

Case Studies in High-Performance Computing

Assignment 2 - Krylov Subspace Methods and GMRES

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1 The GMRES Algorithm

We present the pseudocode of the GMRES algorithm as is given in [BYU \(2025\)](#) with some minor adjustments:

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procedure GMRES( $A, b, m$ )
    Ensure  $A$  is a function that applies the matrix to a vector
     $n \leftarrow \text{length}(b)$ 
     $Q \leftarrow \text{zeros}(n, m + 1)$ 
     $H \leftarrow \text{zeros}(m + 1, m)$ 
     $x_0 \leftarrow \text{zeros}(n)$ 
     $r_0 \leftarrow b - A(x_0)$ 
     $\beta \leftarrow \|r_0\|_2$ 
     $Q[:, 0] \leftarrow r_0/\beta$ 
     $residuals \leftarrow [\beta]$ 
    for  $j = 0$  to  $m - 1$  do
         $q \leftarrow A(Q[:, j])$ 
        for  $i = 0$  to  $j$  do
             $H[i, j] \leftarrow Q[:, i]^T \cdot q$ 
             $q \leftarrow q - H[i, j] \cdot Q[:, i]$ 
        end for
         $H[j + 1, j] \leftarrow \|q\|_2$ 
        if  $H[j + 1, j] > 10^{-12}$  then
             $Q[:, j + 1] \leftarrow q/H[j + 1, j]$ 
        end if
         $e_1 \leftarrow \text{zeros}(j + 2)$ 
         $e_1[0] \leftarrow \beta$ 
        Solve the least squares problem  $\min \|H[0 : j + 2, 0 : j + 1]y - e_1\|_2$  for  $y$ 
         $res \leftarrow \|\beta e_1 - H[0 : j + 2, 0 : j + 1] \cdot y\|_2$ 
         $residuals.append(res)$ 
    end for
     $x \leftarrow x_0 + Q[:, 0 : m] \cdot y$ 
    return  $x, \text{array}(residuals)$ 
end procedure

```

References

BYU. Lab 1 - gmres. <https://acme.byu.edu/0000017a-1bb8-db63-a97e-7bfa0be30000/vol11lab23gmres-pdf>, 2025. A lab handout for a course discussing the GMRES algorithm.