DSC5211C QUANTITATIVE RISK MANAGEMENT

SESSION 5

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Systems of Equations

Objectives

- Measuring the Quality of the Forecasts
- Vector Autoregression
- Cointegration
- Applications to Metals and Oil Prices

References

Enders, Chapters 5, 6.

Franses, Chapters 9, 10.

Diagnostic Evaluation

- Do significance tests of parameter estimates (as in regression) by checking t-stats

- You can compare the fit of several models using

  - Sum of Squared Residuals (*SSR*)

  - Akaike Information Criterion (*AIC*)

$$AIC = 1 + \ln(2\pi) + \ln(SSR/n) + 2k/n$$

where *n* is no. of observations used and *k* is no. of explanatory variables, including constant

- Low values are preferable

Diagnostic Evaluation (Cont.)

- Schwartz Bayesian Criterion *(SBC)*

$$SBC = 1 + \ln(2\pi) + \ln(SSR/n) + k\ \ln(n)/n$$

- Hannan-Quinn Criterion *(HQ)*

$$HQ = 1 + \ln(2\pi) + \ln(SSR/n) + 2k\ \ln[\ln(n)]/n$$

where n is no. of observations used and k is no. of explanatory variables, including constant

- Low values are preferable

Diagnostic Evaluation

- As with regression, residuals should have zero mean, fixed variance, no autocorrelation
- AUTOCORRELATION:
  - We can investigate ACF of the residuals
  - We ***cannot*** use DW when lag Y_t is an explanatory variable

Diagnostic Evaluation: Autocorrelation

- We can test for autocorrelation in several lags together:

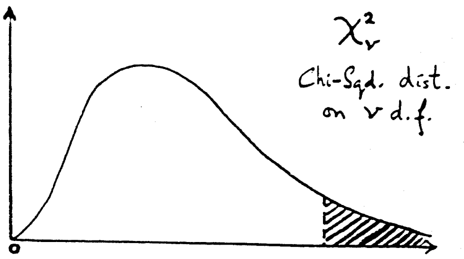
Under null hypothesis of no autocorrelation in first m lags, Ljung-Box Q-statistic has a chi-squared distribution with d.f. = (m-p-q)

$$Q(m) = n(n+2)\sum_{i=1}^{m} \frac{r_i^2}{(n-i)} \sim \chi^2_{m-p-q}$$

where r_i =corr(e_t,e_{t-i})

Chi-Squared Critical Values

Degrees of Freedom	10% Level	5% Level	1% Level	0.1% Level
1	2.706	3.841	6.635	10.828
2	4.605	5.991	9.210	13.816
3	6.251	7.815	11.345	16.266
4	7.779	9.488	13.277	18.467
5	9.236	11.071	15.086	20.515
6	10.645	12.592	16.812	22.458
7	12.017	14.067	18.475	24.322
8	13.362	15.507	20.090	26.124
9	14.684	16.919	21.666	27.877
10	15.987	18.307	23.209	29.588
11	17.275	19.675	24.725	31.264
12	18.549	21.026	26.217	32.909
13	19.812	22.362	27.688	34.528
14	21.064	23.685	29.141	36.123
15	22.307	24.996	30.578	37.697
16	23.542	26.296	32.000	39.252
17	24.769	27.587	33.409	40.790
18	25.989	28.869	34.805	42.312
19	27.204	30.144	36.191	43.820
20	28.412	31.410	37.566	45.315
21	29.615	32.671	38.932	46.797
22	30.813	33.924	40.289	48.268
23	32.007	35.172	41.638	49.728
24	33.196	36.415	42.980	51.179
25	34.382	37.652	44.314	52.620
26	35.563	38.885	45.642	54.052
27	36.741	40.113	46.963	55.476
28	37.916	41.337	48.278	56.892
29	39.087	42.557	49.588	58.301
30	40.256	43.773	50.892	59.703
31	41.422	44.985	52.191	61.098
32	42.585	46.194	53.486	62.487
33	43.745	47.400	54.776	63.870
34	44.903	48.602	56.061	65.247
35	46.059	49.802	57.342	66.619
36	47.212	50.998	58.619	67.985
37	48.363	52.192	59.893	69.346
38	49.513	53.384	61.162	70.703
39	50.660	54.572	62.428	72.055
40	51.805	55.758	63.691	73.402
41	52.949	56.942	64.950	74.745
42	54.090	58.124	66.206	76.084
43	55.230	59.304	67.459	77.419
44	56.369	60.481	68.710	78.749
45	57.505	61.656	69.957	80.077
46	58.641	62.830	71.201	81.400
47	59.774	64.001	72.443	82.720
48	60.907	65.171	73.683	84.037
49	62.038	66.339	74.919	85.351
50	63.167	67.505	76.154	86.661

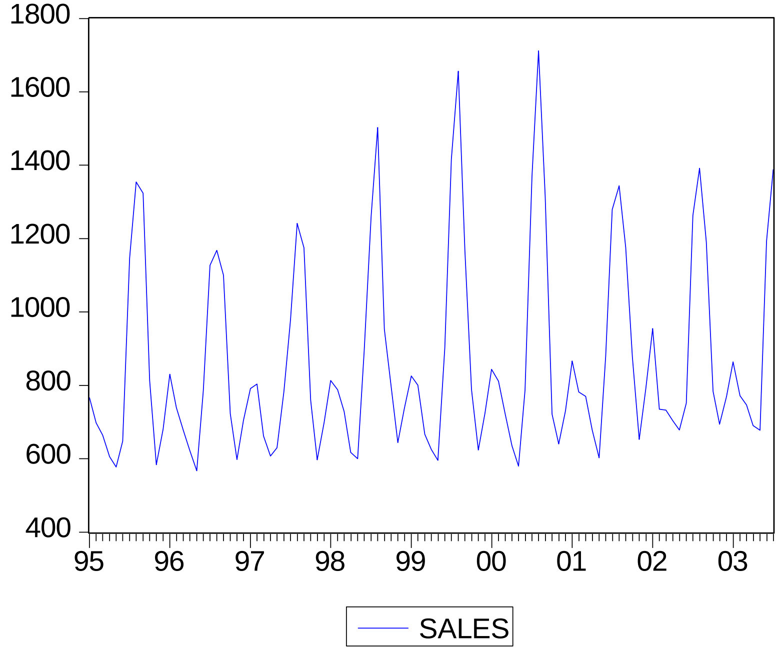


Forecast Evaluation

- Theil Inequality Coefficient: lies between zero and one (ZERO indicates a perfect fit).
- Bias Proportion tells us how far the mean of the forecast is from the mean of the actual series.
- Variance Proportion: tells us how far the variation of the forecast is from the variance of the actual series.
- Covariance Proportion: measures the unsystematic forecasting errors.
- Bias Proportion +Variance Proportion + Covariance Proportion = 1

Mixed ARIMA Regression Modelling

- Monthly residential electricity sales from Jan. 1995 to July 2003



Mixed ARIMA Regression Modelling

- SALES Residential electricity sales (KWh) per customer
- C66 Cooling degree hours at base temperature 66 degrees (a measure of summer heat). The cooling degree hours at base temperature T is: $\sum_{i=1} i \times n_i$ where n_i is the number of hours in the month at temperature $T+i$.
- C76 Cooling degree hours at base temperature 76 degrees (a measure of summer heat)
- H55 Heating degree hours at base temperature 55 degrees (a measure of winter cold). The heating degree hours at base temperature T is: $\sum_{i=1} i \times n_i$ where n_i is the number of hours in the month at temperature T-i.
- DINC Disposable income per household ($)
- AIRC Proportion of households with air conditioning

Mixed ARIMA Regression Modelling for Sales

- We might specify a regression with cooling and heating degree hours, and also airc and inc:

Dependent Variable: D(SALES,0,12)
Method: Least Squares

Sample(adjusted): 1996:01 2003:07
Included observations: 91 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-75.75381	18.60223	-4.072298	0.0001
D(C66,0,12)	0.126993	0.031680	4.008651	0.0001
D(C76,0,12)	0.036265	0.098778	0.367140	0.7144
D(H55,0,12)	0.068661	0.006063	11.32381	0.0000
D(AIRC,0,12)	3314.695	1047.885	3.163223	0.0022
D(DINC,0,12)	0.086061	0.018071	4.762303	0.0000

R-squared	0.772367	Mean dependent var	41.39793
Adjusted R-squared	0.758977	S.D. dependent var	231.2518
S.E. of regression	113.5309	Akaike info criterion	12.36569
Sum squared resid	1095587.	Schwarz criterion	12.53124
Log likelihood	-556.6387	F-statistic	57.68177
Durbin-Watson stat	0.911745	Prob(F-statistic)	0.000000

Correlogram of Residuals

Sample: 1996:01 2003:07
Included observations: 91

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.539	0.539	27.303	0.000
		2	0.627	0.475	64.730	0.000
		3	0.391	-0.078	79.407	0.000
		4	0.361	-0.061	92.102	0.000
		5	0.206	-0.081	96.285	0.000
		6	0.169	-0.021	99.140	0.000
		7	0.123	0.068	100.67	0.000
		8	0.222	0.258	105.71	0.000
		9	0.196	0.083	109.66	0.000
		10	0.192	-0.132	113.51	0.000
		11	0.234	0.044	119.31	0.000
		12	0.121	-0.154	120.87	0.000
		13	0.252	0.208	127.78	0.000
		14	-0.047	-0.303	128.02	0.000
		15	0.020	-0.120	128.06	0.000
		16	-0.149	-0.053	130.56	0.000
		17	-0.062	0.115	131.00	0.000
		18	-0.169	0.030	134.30	0.000
		19	-0.083	-0.010	135.11	0.000
		20	-0.068	0.100	135.67	0.000
		21	-0.004	-0.141	135.67	0.000
		22	-0.070	-0.113	136.26	0.000
		23	-0.024	0.081	136.33	0.000
		24	-0.046	0.079	136.61	0.000

Mixed ARIMA: Electricity Sales

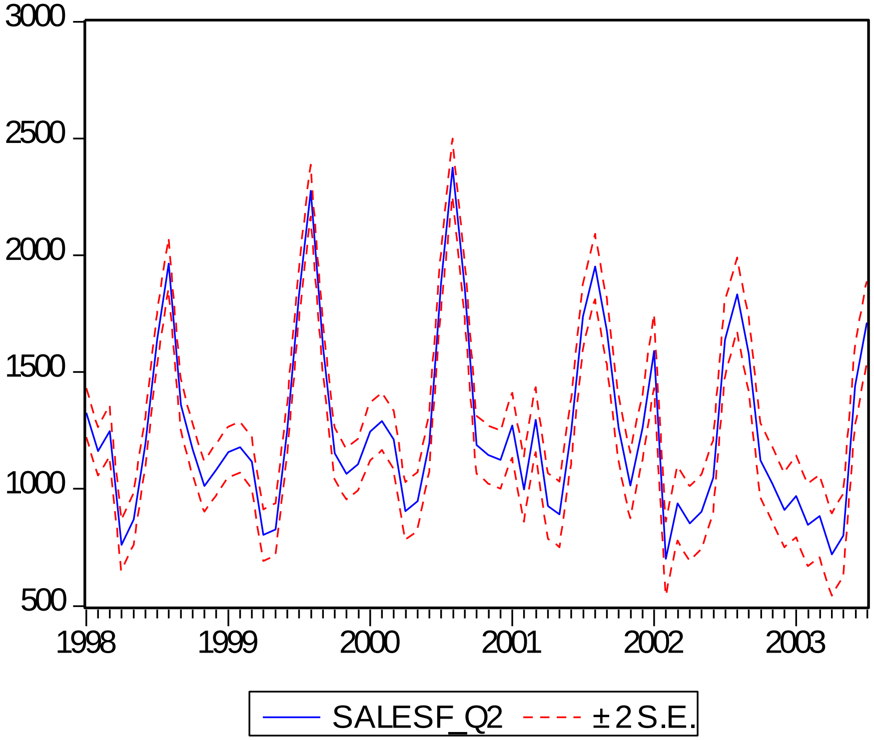
Dependent Variable: D(SALES,0,12)
Method: Least Squares

Sample(adjusted): 1998:01 2003:07
Included observations: 67 after adjusting endpoints
Convergence achieved after 23 iterations
Backcast: 1996:01 1997:12

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-84.46868	7.869247	-10.73402	0.0000
D(C66,0,12)	0.087494	0.014619	5.984825	0.0000
D(C76,0,12)	0.294650	0.061088	4.823343	0.0000
D(H55,0,12)	0.070927	0.003105	22.84501	0.0000
D(AIRC,0,12)	4397.745	67.77073	64.89151	0.0000
AR(2)	0.321440	0.088313	3.639789	0.0006
AR(12)	-0.431910	0.090857	-4.753756	0.0000
SAR(12)	0.352068	0.062388	5.643212	0.0000
MA(12)	0.885671	0.051479	17.20445	0.0000
SMA(12)	-0.885452	0.077071	-11.48879	0.0000

R-squared	0.941213	Mean dependent var	-9.047384
Adjusted R-squared	0.931931	S.D. dependent var	198.4539
S.E. of regression	51.77663	Akaike info criterion	10.86862
Sum squared resid	152806.7	Schwarz criterion	11.19768
Log likelihood	-354.0988	F-statistic	101.4007
Durbin-Watson stat	1.994561	Prob(F-statistic)	0.000000

Mixed ARIMA Electricity Sales: Forecasts



Forecast: SALESF_Q2
Actual: SALES
Forecast sample: 1995:01 2003:
Adjusted sample: 1998:01 2003:
Included observations: 67

Root Mean Squared Error	89.81836
Mean Absolute Error	68.32131
Mean Abs. Percent Error	6.621406
Theil Inequality Coefficient	0.034709
Bias Proportion	0.050404
Variance Proportion	0.003377
Covariance Proportion	0.946218

Quality of Forecasts

- Theil Inequality Coefficient: 3.4% indicates a very good fit.
- Bias Proportion: 5% tells us that the forecast for the mean are very close to the actual values.
- Variance Proportion: 0.3% tells us the variance of the forecasts is almost perfect!
    - This is due to modelling seasonal behaviour.
- Covariance Proportion: 94.6% tells us that a big proportion of forecasting errors are unsystematic.
- Conclusion: this is a very good model.

MULTIPLE EQUATIONS MODELS

- Try to estimate a model with multiple equations that are somehow linked together.

- The most important type of such a system is the simultaneous equations model (simultaneous effect between/within equations)

- Key issues: more than one "dependent" variable as there is more than one equation; and some "dependent" variables are found on the right-hand side of some equations.

MULTIPLE EQUATIONS: System

- A *system* is a group of equations containing unknown parameters.

- The general form of a system is: $f(\mathrm{Y}_\mathrm{t}, \mathrm{X}_\mathrm{t}, e_t, B)$ where Y_t is a vector of endogenous variables, X_t is a vector of exogenous variables, and e_t is a vector of possibly serially correlated disturbances. The task of estimation is to find estimates of the vector of parameters B.

- A ***model*** is a group of known equations describing endogenous variables. Models are used to solve for values of the endogenous variables, given information on other variables in the model.

Estimating the Parameters of a System

- One approach is to estimate each equation in the system separately, using ***one of the single equation methods*** described earlier in this manual.

- A second approach is to estimate, simultaneously, the complete set of parameters of the equations in the system.

- The simultaneous approach allows you to place constraints on coefficients across equations and to employ techniques that account for correlation in the residuals across equations.

MULTIPLE EQUATIONS MODELS II

- If the errors between equations are somehow related, doing OLS equation-by-equation, is not efficient.

- ***The estimated variances and standard errors are understated, causing t-statistics to be overstated (same outcome as serial correlation)***.

- However, if ***all of the right-hand side variables in all equations are the same***, one can show that OLS, equation-by-equation, is an acceptable method (even if errors between equations are related).

- If not, must ***do a system of equations method***, known as Zellner's method (this is available on E-VIEWS).

Vector Autoregression (VAR)

- VAR models are basically a combination of multiple equations models and multi-variate time series techniques.

- Why VAR Models?
    - These models do not have the simultaneous problem.

    - Most are only loosely based on theory: they examine how a collection of variables are related over time.

    - Allows for the possibility that error terms of different equations are related.

Why VAR

- Why VAR Models?
    - VAR uses lagged variables to help describe current and future movements of “endogenous” (dependent) variables. This gives the models a dynamic property.
    - This makes VAR useful for **forecasting** and **simulation**
- While not based on theory, one can impose restrictions to tie VAR models to some theory or previous research (structural VAR approach).

VAR Process: A Two Variable Example

- Two variables: X_t and Z_t.
- First-Order VAR System:

$$X_t = a_0 + a_1 X_{t-1} + b_1 Z_{t-1} + e_{1t}$$

$$Z_t = c_0 + c_1 X_{t-1} + d_1 Z_{t-1} + e_{2t}$$

$Special\ Case: b_1 = c_1 = 0 \quad correlation\ of\ e_{1t}, e_{2t} = 0$

VAR Processes: Higher Order Systems

- Only one lag on X and Z appears in each of the above equations.
    i. systems with one lag are referred to as first order systems.
    ii. higher order systems involve more than one lag in at least one of the variables

- Not necessary to limit the size of the forecasting problem to only two variables
    i. data limitations preclude systems with very large number of variables (say > 10)

How Long for the Lags?

- Long enough to ***get rid of any significant autocorrelation in the residuals*** of each equation (otherwise there is information to improve the forecasts)
- Not so long that the model is “over-parameterized” and there is a loss of forecasting efficiency
- AIC and SIC

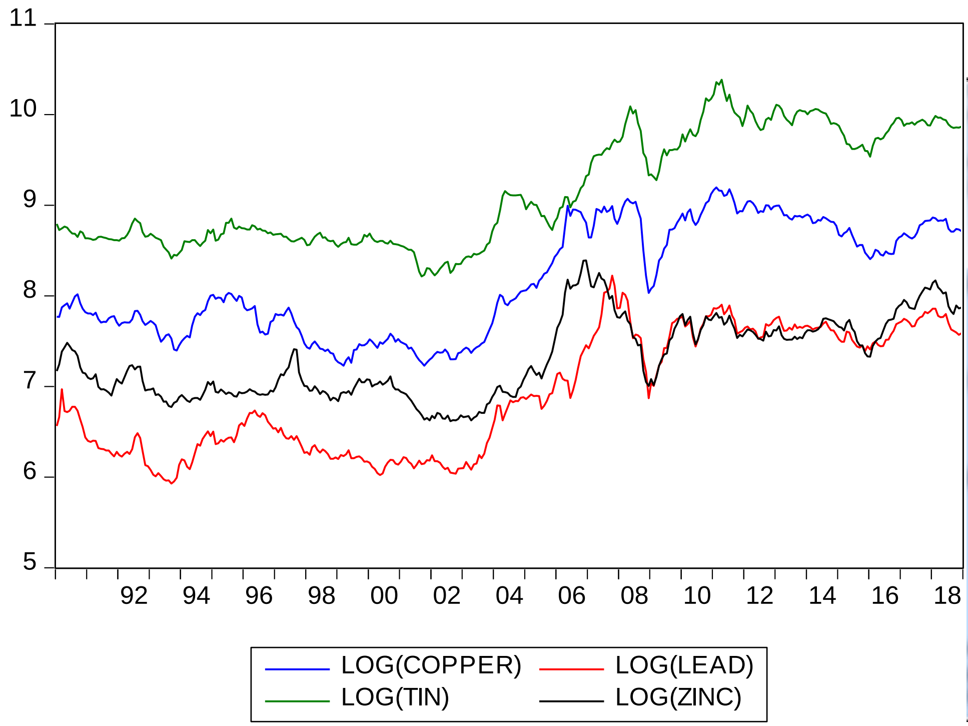
How Long for the Lags?

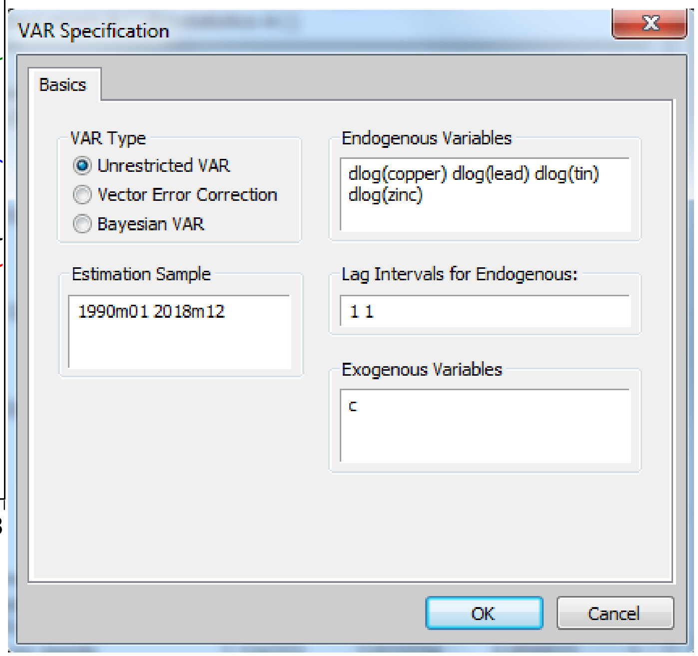
- One strategy:
  - start with a longer lag

  - check that autocorrelations of residuals are small (i.e. you are not wasting information).

  - shorten the lag and re-estimate
    - do AIC and/or SIC increase or decrease? (smaller is better!)
    - check on the stability of the estimated coefficients as the lag length is shortened

VAR – METALS EXAMPLE – Log (Price)





Vector Autoregression Estimates

Sample (adjusted): 1990M03 2018M12
Included observations: 346 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	DLOG(COP...	DLOG(LEAD)	DLOG(TIN)	DLOG(ZINC)
DLOG(COPPER(-1))	0.416617	0.091242	0.085898	0.189932
	(0.06889)	(0.08211)	(0.06327)	(0.06986)
	[6.04795]	[1.11125]	[1.35758]	[2.71860]
DLOG(LEAD(-1))	-0.060277	0.204942	0.058756	-0.045274
	(0.05740)	(0.06841)	(0.05272)	(0.05821)
	[-1.05016]	[2.99558]	[1.11448]	[-0.77773]
DLOG(TIN(-1))	-0.024241	-0.154617	0.197643	-0.222469
	(0.06554)	(0.07812)	(0.06020)	(0.06647)
	[-0.36985]	[-1.97920]	[3.28306]	[-3.34682]
DLOG(ZINC(-1))	0.026251	0.029992	-0.025263	0.301831
	(0.06836)	(0.08148)	(0.06279)	(0.06933)
	[0.38400]	[0.36808]	[-0.40234]	[4.35346]
C	0.001771	0.002246	0.002331	0.001495
	(0.00311)	(0.00370)	(0.00285)	(0.00315)
	[0.57016]	[0.60670]	[0.81687]	[0.47452]
R-squared	0.151917	0.059181	0.083139	0.135929
Adj. R-squared	0.141969	0.048145	0.072384	0.125793
Sum sq. resids	1.134103	1.611234	0.956813	1.166537
S.E. equation	0.057670	0.068739	0.052971	0.058489
F-statistic	15.27084	5.362582	7.730248	13.41086
Log likelihood	498.7105	437.9602	528.1186	493.8323
Akaike AIC	-2.853818	-2.502660	-3.023807	-2.825620
Schwarz SC	-2.798233	-2.447076	-2.968223	-2.770036
Mean dependent	0.002733	0.002682	0.003296	0.001814
S.D. dependent	0.062258	0.070456	0.054999	0.062555

Determinant resid covariance (dof adj.)	4.22E-11
Determinant resid covariance	3.98E-11
Log likelihood	2178.900
Akaike information criterion	-12.47919
Schwarz criterion	-12.25686

Stability Conditions

- Similar to the univariate AR we can investigate the presence of unit roots in the VAR(p).

EXAMPLE

$$y_{1,t} = a_{11} y_{1,t-1} + a_{12} y_{2,t-1} + \varepsilon_{1,t}$$
$$y_{2,t} = a_{21} y_{1,t-1} + a_{22} y_{2,t-1} + \varepsilon_{2,t}$$

Equivalent to

$$y_{1,t} - a_{11} y_{1,t-1} - a_{12} y_{2,t-1} = \varepsilon_{1,t}$$
$$y_{2,t} - a_{21} y_{1,t-1} - a_{22} y_{2,t-1} = \varepsilon_{2,t}$$

Stability Conditions - II

$$\begin{bmatrix} (1-a_{11}L) & -a_{12}L \\ -a_{21}L & (1-a_{22}L) \end{bmatrix} \begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}$$

$$\begin{vmatrix} (1-a_{11}Z) & -a_{12}Z \\ -a_{21}Z & (1-a_{22}Z) \end{vmatrix} = 0$$

- When the roots of the polynomial $Z > 1$ are outside the unit circle the time series converge.

Cointegration

- In the face of non-stationary series with a unit root (e.g., a random walk series), first differencing appears to provide the appropriate solution to ensuring series are weakly stationary.

- First differencing, however, does possess a major limitation as it means we lose useful long-run information.

- Economists are invariably interested in examining long-run steady-state relationships.

- The theory of co-integration allows us to integrate short-run dynamics with long-run equilibrium.

Cointegration: Long-run equilibrium

- If an economic time series [y_t] follows a random walk, its first difference forms a stationary series.

- As we have already encountered, y_t is integrated of order one, and has to be differenced once in order to achieve stationarity. This is usually expressed as $y_t \sim I(1)$.

- We have already analyzed dangers of generating a spurious regression by regressing one I(1) time series on another.

- However, there is a situation when such a regression does not yield a spurious relationship. This was the case when two I(1) series were cointegrated. ***Cointegration is shown to be an exception to a general rule***.

Cointegration: Long-run equilibrium

- The general rule is that if two series, y_t and x_t are both I(1), then any linear combination of the two series will yield a series which is also I(1).

- The exception to this general rule is when a linear combination of two (or more) series are integrated of a lower order. In this case the common stochastic trends have cancelled out yielding a series that is stationary.

- Thus, in the case of the regression of two I(1) series, we do not obtain something that is spurious but something that may be relatively sensible in economic terms.

Cointegration: An Intuition

- Assume we have two variables y_t and x_t such that:

$$y_t = \mu_{yt} + e_{yt} \qquad\qquad x_t = \mu_{xt} + e_{xt}$$

- where μ_{it} is a random walk process representing the trend in variable i at time t and e_{it} is the stationary component of variable i at time t.
- If y_t and x_t are cointegrated there must be non-zero values of β_1 and β_2 such that the linear combination $\beta_1 y_t + \beta_2 x_t$ is stationary:

$$\begin{aligned}\beta_1 y_t + \beta_2 x_t &= \beta_1[\mu_{yt} + e_{yt}] + \beta_2[\mu_{xt} + e_{xt}] \\ &= [\beta_1\mu_{yt} + \beta_2\mu_{xt}] + [\beta_1 e_{yt} + \beta_2 e_{xt}]\end{aligned}$$

- For $\beta_1 y_t + \beta_2 x_t$ to be stationary, the term $[\beta_1\mu_{yt} + \beta_2\mu_{xt}]$ must vanish.

Cointegrating Vector

- In this example, if we set $\mu_{yt} = \mu_{xt} = \mu_t$, and $\beta_1 = 1$ and $\beta_2 = -1$, then we can see that

$$\beta_1 y_t + \beta_2 x_t = [\beta_1\mu_t + \beta_2\mu_t] + [\beta_1 e_{yt} + \beta_2 e_{xt}] \quad \text{becomes}$$
$$y_t - x_t = [\mu_t - \mu_t] + [e_{yt} - e_{xt}] = [e_{yt} - e_{xt}]$$

- which is stationary. The parameter values for β_1 and β_2 are parameters contained in what is called the cointegrating vector.

- The essential insight here is that these parameters must be such that they purge or remove the stochastic trend from the linear combination.

- In this illustrative case $\beta_1 = 1$ and $\beta_2 = -1$ did just that.

Cointegration and Vector Error Correction Models (VECM)

- Variables that are cointegrated can be represented by a special kind of VAR - A Vector Error Correction Model
- Two Variable VECM:

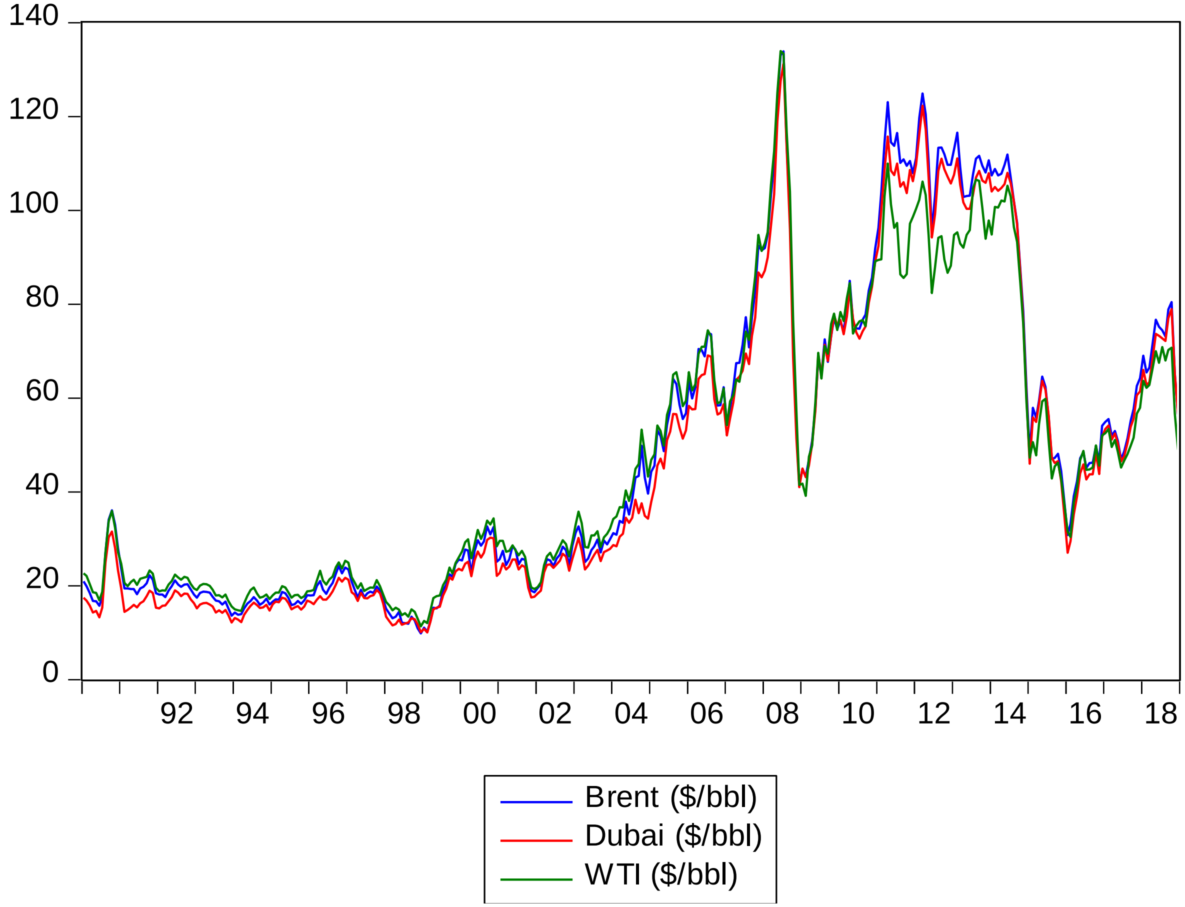
$$\Delta X_t = a_0 + a_1 \Delta X_{t-1} + b_1 \Delta Z_{t-1} + f_1[gX_{t-1} + hZ_{t-1}] + e_{1t}$$
$$\Delta Z_t = c_0 + c_1 \Delta X_{t-1} + d_1 \Delta Z_{t-1} + f_2[gX_{t-1} + hZ_{t-1}] + e_{2t}$$

- $gX_{t-1} + hZ_{t-1}$ is called the cointegrating vector (the linear combination of X and Z that is stationary)
- f_1 and f_2 are called the error correction coefficients
- If f_1 and f_2 are both equal to zero, then the VECM is just an ordinary VAR in first differences of X and Z

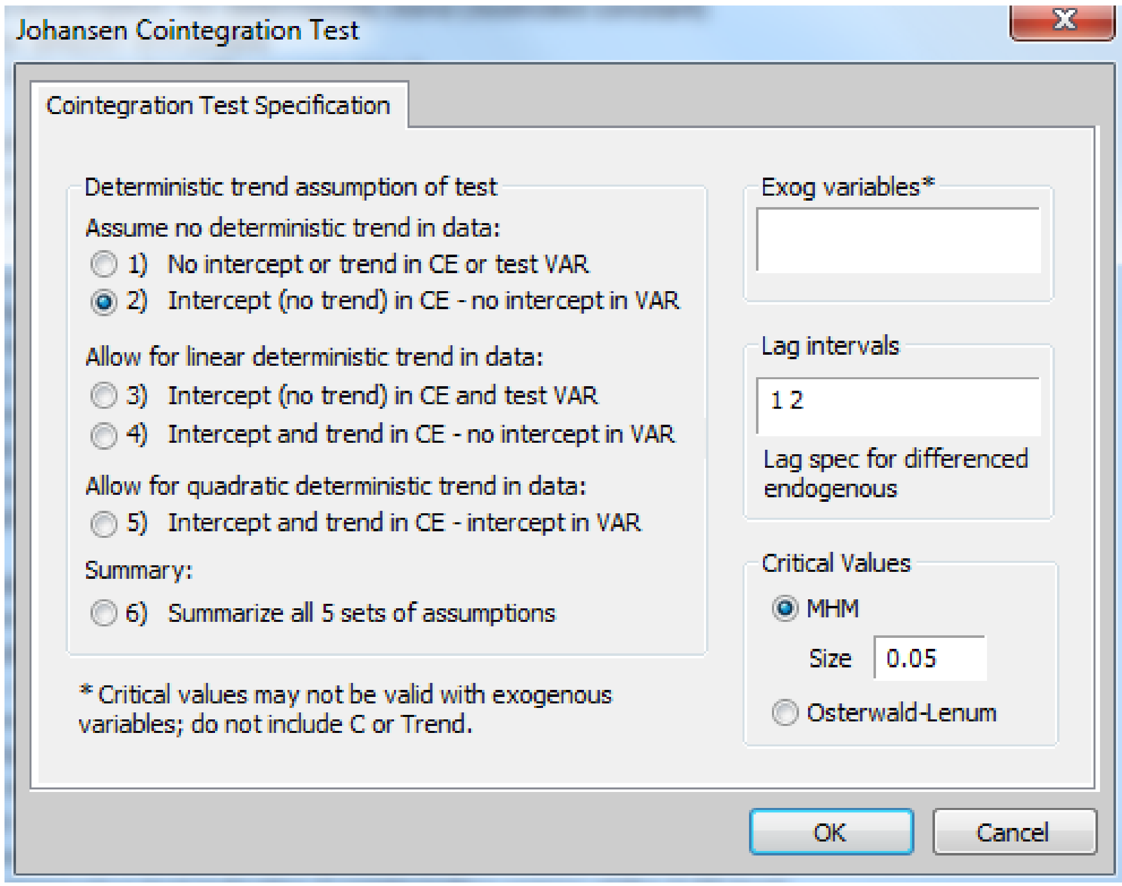
Cointegration and VECM's

- Advantage of VECM specification
  - VAR in differences ignores the information that the levels of the variables cannot wander aimlessly, but are tied together in the long run.
  - *It may* be able to improve forecasts over intermediate to long-run over just VAR in differences.

Cointegration – Crude Prices Example



Cointegration – Johansen Cointegration Test



Johansen Cointegration Test

Sample (adjusted): 1990M04 2018M12
Included observations: 345 after adjustments
Trend assumption: No deterministic trend (restricted constant)
Series: BRENT DUBAI WTI
Lags interval (in first differences): 1 to 2

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.079949	51.17811	35.19275	0.0005
At most 1 *	0.051084	22.43041	20.26184	0.0248
At most 2	0.012502	4.340372	9.164546	0.3638

Trace test indicates 2 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

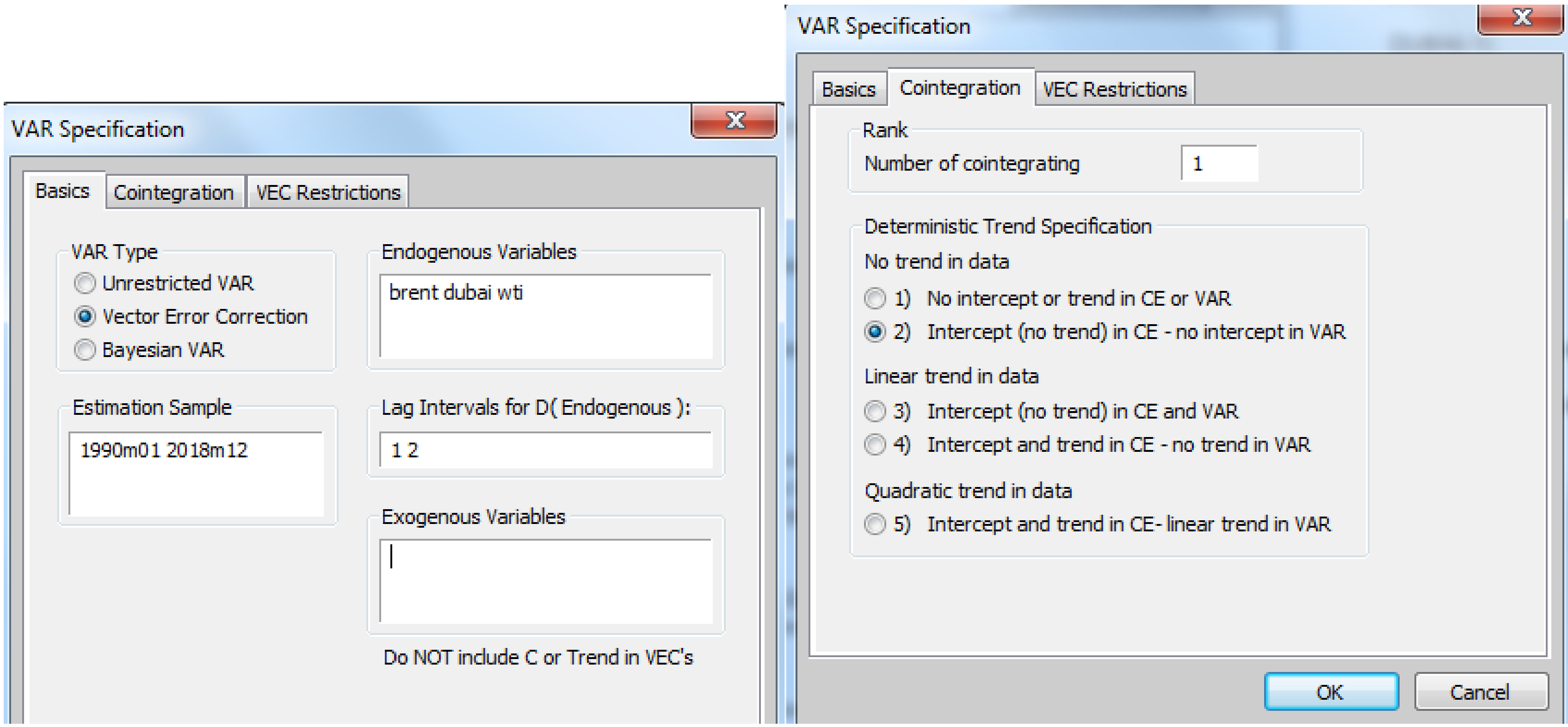
Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.079949	28.74770	22.29962	0.0055
At most 1 *	0.051084	18.09004	15.89210	0.0222
At most 2	0.012502	4.340372	9.164546	0.3638

Max-eigenvalue test indicates 2 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

There are 1 or 2 Cointegrating Equations!!

Cointegration – Define the Parameters



Cointegration – Output - CI

Vector Error Correction Estimates

Sample (adjusted): 1990M04 2018M12
Included observations: 345 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1
BRENT(-1)	1.000000
DUBAI(-1)	-0.829003
	(0.05629)
	[-14.7283]
WTI(-1)	-0.221292
	(0.06207)
	[-3.56495]
C	0.215165
	(0.57798)
	[0.37227]

Cointegration – Output – System of Equations

Error Correction:	D(BRENT)	D(DUBAI)	D(WTI)
CointEq1	0.231939	0.362532	0.381708
	(0.16360)	(0.14968)	(0.15873)
	[1.41771]	[2.42197]	[2.40471]
D(BRENT(-1))	-0.302142	-0.244811	-0.379635
	(0.26359)	(0.24117)	(0.25575)
	[-1.14625]	[-1.01510]	[-1.48441]
D(BRENT(-2))	-0.890368	-0.720756	-0.757957
	(0.25682)	(0.23497)	(0.24917)
	[-3.46695]	[-3.06744]	[-3.04187]
D(DUBAI(-1))	0.702133	0.661004	0.591352
	(0.25517)	(0.23346)	(0.24758)
	[2.75163]	[2.83129]	[2.38855]
D(DUBAI(-2))	0.819074	0.615279	0.829986
	(0.24438)	(0.22359)	(0.23711)
	[3.35164]	[2.75180]	[3.50045]
D(WTI(-1))	0.039042	0.033012	0.181104
	(0.14368)	(0.13146)	(0.13941)
	[0.27172]	[0.25113]	[1.29912]
D(WTI(-2))	0.106846	0.141043	0.034671
	(0.14401)	(0.13176)	(0.13972)
	[0.74196]	[1.07049]	[0.24815]

Cointegration – Output – Evaluation

R-squared	0.194526	0.231140	0.199878
Adj. R-squared	0.180228	0.217491	0.185675
Sum sq. resids	6081.149	5090.570	5724.648
S.E. equation	4.241646	3.880832	4.115438
F-statistic	13.60479	16.93530	14.07258
Log likelihood	-984.5061	-953.8349	-974.0849
Akaike AIC	5.747861	5.570058	5.687449
Schwarz SC	5.825846	5.648042	5.765434
Mean dependent	0.110174	0.117884	0.082754
S.D. dependent	4.684765	4.387128	4.560548

Determinant resid covariance (dof adj.)	35.54587
Determinant resid covariance	33.42582
Log likelihood	-2073.961
Akaike information criterion	12.16789
Schwarz criterion	12.44640

Summary

- System of Multiple Equations
  - Try to estimate a model with multiple equations that are somehow linked together.

- Vector Autoregression
  - VAR uses lagged variables to help describe current and future movements of “endogenous” (dependent) variables.
  - This makes VAR useful for **forecasting** and **simulation**

- Cointegration
  - The theory of co-integration allows us to integrate short-run dynamics with long-run equilibrium.

References

Enders, W., 1995, *Applied Econometric Time Series*, New York: Wiley. Chapters 5, 6.

Franses, P.H., 1998, *Time Series Models for Business and Economic Forecasting*, Cambridge University Press. Chapters 9, 10.

Makridakis, S., S.C. Wheelwright and R.J. Hyndman, 1998, *Forecasting Methods and Applications* (3rd ed.), New York: Wiley. Chapter 8.3, 8.4