turn once the grid was erected. This was done by creating some friction in the nodes when tightening the bolts linking the laths, after the grid was erected. Additional bracing cables were put in the grid.

Finally, the project of Mannheim was a key project in the development of modern lightweight structures. Great engineers were born in touch with Frei Otto, following its footsteps or collaborating with him. This heritage has irrigated for several decades the engineering of lightweight structures in Europe and gave birth, directly or indirectly, to several studios among which we can cite *Buro Happold*, *Schlaich Bergermann & Partner* and *RFR*.

1.2.3 The dry period : 25 years from Mannheim to Hannover

Although the experience of Mannheim proved the feasibility and the potential of gridshell structures for large-scale projects, it also revealed that these projects were subject to an incredible complexity in terms of structural design, geometry, modeling, testing, team work, construction methods, ... At that time, very few people could pretend to master all the knowledge and techniques required to design and built timber gridshells and developed in the bosom of the *Institute for Lightweight Structures* in Stuttgart.

⁷Theoretically, self-weight loads would produce only compression in the members because the (funicular) form of the grid resulted from the inversion of a hanging chain model in pure tension.

This project was obviously well ahead of its time and the engineering cost to design such structures was probably prohibitive considering the tools available at that time. This certainly explains why no elastic gridshells were built during the 25 following years, despite the optimism of the pioneers of the Multihalle.⁸

Note that around 1975 small workshop and experiments lead to the construction of several but small elastic gridshells, as reported in [BHS74]. A non-exhaustive but quite extensive list of known executed gridshell projects is presented in fig. 1.1. The dry period is clearly visible.

1.2.4 The signs of a renewal : Dorset and Doncaster

It is only 20 years later that gridshells started to reappear, in the late 90's mainly in the United Kingdom, and for projects that had interest in environmental problematics.

Westminster Lodge, Dorset, England, 1995

In 1995, a small student residence named *Westminster Lodge* was built in Dorset, England. This dwelling was part of a larger project – Hooke Park – aiming at investigating how the local forest resources, in particular immature roundwood thinnings, could be better utilized. The project was lead by ABK, Frei Otto, Buro Happold and Cullinan Studio. Unlike Mannheim, the timber shell was bent and weaved rod by rod on a scaffold platform. But the structural system exhibited a double-layer gridshell pattern very similar to the one employed for the Multihalle (see fig. 1.6a). The rods were made out of splice-jointed roundwood to form long-length poles of diameter 200 mm. The development of this jointing technique, which could be produced directly in the forest, was part of the project's investigations [BDH98]. The grid was braced by a layer of diagonal boards nailed to the roundwoods. The structure was finally cladded with a planted turf roof (see fig. 1.6b).

Earth Center, Doncaster, England, 1998

At the same time, a project of a similar spirit arose for the *Earth Center* in Doncaster, England.⁹ The project planning started in 1994 and a series of small timber gridshells were designed by Buro Happold and then built in 1998. The landscape structures were single-layer timber gridshells made with oak laths. Once erected with a crane, the grids were braced with crossing diagonal stainless steel cables (see fig. 1.7a). Openings were possibly reinforced with curved timber frames (see fig. 1.7b).

These projects definitely trailed the technique in England and initiated the renewal period

⁸“For many years after its completion, Happold promoted the benefit of the timber gridshell as a construction technique and stated that he could not understand why it had not been adopted more widely. He perceived the benefits to be in the efficiency of the construction method to enable doubly curved (shell) structures to be constructed quickly and cost effectively.” [Har+03].

⁹“The Earth Centre Forest Garden was intended to demonstrate how managed woodland could supply the vast majority of all natural resources needed for human survival.”



Figure 1.6 – Roundwood gridshell built in 1995 in Dorset, England.



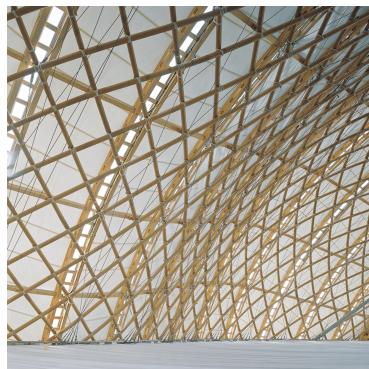
Figure 1.7 – Timber gridshells built in 1998 in Doncaster, England

(see §1.2.5). Although they remained small-scale projects for which modeling was achieved through physical models, they trained and restored partially the operational ability of Buro Happold to design timber gridshells as pointed by Harris et al. [Har+03].

1.2.5 The renewal : Hannover, Downland and Savill

What was missing for elastic gridshells to re-emerge after the major experiment of Mannheim was probably the development of modern numeric tools to ease and speed up the design process.¹⁰ Amongst those tools we should identify two main categories : geometry processing

¹⁰“The key to the modern use of timber gridshells is the development of computer methods in modelling complex three-dimensional shell structures. For the Mannheim structure, the primary method of form finding was the use of physical models. The Earth Centre structures were small and easily modelled using wire mesh, but when Buro Happold were commissioned to design the Japanese Pavilion for Expo 2000 in Hannover (Architect Shigeru Ban), it was apparent that much more sophisticated computer form finding



(a) Interior view



(b) Sky view

Figure 1.8 – Carboard gridshell built for Expo 2000 in Hannover, Germany.

softwares and structural analysis softwares. Recall that in the 70's, geometry processing was done through physical models and photographic measurements [BHS74, pp. 130-135] while structural analysis was conducted through a compound of physical model testing with scaling techniques [Bur+78, pp. 130-135], hand calculations and the very first numerical formfinding calculations [BHS74, pp. 184-193] and finite element calculations [BHS74, pp. 210-217].

In the late 90's, the rise in importance of computer methods offered new possibilities.

Japan Pavilion, Hannover, Germany, 2000

In 1997, architecte Shigeru Ban began to collaborate with Frei Otto and Buro Happold to design the *Japan Pavilion* for *Expo 2000* in Hannover, Germany [MOB06]. This pavilion was a large-scale corrugated gridshell made out of cardboard tubes, about 75 meters long and 25 meters wide. Corrugations bring curvature, and therefore enhance the strength of the shell. The tubes were tied together with a fabric tape, a very low-tech joint (see fig. 1.8a). The structure was covered with a paper membrane specially developed for the project to meet the requirements of the german fire regulations (see fig. 1.8b). For the occasion, a new erection method was set up in which the grid was laid out not at the ground level but at a higher level on a hydraulic scaffold platform. From there, the grid was pushed up into position using the platform's jacks. It was found late that the cardboard tubes were subject to a high level of creep. This required the introduction of new timber arches to reinforce the gridshell and to enlarge the existing timber rafters intended to brace the grid and support the paper membrane (see fig. 1.8a).

and analysis would be necessary." [Har+03]

Weald and Downland, Downland, England, 2002

The design of the *Downland* gridshell began right after the completion of the Westminster Lodge (see §1.2.4) where architects from E. Cullinan Studio became acquainted with the engineers from Buro Happold. At Downland, the project team truly revived the technique of large-scale timber gridshells while bringing lots of improvements to the system. The building opened to the public in 2002. Its corrugated shape recalls the one of the Japan Pavilion from which it was inspired (see fig. 1.9b).

The building is 50 meters long and 12.5 to 16 meters wide, covering an area of about 675 m^2 for a height varying from 7 to 9.5 meters [HK02]. The structure is a double-layer gridshell made of rectangular oak laths of cross-section 50 mm x 35 mm (see fig. 1.9a). To produce high grade timber elements, the continuous laths were re-formed from small carefully selected wood pieces, finger-jointed every 60 cm in 6.0 m length pieces. These pieces of lath were then scarf-jointed on site every 6 m to obtain the desired length, up to 50 m.

The grid pitch is 1.0 m except in weaker areas where it is 0.5 m. There, the grid is twice denser to achieve the required buckling resistance [Har+03]. Rib-lath bracing was preferred to steel cable bracing as ribs were deemed to offer a more convenient support for the cladding elements and to reduce the complexity of the connection. A new connection system was developed to avoid the cost of drilling thousands of slotted holes that would, in addition, reduce the cross-section area, while maintaining the required scissor behavior for the deformation of the timber lattice.¹¹



(a) Interior view



(b) Exterior view

Figure 1.9 – Timber gridshell built in 2003 in Downland, England.

The flat lattice was laid out on a scaffold platform. Unlike the Japan Pavilion, the lattice was progressively lowered down into position. This stage took 6 weeks. Once deformed, the shear blocks were introduced in the grid and bracing rib-laths were installed, giving its full

¹¹This detail was patented by the design team and the client.

strength to the shell. Finally the gridshell was cladded with a mix of polycarbonate plates (to let the light in) and timber boards on top of insulation panels and a rain screen.

It is worth to mentionne that for the first time the form was not found by inverting some sort of hanging chain model that would produce a pure funicular shape where only compression occurred. Instead, the shape was the result of a numerical computation that took into account the bending behavior of the laths.¹² Harris et al. [Har+03] argued that computer models enabled some interactivity in the formfinding process that would not be possible with physical models, leading to a better synergy between architectural and structural requirements. They also argued that physical models contributed invaluable to the development of a creative and efficient design throughout the project.

Lothian Gridshell, Pishwanton, Scotland, 2002

This project deserves somme attention because the developed approach was completely different from the projects exposed until now : “Previous projects have portrayed the method as a highly technical use of a low tech resource. This, however, need not be the case as we see with this project [...]” [Low02]. The structure was the result of : “[...] an unusual collaboration between sole practitioner Christopher Day, engineer David Tasker, a crowd of local volunteers and (more unusually) the philosophies of Rudolf Steiner and Johann Wolfgang Goethe” [Bdo02].¹³.



(a) Interior view



(b) Exterior view

Figure 1.10 – Timber gridshell built in 2002 in Pishwanton, England.

The single-layer gridshell was made out of local larch. Once erected by hands, the dome-like shape covered about 80 m^2 and spanned 10 meters. The grid was braced with timber boards (see fig. 1.10a) and covered with a planted turf roof (see fig. 1.10b). Some calculations were made but in the end, it had to carry load testing to prove its safety and gain its regulation

¹²This software was developed by Chris Williams of the University of Bath.

¹³From the online paper “The other gridshell” : <http://www.bdonline.co.uk/the-other-gridshell/1020435.article>

approval.¹⁴

Woodland Centre, Filmwell, England, 2003

The gridshell of the Woodland Center was built 7 years after the project had started.¹⁵ The building was designed by architect Feilden Clegg and engineers from Atelier One. It was part of a larger research and development project that aimed at developing chestnut – a low grade wood – as a construction material.¹⁶ The building, still existing, is composed of 5 barrel vaults spanning 12 meters and about 5 meters wide. Therefore, it covers about 300 m² [Low04]. Each vault module is a transportable unit composed of two curved arches. A single layer gridshell was then applied to this primary frame and braced with chestnut panels. The grid was made of laths with 75 mm x 25 mm rectangular cross-section, assembled with simple bolts. On top of that, insulation materials and a membrane as rainscreen [Fou03].



(a) Interior view



(b) Exterior view

Figure 1.11 – Timber gridshell built in 2003 in Filmwell, England.

Savill Garden, Savill, England, 2006

This project saw the light of day thanks to the reputation of the gridshell built in Downland. Again, Buro Happold did the structural design while Green Oak Carpentry realized it. But this time, the architect was Glenn Howells.

The *Savill* gridshell is 90 meters long and 25 meters wide. It covers an area of about 2000 m², and is therefore almost three times larger than the gridshell in Downland. Once again, the

¹⁴“There were a lot of calculations but no computer-generated models to show they all added up. In fact, the form was previously established with scale models. When it came to gaining Building Regulations approval, the team needed to prove that the building would be strong enough. So Tasker arranged for the unfinished structure to be loaded with about 18 tonnes of sand from a local quarry – equivalent to the maximum predicted snow load, plus a safety factor.” [Bdo02]

¹⁵More to be found at : [Growing and making Flimwell's chestnut gridshell](#).

¹⁶This project was conducted by the [Building Research Establishment](#).



(a) Interior view



(b) Exterior view

Figure 1.12 – Timber gridshell built in 2006 in Savill, England.

corrugated shape was defined by a parametric equation ($z = f(x, y)$) to enable interactivity between architects and engineers during the formfinding process (see fig. 1.12b). Chris Williams was responsible for this job.

In Savill, the forming strategy was quite different than those employed in Mannheim, Hannover or Downland [HHR08]. Firstly, a single layer gridshell – constituted by the bottom two laths jointed with simple bolts – was deformed into the target shape. Secondly, the shear blocks were screwed on these laths. Thirdly, the upper two laths were positioned and screwed on top of the shear blocks to re-form a double-layer gridshell. Finally, the grid was then braced with two alternate layers of plywood boards, 12 mm thick each. Bracing the grid with continuous panels instead of cables or diagonal elements was a major architectural choice (see fig. 1.12a). Moreover, it gave a well-defined surface for the cladding composed of 160 mm of insulation, covered by a waterproof aluminium layer made with standing-seam profiles supporting the oak boards [Tra06].

Another consequence of this forming process was the drastic simplification of the connexion. The system developed for Downland was of no utility in that case and only simple bolts and screws were required. In this project, the pitch of this grid is 1.0 m. The 20 kms of laths are made from larch and have a 80 mm x 50 mm rectangular cross-section. They are spaced from 100 mm to 150 mm by the shear blocks.

Of course, the steel perimeter is a major component of the project but is not in the scope of this thesis. For further details the reader is invited to refer to Harris et al. [HHR08] and Trada [Tra06].

Chiddingstone Castle Orangery, Kent, England, 2007

The gridshell covering the orangery of Chiddingstone Castle is a very small one. Built in 2007, it is 12 meters long, 5 meters wide and covers about 50 m² (see fig. 1.13b). The

structural system is derived from the one employed in Downland and is, once again, developed by Buro Happold and the Green Oak Caprentery. But this time the architecte is Peter Hulbert.



Figure 1.13 – Timber gridshell built in 2007 in Kent, England.

However this project embed some interesting innovations. Indeed, this time the gridshell is braced with a bidirectional cable network. Twin cables are employed to facilitate clamping on the node connection, which has been adapted from the previous version developed in Downland. Moreover, this connection is equipped with an additional threaded hole which can receive the clamping supports for the glazing (see fig. 1.13a). The timber shell is then glazed with triangular panels (note that the mesh quadrangles are not planar any more in the deformed configuration).

1.2.6 Gridshell in composite materials : a new perspective.

Starting from 2002, the laboratory [Navier](#) from the Ecole des Ponts ParisTech started a research program on elastic gridshells that is still ongoing. It focuses on both the use of new materials such as composite materials and the development of modern computer methods for the shape generation, the formfinding and the structural analysis.

Indeed, composite materials in glass fiber reinforced polymers (GFRP) are very suitable for this type of structures where both flexibility and strength of the profiles are required. On the level of mechanic behavior they surpass wood. And they are easy and cheap to produce in long length when they are manufactured by pultrusion, avoiding complex jointing issues.

GFRP Gridshell Prototypes, Champs-sur-Marne, France, 2007-2008

These developments have been validated by the construction of two prototypes (Figures ??a and ??b) covering an area about 150 m^2 .



(a) First prototype (2007)



(b) Second prototype (2008)

Figure 1.14 – GFRP gridshells built in 2007 and 2008 in Noisy-Champs, France.

Solidays, Champs sur Marne, France, 2011

In 2011, the laboratory used its expert knowledge for a large-scale project [Bav+12], the forum of the Solidays festival (??c). This achievement, built by voluntary workers, has been the first composite material gridshell to house public.



(a) First prototype (2007)



(b) Second prototype (2008)

Figure 1.15 – GFRP gridshell built in 2011 in Paris, France.

Ephemeral Cathedral, Créteil, France, 2013

In 2012, the context is favorable for a new achievement named "*Temporary Cathedral of Créteil*". Although this gridshell has an area very similar to the Solidays one, the project raised new challenges, in particular the challenge of reliability. Indeed, its period of use is at least two years. Additionally, the skills coming from T/E/S/S company made possible important developments such as for doors, lacing edge beam, anchorages and sleeves.

1.2. Elastic gridshells : a thorough project review

Unlike the two first prototypes, the gridshells built for Solidays and at Créteil are based on a new approach regarding shape-structure relationship. Indeed, thanks to a numerical tool performing the compass method, the geometry of the object is no longer defined as the reversal of a hanging net – in this case, only the flat geometry could be mastered [Add13] – but now the flat geometry is straight deducted from the one proposed by the architect. This new approach opens up new architectural horizons, making possible the exploration of new shapes for gridshells.



(a) Interior view



(b) Exterior view

Figure 1.16 – GFRP gridshell built in 2013 in Créteil, France.

1.2.7 Flourishing timber gridshell pavilions

The impetus given by gridshell.it

Starting from Lecce, Italy, 2010 About 12 structures of small size.



(a) Lecce, 2010



(b) Toledo 2.0, 2014

Figure 1.17 – Timber gridshells built by gridshell.it in Italy.

Other similar timber pavilions

Uta, ZA,

The inputs from the laboratory Navier

Same constructive system but a reflexion on bracing (with cables and how to put tension in the cables. Also, generally they were made by a simple grid pattern (cross pattern or square with cutting corners.) Here the grids were exhibited from the compass method and the a formfinding tool developed by Navier. Computation were studied with GSA. Terrell. Wood testing were done by Navier with consultancy over detailing. Wood specification. Cross-section and specification of the system (grid pitch, number of layers, shear blocks, cross section of the laths, formfinding, timber specification).

Fist time that the same grid blocks are recombined in a new shape.



(a) Toulouse



(b) Montpellier

Figure 1.18 – Timber gridshells built in 2016 in France.

Toledo 1 : [DKZ14] Toledo 2 : [DAm+15a]

1.2.8 Recent experiments



(a) Robotic manufacturing



(b) Concrete bracing envelop

Figure 1.19 – Recent experiments built in 2016 in Champs-sur-Marne, France.

1.3 Research works in the field of elastic gridshells : a review

[Lab+16, FKB16]

1.3.1 Formfinding and structural analysis

Les méthodes numériques de Day, Barnes, Adriaenssen, Douthe, Lefevre, Tayeb 3/6/4 DOF, prise en compte de la torsion. Adriaenssens et al. [ABW99] and Adriaenssens and Barnes [AB01] [BAK13] [DKZ14] [DZK16] [Pou15] [Vaulot2016]

[dPel+15] Malek [Mal12]

1.3.2 Stability

Flambement (Malek, Mesnil, Lefevre) **Mesnil2013** Mesnil et al. [MOD15] Mesnil et al. [Mes+17] Lefevre et al. [LDB15] Tayeb et al. [Tay+13]

1.3.3 Gridfinding

Le formfinding vs. grid finding (COmpas Lionel, Compas Baptise, GFFT, Compas Yannick singularité, Lina) Elisa Fuentes

[Pon+16] [dPel+11] [Laf15] [MM17]

1.3.4 Morphology

[Dou+16] [Jensen2013] Mesnil Description analytique de la forme (C. Williams)

1.3.5 New materials

Les matériaux composites

1.3.6 Erection

Les méthodes de montage. L'inflatable(Otto, Pugnale, Quinn) [BHS74] [QG14] [LP15] Liuti et al. [LPD16]

1.3.7 Cladding

Une reflexion sur l'enveloppe (booby ENPC). Lafuente Hernández and Gengnagel [LG14] Cuvilliers et al. [Cuv+17] [FN15]

1.3.8 Rationalization

La simplification (gridshell.it => ZA, UTSA, Cluj, quaternion + Navier, Navier)

1.3.9 Robotisation

La robotisation de la production (ENPC) [MKS16] ZA

1.3.10 Structural optimisation

Optimisation des sections [DAm+15b]

Des pistes ouvertes

N	Nickname	Year	City	Country	Type	Architecte	Enginneer	Ref
1	Berkeley	1962	Berkeley	USA	Prototype	F. Otto	F. Otto	[BHS74]
2	Essen	1962	Essen	Germany	Pavilion	F. Otto	F. Otto	[BHS74]
3	German Pavillion	1967	Montreal	Canada	Pavilion	F. Otto	F. Otto	[BHS74]
4	Seibu	1973	hannover	Japan	Prototype	K. Matsushita	T. Shirayanagi	[Bur+78, p. 245]
5	Ahmedabad	1976	Ahmedabad	India	Building	G. Sarabhai	G. Ramaswamy	[Bur+78, p. 249]
6	Bamboo	1976	London	England	Workshop	J. Park	B. Oleiko	[Bur+78]
7	Mexico	1973	hannover	Japan	Prototype	J. Hennicke	J. Hennicke	[Bur+78]
8	Mexico	1973	hannover	Japan	Workshop	J. Hennicke	J. Hennicke	[Bur+78]
9	Mexico	1977	Zitacuaro	Mexico	House	F. Montero	F. Montero	[Bur+78]
10	Mannheim	1975	Mannheim	Germany	Building	C. Mutschler	Arup + F. Otto	[Bur+78]
11	Japan Pavilion	2000	Hannover	Germany	Pavilion	S. Ban + F. Otto	Buro Happold	[]
12	Downland	2002	Singleton	England	Building	E. Cullinan	Buro Happold	[Har+03]
13	Savill	2006	Englefield	England	Building	G. Howells	Buro Happold	[HHR08]
14	Chiddington	2007	Chiddington	England	Canopy	P. Hulbert	Buro Happold	[]

Table 1.1 – Key numbers.

Chapter 1. Elastic gridshells

N	Year	Nickname	Type	City	Country	Architecte	Engineer	Page	Citation
1	1962	Experimental structure	Workshop	Berkeley	USA	Students	F. Otto	p. 270	[BHS74]
2	1962	Exhibition pavilion	Pavilion	Essen	Germany	F. Otto	F. Otto	p. 272	[BHS74]
3	1967	German Pavilion	Pavilion	Montreal	Canada	F. Otto	Lehonardt	p. 274	[BHS74]
4	1973	Seibu	Experiment	hannover	Japan	IL	IL	p. 245	[Bur+78]
5	1974	Basket shell	Experiment	Amehabad	India	G. Sarabhai	IL	p. 304	[BHS74]
6	1974	Experimental structure	Experiment	London	England	Students	Arup + IL	p. 306	[BHS74]
7	1975	Mannheim Multihalle	Building	Mannheim	Germany	C. Mutshler	Arup + IL	p. 308	[BHS74]
8	1973	Ferrocement gridshell	Building	Ahmedabad	India	G. Sarabhai	G. Ramaswamy + IL	p. 248	[Bur+78]
9	1976	AA Bamboo Latice Shell	Workshop	London	England	Students	J. Park + B. Oleiko	p. 255	[Bur+78]
10	1976	Test structure of a gridshell	Experiment	Stuttgart	Germany	Students	J. Hennicke	p. 298	[BHS74]
11	1977	Small Pavilion	Workshop	Mexico City	Mexico	Students	J. Hennicke	p. 258	[Bur+78]
11	1977	Small Greenhouse	Workshop	Zitacuaro	Mexico	Students	F. Montero	p. 260	[Bur+78]
12	1977	Experimental structure	Workshop	Mexico City	Mexico	Students	F. Montero	p. 270	[Bur+78]
13	1977	Experimental structure	Workshop	Mexico City	Mexico	Students	F. Montero	p. 270	[Bur+78]
14	1995	Westminster Lodge	Building	Dorset	England	E. Cullinan	F. Otto + Happold	p. 90	[BDH98]
15	1998	Earth Center	Building	Doncaster	England	Grant	Happold		
16	2000	Japan Pavilion	Pavilion	Hannover	Germany	S. Ban + F. Otto	Happold		[MOB06]
17	2002	Downland	Building	Downland	England	E. Cullinan	Happold + C. William		[Har+03]
18	2003	Life Science Centre Trust	Building	Pishwanton	England				
19	2006	Savill	Building	Savill	England	G. Howells	Happold + C. William		[HHR08]
20	2007	Chiddingstone Orangery	Roofing	Kent	England	P. Hulbert	Happold		
21	2007	ENPC	Experiment	Noisy-Champs	France	Navier	Navier		[DBC06]
22	2011	Solidays	Pavilion	Paris	France	ENPC	Navier + T/E/S/S		[Bav+12]
23	2012	Toledo	Workshop	Naples	Italy	gridshell.it	B. D'Amico		[DKZ14]
24	2013	Créteil	Building	Créteil	France	T/E/S/S	T/E/S/S + Navier		[dPel+16]
25	2014	Toledo 2.0	Workshop	Naples	Italy	gridshell.it	B. D'Amico		[DAm+15a]
26	2016	JPO	Pavilion	Toulouse	France	Quaternion	Navier + Terrell		
27	2016	FAV	Pavilion	Montpellier	France	Quaternion	Navier + Terrell		
28	2016	CLC	Workshop	Noisy-Champs	France	ENPC	Navier		
27	2016	Trondheim	Workshop	Trondheim	Norway	Students	NTNU		[Had+16]

Table 1.2 – Key numbers.

Year	Nickname	Project			Structure	
		Country	Duration	Material	Layer	Pitch
1975	Mannheim	Germany	LT	hemlock	double	0.5 m

Table 1.3 – Key numbers.

1.4 Gridshell structure : definition and classification

1.4.1 Historic overview

1.4.2 Rigid gridshell



(a) Forum Café, Solidays (2011).



(b) Prototype (2007).



(c) Prototype (2008).

Figure 1.20 – Prototypes and projects of GFRP elastic gridshells.

1.5 Elastic gridshells : revisiting Mannheim

1.6 Elastic gridshells : the benefits of composite materials

Glass fiber reinforced polymer (GFRP) tubes are at the heart of the presented technology. They can favorably replace wood where both resistance and bending ability of the material is sought [DCB10].

¹⁷German pavilion 1967 : <https://www.youtube.com/watch?v=Z0mtFMoseUk>

¹⁸German pavilion 1967 : <http://www.uncubemagazine.com/magazine-33-15508949.html/page1>

1.6. Elastic gridshells : the benefits of composite materials

The tubes are made by pultrusion, “a continuous molding process whereby reinforcing fibers are saturated with a liquid polymer resin and then carefully formed and pulled through a heated die to form a part. Pultrusion results in straight constant cross section parts of virtually any shippable length”.¹⁹ This process is very economic and its standardization guarantees very stable material and mechanical properties. It frees designers from the problem of joining wood pieces with finger joints to obtain long and continuous members and of wood durability.

1.6.1 Recent developments

The Faraday Pavilion : [NLG13] The Pishwanton / lothian : [Pishwanton2003] The laboratory Navier tested various numerical methods to generate such grids [BBC09] Masson : [MM17].

1.6.2 Gridshell in composite material

The benefits of GFRP gridshells have been covered previously. We just recall the main aspects.

The gridshells built in composite material, being at the heart of this paper, are consistent with the framework defined previously, that is to say :

Glass fiber reinforced polymer (GFRP) tubes are at the heart of the presented technology. They can favorably replace wood where both resistance and bending ability of the material is sought [DCB10].

The tubes are made by pultrusion, “a continuous molding process whereby reinforcing fibers are saturated with a liquid polymer resin and then carefully formed and pulled through a heated die to form a part. Pultrusion results in straight constant cross section parts of virtually any shippable length”.²⁰ This process is very economic and its standardization guarantees very stable material and mechanical properties. It frees designers from the problem of joining wood pieces with finger joints to obtain long and continuous members and of wood durability.

Structural Typology

Their mechanical behaviour is very similar to the one of real shells even if the material is discrete and located in a grid more or less open. In spite of that, gridshells benefit from the same advantages as the ones showed by an eggshell : they can cross large span using a low amount of material. Their stiffness is mainly linked to their double-curved shape.

¹⁹Video explaining the pultrusion process : https://www.youtube.com/watch?v=4MoHNZB5b_Y

²⁰Video explaining the pultrusion process : https://www.youtube.com/watch?v=4MoHNZB5b_Y

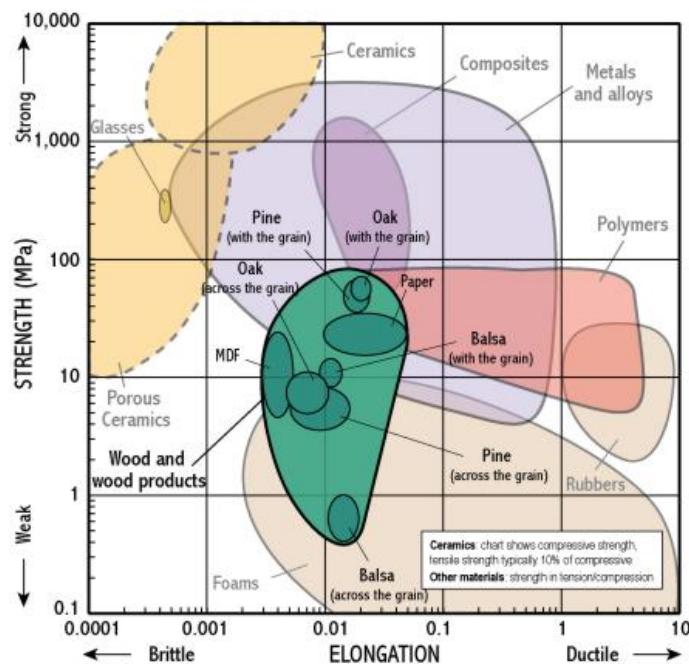
Material Flexibility for Structural Rigidity

In this field of application, composite materials like glass fibre reinforced polymer (GFRP) could favourably replace wood, where both resistance and bending ability of the material is sought [DCB10]. The stiffness of the structure does not derive from the intrinsic material rigidity but principally from its geometric curvature. Ideally, the composite profiles are produced by pultrusion, an economic continuous moulded process. The standardization of the process guarantees very stable material and mechanical properties. It frees designers from the painful problematic of wood joining and wood durability. The characterization of this material is presented further in the paper.

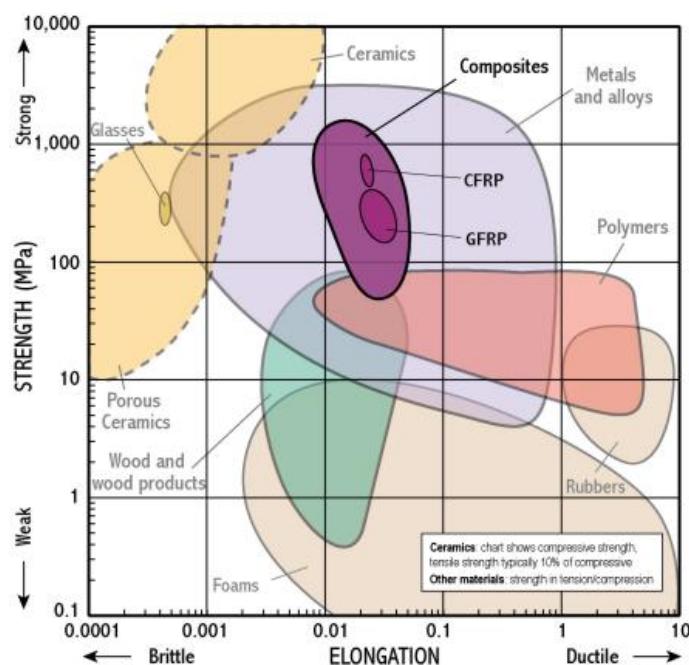
Erection Process

Usually, the grid morphology is not trivial and leads to design numerous costly and complex joints. To overcome this issue, an original and innovative erection process was developed that takes advantage of the flexibility inherent to slender elements. A regular planar grid made of long continuous linear members is built on the ground (Figure 1.23a). The elements are pinned together so the grid has no in-plane shear stiffness and can accommodate large-scale deformations during erection. Then, the grid is bent elastically to its final shape (Figure 1.22). Finally, the grid is frozen in the desired shape with a third layer of bracing members (Figure 1.23b) and the structure becomes a shell.

1.6. Elastic gridshells : the benefits of composite materials



(a) Forum Café, Solidays (2011).



(b) Prototype (2007).

Figure 1.21 – Prototypes and projects of GFRP elastic gridshells.



Figure 1.22 – Erection of the primary grid by two cranes

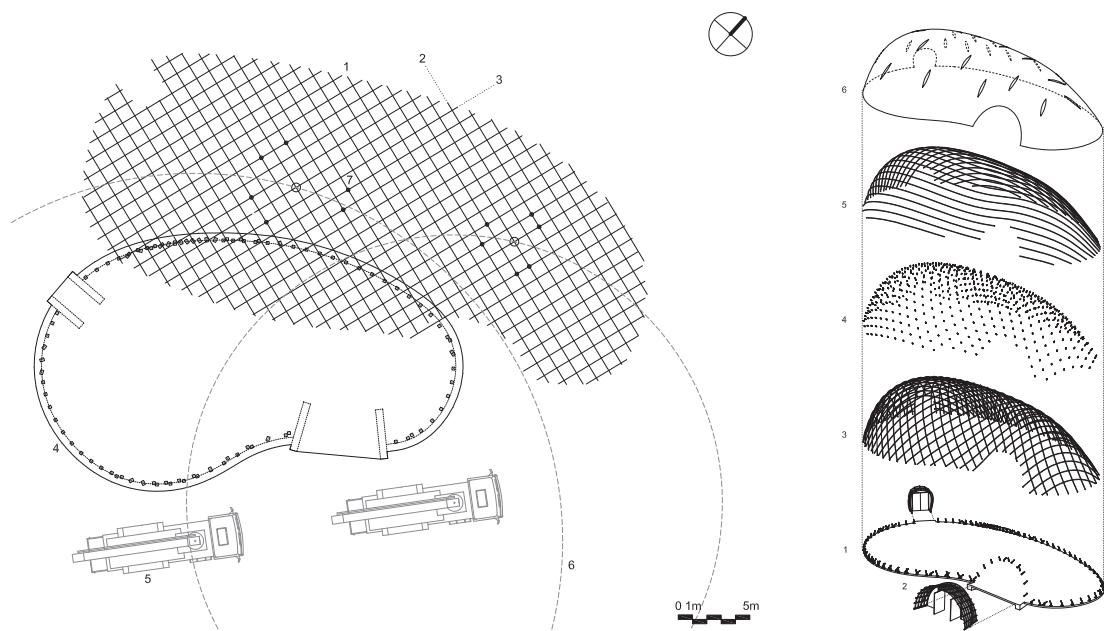


Figure 1.23 – Erection plan and construction stages

1.7 References

- [AB01] S. Adriaenssens and M. Barnes. “Tensegrity spline beam and grid shell structures”. In: *Engineering Structures* 23.1 (2001), pp. 29–36.
- [ABW99] S. Adriaenssens, M. Barnes, and C. Williams. “A new analytic and numerical basis for the form-finding and analysis of spline and gridshell structures”. In: *Computing Developments in Civil and Structural Engineering*. Ed. by B. Kumar and B. H. V. Topping. Edinburgh: Civil-Comp Press, 1999, pp. 83–91.
- [Add13] B. Addis. “‘Toys that save millions’ - A history of using physical models in structural design”. In: *The Structural Engineer* 91.4 (2013), pp. 12–27.
- [BAK13] M. Barnes, S. Adriaenssens, and M. Krupka. “A novel torsion/bending element for dynamic relaxation modeling”. In: *Computers & Structures* 119 (Apr. 2013), pp. 60–67.
- [Bav+12] O. Baverel, J.-F. Caron, F. Tayeb, and L. du Peloux. “Gridshells in composite materials : construction of a 300 m² forum for the solidays’ festival in Paris”. In: *Structural Engineering International* 22.3 (2012), pp. 408–414.
- [BBC09] L. Bouhana, O. Baverel, and J.-F. Caron. “Mapping two-way continuous elastic grid on an imposed surface : application to grid shells”. In: *IASS 2009*. Valencia, Spain, 2009, pp. 989–998.
- [BDH98] R. Burton, M. Dickson, and R. Harris. “The use of roundwood thinnings in buildings : a case study”. In: *Building Research & Information* 26.2 (1998), pp. 76–93.
- [Bdo02] Bdonline.co.uk. *The other gridshell*. 2002.
- [BHS74] B. Burkhardt, J. Hennicke, and E. Schauer, eds. *IL10 Grid Shells*. Institut für leichte Flächentragwerke (IL). Stuttgart, 1974.
- [Bou10] L. Bouhana. “Optimisation structurelle des gridshells”. PhD thesis. Université Paris-Est, 2010, p. 154.
- [Bur+78] B. Burkhardt, M. Chaitos, J. Langner, W. Langner, and G. Lubberger, eds. *IL13 Multihalle Mannheim*. Institut für leichte Flächentragwerke (IL). Stuttgart, 1978.
- [CC16] M. Collins and T. Cosgrove. “A Review of the State of the Art of Timber Gridshell Design and Construction”. In: *Civil Engineering Research in Ireland*. 2016.
- [CT17] J. Chilton and G. Tang. *Timber Gridshells: Architecture, structure and craft*. Routledge, 2017.
- [Cuv+17] P. Cuvilliers, C. Douthe, L. du Peloux, and R. Le Roy. “Hybrid structural skin : prototype of an elastic gridshell braced by a fibre-reinforced concrete envelope”. In: *Journal of The International Association for Shell and Spatial Structures* (2017), pp. 1–12.
- [DAm+15a] B. D’Amico, A. Kermani, H. Zhang, A. Pugnale, S. Colabella, and S. Pone. “Timber gridshells: Numerical simulation, design and construction of a full scale structure”. In: *Structures* 3.May 2016 (2015), pp. 227–235.

- [DAm+15b] B. D'Amico, A. Kermani, H. Zhang, P. Sheperd, and C. Williams. “Optimisation of cross-section of actively bent grid shells with strength and geometric compatibility constraints”. In: *Computers & Structures* 154 (2015), pp. 163–176.
- [DBC06] C. Douthe, O. Baverel, and J.-F. Caron. “Formfinding of a grid shell in composite materials”. In: *Journal of the IASS* 47 (2006), pp. 53–62.
- [DCB10] C. Douthe, J.-F. Caron, and O. Baverel. “Gridshell structures in glass fibre reinforced polymers”. In: *Construction and Building Materials* 24.9 (2010), pp. 1580–1589.
- [DKZ14] B. D'Amico, A. Kermani, and H. Zhang. “Form finding and structural analysis of actively bent timber grid shells”. In: *Engineering Structures* 81 (2014), pp. 195–207.
- [Dou+16] C. Douthe, R. Mesnil, H. Orts, and O. Baverel. “New shapes for elastic gridshells covered by planar facets”. In: *Proceedings of the IASS Annual Symposium*. Tokyo, Japan, 2016, pp. 1–9.
- [Dou07] C. Douthe. “Etude de structures élancées précontraintes en matériaux composites : application à la conception des gridshells”. PhD thesis. Université Paris Est, 2007, p. 274.
- [dPel+11] L. du Peloux, O. Baverel, J.-F. Caron, and F. Tayeb. “From shape to shell : a design tool to materialize freeform shapes using gridshell structures”. In: *Design Modelling Symposium*. Berlin, Deutschland, 2011.
- [dPel+15] L. du Peloux, F. Tayeb, B. Lefevre, O. Baverel, and J.-F. Caron. “Formulation of a 4-DoF torsion / bending element for the formfinding of elastic gridshells”. In: *Proceedings of the International Association for Shell and Spatial Structures*. August. Amsterdam, 2015, pp. 1–14.
- [dPel+16] L. du Peloux, F. Tayeb, O. Baverel, and J.-F. Caron. “Construction of a large composite gridshell structure: a lightweight structure made with pultruded glass fibre reinforced polymer tubes”. In: *Structural Engineering International* 26.2 (2016), pp. 160–167.
- [DZK16] B. D'Amico, H. Zhang, and A. Kermani. “A finite-difference formulation of elastic rod for the design of actively bent structures”. In: *Engineering Structures* 117 (2016), pp. 518–527.
- [FKB16] J. G. Fernandes, P. H. Kirkegaard, and J. M. Branco. “Tectonic Design of Elastic Timber Gridshells”. In: *World Conference on Timber Engineering*. Vienna, Austria, 2016.
- [FN15] G. Filz and D. Naicu. “2 Landscapes – interaction of 2 gridshells based on a modified Stewart - Gough - principle”. In: *Proceedings of the IASS Working Groups 12 + 18*. Tokyo, Japan, 2015.
- [Fou03] Fourthdoor.org. *Growing and making Flimwell's chestnut gridshell*. 2003.

- [Had+16] J. Haddal Mork, S. Dyvik Hillersøy, B. Manum, A. Rønnquit, and N. Labonne. “Introducing the Segment Lath - a Simplified Modular Timber Gridshell Built in Trondheim Norway”. In: *World Conference on Timber Engineering*. 3. Vienna, Austria, 2016.
- [Har+03] R. Harris, J. Romer, O. Kelly, and S. Johnson. “Design and construction of the Downland Gridshell”. eng. In: *Building research and information* 31.6 (2003), pp. 427–454.
- [HHR08] R. Harris, S. Haskins, and J. Roynon. “The Savill Garden gridshell : Design and construction”. In: *The Structural Engineer* 86.17 (2008), pp. 27–34.
- [HK02] R. Harris and O. Kelly. “The structural engineering of the Downland gridshell”. In: *Space Structures 5* 1.April 2002 (2002), pp. 161–172.
- [HL75] E. Happold and I. Liddell. “Timber lattice roof for the Mannheim Bundesgartenschau”. In: *The structural engineer* 53.3 (1975), pp. 99–135.
- [Lab+16] N. Labonne, J. H. Mork, S. H. Dyvik, A. Rønnquist, and B. Manum. “Experimental and Numerical Study of the Structural Performance of a Timber Gridshell”. In: *World Conference on Timber Engineering*. 2. Vienna, Austria, 2016, pp. 3–10.
- [Laf15] E. Lafuente Hernández. “Design and Optimisation of Elastic Gridshells”. PhD thesis. Universität der Künste Berlin, 2015.
- [LDB15] B. Lefevre, C. Douthe, and O. Baverel. “Buckling of elastic gridshells”. In: *Journal of the IAASS* September (2015).
- [LG14] E. Lafuente Hernández and C. Gengnagel. “A new hybrid: Elastic gridshells braced by membranes”. In: *WIT Transactions on the Built Environment* 136 (2014), pp. 157–170.
- [Lid15] I. Liddell. “Frei Otto and the development of gridshells”. In: *Case Studies in Structural Engineering* 4 (2015), pp. 39–49.
- [Low02] O. Lowenstein. “Lothian Gridshell”. In: *Building for a Future Winter* (2002), p. 29.
- [Low04] O. Lowenstein. “Gridshells in England. Ideas from the U.K. make stunning use of small timber”. In: *Wood Design & Building* (2004).
- [LP15] A. Liuti and A. Pugnale. “Erection of post-formed gridshells by means of inflatable membrane technology membrane technology”. In: *International Conference of the Architectural Science Association*. 2015, pp. 678–687.
- [LPD16] Liuti, A. Pugnale, and B. D’Amico. “Building timber gridshells with air : Numerical simulations and technique challenges”. In: *Structures and Architecture* (2016).
- [Mal12] S. R. Malek. “The effect of geometry and topology on the mechanics of grid shells”. PhD thesis. MIT, 2012.
- [Mes+17] R. Mesnil, C. Douthe, O. Baverel, and B. Léger. “Linear buckling of quadrangular and kagome gridshells: A comparative assessment”. In: *Engineering Structures* 132 (2017), pp. 337–348.

- [MKS16] A. Menges, O. D. Krieg, and T. Schwinn. *Advancing Wood Architecture. A computational approach*. Routledge, July 2016, p. 256.
- [MM17] Y. Masson and L. Monasse. “Existence of global Chebyshev nets on surfaces of absolute Gaussian curvature less than 2π ”. In: *Journal of Geometry* 108.1 (2017), pp. 25–32.
- [MOB06] M. McQuaid, F. Otto, and S. Ban. “Building the Japan pavilion”. In: *Shigeru Ban*. Phaidon Press, 2006, pp. 8–11.
- [MOD15] R. Mesnil, J. Ochsendorf, and C. Douthe. “Stability of Elastic Grid Shells”. In: *International Journal of Space Structures* 30.1 (2015), pp. 27–36.
- [NLG13] P. Nicholas, E. Lafuente Hernández, and C. Gengnagel. “The Faraday Pavilion : activating bending in the design and analysis of an elastic gridshell”. In: *Proceedings of the Symposium on Simulation for Architecture & Urban Design*. April. San Diego, CA, USA, 2013, Article No. 21.
- [Pon+16] S. Pone, G. Mirra, E. Pignatelli, D. Lancia, and S. Colabella. “Specialised algorithms for different project stages in a post-formed timber gridshell design”. In: *Structures and Architecture* September (2016), pp. 259–266.
- [Pou15] E. Poulsen. “Structural design and analysis of elastically bent gridshells”. PhD thesis. Chalmers University of Technology, Gothenburgn Sweden, 2015.
- [QG14] G. Quinn and C. Gengnagel. “A review of elastic grid shells, their erection methods and the potential use of pneumatic formwork”. In: *WIT Transactions on the Built Environment* 136 (2014), pp. 129–144.
- [Tay+13] F. Tayeb, J.-F. Caron, O. Baverel, and L. du Peloux. “Stability and robustness of a 300m² composite gridshell structure”. In: *Construction & Building Materials* (2013).
- [Tay15] F. Tayeb. “Simulation numérique du comportement mécanique non linéaire de gridshells composés de poutres élancées en matériaux composites et de sections quelconques”. PhD thesis. 2015.
- [Tra06] Trada. “The Savill Building. A visitor centre with a timber gridshell roof Gridshell structures”. In: *TRADA* (2006), pp. 1–7.