BDC5101

Deterministic Operations Research Models

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The Retailer Game

http://retailer2.net/

Selling Seasonal Products

- **Challenges in selling fashion products**
  - **Short selling season**
    - *About several weeks (and it is getting shorter and shorter)*
    - *Obsolescence after the season*
  - **Volatile market conditions**
    - *Difficult to predict the trend in advance*
    - *Erratic demand: “hit” items sell out quickly while “dogs” sell slowly*
    - *Lack of experience or historical data*
  - **Rigid supply chains**
    - *Long procurement lead time (several months) due to out-sourcing, etc.*
    - *Product line and quantity commitment well in advance of the selling season*
- **Examples of seasonal products**
  - **Apparel, IT Products**

Markdown

- **Inventory is determined before the selling season begins**
- **Once the season begins, the only thing we can control is <u>price</u>**

- **Markdown: dynamically *reduce* price to boost demand and increase revenue**

"Retailers hate markdowns. Discount an item too late, and stores are stuck with truckloads of inventory. Too early, and they loose profits as people snap up items thrown on the bargain table prematurely …"

Wall Street Journal, Aug. 7, 2001

The Retailer Game

- **A stock of <u>2000</u> units of a single fashion item, no replenishment**
- **Demand is hard to predict, limited historical data available**
- **Pricing**
  - **Four allowable prices: <u>$60</u> (full price), <u>$54</u> (90%), <u>$48</u> (80%), <u>$36</u> (60%)**
  - **Start with $60 in Week 1, then mark down over time (no price increase)**
- **<u>15-week</u> selling season**
  - **Left over items are sold to discounters for <u>$25</u> per unit (salvage value)**
- **Costs are paid (hence they are *sunk*)**
- **GOAL: maximize revenue by determining the *timing* and *magnitude* of markdowns**

The Retailer Game

- **Two key steps**

  - **Estimation: build the demand model**
    - *Study the historical data*
    - *Choose a demand model and estimate parameters*

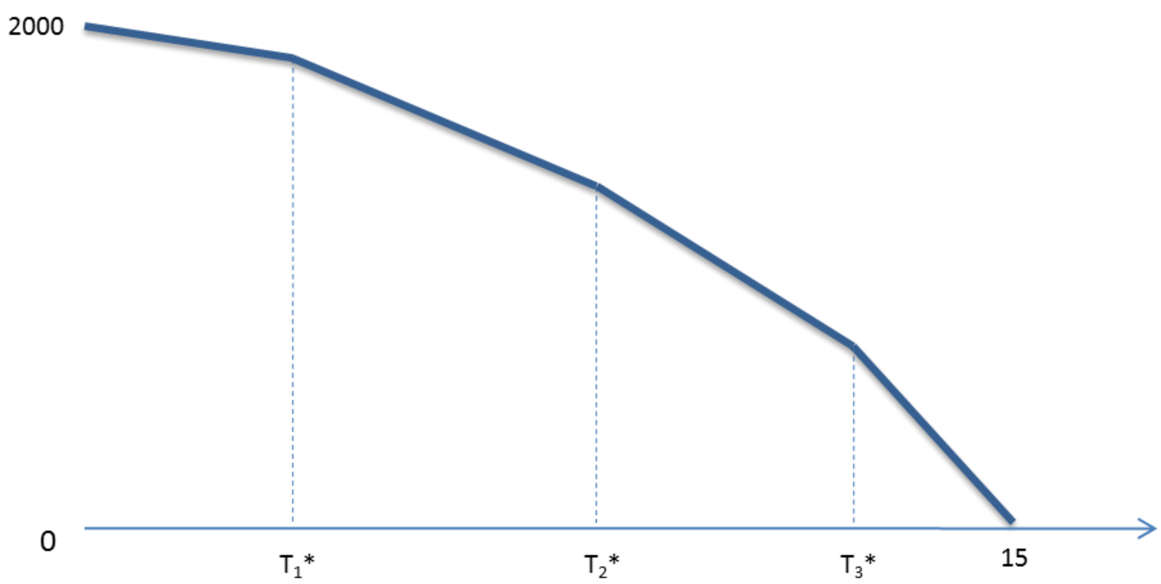
  - **Optimize prices**

Data $\Rightarrow$ Information $\Leftrightarrow$ Model $\Rightarrow$ Prescription

Optimization Step

- **Suppose we have an estimate of demands at the four price points:**

Price	$60	$54	$48	$36
Demand	125	162.5	217.5	348.8



Linear Programming Formulation

- **Decision variables:**

  x_1: **# of *effective* weeks the item is sold at price $60**

  x_2: **# of *effective* weeks the item is sold at price $54**

  x_3: **# of *effective* weeks the item is sold at price $48**

  x_4: **# of *effective* weeks the item is sold at price $36**

- **Objective function:**

  Revenue = Sales Revenue + Salvage Revenue

$$125 * 60 * x_1 + 162.5 * 54 * x_2 + 217.5 * 48 * x_3 + 348.8 * 36 * x_4$$
$$+ 25*(2000 - 125 * x_1 - 162.5 * x_2 - 217.5 * x_3 - 348.8 * x_4)$$

Linear Programming Formulation

- **Constraints:**
  - **Capacity constraint: sales ≤ inventory**

$$125 * x_1 + 162.5 * x_2 + 217.5 * x_3 + 348.8 * x_4 \leq 2000$$

  - **Time constraint: total effective weeks at all prices ≤ the selling season**

$$x_1 + x_2 + x_3 + x_4 \leq 15$$

  - **Full price in week 1:**

$$x_1 \geq 1$$

  - **Non-negativity:** $x_1, x_2, x_3, x_4 \geq 0$

Sensitivity Analysis

- **Effects of initial inventory**
- **Effects of selling horizon**
- **Effects of good market / bad market**

Improving the Markdown Strategy

- **Dynamic adjustment**
- **More discount levels**
- **Consider future uncertainties**

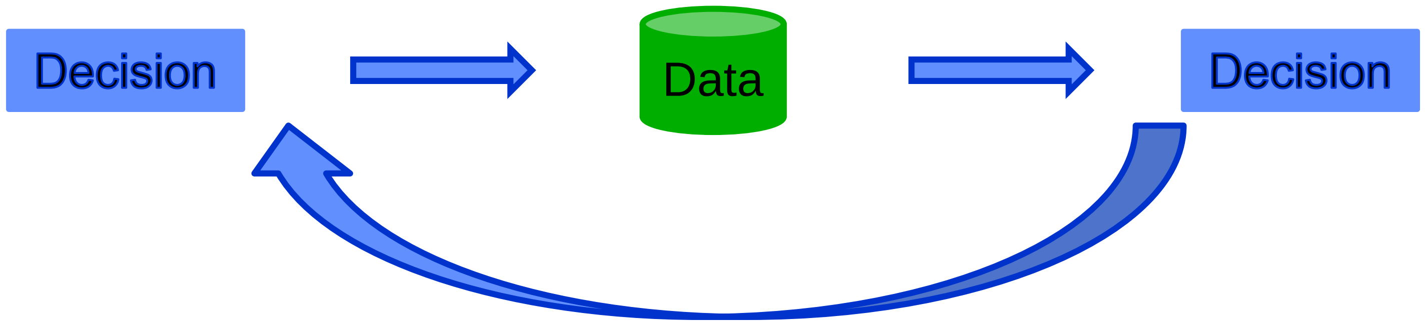
Uncertain Linear Program

Incomplete Knowledge of Data

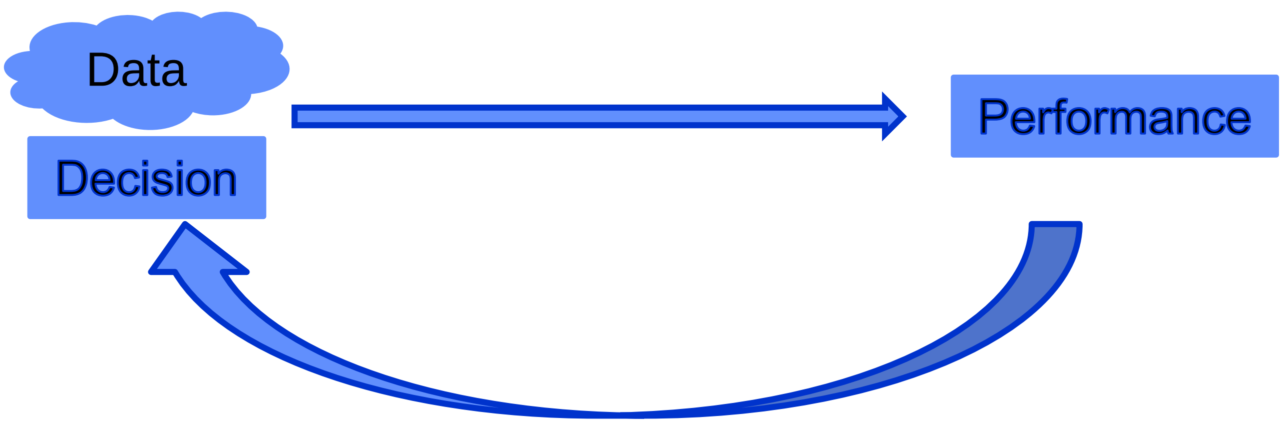
- Available resources, prices
- Coefficient matrix (technology matrix / bill of material)

What's Different?

- In previous two aspects, we have control over data:



- Decision making under uncertainty: immunize against data uncertainty



Production Example Revisited Again

- What type of industry faces lots of uncertainty in production?
  - Bio-chemical
  - Pharmaceutical
  - Agricultural
- Source of uncertainty
  - Supply
  - Production process

Production Problem

- **Product characteristic**

	Price	Content of M	Manpower	Equip	Cost
Drug M	$6200	50%	90h	40h	$700

- **Supply characteristic**

Raw Material	Cost	Content of M
Raw 1	$100.00	1%
Raw 2	$199.90	2%

- **Resource characteristic**

Budget	Manpower	Equip	Storage Capacity
$100,000	2000h	800h	1,000kg

Model

- x: amount of Drug M produced (report to retailer)
- y_1: amount of raw material 1 to be procured
- y_2: amount of raw material 2 to be procured

$$\max \quad (6200 - 700)x - 100y_1 - 199.9y_2$$

$$s.t. \quad 90x \leq 2000, \text{ (manpower)}$$

$$40x \leq 800, \text{ (equip)}$$

$$y_1 + y_2 \leq 1000, \text{ (storage)}$$

$$700x + 100y_1 + 199.9y_2 \leq 100000, \text{ (budget)}$$

$$0.01y_1 + 0.02y_2 - 0.5x \geq 0, \text{ (ingredient)}$$

$$x, y_1, y_2 \geq 0$$

Solution: $8819.66 (8.8%)

- **Primal Solution**

Drug M	Raw 1	Raw 2
17.55	0	438.79

- **Dual Solution**

Budget	Manpower	Equip	Storage Capacity	Ingredient M
0.088	0	0	0	**-10876.5**

Data Uncertainty

- **Supply characteristic**

Raw Material	Content of M	Variability Range
Raw 1	1%	[0.995%, 1.005%]
Raw 2	2%	[1.96%, 2.04%]

  - **The exact value of the percentage of M is unknown until production is finished.**

- **Original Solution:**

Drug M	Raw 1	Raw 2
17.55	0	438.79

- **If raw 1 realizes to be 0.995% and raw 2 realizes to be 1.96%, then 438.79*1.96%=8.6 < 17.55*50%=8.775!**

Remedy

- **Ad-hoc remedy: since we only have 438.79*1.96% amount of ingredient M, we adjust production quantities to 17.2 < 17.55.**

- **Drawbacks:**
  - **Commitment to demands**
  - **Profit loss: $6889, 22% less than $8820.**

A Robust Optimization Perspective

- **Prepare for the worst.**
  - **When formulating the problem, use the realization raw 1: 0.995% and raw 2: 1.96%.**

$$\max \quad (6200-700)x - 100y_1 - 199.9y_2$$

$$s.t. \quad 90x \leq 2000, \text{ (manpower)}$$

$$40x \leq 800, \text{ (equip)}$$

$$y_1 + y_2 \leq 1000, \text{ (storage)}$$

$$700x + 100y_1 + 199.9y_2 \leq 100000, \text{ (budget)}$$

$$0.00995y_1 + 0.0196y_2 - 0.5x \geq 0, \text{ (ingredient)}$$

$$x, y_1, y_2 \geq 0$$

A Robust Optimization Perspective

Drug M	Raw 1	Raw 2
17.47	877.732	0

- **For any realization of raw 1 and raw 2, solution is always feasible:**
  - **Suppose content of M in raw 1 turns out to be 1% and content of M in raw 2 turns out to be 2%. Then**

$$877.732*1\% = 8.777 > 17.47*50\% = 8.735$$

- **Prevent loss:**
  - **In the worst case: $8294.57, only 5.95% less than $8820**

Controlling Conservativeness

- **Prepare for the worst.**
  - **Is it highly likely that the content in raw 1 and raw 2 are 0.995% and 1.96% simultaneously?**
- **Alternatively:**

Raw Material	Content of M	Range of	Range of Content
Raw 1	$(1 + z_1 * 0.005)\%$	[-1, 1]	[0.995%, 1.005%]
Raw 2	$(2 + z_2 * 0.04)\%$	[-1, 1]	[1.96%, 2.04%]

  - **Is it highly likely that** $|z_1|+|z_2| = 2$**?**
- **Better Model:**
  - $|z_1|+|z_2| \le 2 \times \alpha$ **with** $0 \le \alpha \le 1$
  - $\alpha = 0$:
  - $\alpha = 1$:

Controlling Conservativeness

- **Ingredient constraint:**
  - $(1 + z_1 * 0.005)\% \times\ y_1 + (2 + z_2 * 0.04)\% \times\ y_2 - 0.5x \geq 0$
- **Worst case input?**

$$\mathrm{g}(y_1, y_2) = \min\ (1 + z_1 * 0.005)\% \times\ y_1 + (2 + z_2 * 0.04)\% \times\ y_2$$
$$s.t. \quad |z_1| + |z_2| \leq 2 \times \alpha$$
$$|z_1| \leq 1, |z_2| \leq 1,$$

$$\Updownarrow$$

$$\mathrm{g}(y_1, y_2) = 1\% y_1 + 2\% y_2 + \min\ (0.005\% y_1) z_1 + (0.04\% y_2) z_2$$
$$s.t. \quad |z_1| + |z_2| \leq 2 \times \alpha$$
$$|z_1| \leq 1, |z_2| \leq 1,$$

Model with Conservativeness Control

$$
\begin{aligned}
\max \quad & (6200 - 700)x - 100y_1 - 199.9y_2 \\
s.t. \quad & 90x \le 2000, \text{ (manpower)} \\
& 40x \le 800, \text{ (equip)} \\
& y_1 + y_2 \le 1000, \text{ (storage)} \\
& 700x + 100y_1 + 199.9y_2 \le 100000, \text{ (budget)} \\
& g(y_1, y_2) - 0.5x \ge 0, \text{ (ingredient)} \\
& x, y_1, y_2 \ge 0
\end{aligned}
$$

LP Formulation

- **Worst case input**

$$g(y_1, y_2) = 1\% y_1 + 2\% y_2 + \min\ (0.005\% y_1) z_1 + (0.04\% y_2) z_2$$

$$s.t. \quad |z_1| + |z_2| \leq 2 \times \alpha$$

$$|z_1| \leq 1, |z_2| \leq 1,$$

$$\Updownarrow$$

$$g(y_1, y_2) = 1\% y_1 + 2\% y_2 + \min\ (0.005\% y_1) z_1 + (0.04\% y_2) z_2$$

$$\begin{aligned}
s.t. \quad & x_1 + x_2 \leq 2 \times \alpha & p_1 \\
& z_1 - x_1 \leq 0 & p_2 \\
& -z_1 - x_1 \leq 0 & p_3 \\
& z_2 - x_2 \leq 0 & p_4 \\
& -z_2 - x_2 \leq 0 & p_5 \\
& x_1 \leq 1 & p_6 \\
& x_2 \leq 1, & p_7 \\
& x_1 \geq 0, x_2 \geq 0 &
\end{aligned}$$

LP Formulation

- **Using duality**

$$g(y_1, y_2) = 1\%y_1 + 2\%y_2 + \max\ 2\alpha p_1 + p_6 + p_7$$
$$\begin{aligned} s.t. \quad p_2 - p_3 &= 0.005\%y_1 \\ p_4 - p_5 &= 0.04\%y_2 \\ p_1 - p_2 - p_3 + p_6 &\le 0 \\ p_1 - p_4 - p_5 + p_7 &\le 0 \\ p_i &\le 0, i = 1, \dots, 7 \end{aligned}$$

- **Ingredient constraint:**
  - $g(y_1, y_2) - 0.5x \ge 0$
  - $1\%y_1 + 2\%y_2 + 2\alpha p_1 + p_6 + p_7 - 0.5x \ge 0$ **for some feasible dual solution:** $\boldsymbol{p}$

LP Formulation

$$\max \quad (6200 - 700)x - 100y_1 - 199.9y_2$$

$$s.t. \quad 90x \leq 2000, \text{ (manpower)}$$

$$40x \leq 800, \quad \text{(equip)}$$

$$y_1 + y_2 \leq 1000, \text{ (storage)}$$

$$700x + 100y_1 + 199.9y_2 \leq 100000, \text{(budget)}$$

$$1\%y_1 + 2\%y_2 + 2\alpha p_1 + p_6 + p_7 - 0.5x \geq 0,$$

$$p_2 - p_3 = 0.005\%y_1$$

$$p_4 - p_5 = 0.04\%y_2$$

$$p_1 - p_2 - p_3 + p_6 \leq 0$$

$$p_1 - p_4 - p_5 + p_7 \leq 0$$

$$x, y_1, y_2 \geq 0, p_i \leq 0, i = 1, \dots, 7$$

Ingredient

Reference: Bertsimas and Sim, "The Price of Robustness", 2004, *Operations Research* Vol. 52, No.1