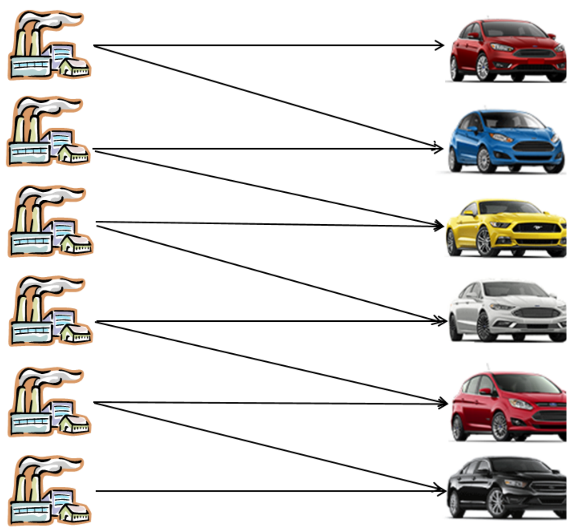
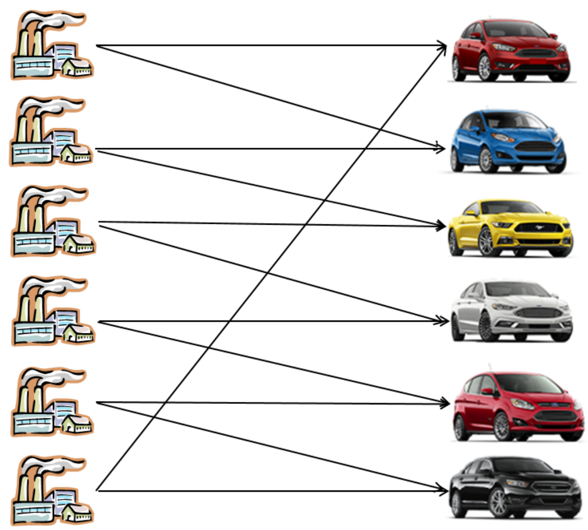
BDC5101: Deterministic Operations Research Models Semester II, 2018/2019, NUS

Assignment 3: Due on Apr.1, 2019

1. (10') **Process Flexibility:** Consider the case of the Ford Motor Company discussed in class. Suppose the company owns 6 production plants and is selling 6 models. The capacity at each plant is 100 per week, and the demands for each model are assumed to be independent and following $\mathcal{N}(100, 30^2)$ truncated above zero. Recall that a dedicated design results in an average sales of around 525.26 while a full flexible design results in an average sales of around 569.66. Evaluate the performance of the following two chain designs.



(a) Open Chain Design



(b) Long Chain Design

2. (10') **Multi-Location Newsvendor:** Consider a fashion retailer in Singapore that manages three retail stores at Jurong West, Orchard and Harbour Front respectively. The retailer faces the problem of how to manage the inventories of its best-selling product at these three locations for the new selling season. The product of concern has a retail price of S$100 at all three locations and costs S$50 per unit to procure from the manufacturer. The demands for the product at location i, $i \in \{\text{Jurong West}, \text{Orchard}, \text{Harbour Front}\}$ are assumed to be normal truncated above zero, i.e., $\max\{\mathcal{N}(\mu_i, \sigma_i^2), 0\}$ and the corresponding parameters, estimated from historical data, are summarized in the first two rows in the table below. The demands across different locations are assumed to be

Table 3.1: Estimation of Demand Parameters and Inventory Position

	Jurong West	**Orchard**	**Harbour Front**
μ_i	300	500	500
σ_i	20	20	40
Inventory	300	500	500

independent. The management team decides to procure in total 1300 products from the manufacturer and put 300, 500 and 500 units at Jurong West, Orchard and Harbour Front respectively (see last row in Table 3.1). That is, the inventory at a particular location is matched with the mean demand (before truncation). No inventory can be replenished from the manufacturer during the selling season and unsold products are worthless after the selling season.

(a) Compute the total expected profit aggregated over the three locations (the model of process flexibility example discussed in class can be used to compute this with minor modification).

(b) What is the total expected profit if there is no demand uncertainty at all (i.e., $\sigma_i = 0$)?

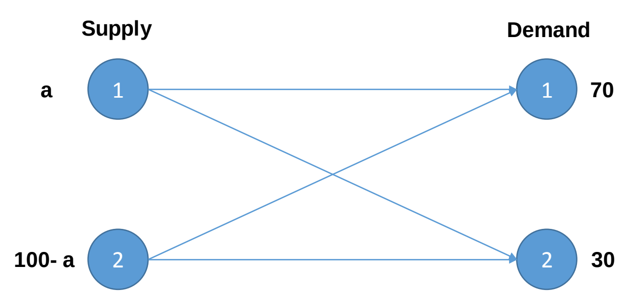
(c) To deal with the demand uncertainty, the management team is considering employing a logistic service company to help transship its products from one store to another. For example, after the selling season begins, the demand is realized to be 250, 550 and 450 units at Jurong West, Orchard and Harbour Front respectively. In this case, Orchard is running out of inventory but both Jurong West and Harbour Front have some leftover. The logistic company can then transship either 50 units of leftover at Jurong West or Harbour Front to satisfy the remaining demand at Orchard. The logistic company charges a fixed transaction cost of S$200 (which is incurred even if nothing is shipped) and a unit transshipment cost depending on the origin and destination summarized in the table below. That is, if 50 units of product were to be shipped from Harbour Front to Orchard, then the total cost to the retailer is S$$200 + 7 \times 50$. Should the retailer adopt the service

Table 3.2: Transshipment Cost (S$/unit)

	Jurong West	**Orchard**	**Harbour Front**
Jurong West	0	22	19
Orchard	22	0	7
Harbour Front	19	7	0

provided by the logistic company (in simulation, you should check the robustness of your answer by looking at larger sample size and running simulations multiple times)?

3. (10') Consider a transportation problem with two supply nodes and two demand nodes. The supply nodes have a and $100 - a$ supplies respectively with $0 \leq a \leq 100$. The demand nodes require 70 and 30 respectively. The figure below illustrates the graph of the transportation problem. Let c_{ij} be the unit transportation cost from supply i to demand j, $1 \leq i, j \leq 2$.



(a) Write down a linear programming formulation of the above problem and present your **A** matrix.

(b) Consider a spanning tree of the above graph generated by deleting the arc $(1, 1)$ (or forcing $x_{11} = 0$). Find the flow solution that satisfies all the flow-balancing constraints (i.e., the solution to $\mathbf{Ax} = \mathbf{b}$).

(c) Provide the range of values on the supply a such that the flow solution you found in (b) is a feasible solution to the transportation problem.

4. (10') **TSP with Time Window:** Consider the problem faced by an online food delivery company (e.g., Deliveroo). The following map illustrates the location for picking up the food (location 1) and the locations of each orders (locations 2-6) around the area of Vivo City in Singapore. The data for coordinates of each location is presented in Table 3.3. The last column in Table 3.3 provides

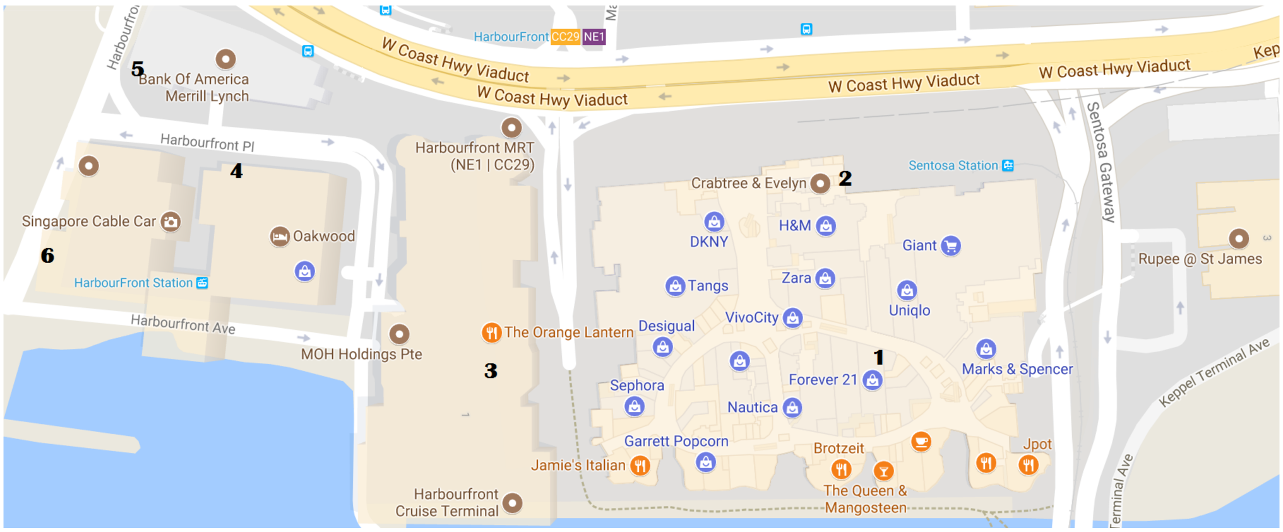


Table 3.3: Data on locations

	Latitude	Longitude	Maximum Waiting Time (in minute)
1	1.264306	103.822222	-
2	1.265270	103.821762	5
3	1.264181	103.820403	10
4	1.265067	103.818683	15
5	1.265655	103.818049	13
6	1.264998	103.817868	14

information on the maximum number of minutes a customer at a particular location is willing to wait before filing a complaint. With the data on latitude and longitude, one can use Google map to compute the pairwise walking time (in minute) summarized in the following table. Suppose now the delivery man is already at location 1 to pick up the food. The delivery man needs to deliver the food to customers at all the other locations and then return back to location 1. Assuming the time for consumers to pick up their delivery is zero, the objective of the delivery problem is to minimize the total traveling time (or waiting time of the consumers).

(a) Without considering the maximum waiting time of the consumer, what is the minimum traveling time and route to traverse the locations of all customers and returning to location 1? (the route may not be unique)

(b) To ensure a good service quality, the delivery company commits to the delivery before consumers' maximum waiting time. Can you still find a feasible route in this case? If yes, what is the minimum traveling time and route?

Table 3.4: Pairwise Walking Time

	1	2	3	4	5	6
1	-	3	3	10	9	10
2	3	-	3	7	6	7
3	3	3	-	7	6	7
4	10	7	7	-	1	2
5	9	6	6	1	-	1
6	10	7	7	2	1	-

5. (10') **Economic Lot-Sizing Problem:** An airplane manufacturer specializes in producing small airplanes. It has just received an order from a major corporation for 10 customized executive jet airplanes. The order calls for three of the airplanes to be delivered in the first season, two more to be delivered during the second season, three more during the season 3, and the last two during the season 4. Setting up the production facilities in any season for production requires a setup cost of \$2 million. Once the production facilities are set up, the manufacturer can produce any number of airplanes in that season. For example, the manufacturer can produce all 10 airplanes in the first season. However, only 3 airplanes will be delivered to the corporation in the season 1 and the rest 7 airplanes need to be held in inventory and maintained until their scheduled delivery times. The cost of holding and maintaining is $\$200,000$ per airplane per season. The demand and cost information is summarized in the following table according to seasons. The production cost for each airplane is ignored. To illustrate, one feasible

Table 3.5: Demand and cost information (in million)

Season	1	2	3	4
Demand (d)	3	2	3	2
Setup Cost (K)	2	2	2	2
Holding Cost (h)	0.2	0.2	0.2	0.2

production schedule is given below. The total cost for this production schedule is computed as follows:

Table 3.6: One feasible production schedule

Season	1	2	3	4
Production	3	6	0	1

- In season 1, three airplanes are produced according to Table 3.6, which incurs \$2 million setup cost. All airplanes produced are delivered and hence no holding cost incurred for this season.
- In season 2, six airplanes are produced according to Table 3.6, which again incurs \$2 million setup cost. Only two airplanes are delivered and the rest four airplanes sitting in the inventory incurs a holding cost of $4 \times 0.2 = 0.8$ million dollars.
- In season 3, nothing is produced and hence there is no setup cost. Three airplanes are delivered in this season and only one airplane left in inventory resulting in a holding cost of $1 \times 0.2 = 0.2$ million dollars.
- In season 4, one airplane is produced according to Table 3.6, which incurs \$2 million setup cost. Combined with the airplane left in inventory, the two airplanes are all delivered and hence no holding cost incurred for this season.

The total cost for this production schedule is then $2 + (2 + 0.8) + 0.2 + 2 = 7$ million dollars. The problem is then to determine how many airplanes to produce (if any) at the beginning of each season in order to minimize the total cost.

(a) Formulate the problem as an integer programming problem. (**Hint:** Use x_t to denote the inventory of airplanes at the end of season t, y_t to denote the number of airplanes produced in season t and z_t to denote the binary decision that whether the production facilities are set up or not in season t.)

(b) Solve the optimization problem you developed in (a) using the software you prefer.

(c) What is the minimum cost for the linear relaxation of your formulation in (a)? Is the relaxation tight?