PDE Project on Numeric Methods

Name: Lior Cohen

ID: 314818345

n_1: 1 n_2: 1 n_3: 3 n_4: 3 n_5: 4
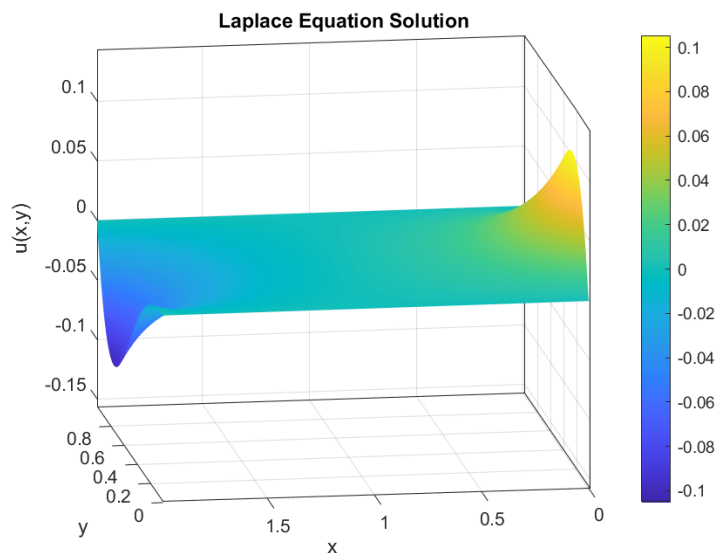
Project #1

Part 1: Laplace Equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < 2 \ , 0 < y < 1,$$

Boundary conditions:

$$u(x,0) = u(x,1) = 0, \qquad u(0,y) = y^{n_1}(1-y)^{n_3}, \qquad u(2,y) = -y^{n_4}(1-y)^{n_2}$$

The x's that $u(x,y)$ as a function of y have both maximum and minimum:

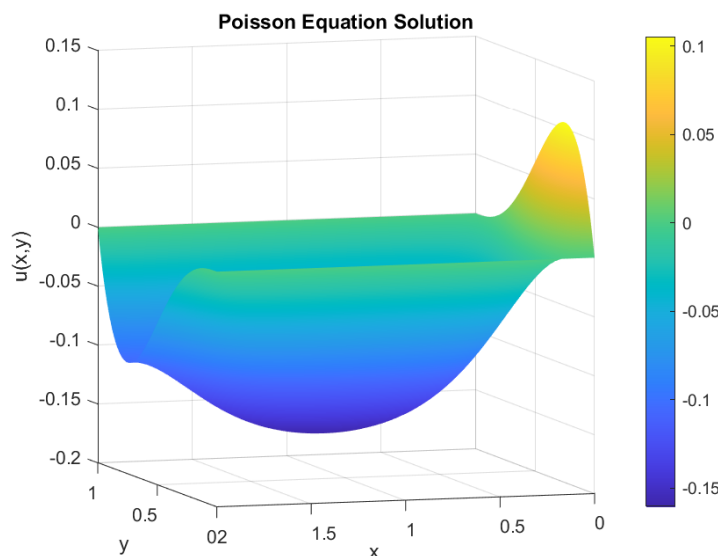*1.* $X = 1.0000$
2. $X = 1.0200$

The values of h and k and why I chose them:

h = 0.02   k = 0.02

h and k are simply the grid spacings in the x and y directions. Smaller values mean a finer grid (more points in each direction), which typically gives more accurate finite difference approximations but requires solving a larger system of equations.

Part 2: Poisson Equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 1 + \frac{n_5}{10},$$
$$0 < x < 2 \ , 0 < y < 1$$

With the same boundary conditions as the Laplace equation.

The axis values where the solution has its ***minimum*** value:

$$u = -0.1607$$
$$x = 1.1400$$
$$y = 0.5000$$

Project #2

n_1: 1 n_2: 1 n_3: 3 n_4: 3 n_5: 4 | **Grid variables:** $h = 0.01$ and $k = r \times h^2$ with $r = 0.4$, thus $k = 4 \times 10^{-5}$

Part 1: Homogeneous Heat Equation
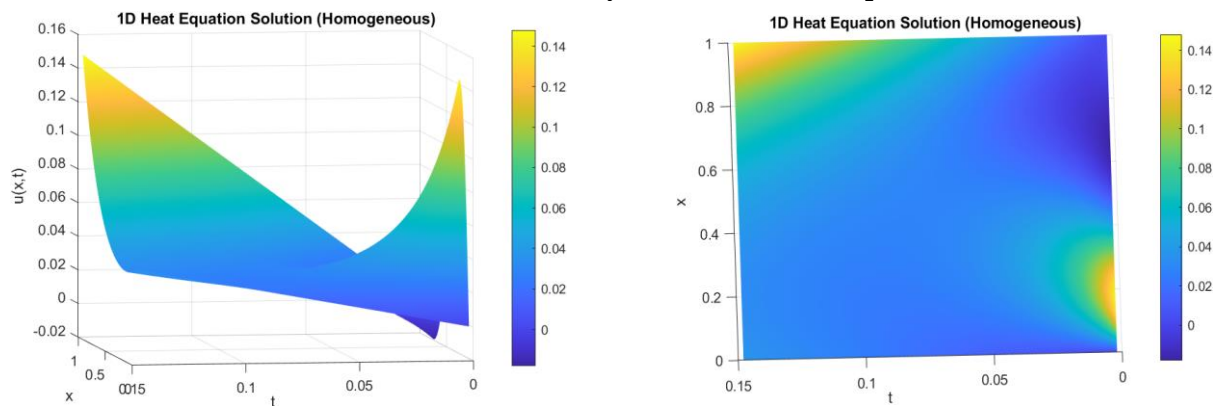
$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} , \qquad 0 < x < 1 \ , t > 0$$

Initial condition:

$$u(x,0) = 5x^{n_1}(1-x)^{n_3}\left(\frac{1}{2} - x\right)$$
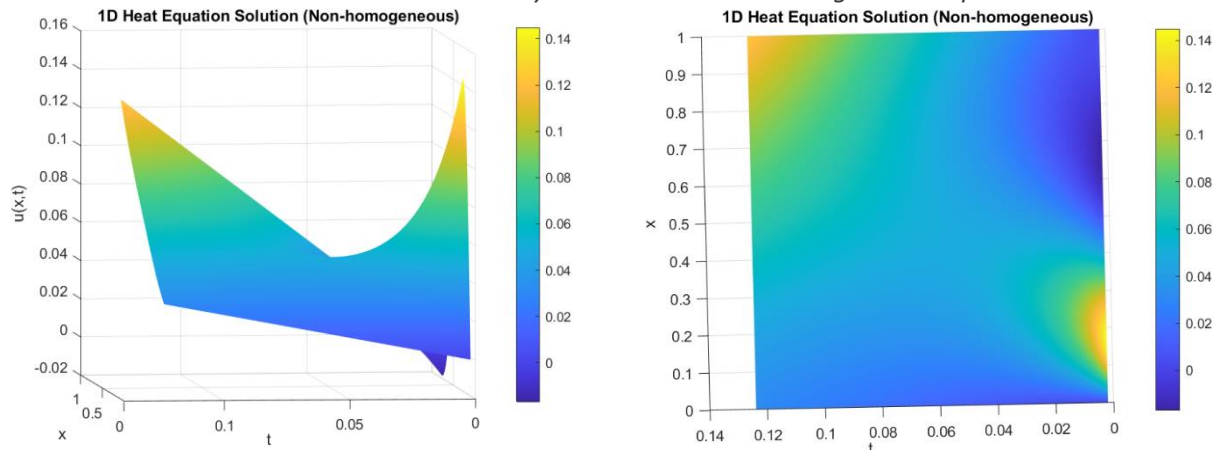
Boundary conditions:

$$u(0,t) = \frac{t}{1+n_4} , \qquad u(1,t) = \frac{t}{n_2} .$$

Part 2: Non-Homogeneous Heat Equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{n_5 x}{5} , \qquad 0 < x < 1 \ , t > 0$$

With the same initial condition and boundary conditions as the Homogeneous equation.

Time T, such that for all fixed $t > T$ the solution $u(x,t)$, as function of x, is monotonic increasing:

Homogeneous Solution T: $T = 0.0744 \, [s]$

Non-Homogeneous Solution T: $T = 0.0624 \, [s]$

Grid variables: h is the spatial step size and k is the time step size. A finer spatial grid and a smaller time step both improve accuracy but increase computational effort. (and r is for Euler scheme we used; it ensures the solution is stable and prevents numerical errors from growing and blow up (as it remains below the critical value of $r < 0.5$)).