LI LIPING
A0186040M

a. demand curve: $P = A - Q = A - nq$

profit: $\pi(q) = P \cdot q - C(q)$

$$= (A - nq)\cdot q - (0.5bq^2 + F)$$

$$= Aq - nq^2 - 0.5bq^2 - F$$

To maximize the profit:

$$\frac{\partial \pi(q)}{\partial q} = A - 2nq - bq = 0$$

$$q = \frac{A}{2n+b}$$

b. if each firm maximizes profit

the equilibrium price should be:

$$P = A - nq = A - n\cdot\frac{A}{2n+b}$$

$$= \frac{(n+b)A}{2n+b}$$

c. if n is endogenous, when profit $= 0$

the market will reach equilibrium

$$\pi(q) = P\cdot q - C(q) = 0$$

and each firm also maximizes its profit.

so $\begin{cases} P = AR = AC \\ MR = MC \end{cases}$

$$MR = \frac{\partial (Pq)}{\partial q} = A - 2nq$$

$$MC = bq$$

$$\begin{cases} A - 2nq = bq \\ A - nq = \dfrac{0.5bq^2 + F}{q} \end{cases} \Rightarrow \begin{cases} q = \dfrac{2F}{A} \\ n = \dfrac{A^2 - 2bF}{4F} \end{cases}$$

d. $A = \$100$, $b = 1$, $F = \$50$

$$\begin{cases} P = 100 - nq \\ C(q) = 0.5q^2 + 50 \end{cases}$$

$$n = \frac{A^2 - 2bF}{4F} = \frac{100^2 - 2\times1\times50}{200} = 49.5$$

$$q = \frac{2F}{A} = \frac{100}{100} = 1$$

$$P = 100 - 49.5\times1 = \$50.5$$

but n should be integer in this case

so: $n = 49$, $P = 51$