HW1: Due September 7 at 6pm

Please submit by IVLE and include your name and student ID on the cover page.

Let inverse demand be given by $P = A - Q$, where total industry output is equal to the individual output q of each of n firms. Let each firm have a cost function given by $C(q) = 0.5bq^2 + F$, where $b > 0$ and $F > 0$.

a. Derive the profit maximizing output choice for each firm taking n as given.
b. Derive the equilibrium price taking n as given and assuming each firm maximizes profit.
c. Now let n be endogenous and solve for the equilibrium value of n.
d. Solve for the long-run equilibrium values of n, P, and q for the specific case of: $A =$ \$100; $b = 1$; and $F =$ \$50.

a. set r as the output of other $(n-1)$ firms

$$P = A - r - q$$

$$\text{profit}: \pi(q) = P \cdot q - C(q)$$

$$= (A - r - q)q - (0.5bq^2 + F)$$

$$= Aq - rq - q^2 - 0.5bq^2 - F$$

$$\frac{\partial \pi(q)}{\partial q} = A - r - 2q - bq = 0$$

$$q^* = \frac{A - r}{2 + b}$$

Since each firm wants to maximize its profit

so $r = (n-1)q^*$

$$q^* = \frac{A - (n-1)q^*}{2+b} \Rightarrow q^* = \frac{A}{1+b+n}$$

b. the equilibrium price

$$P = A - nq^* = A - n \cdot \frac{A}{1+n+b} = \frac{(1+b)A}{1+n+b}$$

c $\pi = 0$

$$(A - nq^*)q^* = 0.5bq^2 + F$$