

Case Studies in High-Performance Computing

Assignment 2 - Krylov Subspace Methods and GMRES

Ion Lipsiuc

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1 The Arnoldi Iteration

We present the pseudocode of the Arnoldi iteration algorithm exactly as is given in BYU (2025):

```
procedure ARNOLDI( $A, u, m$ )  
   $n \leftarrow \text{length}(u)$   
   $Q \leftarrow \text{zeros}(n, m + 1)$ , with dtype=complex       $\triangleright$  Initialise matrix to hold  
  orthonormal basis vectors  
   $H \leftarrow \text{zeros}(m + 1, m)$ , with dtype=complex     $\triangleright$  Initialise Hessenberg matrix  
   $Q[:, 0] \leftarrow u / \|u\|_2$   
  for  $j = 0$  to  $m - 1$  do  
     $Q[:, j + 1] \leftarrow A \cdot Q[:, j]$   
    for  $i = 0$  to  $j$  do  
       $H[i, j] \leftarrow \langle Q[:, i], Q[:, j + 1] \rangle$        $\triangleright$  Use complex inner product  
       $Q[:, j + 1] \leftarrow Q[:, j + 1] - H[i, j] \cdot Q[:, i]$   $\triangleright$  Orthogonalise against  $Q[:, i]$   
    end for  
     $H[j + 1, j] \leftarrow \|Q[:, j + 1]\|_2$   
     $Q[:, j + 1] \leftarrow Q[:, j + 1] / H[j + 1, j]$   
  end for  
  return  $Q, H$   
end procedure
```

References

BYU. Lab 1 - krylov subspaces. <https://acme.byu.edu/0000017a-1bb8-db63-a97e-7bfa0bdf0000/krylov1-pdf>, 2025. A lab handout for a course discussing Krylov subspaces.