Homework #3

2019042497 송정명

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wjdaud@LAPTOP-1NG9U1J0:~/NA/homework3$ ./homework3

=================== Equation 1 ===================
                        matrix A / vector b
       4.000000        2.000000        3.000000       -1.000000    /       4.000000
      -2.000000       -1.000000       -2.000000        2.000000    /      -3.000000
       5.000000        3.000000        4.000000       -1.000000    /       4.000000
      11.000000        4.000000        6.000000        1.000000    /      11.000000

---------- 1) Gauss-Jordan Elimination ----------

Matrix A is Singular matrix

---------- 2) LU Decomposition ----------

Solution x by using LU Decomposition:
       1.000000
      -3.000000
       2.000000
       0.000000

Improved Solution x by using LU Decomposition:
       1.000000
      -3.000000
       2.000000
       0.000000

Inverse of matrix A by using LU Decomposition:
-1000000002004087734272.000000-375000000751532900352.000000375000000751532900352.000000125000001350022594560.000000
-2000000567847336017 92.000000-750000190952518451 20.000000750000102991588229 12.000000250000048990684446 72.000000
3000000411966352916 48.000000112500015448738234368.000000-1125000066526452121 60.000000-375000051495794114 56.000000
1000000002004087734272.000000375000000751532900352.000000-374999963534863892 48.000000-125000001350022594560.000000
---------- 3) Singular Value Decomposition ----------

Solution x by using Singular Value Decomposition:
       1.375000
      -2.750000
       1.750000
      -0.250000

Inverse of matrix A by using Singular Value Decomposition:
 -809912.687500 -303717.250000  303717.343750  101239.179688
-1619824.625000 -607432.875000  607436.125000  202477.515625
 2429737.000000  911150.437500 -911152.687500 -303716.781250
  809912.750000  303717.500000 -303717.625000 -101238.937500

Determinant by using LU decomposition:       -0.000000


==================== Equation 2 ====================
                    matrix A / vector b
       2.000000      -4.000000      -5.000000       5.000000       0.000000     /      -5.000000
      -1.000000       1.000000       2.000000       0.000000       4.000000     /       2.000000
      -1.000000       6.000000       0.000000       3.000000       2.000000     /       0.000000
       0.000000       1.000000       3.000000       7.000000       5.000000     /       4.000000
       5.000000       0.000000       8.000000       7.000000      -2.000000     /      -1.000000

---------- 1) Gauss-Jordan Elimination ----------

Solution x by using Gauss-Jordan Elimination:
      -2.873567
      -0.612357
       0.976277
       0.635819
      -0.553441
```

```
Inverse of matrix A by using Gauss-Jordan Elimination:
       0.354536       0.766945       0.207769      -0.595412       0.253128
       0.035454       0.126695       0.195777      -0.159541       0.050313
      -0.138686      -0.098540      -0.096715       0.124088       0.016423
      -0.052138      -0.303963      -0.023201       0.234619      -0.044578
       0.149114       0.459333       0.051356      -0.171012       0.042492

---------- 2) LU Decomposition ----------

Solution x by using LU Decomposition:
      -2.873566
      -0.612357
       0.976277
       0.635819
      -0.553441

Improved Solution x by using LU Decomposition:
      -2.873566
      -0.612357
       0.976277
       0.635818
      -0.553441

Inverse of matrix A by using LU Decomposition:
       0.354536       0.766945       0.207769      -0.595412       0.253128
       0.035454       0.126695       0.195777      -0.159541       0.050313
      -0.138686      -0.098540      -0.096715       0.124088       0.016423
      -0.052138      -0.303962      -0.023201       0.234619      -0.044578
       0.149114       0.459333       0.051356      -0.171011       0.042492

---------- 3) Singular Value Decomposition ----------

Solution x by using Singular Value Decomposition:
      -2.873566
      -0.612357
       0.976278
       0.635819
      -0.553441
```

```
Inverse of matrix A by using Singular Value Decomposition:
       0.354536       0.766945       0.207769      -0.595412       0.253128
       0.035454       0.126695       0.195777      -0.159541       0.050313
      -0.138686      -0.098540      -0.096715       0.124088       0.016423
      -0.052138      -0.303963      -0.023201       0.234620      -0.044578
       0.149114       0.459333       0.051356      -0.171012       0.042492

Determinant by using LU decomposition:     3835.999512


==================== Equation 3 ====================
                        matrix A / vector b
       0.400000       8.200000       6.700000       1.900000       2.200000       5.300000      /      -2.900000
       7.800000       8.300000       7.700000       3.300000       1.900000       4.800000      /      -8.200000
       5.500000       8.800000       3.000000       1.000000       5.100000       6.400000      /       7.700000
       5.100000       5.100000       3.600000       5.800000       5.700000       4.900000      /      -1.000000
       3.500000       2.700000       5.700000       8.200000       9.600000       2.900000      /       5.700000
       3.000000       5.300000       5.600000       3.500000       6.800000       5.700000      /       3.000000

---------- 1) Gauss-Jordan Elimination ----------

Solution x by using Gauss-Jordan Elimination:
      -0.326608
       1.532293
      -1.044825
      -1.587447
       2.928480
      -2.218931

Inverse of matrix A by using Gauss-Jordan Elimination:
      -0.162205       0.122801       0.024068      -0.016431      -0.022840       0.046132
       0.169407      -0.041117       0.228313      -0.087624       0.180306      -0.395655
      -0.011636       0.122745      -0.117407      -0.180981       0.015910       0.186766
       0.105669      -0.051726      -0.108916       0.299774       0.000859      -0.190541
      -0.053026      -0.042361       0.160508      -0.224034       0.161811       0.015024
      -0.062341      -0.064694      -0.234216       0.351126      -0.364828       0.434633
```

```
---------- 2) LU Decomposition ----------

Solution x by using LU Decomposition:
      -0.326608
       1.532292
      -1.044826
      -1.587447
       2.928480
      -2.218930

Improved Solution x by using LU Decomposition:
      -0.326608
       1.532292
      -1.044825
      -1.587448
       2.928480
      -2.218930

Inverse of matrix A by using LU Decomposition:
      -0.162205       0.122801       0.024068      -0.016431      -0.022840       0.046132
       0.169407      -0.041117       0.228313      -0.087624       0.180306      -0.395655
      -0.011636       0.122745      -0.117407      -0.180981       0.015910       0.186766
       0.105669      -0.051726      -0.108916       0.299774       0.000859      -0.190541
      -0.053026      -0.042362       0.160508      -0.224034       0.161811       0.015024
      -0.062341      -0.064694      -0.234216       0.351126      -0.364828       0.434633

---------- 3) Singular Value Decomposition ----------

Solution x by using Singular Value Decomposition:
      -0.326609
       1.532292
      -1.044825
      -1.587447
       2.928480
      -2.218930
```

```
Inverse of matrix A by using Singular Value Decomposition:
      -0.162205       0.122801       0.024068      -0.016431      -0.022840       0.046132
       0.169407      -0.041117       0.228313      -0.087624       0.180306      -0.395655
      -0.011636       0.122745      -0.117407      -0.180981       0.015910       0.186766
       0.105669      -0.051726      -0.108916       0.299774       0.000859      -0.190541
      -0.053026      -0.042361       0.160508      -0.224034       0.161811       0.015024
      -0.062341      -0.064694      -0.234216       0.351126      -0.364828       0.434633

Determinant by using LU decomposition:    16178.401367
```

Equation에서 읽어온 matrix A와 vector b를 출력해 잘 읽어왔는지 확인하도록 했습니다.

이후 이 Ax = b에 대해 각각 Gauss-Jordan Elimination (gaussj()), LU Decomposition (ludcmp()), Singular Value Decomposition (svdcmp())를 적용하여 해와 inverse를 구하고 출력하도록 했습니다.

추가로, 2번 문제를 위해 LU Decomposition (ludcmp())에서는 iterative improvement(mprove())을 진행하고 출력했습니다.

마지막으로 LU Decomposition (ludcmp()) 하는 과정에서 나온 lu 결과를 통해 대각선의 원소 값을 모두 곱하여 determinant를 구해주었습니다.

1. 각 method (gaussj(), ludcmp(), svdcmp())에 대한 경험적인 장단점

1) Gauss-Jordan Elimination (gaussj())의 장단점

장점: x와 inverse matrix를 구하기 쉬운 것입니다. (함수를 사용했을 때 바로 구해집니다.)

단점: Singular matrix일때는 작동하지 않으며, determinant를 구하는 것과는 관련이 없어 원래 구하듯이 구하여야 합니다.

2) LU Decomposition (ludcmp())의 장단점

장점: determinant를 구하기 쉽습니다. Singular matrix일 때 함수 내부에서 pivot을 0이 아닌 TINY(1.0e-20)로 바꾸어 근사해서 해를 구할 수 있습니다.

단점: 역행렬을 구하기 위한 계산이 복잡한 편입니다.

3) Singular Value Decomposition (svdcmp())의 장단점

장점: Singular matrix인 경우에도 근사해서 해를 구할 수 있습니다.

단점: 해를 구하기 위해 직접 inverse(A^{-1}) = V [diag(1/w_j)] * U^T를 구현해야 했습니다. 근사해서 구한 해이기 때문에 다른 method의 해와 미세하게 차이가 납니다.

2. iterative improvement(mprove()) 적용, 결과 분석

LU Decomposition (ludcmp())을 통해 구한 해에 적용하였습니다. 이번 실습에서는 결과값이 유의미하게 변하지는 않았지만, 계산 정밀도와 안정성을 향상할 수 있는 방법이라고 생각합니다.

3. 각 행렬에 대한 inverse와 determinant 계산

Gauss-Jordan Elimination, LU Decomposition, SVD 3가지 방법으로 inverse를 구했습니다. 또한, determinant의 경우 LU Decomposition을 통해 쉽게 구했습니다.