

Life-cycle model with nonlinear household earnings

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1 Data

Stephane suggests not to take out the age-income profile in the first stage regression, so

$$\log Y_t = y_t = \eta_t + \epsilon_t$$

where η_t and ϵ_t are the permanent and transitory component. Earnings Y_{it} are total pre-tax household labor earnings. We construct y_{it} as residuals of log household earnings on a set of demographics, which include birth year dummies, family size and composition (including dummies for income recipients other than husband and wife, and for kids out of home), race, state and big city dummies, and education which is interacted with birth year dummies. Besides, we also take out the covariates effect in the residuals controlling for birth year and education dummies. We proceed similarly for consumption and asset holdings.

We focus on a balanced subsample of married male heads aged between 25 and 64. The reason we stop the sample at age 64 because we assume retirement occurs at age 65 in the structural model. The sample size $N = 889$ households. Table 1 shows mean total earnings, consumption and asset holdings, by year.

Things need to be done:

1. Estimate an initial asset distribution conditional on age from 25 to 56, where 25 is the initial age of the youngest household, and 56 is the initial age of the oldest household.

Table 1: Descriptive statistics (means)

	1999	2001	2003	2005	2007	2009
Earnings	87040.53	93086.36	94777.14	96391.41	99908.33	98904.51
Consumption	30889.57	34600.91	37083.53	42568.51	43849.23	39933.18
Assets	250318.9	302756	358051.3	404055.7	488675.1	446176

Notes: Balanced subsample from PSID, $N = 889$, $T = 6$

2. Estimate the conditional distribution of initial permanent component for each age from 25 to 56. This distribution is also conditional on asset.
3. Estimate conditional quantile functions. The permanent component is conditional on age and its lag, and the transitory component is only conditional on age.
4. Using initial asset distribution, we can get initial permanent component distribution for each age. Then using the distributions of initial asset and permanent component, we can simulate the lifecycle of a household starting from each age from 25 to 56.
5. So we simulate the model for 32 times with a different starting age each time. For each simulation, we pick the first 6 periods. Then we can construct a simulated sample in the same way as the PSID sample.

2 The model

Consumers live for T^* periods and work for $T < T^*$, where both T^* and T are exogenous and fixed. Households maximize expected life-time utility with discount factor β :

$$\max_{c_t} \mathbb{E}_1 \left[\sum_{t=1}^{T^*} \beta^{t-1} u(C_t) \right]$$

During working years $1 \leq t \leq T$, the consumer receives labor income Y_t , which is decomposed into a deterministic experience profile μ_t , a permanent component η_t , and a transitory component ϵ_t :

$$\log Y_t = \mu_t + y_t$$

$$y_t = \eta_t + \epsilon_t$$

There is one asset in the economy, with a constant interest rate R . So consumers subject to the constraints

$$A_t = RA_{t-1} + Y_t - C_t$$

After retirement, consumers receive social security transfers Y^{ss} from the government, which is a function of entire realizations of individual labor income. Income is not subject to risk during retirement.

3 Calibration

Demographics Individuals enter the model at age 30, and leave the model at age 90. In the model, each period is 2 years, so $T^* = (90 - 30)/2 + 1 = 31$. Individuals retire at age 65, so their working periods $T = (64 - 30)/2 + 1 = 18$.

Preferences We assume the utility function is of the CRRA form, with intertemporal elasticity of substitution $1/\rho$,

$$u(C) = C^{1-\rho} / (1 - \rho)$$

where the relative risk aversion ρ is 2.

Discount factor and interest rate The interest rate R is assumed to equal 1.06, and the discount factor β is 0.93.

Initial wealth An individual's initial asset is set to either 0.17, 0.5, or 0.83 of her initial labor income, each with probability $1/3$.

Income process The deterministic age profile for log income μ_t is taken from Carroll and Samwick (1997). The permanent and the transitory components evolve as following:

$$\begin{aligned}\eta_t &= Q_t(\eta_{t-1}, u_{\eta t}) \\ \epsilon_t &= Q_t(u_{\epsilon t})\end{aligned}$$

Borrowing limit Consumer's net worth can not fall below zero, i.e. no borrowing constraint $\underline{A} = 0$.

Social security benefits Social security benefits Y^{ss} is 50% of lifetime average individual earnings $Y^{ss} = 0.5 \sum_{t=1}^T Y_t$.

4 Simulations

The grid nodes for permanent and transitory component are both 100. Figure 1 shows the life cycle profile of income and consumption by quantiles. Income increases faster in higher quantiles. The decrease of income is earlier in lower quantiles. Consumers start to reduce consumption before retirement, and the decrease is also earlier in lower quantiles.

Figure 2 shows the life cycle profile of asset by quantiles. Individuals consume their initial assets, so the asset is decreasing at the beginning. It shows much more precautionary saving in higher quantiles.

Figure 3a plots the nonlinear persistence calculated from the data, and Figure 3b plots that from the model. The range of the variation of the nonlinear persistence for the data is $0.3034 \sim 1.0270$, and that for the model is $0.4218 \sim 0.9976$.

References

Carroll, C. D. and Samwick, A. (1997). The nature of precautionary wealth. *Journal of monetary Economics*.

Figure 1: Life cycle profile of income and consumption

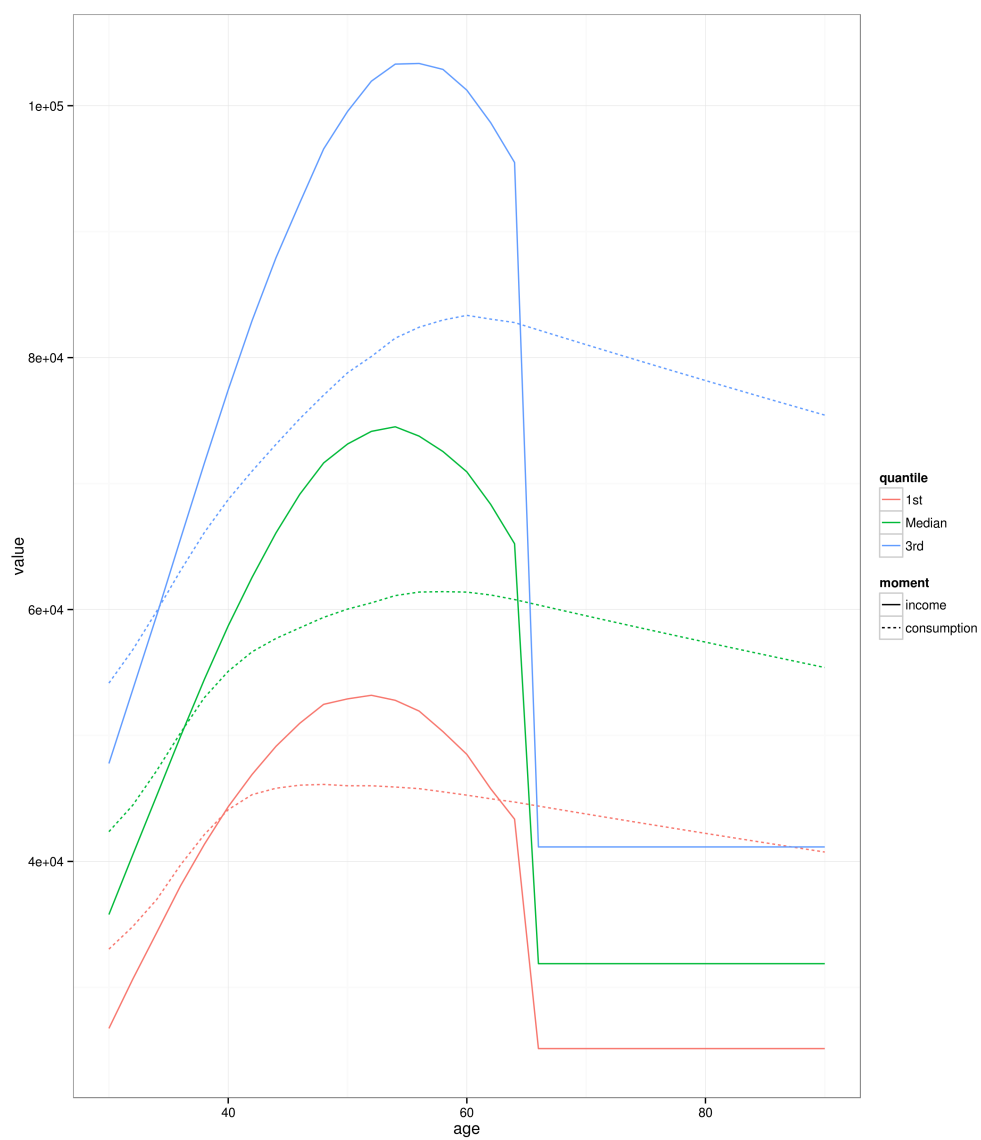


Figure 2: Life cycle profile of asset

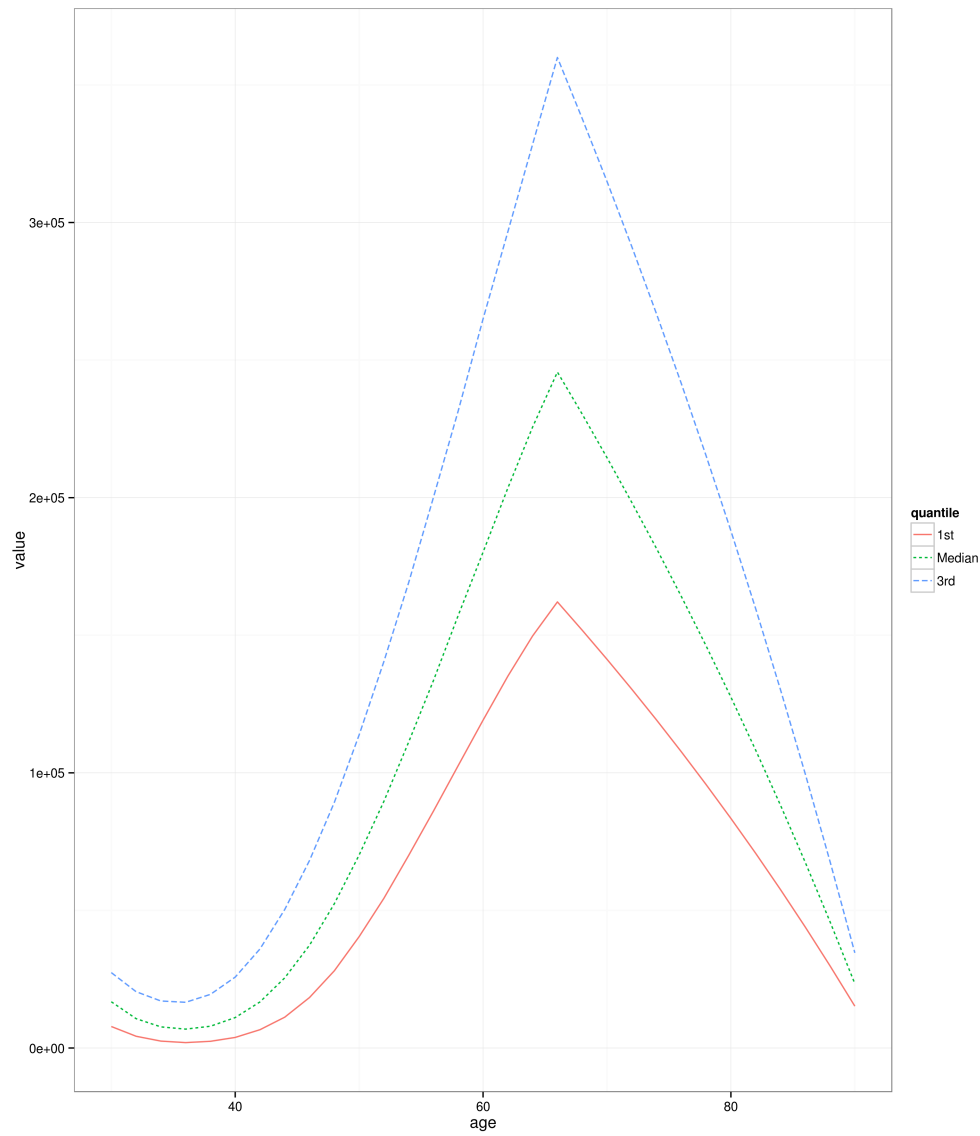
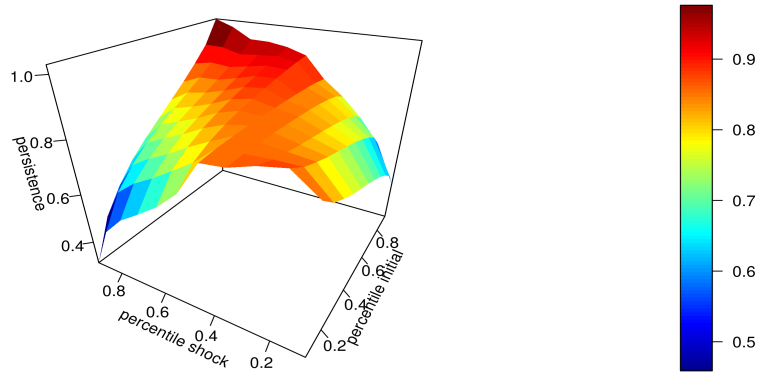


Figure 3: Nonlinear persistence

(a) Earnings, PSID data



(b) Earnings, model simulation

