14,16,17,18,19.1,19.3

14.1) לא נכון, אם $A=\begin{pmatrix}1&0\\0&1\end{pmatrix}$ ו- $B=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$ אז $A+B=\begin{pmatrix}0&0\\0&0\end{pmatrix}$ לא הפיכה!

14.2) לא נכון, אם $A=\begin{pmatrix}4&-5\\0&0\end{pmatrix}$ ו- $B=\begin{pmatrix}5&0\\4&0\end{pmatrix}$ אז $AB=\begin{pmatrix}0&0\\0&0\end{pmatrix}$ וזה בסתירה לטענה.

14.3) לא נכון, נניח ש- $k\neq 0$ אמרת נתת את מי הנפח ומיד לא הפיכה.

14.4) A סימטרית לכן $A=A^t$

$$A=A^t \Rightarrow (A)^{-1}=(A^t)^{-1}=(A^{-1})^t \Rightarrow (A)^{-1}=(A^{-1})^t \Big\} \Rightarrow A$$

$$A\cdot A^{-1}=I \Rightarrow A^t\cdot A^{-1}=I$$

14.5) לא נכון. נניח $A=\begin{pmatrix}1&0&0\\1&1&0\\1&1&1\end{pmatrix}$ אזי $A^t=\begin{pmatrix}1&1&1\\0&1&1\\0&0&1\end{pmatrix}$

תמן: $A^t\cdot A=\begin{pmatrix}3&2&1\\2&1&1\\1&1&1\end{pmatrix}$

14.7) $A^2-AB+2I=0 \Rightarrow A^2-AB=-2I \Rightarrow A(A-B)=-2I$

$\Rightarrow -\frac{1}{2}A(A-B)=I$

הוכחנו ש- $(A-B)$ הוא מטריצה הפיכה כי הכפלה שלה עם $-\frac{1}{2}A$ מביאה את I.

16) $A=\begin{pmatrix}1&-2\\2&-4\end{pmatrix}\approx\begin{pmatrix}1&-2\\0&0\end{pmatrix} \Rightarrow$ לא הפיכה!

$$B=\begin{pmatrix}2&0&0\\-1&4&0\\3&-3&5\end{pmatrix}\approx\begin{pmatrix}1&0&0\\0&1&0\\3&-3&5\end{pmatrix}\approx\begin{pmatrix}1&0&0\\0&1&0\\0&-3&5\end{pmatrix}\approx\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} \Rightarrow$$ הפיכה!

$$C=\begin{pmatrix}1&3&1\\2&2&4\\3&1&8\end{pmatrix}\approx\begin{pmatrix}1&3&1\\0&-4&2\\0&-8&5\end{pmatrix}\approx\begin{pmatrix}1&3&1\\0&-4&2\\0&0&1\end{pmatrix}\approx\begin{pmatrix}1&3&0\\0&-4&0\\0&0&1\end{pmatrix}\approx\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} \Rightarrow$$ הפיכה!

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17)

$$A=\begin{pmatrix}1&-1&2\\2&1&0\\-1&0&1\end{pmatrix}\ \begin{matrix}R_2-2R_1\to R_2\\R_3+R_1\to R_3\end{matrix}\ \left(\begin{array}{ccc|ccc}1&-1&2&1&0&0\\0&3&-3&-2&1&0\\0&-1&3&1&0&1\end{array}\right)\ \begin{matrix}R_2/3\to R_2\\R_3/(-1)\to R_3\end{matrix}\ \left(\begin{array}{ccc|ccc}1&-1&2&1&0&0\\0&1&-1&-2/3&1/3&0\\0&1&-3&-1&0&1\end{array}\right)\ R_3-R_2\to R_3$$

$$\left(\begin{array}{ccc|ccc}1&-1&2&1&0&0\\0&1&-1&-2/3&1/3&0\\0&0&-2&-1/3&-1/3&-1\end{array}\right)\ R_3/-2\to R_3\ \left(\begin{array}{ccc|ccc}1&-1&2&1&0&0\\0&1&-1&-2/3&1/3&0\\0&0&1&1/6&1/6&1/2\end{array}\right)\ \begin{matrix}R_1-2R_3\to R_1\\R_2+R_3\to R_2\end{matrix}\ \left(\begin{array}{ccc|ccc}1&-1&0&4/3&-1/3&-1\\0&1&0&-1/2&1/2&1/2\\0&0&1&1/6&1/6&1/2\end{array}\right)\ R_1+R_2\to R_1$$

$$\left(\begin{array}{ccc|ccc}1&0&0&1/6&1/6&-1/2\\0&1&0&-1/2&1/2&1/2\\0&0&1&1/6&1/6&1/2\end{array}\right)\qquad A^{-1}=\begin{pmatrix}1/6&1/6&-1/2\\-1/2&1/2&1/2\\1/6&1/6&1/2\end{pmatrix}$$

$-\frac{3}{2}+\frac{2}{3}$

$\frac{-9+4}{6}=-\frac{5}{6}$

$$B=\left(\begin{array}{ccc|ccc}-1&1&-2&1&0&0\\3&-3&4&0&1&0\\2&1&1&0&0&1\end{array}\right)\ \begin{matrix}R_2+3R_1\to R_2\\R_3+2R_1\to R_3\end{matrix}\ \left(\begin{array}{ccc|ccc}-1&1&-2&1&0&0\\0&0&-2&3&1&0\\0&3&-3&2&0&1\end{array}\right)\ \begin{matrix}R_2/:(-2)\to R_2\\R_3/3\to R_3\end{matrix}\ \left(\begin{array}{ccc|ccc}-1&1&-2&1&0&0\\0&0&1&-3/2&-1/2&0\\0&1&-1&2/3&0&1/3\end{array}\right)$$

$$\begin{matrix}R_2\leftrightarrow R_3\\R_3+R_3\to R_2\end{matrix}\ \left(\begin{array}{ccc|ccc}-1&1&-2&1&0&0\\0&1&0&-5/6&1/2&1/3\\0&0&1&-3/2&-1/2&0\end{array}\right)\ R_1+2R_3\to R_1\ \left(\begin{array}{ccc|ccc}-1&1&0&-2&-1&0\\0&1&0&-5/6&-1/2&1/3\\0&0&1&-3/2&-1/2&0\end{array}\right)\ \begin{matrix}\cancel{R_2\leftrightarrow R_3}\\R_1\cdot(-1)\to R_1\end{matrix}\ \left(\begin{array}{ccc|ccc}1&-1&0&2&1&0\\0&1&0&-5/6&-1/2&1/3\\0&0&1&-3/2&-1/2&0\end{array}\right)$$

$$R_1+R_2\to R_1\ \left(\begin{array}{ccc|ccc}1&0&0&1/6&1/2&1/3\\0&1&0&-5/6&-1/2&1/3\\0&0&1&-3/2&-1/2&0\end{array}\right)\qquad B'=\begin{pmatrix}1/6&1/2&1/3\\-5/6&-1/2&1/3\\-3/2&-1/2&0\end{pmatrix}$$

$$C=\left(\begin{array}{ccc|ccc}0.01&-0.01&0.02&1&0&0\\0.02&0.01&0.01&0&1&0\\-0.01&0&0.01&0&0&1\end{array}\right)\ \begin{matrix}R_2-2R_1\to R_2\\R_3+R_1\to R_3\end{matrix}\ \left(\begin{array}{ccc|ccc}0.01&-0.01&0.02&1&0&0\\0&0.03&-0.03&-2&1&0\\0&-0.01&0.03&1&0&1\end{array}\right)\ R_3+R_2\to R_3$$

$$\left(\begin{array}{ccc|ccc}0.01&-0.01&0.02&1&0&0\\0&0.03&-0.03&-2&1&0\\0&0.02&0&-1&0&1\end{array}\right)\ \cdot 100\ \Rightarrow\ \left(\begin{array}{ccc|ccc}1&-1&2&100&0&0\\0&3&-3&-200&100&0\\0&2&0&-100&0&100\end{array}\right)\ \begin{matrix}R_2:3\to R_2\\R_3:2\to R_3\end{matrix}$$

$$\left(\begin{array}{ccc|ccc}1&-1&2&100&0&0\\0&1&-1&-200/3&100/3&0\\0&1&0&-50&0&50\end{array}\right)\ R_2\leftrightarrow R_3\ \left(\begin{array}{ccc|ccc}1&-1&2&100&0&0\\0&1&0&-50&0&50\\0&1&-1&-200/3&100/3&0\end{array}\right)\ \begin{matrix}\cancel{R_2\leftrightarrow R_3}\\R_3-R_2\to R_3\end{matrix}\ \left(\begin{array}{ccc|ccc}1&-1&2&100&0&0\\0&1&0&-50&0&50\\0&0&-1&-50/3&100/3&-50\end{array}\right)$$

$$\begin{matrix}R_1+2R_3\to R_1\\R_3\cdot(-1)\to R_3\end{matrix}\ \left(\begin{array}{ccc|ccc}1&-1&0&100/3&200/3&-100\\0&1&0&-50&0&50\\0&0&1&50/3&-100/3&50\end{array}\right)\ R_1+R_2\to R_1\ \left(\begin{array}{ccc|ccc}1&0&0&-50/3&100/3&\\0&1&0&&&\\0&0&1&&&\end{array}\right)\ \left(\begin{array}{ccc|ccc}1&-1&2&1&0&0\\0&1&-1&-2/3&1/3&0\\0&1&0&-1/2&1/2&1/2\end{array}\right)\ R_2\leftrightarrow R_3$$

$$\left(\begin{array}{ccc|ccc}1&-1&2&1&0&0\\0&1&0&-1/2&1/2&1/2\\0&1&-1&-2/3&1/3&0\end{array}\right)\ -(R_3+R_2)\to R_3\ \left(\begin{array}{ccc|ccc}1&-1&2&1&0&0\\0&1&0&-1/2&1/2&1/2\\0&0&1&1/6&1/6&1/2\end{array}\right)\ R_1-2R_3\to R_1\ \left(\begin{array}{ccc|ccc}1&-1&0&2/3&-1/3&-1\\0&1&0&-1/2&1/2&1/2\\0&0&1&1/6&1/6&1/2\end{array}\right)\ R_1+R_2\to R_1$$

$$\left(\begin{array}{ccc|ccc}1&0&0&1/6&1/6&1/2\\0&1&0&-1/2&1/2&1/2\\0&0&1&1/6&1/6&1/2\end{array}\right)\qquad C^{-1}=100\cdot\begin{pmatrix}1/6&1/6&1/2\\-1/2&1/2&1/2\\1/6&1/6&1/2\end{pmatrix}$$

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$$D = \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 0 & 0 & 1 \end{array}\right) \quad \begin{array}{l} R_1: 2 \to R_1 \\ R_2: 3 \to R_2 \\ R_3: 4 \to R_3 \end{array} \quad \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/4 \end{array}\right) \quad D^{-1} = \begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{pmatrix}$$

$$(A^t)^{-1} = (A^{-1})^t \qquad (A^t)^{-1} = \begin{pmatrix} 1/6 & -1/2 & 1/6 \\ 1/6 & 1/2 & 1/6 \\ -1/2 & 1/2 & 1/2 \end{pmatrix}$$

$$(AB)^{-1} = B^{-1} \cdot A^{-1}$$

$$= \begin{pmatrix} 0 & 1/2 & -1/6 \\ 1/6 & -1/3 & 1/3 \\ 0 & -1/2 & 1/2 \end{pmatrix}$$

$$\cancel{1/6 \cdot 1/6 + 1/2 \cdot 1/6 - 1/2 \cdot 1/3 = \frac{7}{36} + \frac{3}{36} - \frac{6}{36}} = 1/6 \cdot 1/6 - 1/2 \cdot 1/2 + 1/3 \cdot 1/6 = 1/36 - 9/36 + 2/36 = 0$$

$$\cancel{1/6 \cdot 1/6 + 1/6 \cdot 1/2 + 1/6 \cdot 1/3 = \frac{7}{36} + \frac{3}{36} + \frac{2}{36}} \quad 1/6 \cdot 1/6 + 1/2 \cdot 1/2 + 1/3 \cdot 1/6 = 1/36 + 9/36 + 2/36 = 1/2$$

$$-5/6 \cdot 1/6 + 1/2 \cdot 1/2 + 1/3 \cdot 1/6 = -5/36 + 9/36 + 2/36 = 1/6 \quad 1/6 \cdot -1/2 + 1/2 \cdot 1/2 + 1/3 \cdot 1/2 = -\frac{1}{12} + \frac{3}{12} + \frac{2}{12} = -\frac{1}{6}$$

$$-5/6 \cdot 1/6 - 1/2 \cdot 1/2 + 1/3 \cdot 1/6 = -5/36 - 9/36 + 2/36 = -1/3 \quad -5/6 \cdot -1/2 - 1/2 \cdot 1/2 + 1/3 \cdot 1/2 = \frac{5}{12} - \frac{3}{12} + \frac{2}{12} = \frac{1}{3}$$

$$-3/2 \cdot 1/6 - 1/2 \cdot -1/2 = -\frac{3}{12} + \frac{3}{12} = 0 \quad -3/2 \cdot \frac{1}{6} - \frac{1}{2} \cdot \frac{1}{2} = -\frac{3}{12} - \frac{3}{12} = -\frac{1}{2} \quad -3/2 \cdot -1/2 - 1/2 \cdot 1/2 = \frac{3}{4} - \frac{1}{4}$$

$$(2B^{-1}A^tB)^{-1} \qquad 2B^{-1} = \begin{pmatrix} 2/3 & 1 & 2/3 \\ -5/3 & -1 & 2/3 \\ -3 & -1 & 0 \end{pmatrix} \quad A^t = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \quad (2B^{-1}) \cdot A =$$

18) $$\left(\begin{array}{ccc|ccc} 2 & k & -1 & 1 & 0 & 0 \\ 1 & 0 & -3 & 0 & 1 & 0 \\ 1 & 2 & k & 0 & 0 & 1 \end{array}\right) \approx \begin{pmatrix} 1 & 0 & -3 \\ 2 & k & -1 \\ 1 & 2 & k \end{pmatrix} \approx \begin{pmatrix} 1 & 0 & -3 \\ 0 & k & 5 \\ 0 & 2 & k+3 \end{pmatrix} \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & \frac{k+3}{2} \\ 0 & k & 5 \end{pmatrix} \approx \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & \frac{k+3}{2} \\ 0 & 0 & 5 - \frac{k(k+3)}{2} \end{pmatrix}$$

$$10 - k^2 - 3k \to k^2 + 3k - 10 \neq 0 \qquad k_{1,2} = \frac{-3 \pm \sqrt{9+40}}{2} \quad \begin{cases} \frac{-3+7}{2} = 2 \\ \frac{-3-7}{2} = -\frac{10}{2} = -5 \end{cases}$$

$$\boxed{k \neq 2, -5}$$

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19.1

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{pmatrix} \quad \left(\begin{array}{ccc|ccc} 2 & 1 & -1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 5 & 2 & -3 & 0 & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 1/2 & -1/2 & 1/2 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 5 & 2 & -3 & 0 & 0 & 1 \end{array}\right) \quad \left(\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 1/2 & 0 & 1/2 & 0 \\ 0 & -1/2 & 1/2 & -5/2 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 3/4 & -5/2 & 1/4 & 1 \end{array}\right) \quad \left(\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & -10/3 & 1/3 & 4/3 \end{array}\right) \quad \left(\begin{array}{ccc|ccc} 1 & 1/2 & 0 & 11/6 & -1/6 & -2/3 \\ 0 & 1 & 0 & 5/3 & 1/3 & -2/3 \\ 0 & 0 & 1 & -10/3 & 1/3 & 4/3 \end{array}\right) \quad \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1/3 & -1/3 \\ 0 & 1 & 0 & 5/3 & 1/3 & -2/3 \\ 0 & 0 & 1 & -10/3 & 1/3 & 4/3 \end{array}\right)$$

$$1 - \frac{2}{3} + \frac{1}{3} = \frac{2}{3}$$

$$\underline{x} = A^{-1} \cdot \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} =$$

19.3)

$$\left(\begin{array}{ccc|ccc} 1 & i & 0 & 1 & 0 & 0 \\ -i & 0 & 1 & 0 & 1 & 0 \\ 0 & (1+i) & (1-i) & 0 & 0 & 1 \end{array}\right) \quad R_2 + R_1 \cdot i \to R_2 \quad \left(\begin{array}{ccc|ccc} 1 & i & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & i & 1 & 0 \\ 0 & (1+i) & (1-i) & 0 & 0 & 1 \end{array}\right) \quad R_3 + R_2 \to R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & i & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & i & 1 & 0 \\ 0 & i & (2-i) & i & 1 & 1 \end{array}\right) \quad R_3 + iR_2 \quad \left(\begin{array}{ccc|ccc} 1 & i & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & i & 1 & 0 \\ 0 & 0 & 2 & i-1 & 1+i & 1 \end{array}\right) \quad \begin{array}{l} R_2 \cdot (-1) \to R_2 \\ R_3 : 2 \to R_3 \end{array} \quad \left(\begin{array}{ccc|ccc} 1 & +i & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -i & -1 & 0 \\ 0 & 0 & 1 & \frac{i-1}{2} & \frac{1+i}{2} & \frac{1}{2} \end{array}\right)$$

$$R_2 + R_3 \to R_2 \quad \left(\begin{array}{ccc|ccc} 1 & i & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{-1-i}{2} & \frac{i-1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & \frac{i-1}{2} & \frac{1+i}{2} & \frac{1}{2} \end{array}\right) \quad R_1 - iR_2 \quad \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1+i}{2} & \frac{1+i}{2} & -\frac{1}{2}i \\ 0 & 1 & 0 & \frac{-1-i}{2} & \frac{i-1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & \frac{i-1}{2} & \frac{1+i}{2} & \frac{1}{2} \end{array}\right)$$

$$1 - \left(\frac{-i+1}{2}\right) \quad \frac{1-i}{2}$$

$$-\left(\frac{-1-i}{2}\right) = \frac{1+i}{2}$$

$$\begin{pmatrix} \frac{1+i}{2} & \frac{1+i}{2} & -\frac{1}{2}i \\ \frac{-i-1}{2} & \frac{i-1}{2} & \frac{1}{2} \\ \frac{i-1}{2} & \frac{i+1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{-i}{2} \\ \frac{1+2i}{2} \\ \frac{3}{2} \end{pmatrix}$$

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