BDC5101: Deterministic Operations Research Models **Semester II, 2018/2019, NUS**

Assignment 2: Due on Mar.11, 2019

1. (10') Consider the problem

$$\begin{aligned} \min \quad & 2x_1 + 15x_2 + 5x_3 + 6x_4 \\ s.t. \quad & x_1 + 6x_2 + 3x_3 + x_4 \geq 2 \\ & 2x_1 - 5x_2 + x_3 - 3x_4 \geq 3 \\ & x_1, x_2, x_3, x_4 \geq 0. \end{aligned}$$

   (a) Give the dual linear problem.

   (b) Solve the dual problem using any method you prefer.

   (c) Use your solution for the dual problem in (b) and complementary slackness condition to solve the primal problem above. (**Hint: At the optimal solution, complementary slackness condition implies both (i): the dual solution multiplied by the slackness in the corresponding primal constraint equals to zero; (ii): the primal solution multiplied by the slackness in the corresponding dual constraint equals to zero.**)

2. (10') **Portfolio Management Problem:** Consider the portfolio management problem discussed in class. There are 100 stocks. The expected return of stock i is r_i per dollar invested and the total budget is b dollars, where $b > 0$. Ignoring the risk of the return, the problem of maximizing the total expected return is

$$\begin{aligned} \max \quad & \sum_{i=1}^{100} r_i x_i \\ \text{s.t.} \quad & \sum_{i=1}^{100} x_i \leq b \\ & x_i \geq 0, \ i = 1, ..., 100, \end{aligned}$$

   where x_i denotes the amount of money invested in stock i.

   (a) Write down the dual problem to the above linear programming problem.

   (b) What is the optimal solution to the primal and the dual respectively?

   (c) Verify the solution you provided in (b) is indeed optimal using the weak duality.

3. (10') A potter manufacturer can make four different types of dining room service sets: JJP English, Currier, Primrose, and Bluetail. Furthermore, Primrose can be made by two different methods. Each set uses clay, enamel, dry room time, and kiln time, and results in a profit shown in Table 2.1.

   The manufacturer is currently committed to making the same amount of Primrose using method 1 and method 2. The formulation of the profit maximization problem is given below. The decision variables E, C, P_1, P_2, B are the number of sets of type English, Currier, Primrose Method 1, Primrose Method

Table 2.1: Profit and resources consumed for each product

	E	**C**	**P_1**	**P_2**	**B**	**Capacity**
Clay (lbs)	10	15	10	10	20	130
Enamel (lbs)	1	2	2	1	1	13
Dry room (hours)	3	1	6	6	3	45
Klin (hours)	2	4	2	5	3	23
Profit	51	102	66	66	89	

2, and Bluetail, respectively. We assume, for the purposes of this problem, that the number of sets of each type can be fractional.

$$\begin{aligned} \max \quad & 51E + 102C + 66P_1 + 66P_2 + 89B \\ s.t. \quad & 10E + 15C + 10P_1 + 10P_2 + 20B \leq 130 \\ & E + 2C + 2P_1 + P_2 + B \leq 13 \\ & 3E + C + 6P_1 + 6P_2 + 3B \leq 45 \\ & 2E + 4C + 2P_1 + 5P_2 + 3B \leq 23 \\ & P_1 - P_2 = 0 \\ & E, C, P_1, P_2, B \geq 0. \end{aligned}$$

The optimal solution to the primal is given in Table 2.2 and the optimal solution to the dual together with sensitivity information is given in Table 2.3. Use the information to answer the questions that follow.

Table 2.2: The optimal solution to the primal

	Optimal Value	**Reduced Cost**	**Objective Coefficient**
E	0	-3.571	51
C	2	0	102
P_1	0	0	66
P_2	0	-37.571	66
B	5	0	89

Table 2.3: The optimal solution to the dual and the sensitivity. The last two columns describe the allowed changes in the components of **b** for which the optimal dual solution remains the same

	Dual Variable	**Const. RHS**	**Allowable Increase**	**Allowable Decrease**
Clay	1.429	130	23.33	43.75
Enamel	0	13	∞	4
Dry room	0	45	∞	28
Klin	20.143	23	5.60	3.50
Primrose Methods	11.429	0	3.50	0

(a) What is the optimal quantity of each service set, and what is the total profit?

(b) Give an economic interpretation of the optimal dual variables appearing in the sensitivity report, for each of the five constraints.

(c) Should the manufacturer buy an additional 20 lbs. of Clay at \$1.1 per pound?

(d) In the current model, the number of Primrose produced using method 1 was required to be the same as the number of Primrose produced by method 2. Consider a revision of the model in which this constraint is replaced by the constraint $P_1 - P_2 \geq 0$. In the reformulated problem would the amount of Primrose made by method 1 be positive?

4. (10') **Diet Problem:** The "diet problem" concerns with finding a mix of foods that satisfies requirements on the amount of nutrition. We consider here the problem of choosing fruits to meet certain nutritional requirements. The nutrition information of five selected fruits is provided in Table 2.4 along with a minimum required and maximum allowed intake of each type of nutrition in the last two columns. Your goal is to find the cheapest combination of fruits that will meet nutritional requirements.

Table 2.4: Nutrition and price information per 100 grams

	Apple	**Banana**	**Blueberries**	**Durian**	**Tangerine**	**Min Reqd**	**Max Allowed**
Calories	52	89	57	147	53	500	3000
Carbohydrate (g)	14	23	14	27	13	50	400
Fiber (g)	2.4	2.6	2.4	3.8	1.8	20	30
Vitamin A (IU)	54	64	54	44	681	2000	3500
Vitamin C (mg)	4.6	8.7	9.7	19.7	26.7	75	150
Price (S$/100g)	0.5	0.3	2.5	10	0.5		

(a) Formulate a mathematical model for this problem. Solve the problem using the software you prefer and attach your code. Present and interpret the results.

(b) Without solving the problem using the software, can you determine how your optimal solution and optimal spending in (a) will change if you are also concerned with the total fat intake. The fat contained in each of the fruit and your nutrition requirement on fat is provided in the table below. Verify your solution by solving the problem numerically.

	Apple	**Banana**	**Blueberries**	**Durian**	**Tangerine**	**Min Reqd**	**Max Allowed**
Fat (g)	0.2	0.3	0.3	5	0.3	0	10

5. (10') **Retailer Game:** Consider the benchmark model and solution for the retailer game discussed in class. The initial inventory is 2000 units. The retailer needs to sell his inventory over a time horizon of 15 weeks by employing in order the four price levels: \$60, \$54, \$48, \$36 over time. The estimated average demands under these price levels are summarized in the following table: We have shown in

Table 2.5: Benchmark model

Price	\$60	\$54	\$48	\$36
Demand	125	162.5	217.5	348.8

class the optimal policy is to use for the first 11.67 weeks of full price \$60 and then 3.33 weeks of \$54 (ignoring integrality constraints). The revenue collected under this policy is \$116750. Consider the following three scenarios **separately** and answer the corresponding questions. Attach the codes you used to derive the solutions.

(a) Suppose in order to maintain the brand reputation, the retailer is only allowed to discount from week 4. Will the retailer change his markdown strategy in this case?

(b) Due to bad weather, only 25 units were sold in the first week. How should the retailer adjust his markdown policy? What is the improvement in revenue compared to the case when he sticks with the original markdown policy (specified above)?

(c) In the traditional brick-and-mortar business, menu cost and consumer price impression are the two major reasons behind the very few number of price levels. With the rise of e-commerce and other technologies, it is possible for the retailer to employ more price levels and change the price more frequently. In the retailer game, what happens if the retailer employs the following price grids (with a grid of \$2) instead of a few discount levels? For simplicity, the demand estimates at the additional price levels are taken as a linear interpolation of the original demands. What is the improvement in revenue compared to the benchmark? What is the new markdown policy? How do you compare with the original policy?

Table 2.6: Price grids model

Price	\$60	\$58	\$56	\$54	\$52	\$50	\$48	\$46	\$44	\$42	\$40	\$38	\$36
Demand	125	137.5	150	162.5	180.8	199.1	217.5	239.4	261.3	283.2	305.1	327	348.8