# Technology and Cost

# The Neoclassical View of the Firm

- **Concentrate upon a neoclassical view of the firm**
  - **the firm *transforms* inputs into outputs**


- **There is an alternative approach (Coase)**
  - **What happens *inside* firms?**
  - **How are firms structured? What determines size?**
  - **How are individuals organized/motivated?**

# The Single-Product Firm

- **Profit-maximizing firm must solve a related problem**
  - **minimize the cost of producing a given level of output**
  - **combines two features of the firm**
    - ***production function*: how inputs are transformed into output**

**Assume that there are n inputs at levels $x_1$ for the first, $x_2$ for the second,…, $x_n$ for the nth. The production function, assuming a single output, is written:**

$$q = f(x_1, x_2, x_3, \ldots, x_n)$$

- ***cost function*: relationship between output choice and production costs. Derived by finding input combination that minimizes cost**

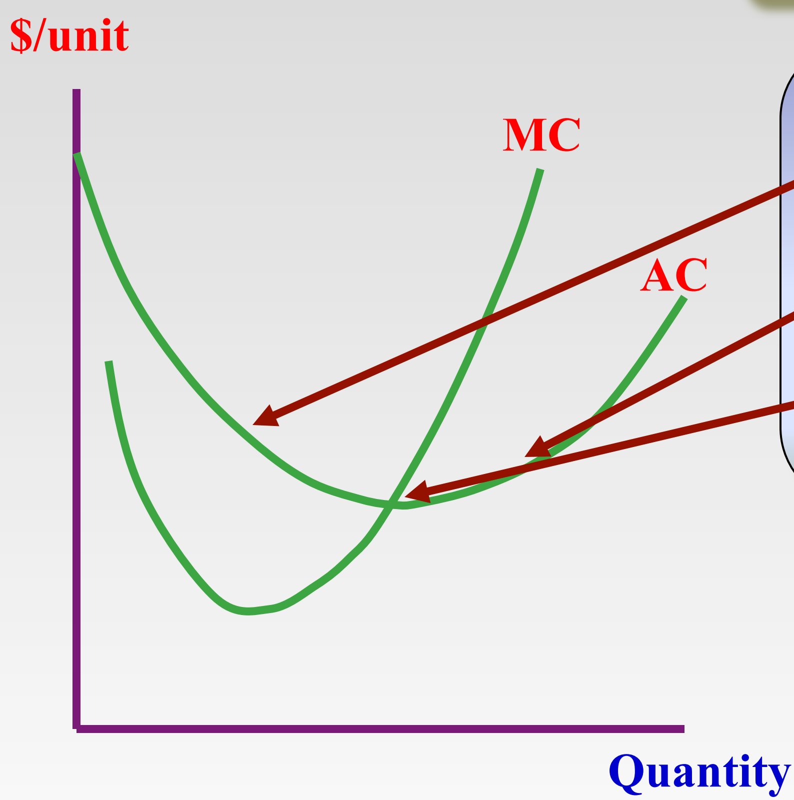
$$\underset{x_i}{\text{Minimize}} \sum_{i=1}^{n} w_i x_i \quad \text{subject to } f(x_1, x_2, \ldots, x_n) = q_1$$

# Cost Relationships

- **This analysis has interesting implications**
  - **different input mix across**
    - **time: as capital becomes relatively cheaper**
    - **space: difference in factor costs across countries**
- **Analysis gives formal definition of the *cost function***
  - **denoted C(Q): total cost of producing output Q**
  - **average cost = AC(Q) = C(Q)/Q**
  - **marginal cost: cost of one more unit**
    - **formally: MC(Q) = dC(Q)/d(Q)**
- **Also consider sunk cost**
  - **incurred on entry independent of output**
  - **cannot be recovered on exit**

# Cost curves: an illustration

Typical average and marginal cost curves




Relationship between AC and MC

If MC < AC then AC is falling

If MC > AC then AC is rising

MC = AC at the minimum of the AC curve

minimazing AC

# Cost and Output Decisions

- Firms maximizes profit where MR = MC *provided*
  - output should be greater than zero
  - implies that price is greater than average variable cost
  - *shut-down decision*
- Enter if price is greater than average total cost
  - must expect to cover sunk costs of entry

还有利润空间存在的时候

marginal revenue = marginal cost 就会 enter

# Economies of scale

- **Definition: average costs fall with an increase in output**
- **Represented by the *scale economy index***

$$S = \frac{AC(Q)}{MC(Q)}$$

- **S > 1: economies of scale** 正在变好
- **S < 1: diseconomies of scale** 正在变差
- **S is the inverse of the elasticity of cost with respect to output**

$$\eta_C = \frac{dC(Q)}{C(Q)} \Big/ \frac{dQ}{Q} = \frac{dC(Q)}{dQ} \Big/ \frac{C(Q)}{Q} = \frac{MC(Q)}{AC(Q)} = \frac{1}{S}$$

# Economies of scale 2

- **Sources of economies of scale**
  - **"the 60% rule": capacity related to volume while cost is related to surface area**
  - **product specialization and the division of labor**
  - **"economies of mass reserves": economize on inventory, maintenance, repair**
  - **indivisibilities**

# Indivisibilities, sunk costs and entry

- **Indivisibilities make *scale of entry* an important strategic decision:**
  - ***enter large with large-scale indivisibilities: heavy overhead***
  - ***enter small with smaller-scale cheaper equipment: low overhead***
- **Some indivisible inputs can be redeployed** 重新部署
  - **aircraft**
- **Other indivisibilities are highly specialized with little value in other uses**
  - ***market research expenditures***
  - ***rail track between two destinations***
- **Latter are *sunk costs*: nonrecoverable if production stops**
- **Sunk costs affect market structure by affecting entry**

# Sunk Costs and Market Structure

- **The greater are sunk costs the more concentrated is market structure**
- **An example:**

**Suppose that elasticity of demand $\eta = 1$**
**Then total expenditure $E = PQ$**
**If firms are identical then $Q = Nq_i$**
**Suppose that $LI = (P - c)/P = A/N^a$**

Lerner Index is inversely related to the number of firms

**Suppose firms operate in only one period: then $(P - c)q_i = K$**

**As a result:**

$$N^e = \left[\frac{AE}{K}\right]^{1/(1+\alpha)}$$

elasticity

# Multi-Product Firms

- **Many firms make multiple products**
  - ***Ford, General Motors, 3M etc.***
- **What do we mean by costs and output in these cases?**
- **How do we define *average costs* for these firms?**
  - ***total cost for a two-product firm is $C(Q_1, Q_2)$***
  - ***marginal cost for product 1 is $MC_1 = \partial C(Q_1,Q_2)/\partial Q_1$*** 偏导.
  - ***but average cost cannot be defined fully generally***
  - ***need a more restricted definition: ray average cost***

# Ray average cost

- *Assume that a firm makes two products, 1 and 2 with the quantities $Q_1$ and $Q_2$ produced in a constant ratio of 2:1.*
- *Then total output Q can be defined implicitly from the equations $Q_1 = 2Q/3$ and $Q_2 = Q/3$*
- *More generally: assume that the two products are produced in the ratio $\lambda_1/\lambda_2$ (with $\lambda_1 + \lambda_2 = 1$).*
- *Then total output is defined implicitly from the equations $Q_1 = \lambda_1 Q$ and $Q_2 = \lambda_2 Q$*
- ***Ray average cost*** **is then defined as:**

$$RAC(Q) = \frac{C(\lambda_1 Q, \lambda_2 Q)}{Q}$$

# An example of ray average costs

- **Assume that the cost function is:**

$$C(Q_1, Q_2) = 10 + 25Q_1 + 30Q_2 - 3Q_1Q_2/2$$

- **Marginal costs for each product are:**

$$MC_1 = \frac{\partial C(Q_1,Q_2)}{\partial Q_1} = 25 - \frac{3Q_2}{2}$$

$$MC_2 = \frac{\partial C(Q_1,Q_2)}{\partial Q_2} = 30 - \frac{3Q_1}{2}$$

# Ray Average Cost 2

- Ray average costs: assume $\lambda_1 = \lambda_2 = 0.5$

  $C(Q_1, Q_2) = 10 + 25Q_1 + 30Q_2 - 3Q_1Q_2/2$

  $Q_1 = 0.5Q;\ Q_2 = 0.5Q$

$$RAC(Q) = \frac{C(0.5Q,\ 0.5Q)}{Q}$$

$$= \frac{10 + 25Q/2 + 30Q/2 - 3Q^2/8}{Q} = \frac{10}{Q} + \frac{55}{2} - \frac{3Q}{8}$$

# Ray Average Cost 3

**Now assume $\lambda_1 = 0.75; \lambda_2 = 0.25$**

$$RAC(Q) = \frac{C(0.75Q, 0.25Q)}{Q}$$

$$= \frac{10 + 75Q/4 + 30Q/4 - 9Q^2/32}{Q}$$

$$= \frac{10}{Q} + \frac{105}{4} - \frac{9Q}{32}$$

# Economies of scale and multiple products

- **Definition of economies of scale with a single product**

$$S = \frac{AC(Q)}{MC(Q)} = \frac{C(Q)}{QMC(Q)}$$

- **Definition of economies of scale with multiple products**

$$S = \frac{C(Q_1, Q_2, \ldots, Q_n)}{MC_1Q_1 + MC_2Q_2 + \ldots + MC_nQ_n}$$

- **This is by analogy to the single product case**
  - **relies on the implicit assumption that output proportions are *fixed***
  - **so we are looking at *ray average costs* in using this definition**

# Ray Average Cost Example Once again

$$C(Q_1, Q_2) = 10 + 25Q_1 + 30Q_2 - 3Q_1Q_2/2$$

$$MC_1 = 25 - 3Q_2/2 \; ; \; MC_2 = 30 - 3Q_1/2$$

**Substitute into the definition of S:**

$$S = \frac{C(Q_1,Q_2,\ldots,Q_n)}{MC_1Q_1 + MC_2Q_2 + \ldots + MC_nQ_n}$$

$$= \frac{10 + 25Q_1 + 30Q_2 - 3Q_1Q_2/2}{25Q_1 - 3Q_1Q_2/2 + 30Q_2 - 3Q_1Q_2/2}$$

**It should be obvious in this case that S > 1**

**This cost function exhibits *global* economies of scale**

# Economies of Scope

- **Formal definition**

$$S_C = \frac{C(Q_1, 0) + C(0, Q_2) - C(Q_1, Q_2)}{C(Q_1, Q_2)}$$

- **The critical value in this case is $S_C = 0$**
  - *$S_C < 0$ : no economies of scope; $S_C > 0$ : economies of scope.*
- **Take the example:**

$$S_C = \frac{10 + 25Q_1 + 10 + 30Q_2 - (10 + 25Q_1 + 30Q_2 - 3Q_1Q_2/2)}{10 + 25Q_1 + 30Q_2 - 3Q_1Q_2/2} > 0$$

economies of scope

# Economies of Scope 2

- **Sources of economies of scope**
- **shared inputs**
  - **same equipment for various products**
  - ***shared advertising creating a brand name***
  - ***marketing and R&D expenditures that are generic***
- **cost complementarities**
  - ***producing one good reduces the cost of producing another***
  - ***oil and natural gas***
  - ***oil and benzene***
  - ***computer software and computer support***
  - ***retailing and product promotion***

# Flexible Manufacturing

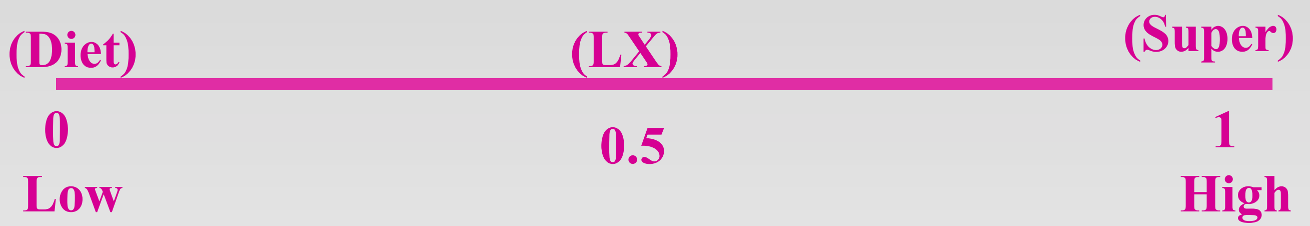
- **Extreme version of economies of scope**
- **Changing the face of manufacturing**
- **"Production units capable of producing a range of discrete products with a minimum of manual intervention"**
  - *Benetton*
  - *Custom Shoe*
  - *Levi's*
  - *Mitsubishi*
- **Production units can be switched easily with little if any cost penalty**
  - *requires close contact between design and manufacturing*

# Flexible Manufacturing 2

- **Take a simple model based on a spatial analogue.**
  - **There is some characteristic that distinguishes different varieties of a product**
    - **sweetness or sugar content**
    - **color**
    - **texture**
  - **This can be measured and represented as a line**
  - **Individual products can be located on this line in terms of the quantity of the characteristic that they possess**
  - **One product is chosen by the firm as its *base product***
  - **All other products are variants on the base product**

# Flexible Manufacturing 3

- **An illustration: soft drinks that vary in sugar content**




Each product is located on the line in terms of the amount of the characteristic it has

This is the characteristics line

# Flexible Manufacturing 4

(Diet) (LX) (Super)

0 0.5 1

Low High

- Assume that the process is centered on LX as base product. A switching cost *s* is incurred in changing the process to either of the other products.

There are additional marginal costs of making Diet or Super -- from adding or removing sugar. These are *r* per unit of "distance" between LX and the other product.

There are shared costs *F*: design, packaging, equipment.

# Flexible Manufacturing 5

- **In the absence of shared costs there would be specialized firms.**
- **Shared costs introduce *economies of scope*.**

**Total costs are:** $C(z_j, q_j) = F + (m - 1)s + \sum_{j=1}^{m} [(c + r|z_j - z_1|)q_j]$

**If production is 100 units of each product:**

**one product per firm with three firms** $C_3 = 3F + 300c$

**one firm with all three products** $C_1 = F + 2s + 300c + 100r$

$C_1 < C_3$ if $2s + 100r < 2F \Rightarrow F > 50r + s$

**This implies a constraint on set-up costs, switching costs and marginal costs for multi-product production to be preferred.**

# Determinants of Market Structure

- **Economies of scale and scope affect market structure but *cannot be looked at in isolation*.**
- **They must be considered *relative to market size*.**
- **Should see concentration decline as market size increases**
  - **Entry to the medical profession is going to be more extensive in Chicago than in Oxford, Miss**
  - **Find more extensive range of financial service companies in Wall Street, New York than in Frankfurt**

# Network Externalities

- **Market structure is also affected by the presence of *network externalities***
  - **willingness to pay by a consumer increases as the number of current consumers increase**
    - **telephones, fax, Internet, Windows software**
    - **utility from consumption increases when there are more current consumers**
- **These markets are likely to contain a small number of firms**
  - **even if there are limited economies of scale and scope**

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# The Role of Policy

- **Government can directly affect market structure**
  - **by limiting entry**
    - **taxi medallions in Boston and New York**
    - **airline regulation**
  - **through the patent system**
  - **by protecting *competitors* e.g. through the Robinson-Patman Act**

# *Empirical Application: Cost Minimization and Cost Function Estimates*

Consider simple cost minimization problem:

- **Minimize:** $C = wL + rK$ **;**
- **Subject to:** $Q = K^{\alpha}L^{\beta}$

**From Production Constraint:** $L = Q^{1/\beta}K^{\alpha/\beta}$

**Substitution yields:** $C = wQ^{1/\beta}K^{\alpha/\beta} + rK$

**Minimizing for given $Q$ with respect to $K$ and then substituting into the cost equation yields:**

$$C = \left\{\left[\frac{\alpha}{\beta}\right]^{\beta/(\alpha+\beta)} + \left[\frac{\beta}{\alpha}\right]^{\alpha/(\alpha+\beta)}\right\} r^{\alpha/(\alpha+\beta)} w^{\beta/(\alpha+\beta)} Q^{1/(\alpha+\beta)}$$

# *Empirical Application: Cost Minimization and Cost Function Estimates 2*

In logs, we have:

$$\ln C = \text{Constant} + \frac{\alpha}{\alpha+\beta}\ln r + \frac{\beta}{\alpha+\beta}\ln w + \frac{1}{\alpha+\beta}\ln Q$$

**In general, we have:**

$$\ln C = \text{Constant} + \delta_1 \ln r + \delta_2 \ln w + \delta_3 \ln Q$$

**A more flexible specification is the translog form**

$$\ln C = \text{Constant} + \delta_1 \ln r + \delta_2 \ln w + 0.5[\delta_{11}(\ln r)^2 + \delta_{12}(\ln w)(\ln r) + \delta_{21}(\ln w)(\ln r) + \delta_{22}(\ln w)^2] + \delta_3 \ln Q + \delta_{31}(\ln Q)(\ln r) + \delta_{32}(\ln Q)(\ln w) + 0.5\delta_{33}(\ln Q)^2$$

# *Empirical Application: Cost Minimization and Cost Function Estimates* 3

- **The translog function is more flexible because it does not restrict the underlying production technology to be Cobb-Douglas. Its general form is consistent with many other plausible technologies**
- **The scale economy index is now $S = 1/\frac{\ln C}{\ln Q}$**

$$= 1/(\delta_3 + \delta_{33}\ln Q + \delta_{31}\ln r + \delta_{32}\ln w)$$

**So long as $\delta_{31}$, $\delta_{32}$, and $\delta_{33}$ do not all equal zero, $S$ will depend on the level of output $Q$**

ball ball you

放过我

**This is one of the many restrictions on the data that can be tested empirically with the translog functional form**

# *Empirical Application: Cost Minimization and Cost Function Estimates* 4

- A pioneering use of the translog approach was the study by Christensen and Greene (1976) on scale economies in electric power generation
  - They assume three inputs: Labor (paid *w*); capital (paid *r*); and Fuel (paid *F*). So, they have five explanatory or right-hand-side variables
    - a pure output term
    - an interaction term of output and *r*
    - an interaction term of output and *w*
    - an interaction term of output and *F*
    - a pure output squared term
  - Results shown on next slide

# *Empirical Application: Cost Minimization and Cost Function Estimates 5*

| Variable | Coefficient | t-statistic |
|---|---|---|
| (ln $Q$) | 0.587 | 20.87 |
| (ln $Q$)(ln $r$) | –0.003 | –1.23 |
| (ln $Q$)(ln $w$) | –0.018 | –8.25 |
| (ln $Q$)(ln $F$) | 0.021 | 6.64 |
| (ln $Q$)$^2$ | 0.049 | 12.94 |

- **All the variables are statistically significant indicating among other things that the scale economies depend on the output level and disappear after some threshold is reached**
- **Christensen and Greene (1976) find that very few firms operate below this threshold**

# Illustration of ray average costs