

Assignment 1: Due on Feb.11, 2019

1. (5') Consider the problem

$$\begin{aligned} \min \quad & 2x_1 + 3|x_2 - 10| \\ \text{s.t.} \quad & |x_1 + 2| + |x_2| \leq 5, \end{aligned}$$

and reformulate it as a linear programming problem.

2. (15') The Primo Insurance Company is introducing two new product lines: special risk insurance and mortgages. The expected profit is \$5 per unit on special risk insurance and \$2 per unit on mortgages. Management wishes to establish sales quotas for the new product lines to maximize total expected profit. The work requirements are as follows:

Department	Work-Hours per Unit		Working-Hours Available
	Special Risk	Mortgage	
Underwriting	3	2	2400
Administration	0	1	800
Claims	2	0	1200

- (a) (5') Formulate a linear programming model for this problem.
- (b) (5') Use the graphical method to solve this model. Numerically verify your solution using the software you prefer.
- (c) (5') Identify the two equations in the constraints, whose solution gives the optimal solution.
3. (10') Find all extreme points in the following polyhedra set:
- (a) (5') $P = \{(x_1, x_2, x_3) | x_1 + x_2 + x_3 \leq 1, x_1, x_2, x_3 \geq 0\}$.
- (b) (5') $P = \{(x_1, x_2, x_3, x_4) | x_1 + x_2 + \frac{1}{2}x_3 \leq 1, x_1, x_2, x_3, x_4 \geq 0\}$.
4. (5') Consider the problem

$$\begin{aligned} \min \quad & x_1 \\ \text{s.t.} \quad & x_1 = 1, \\ & x_1, x_2 \geq 0. \end{aligned}$$

Find all the extreme points and optimal solutions to the above problem.

5. (5') Consider the problem

$$\begin{array}{ll}\min & -x_1 - x_2 \\s.t. & x_1 - x_2 \leq 3 \\& x_1 + x_2 \leq 6 \\& x_1, x_2 \geq 0.\end{array}$$

Convert the problem into standard form and construct a basic feasible solution at which $(x_1, x_2) = (0, 0)$. Is this basic feasible solution optimal?

6. **Investment under Taxation:** (10') An investor has a portfolio of n different stocks. He has bought s_i shares of stock i at price $p_i, i = 1, \dots, n$. The current price of one share of stock i is q_i . The investor expects that the price of one share of stock i in one year will be r_i . If he sells shares, the investor pays transaction costs at the rate of 1% of the amount transacted. In addition, the investor pays taxes at the rate of 30% on capital gains. For example, suppose that the investor sells 1,000 shares of a stock at \$50 per share. He has bought these shares at \$30 per share. Upon selling, he receives $1,000 \times 50 = \$50,000$. However, he owes $0.30 \times (50,000 - 30,000) = \$6,000$ on capital gain taxes and $0.01 \times 50,000 = \$500$ on transaction costs. So, by selling 1,000 shares of this stock he nets $50,000 - 6,000 - 500 = \$43,500$.
- (a) Formulate the problem of selecting how many shares the investor needs to sell in order to raise an amount of money at least K , net of capital gains and transaction costs, while maximizing the expected value of his (remaining) portfolio next year.
- (b) Using the data for the portfolio in `investment.csv`, solve the problem for $K = \$9,000$ and attach your code.