

1.  $\int \frac{\arcsin \sqrt{x}}{\sqrt{x-x^2}} dx$  较复杂部分

解: 法①:  $(\arcsin \sqrt{x})' = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot (\sqrt{x})' = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}}$   
 $= \frac{1}{2} \cdot \frac{1}{\sqrt{x-x^2}}$

$\therefore$  原积分  $= 2 \int \arcsin \sqrt{x} d \arcsin \sqrt{x}$   
 $= 2 \cdot \frac{1}{2} (\arcsin \sqrt{x})^2 + C$   
 $= \arcsin^2 \sqrt{x} + C. (C \text{ 为任意常数})$

法②: 原积分  $= \int \frac{\arcsin \sqrt{x}}{\sqrt{x-x^2}} dx$  有公因子  $x$ .

$= \int \frac{\arcsin \sqrt{x}}{\sqrt{x(1-x)}} dx$

$= \int \frac{\arcsin \sqrt{x}}{\sqrt{x} \cdot \sqrt{1-x}} dx$  凑微分

$= 2 \int \frac{\arcsin \sqrt{x}}{\sqrt{1-(\sqrt{x})^2}} d\sqrt{x}$

$= 2 \int \frac{\arcsin \sqrt{x}}{\sqrt{1+(\sqrt{x})^2}} d(\sqrt{x})$  凑微分

$$= 2 \int \arcsin \sqrt{x} d \arcsin \sqrt{x}$$

$$= 2 \cdot \frac{1}{2} \arcsin^2 \sqrt{x} + C = \arcsin^2 \sqrt{x} + C$$

(C 为任意常数)

2.  $\int x^{x^2} x \cdot (2 \ln x + 1) dx$ . 较复杂部分

解:  $\because x^{x^2} = e^{\ln x^{x^2}} = e^{x^2 \ln x}$

$$\therefore (x^{x^2})' = (e^{x^2 \ln x})'$$

$$= e^{x^2 \ln x} \cdot (2x \ln x + x^2 \cdot \frac{1}{x})$$

$$= x^{x^2} \cdot (2x \ln x + x)$$

$$= \underline{x^{x^2} \cdot x (2 \ln x + 1)}$$

求导后恰好为整个被积函数.

$$\therefore \text{原积分} = \int d x^{x^2}$$

$$= x^{x^2} + C. \quad (C \text{ 为任意常数})$$