PDE Project on Numeric Methods

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$\boldsymbol{n_1}$: 1 $\boldsymbol{n_2}$: 1 $\boldsymbol{n_3}$: 3 $\boldsymbol{n_4}$: 3 $\boldsymbol{n_5}$: 4

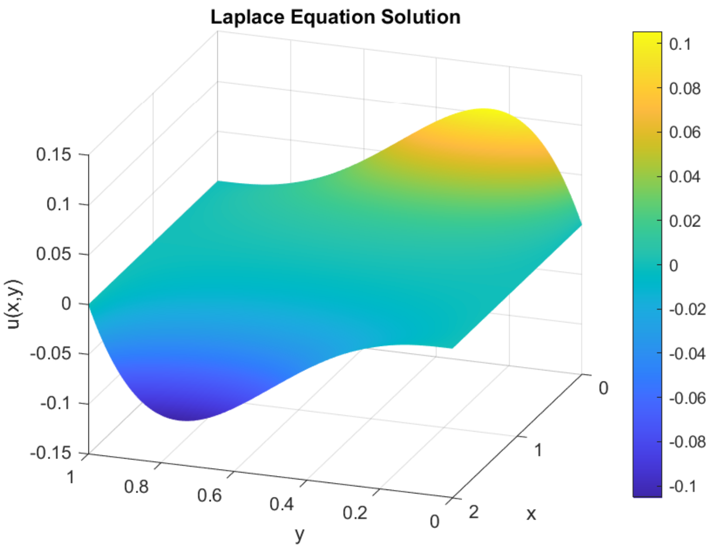
Project #1

Part 1: Laplace Equation

$$\frac{\partial^2 \mathrm{u}}{\partial \mathrm{x}^2} + \frac{\partial^2 \mathrm{u}}{\partial \mathrm{y}^2} = 0\,, \quad 0 < x < 2 \;\; , 0 < y < 1\,,$$

Boundary conditions:

$$u(x,0) = u(x,1) = 0\,, \qquad u(0,y) = y^{n_1}(1-y)^{n_3}\,, \qquad u(2,y) = -y^{n_4}(1-y)^{n_2}$$



The x's that $u(x,y)$ as a function of y have both maximum and minimum:

1. $\boldsymbol{X} = \mathbf{0.9900}$
2. $\boldsymbol{X} = \mathbf{1.0000}$
3. **X = 1.0100**
4. **X = 1.0200**

Values of h and k and why I chose them:

h = 0.01 k = 0.01

h and k are simply the grid spacings in the x and y directions. Smaller values mean a finer grid ,which typically gives more accurate finite difference approximations.

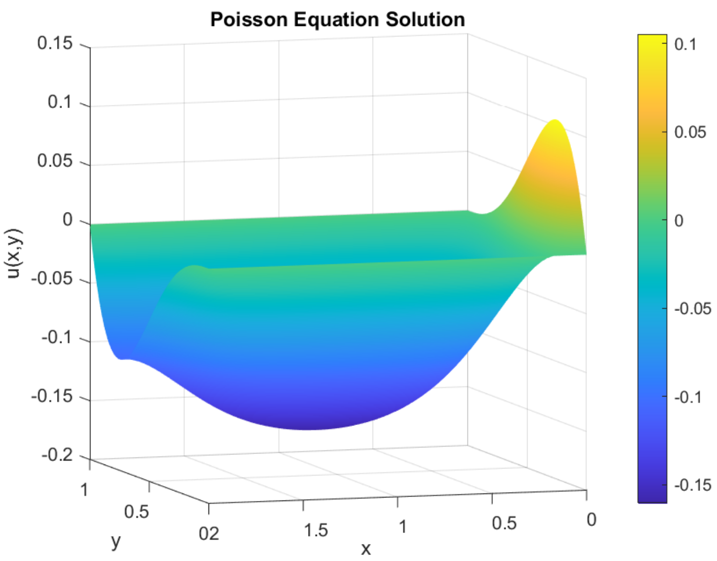
Part 2: Poisson Equation

$$\frac{\partial^2 \mathrm{u}}{\partial \mathrm{x}^2} + \frac{\partial^2 \mathrm{u}}{\partial \mathrm{y}^2} = 1 + \frac{\mathrm{n}_5}{10}\,,$$
$$0 < x < 2 \;\; , 0 < y < 1$$

With the same boundary conditions as the Laplace equation.

The axis values where the solution has its ***minimum*** value:

$\boldsymbol{u} = \mathbf{-0.1602}$
$\boldsymbol{x} = \mathbf{1.1400}$
$\boldsymbol{y} = \mathbf{0.5000}$



Project #2

$\boldsymbol{n_1}$: 1 $\boldsymbol{n_2}$: 1 $\boldsymbol{n_3}$: 3 $\boldsymbol{n_4}$: 3 $\boldsymbol{n_5}$: 4 | ***Grid variables:*** $\boldsymbol{h} = 0.01$ and $\boldsymbol{k} = r \times h^2$ with $\boldsymbol{r} = 0.4$, thus $\boldsymbol{k} = 4 \times 10^{-5}$

Part 1: Homogeneous Heat Equation

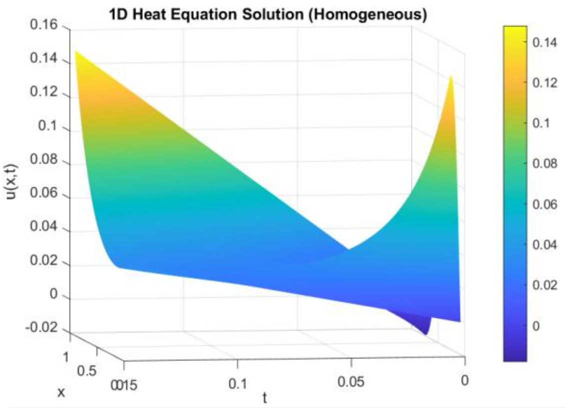
$$\frac{\partial \mathrm{u}}{\partial t} = \frac{\partial^2 \mathrm{u}}{\partial \mathrm{x}^2}, \qquad 0 < x < 1 \quad , t > 0$$

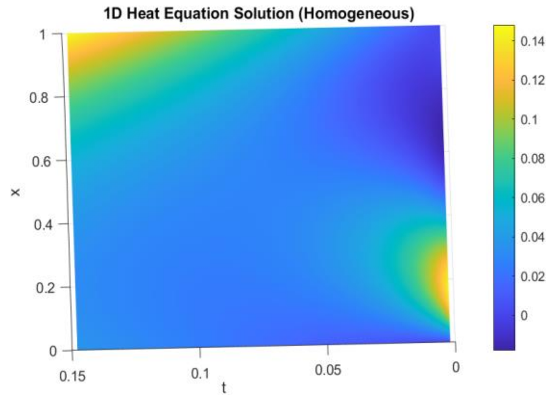
Initial condition:

$$u(x,0) = 5x^{n_1}(1-x)^{n_3}\left(\frac{1}{2} - x\right)$$

Boundary conditions:

$$u(0,t) = \frac{t}{1+n_4}\ , \qquad u(1,t) = \frac{t}{n_2}\ .$$

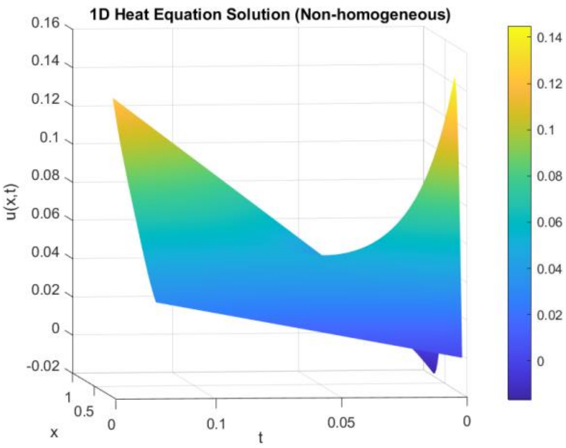


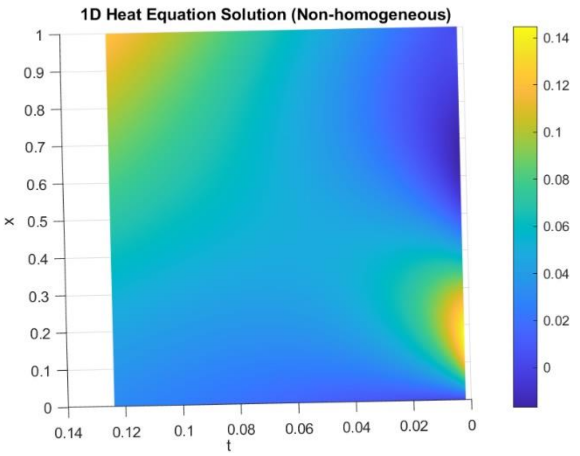


Part 2: Non-Homogeneous Heat Equation

$$\frac{\partial \mathrm{u}}{\partial t} = \frac{\partial^2 \mathrm{u}}{\partial \mathrm{x}^2} + \frac{n_5 x}{5}, \qquad 0 < x < 1 \quad , t > 0$$

With the same initial condition and boundary conditions as the Homogeneous equation.





Time T, such that for all fixed $t > T$ the solution $u(x,t)$, as function of x, is monotonic increasing:

Homogeneous Solution T: $\boldsymbol{T = 0.0744}\ [\boldsymbol{s}]$

Non-Homogeneous Solution T: $\boldsymbol{T = 0.0624}\ [\boldsymbol{s}]$

Grid variables: h is the spatial step size and k is the time step size. A finer spatial grid and a smaller time step both improve accuracy but increase computational effort. (and r is for Euler scheme we used; it ensures the solution is stable and prevents numerical errors from growing and blow up (as it remains below the critical value of $r < 0.5$)).