OED TASK

Filipa Costa, 92626; Maximilian Mandelt, 101475 (Group 4)

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1 Question 1 - Linearize the model and compute the regressor vector

Let $(\theta_{1_0}, \theta_{2_0}, \theta_{3_0}) = (1, 1/2, 1)$ and $X = [0, 1]$ and let $y(x) = \theta_3 \left[\exp(-\theta_2 x) - \exp(-\theta_1 x)\right] + \varepsilon(x)$, $\varepsilon \sim \mathcal{N}\left(0, \sigma^2\right)$

$$\left(\frac{\partial y(x)}{\partial \theta_1}\right)_{(\theta_{1_0}, \theta_{2_0}, \theta_{3_0})} = \left(\theta_3 x e^{-\theta_1 x}\right)_{(\theta_{1_0}, \theta_{2_0}, \theta_{3_0})} = x e^{-x}$$
$$\left(\frac{\partial y(x)}{\partial \theta_2}\right)_{(\theta_{1_0}, \theta_{2_0}, \theta_{3_0})} = \left(-\theta_3 x e^{-\theta_2 x}\right)_{(\theta_{1_0}, \theta_{2_0}, \theta_{3_0})} = x e^{-\frac{1}{2}x}$$
$$\left(\frac{\partial y(x)}{\partial \theta_3}\right)_{(\theta_{1_0}, \theta_{2_0}, \theta_{3_0})} = \left(e^{-\theta_2 x} - e^{-\theta_1 x}\right)_{(\theta_{1_0}, \theta_{2_0}, \theta_{3_0})} = e^{-\frac{1}{2}x} - e^{-x}$$

Regressor Vector: $f(x) = \left(x e^{-x}, x e^{-\frac{1}{2}x}, e^{-\frac{1}{2}x} - e^{-x}\right)^t$

2 Question 2 - Compute the information matrix for a one single point design

The information matrix for the single point $x = 1/2$ is:

$$M(x) = f(x) f(x)^t = \begin{pmatrix} 0.09196986 & -0.11809164 & 0.05224356 \\ -0.11809164 & 0.15163266 & -0.06708205 \\ 0.05224356 & -0.06708205 & 0.02967700 \end{pmatrix}$$

3 Question 3 - Compute the D-optimal approximate design and check GET

Using the REX computational method routine in R,

$\xi^* = \left\{ \begin{array}{ccc} 0.23 & 0.67 & 1.00 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array} \right\}$, was obtained. Using GET, it is proven that it is the D-optimum.

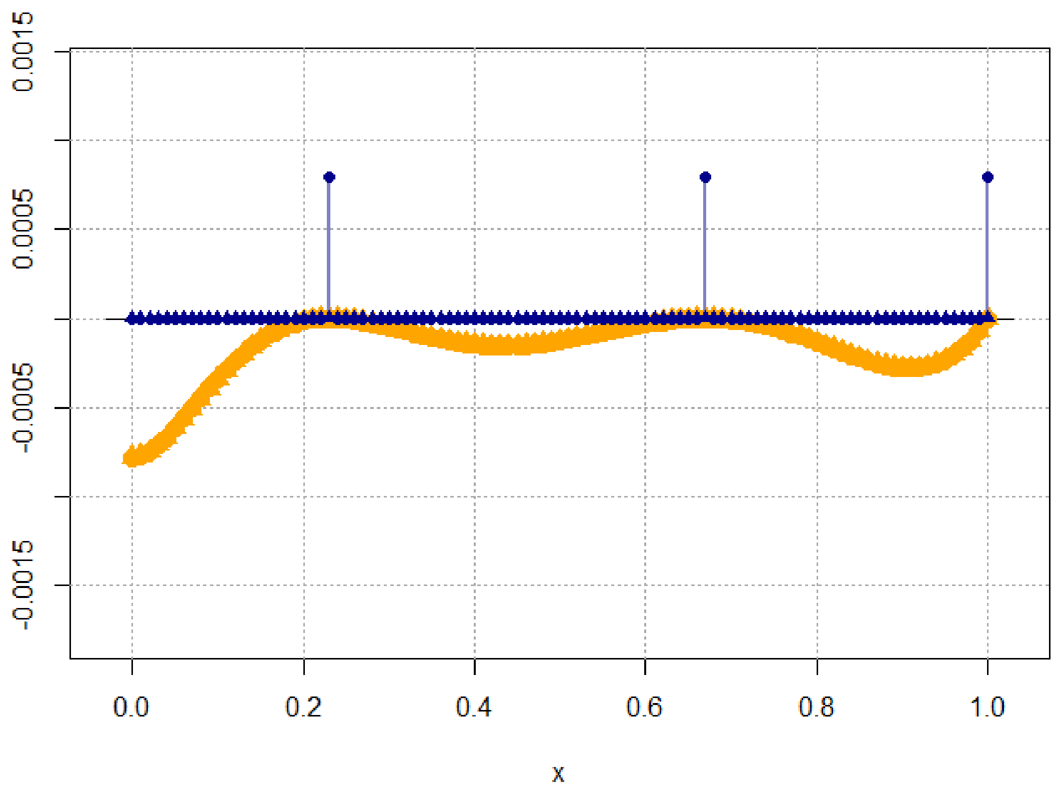

Figure 1: The GET plot of the design

4 Question 4 - Compute the saturated D-optimal exact design

The saturated design has three design points, since three parameters are used in the model. Using the KL computational method routine in R,

$\xi^* = \left\{ \begin{array}{ccc} 0.23 & 0.67 & 1.00 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array} \right\}$ was obtained.