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1. Elastic gridshell

# A neat way to build free-form architecture

## Building free-forms

### Non-standard forms

### Importance of free-forms in modern architecture

### Canonical approaches to build free-forms

### Main challenges

## Gridshell structure : definition and classification

### Historic overview

### Rigid gridshell

### Elastic gridshell

## Elastic gridhshells : revisiting Mannheim

# Experimenting elastic gridshells

## Overall presentation

## Tchebychev nets

* Formfinding vs gridfinding
* Compass method
* Citer Yannick / Patch

## Composite gridshells

## Wooden gridshells

### Double layer

* Méthode de combinaison des contraintes
* Raideur de la poutrelle echelle
* Glissement du noeud

### Bracing

* Détail du pion
* Détail de l’écarteur
* Méthodologie de mise en place et phasage dans le projet

### Wood testing

* Small section
* Présence de defaults

### Mise en oeuvre

* Contrôle de forme
* Phasage : assemblage de la grille

1. Rich Kirchhoff beam model

# Geometry of smooth and discret curves

## Introduction

### Goals and contributions

### Related works

### Overview

## Paramectric Curves

### Definition

### Regularity

### Reparametrization

### Natural parametrization

### Curve length

### Arc-length parametrization

## Frenet’s Trihedron

### Tangent vector

### Normal vector

### Binormal vector

## Curvature

### Osculating circle

### Curvature binormal vector

## Torsion

## Curve Framing

### Moving frame

### Adapted frame

### Frenet frame

### Bishop frame

### Comparison between Frenet and Bishop frames

## Discrete Curvature

### Definitions

### Variability of discrete curvature regarding α

### Convergence benchmark κ1vs.κ2

# kirchhoff rod: variational approach

## Introduction

### Goals and contributions

### Related works

### Overview

## Elastic energy

## Curve-angle representation

### Zero-twistingframe

## Strains

### Axial strain

### Bending strain

### Torsional strain

## Elastic energy

## Quasistatic assumption

## Energy gradient with respect to θ : moment of torsion

### Derivative of material directors with respect to θ

### Derivative of the material curvatures vector with respect to θ

### Computation of the moment of torsion

## Energy gradient with respect to x: internal forces

### Derivative of material directors with respect to **x**

### Derivative of the material curvatures vector with respect to **x**

### Computation of the forces acting on the centerline

## Conclusion

# kirchhoff rod: equilibrium approach

## Introduction

### Goals and contributions

* This is a more straightforward way to achieved the same goal
* This way of doing things is more elegant, can be understand purely in terms of equilibrium, and is more conform to the approach of Days and Barnes
* We write the Kirchhoff dynamic equations for a slender beam (not common in the literature)

### Related works

### Overview

## Dynamic Kirchhoff equations

### Balance of the linear momentum

### Balance of the angular momentum

## 4.3  Equations of motion

### Constitutive equations

### Internal forces and moments

### Rod dynamic

## Geometric interpretation

## forces acting on the centerline

## moment acting on the centerline

## Conclusion

# A rich discret Kirchhoff beam element for numerical analysis

## Introduction

### Goals and contributions

* Bi-arc kinematic accounting for discontinuities (EI or Fext or Mext)
* Interpolation of internal forces and moments
* Time vs Space parallel transport of Bishop Frames
* Boundary conditions : forces vs speed way of dealing with it

### Related works

### Overview

## Kinematic description

### Centerline

### Section

### Hinged bi-arcs

## Quasi-static actions on the centreline

### Forces acting on the centerline (**x**)

### Torsion acting on the sections (θ)

## Boundary Conditions

* Soit géré en terme de vitesse soit en terme d’effort
* Calcul de la courbure aux extrémitées
* soit le repère est imposé et l’on cherche l’effort resultant (moment nule si rotule)
* soit l’effort en bout est connu (moment appliqué) et l’on cherche la courbure
* a chaque fois on part de la condition d’équilibre statique pour obtenir une info supplémentaire

## Connexions : making grids

### Infinitely stiff connexion

### Spring connection

## Integration with dynamic explicit solver

* Cas general vs dynamic relaxation

### Fictious lumped mass

Translational / Rotational

### Double dynamic (x / θ)

Quasistatic assumption

1. Application & Conclusion
2. Appendix

# Calculus of variations

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