

$$1. \lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{\frac{1}{x}} \quad |^\infty \frac{0}{0}|$$

$$= \lim_{x \rightarrow 0} e^{\frac{1}{x} \ln \left(\frac{a^x + b^x + c^x}{3} \right)} \rightarrow 1 \quad (\text{凑等价无穷小代换})$$

$$= e^{\lim_{x \rightarrow 0} \frac{\frac{a^x + b^x + c^x}{3} - 1}{x}} = e^{\lim_{x \rightarrow 0} \frac{a^x + b^x + c^x - 3}{3x}}$$

$$= e^{\frac{1}{3} \lim_{x \rightarrow 0} \frac{a^x + b^x + c^x - 3}{x}} \quad \frac{0}{0} \text{ 型 (洛必达)}$$

$$= e^{\frac{1}{3} \lim_{x \rightarrow 0} \frac{a^x \ln a + b^x \ln b + c^x \ln c}{1}}$$

$$= e^{\frac{1}{3} (\ln a + \ln b + \ln c)} = e^{\frac{1}{3} \ln abc} = e^{\ln(abc)^{\frac{1}{3}}} = \sqrt[3]{abc}.$$

$$2. \lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\tan x} \quad |^\infty \frac{0}{0}|$$

$$= e^{\lim_{x \rightarrow \frac{\pi}{2}} \tan x \cdot \ln(\sin x)} \quad \text{非零因式} = e^{\lim_{x \rightarrow \frac{\pi}{2}} \tan x \cdot (\sin x - 1)}$$

$$= e^{\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x}{\cos x} (\sin x - 1)} = e^{\lim_{x \rightarrow \frac{\pi}{2}} \left[\frac{\sin x - 1}{\cos x} \right]} \quad \frac{0}{0} \text{ 型}$$

$$= e^{\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{-\sin x}} = e^0 = 1.$$