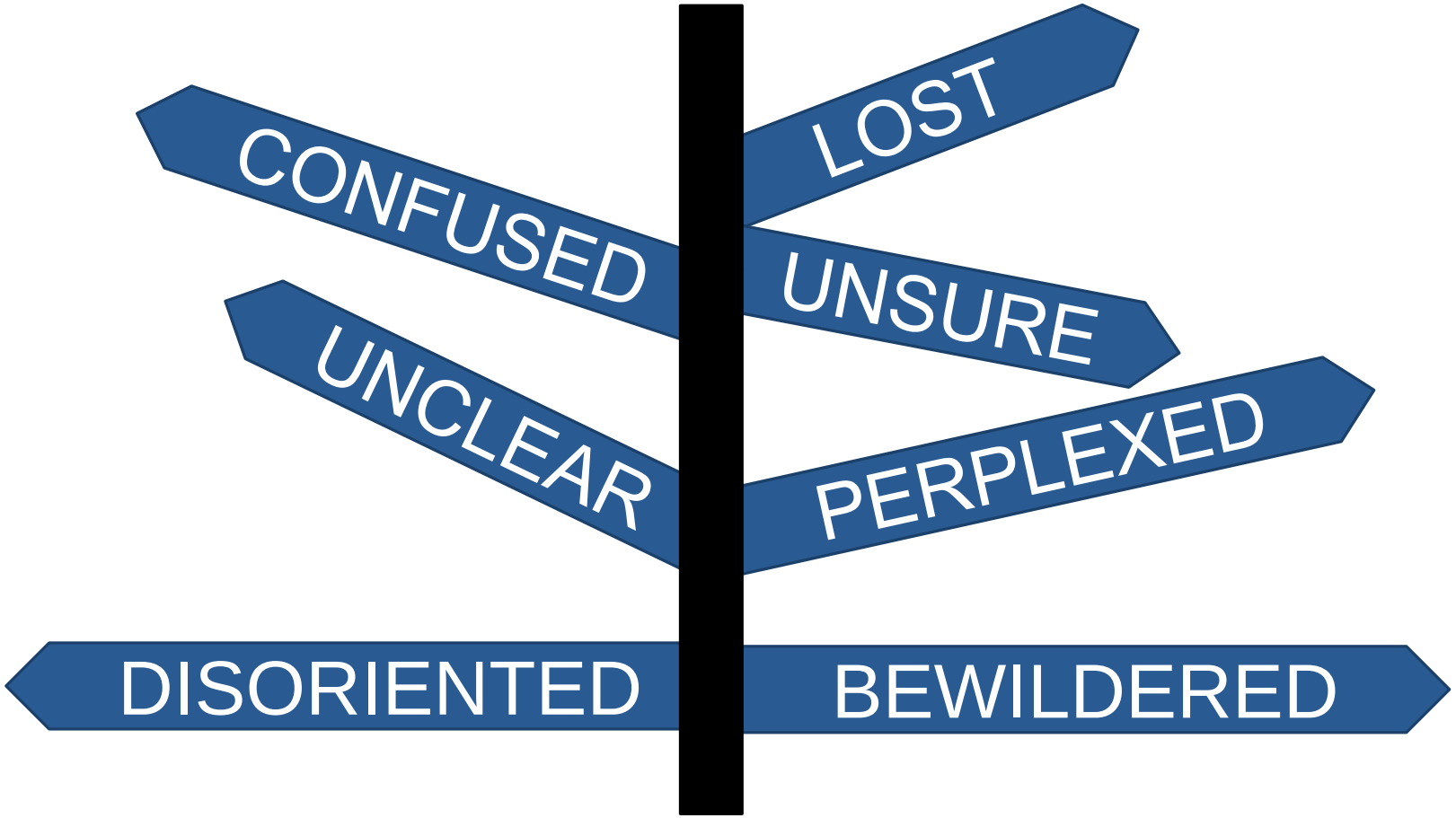
Stochastic Thinking and Random Walks, Segment 1

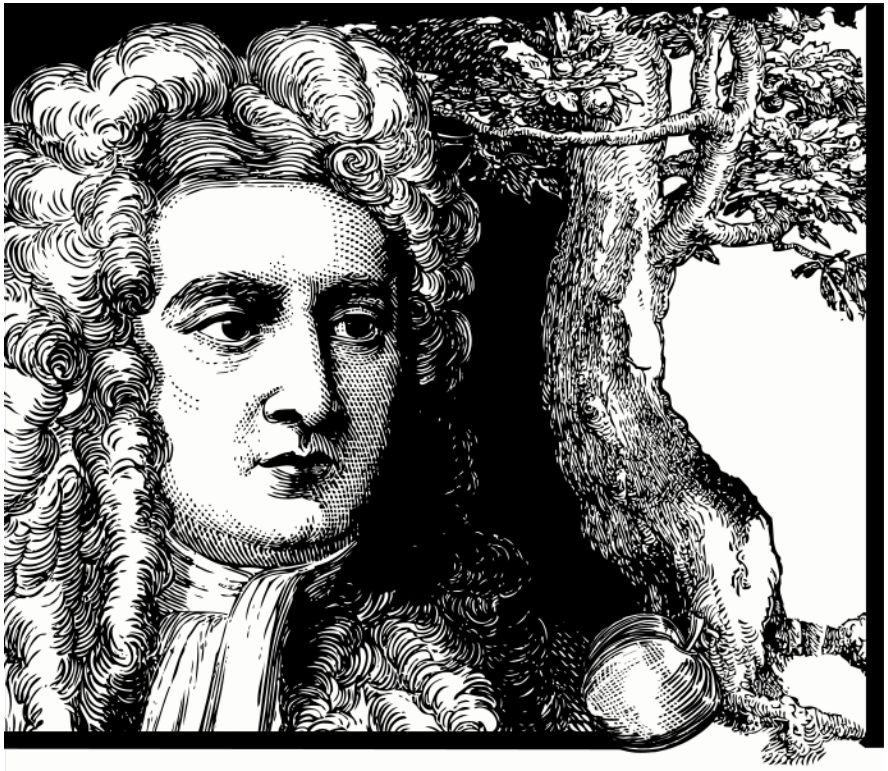
The World is Hard to Understand

- Uncertainty is uncomfortable
- But certainty is usually unjustified



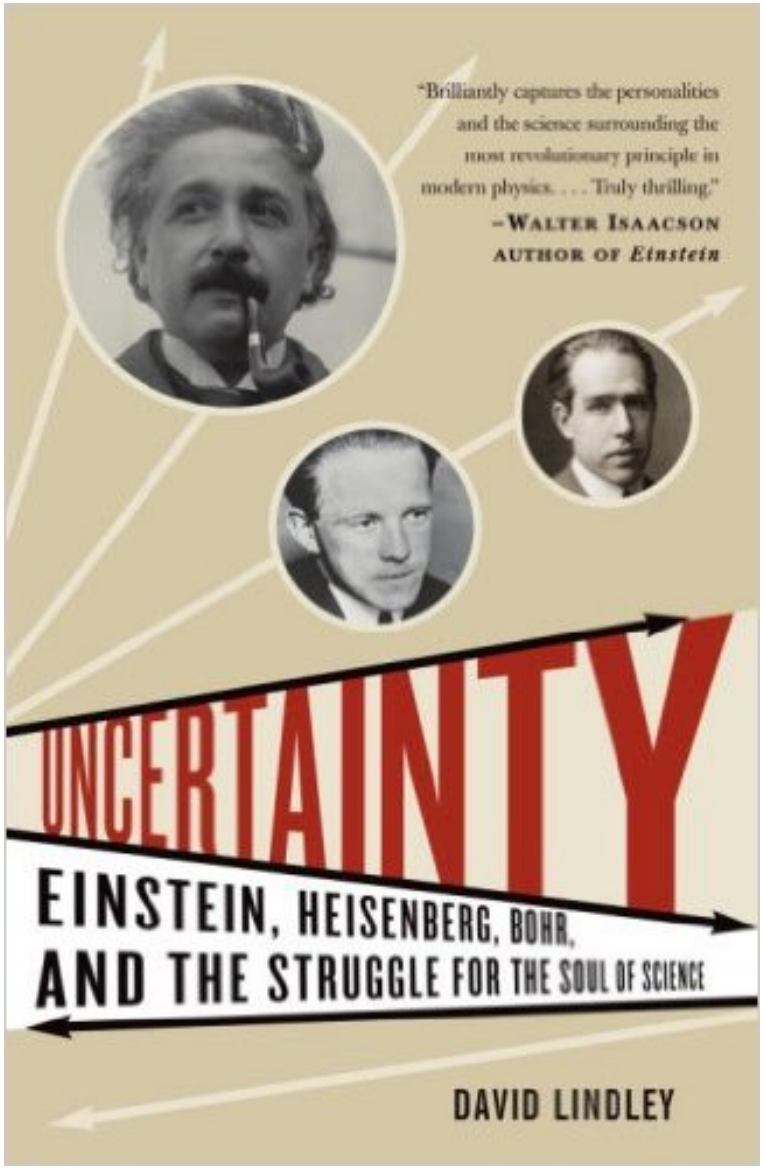
Newtonian Mechanics

- Every effect has a cause
- The world can be understood causally



1643 - 1727

Two Centuries Years Later



Copenhagen Doctrine

- Copenhagen Doctrine (Bohr and Heisenberg) of causal nondeterminism
  - At its most fundamental level, the behavior of the physical world cannot be predicted.
  - Fine to make statements of the form “x is highly likely to
  - occur,” but not of the form “x is certain to occur.”
- Einstein and Schrödinger objected
  - “God does not play dice.” -- Albert Einstein

Does It Really Matter?

- The world may or may not be inherently unpredictable
- But our lack of knowledge does not allow us to make accurate predictions
- Therefore we might as well treat the world as inherently unpredictable
- Predictive nondeterminism

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Stochastic Processes

- An ongoing process where the next state might depend on both the previous states and some random element

```
def rollDie():
    """ returns an int between 1 and 6"""

def rollDie():
    """ returns a randomly chosen int
        between 1 and 6"""
```

Specifications and Implementations

```
def squareRoot(x, epsilon):
    """Assumes x and epsilon are of type float
                x >= 0 and epsilon > 0
    Returns float y such that
            x-epsilon <= y*y <= x+epsilon"""
```

- Specification allows but does not require, a nondeterministic implementation
- Can be tricky when debugging a program that uses it

Implementing a Random Process

```
import random

def rollDie():
    """returns a random int between 1 and 6"""
    return random.choice([1,2,3,4,5,6])

def testRoll(n = 10):
    result = ''
    for i in range(n):
        result = result + str(rollDie())
    print(result)
```

For the following explanations of different types of programmatic models, fill in the blank with the appropriate model the definition describes.

1. A _____ model is one whose behavior is entirely predictable. Every set of variable states is uniquely determined by parameters in the model and by sets of previous states of these variables. Therefore, these models perform the same way for a given set of initial conditions, and it is possible to predict precisely what will happen.

   deterministic ✔

2. A _____ model is one in which randomness is present, and variable states are not described by unique values, but rather by probability distributions. The behavior of this model cannot be entirely predicted.

   stochastic ✔

3. A _____ model does not account for the element of time. In this type of model, a simulation will give us a snapshot at a single point in time.

   static ✔

4. A _____ model does account for the element of time. This type of model often contains state variables that change over time.

   dynamic ✔

5. A _____ model does not take into account the function of time. The state variables change only at a countable number of points in time, abruptly from one state to another.

   discrete ✔

6. A _____ model does take into account the function of time, typically by modelling a function f(t) and the changes reflected over time intervals. The state variables change in an unbroken way through an infinite number of states.

   continuous ✔

If you are using differential equations to model a simulation, are you more likely to be doing a discrete or continuous model?

- ○ Discrete
- ◉ Continuous

✔