DSC5211C QUANTITATIVE RISK MANAGEMENT

SESSION 12

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Project Management

Objectives

- Review the Critical Path Methods in Project Management
- Review the Program Evaluation and Review Technique (PERT)
- Re-write the PERT as a Linear Program
- Incorporate CV@R to Analyze Project Management.

Reading:

Stevenson, W. J., and S. C. Chuong. Operations Management. McGraw Hill, 2012, **Ch. 17**.

Cachon, G., and C. Terwiesch. Matching Supply with Demand. An Introduction to Operations Management. McGraw Hill, 2013, **Ch. 5**.

What is a Project

- A Project it is a unique operational effort, a special type of process that is performed infrequently, with a clear specification of the desired objective.

- This endeavour is composed of multiple activities, with defined precedent relationships, with a specific period of completion.

Project Scheduling Techniques

- Gantt Chart
    - It is chart with a bar for each project activity associated with time.
    - It provides a visual display of a project schedule

- Critical Path Method (CPM)

- Program Evaluation and Review Technique (PERT)

CPM vs. PERT

- Critical Path Method (CPM):
  - We need to find the longest path in the network of activities.
  - The Critical Path represents the minimum project completion time.
  - It estimates the slack (possible delay) associated with non-critical activities.
  - The duration of activities is ***assumed to be known***.

- Program Evaluation and Review Technique (PERT):
  - It was developed to manage the Polaris missile project.
  - It aims to find the longest path in the network of activities.
  - It estimates the slack (possible delay) associated with non-critical activities.
  - The duration of activities is ***assumed uncertain***.

Example

- NUS -Singapore is organizing a Forum on Global Supply Chain Management.

- The Forum brings together top managers and academics in the area of Supply Chain Risk Management.

- In order to put together the Forum the following activities will need to be completed:

Activity	Predecessors	Duration (Months)
A - Define topic and main goals	--	1
B - Budgeting	--	2
C - Find Speakers	A	5
D - Find Local Partners	A	7
E - Find Sponsors	B, C	4
F - Advertising	D, E	2
G - Handle Registrations	D, E	3
H - Coordinate Event	F, G	0.25

Drawing a Project Network – Activity on Arc

- Node 1 represents the start of the project. An arc should lead from node 1 to represent each activity that has no predecessors.

- A node (called the finish node) representing the completion of the project should be included in the network.

- Number the nodes in the network so that the node representing the completion of an activity always has a larger number than the node representing the beginning of an activity.

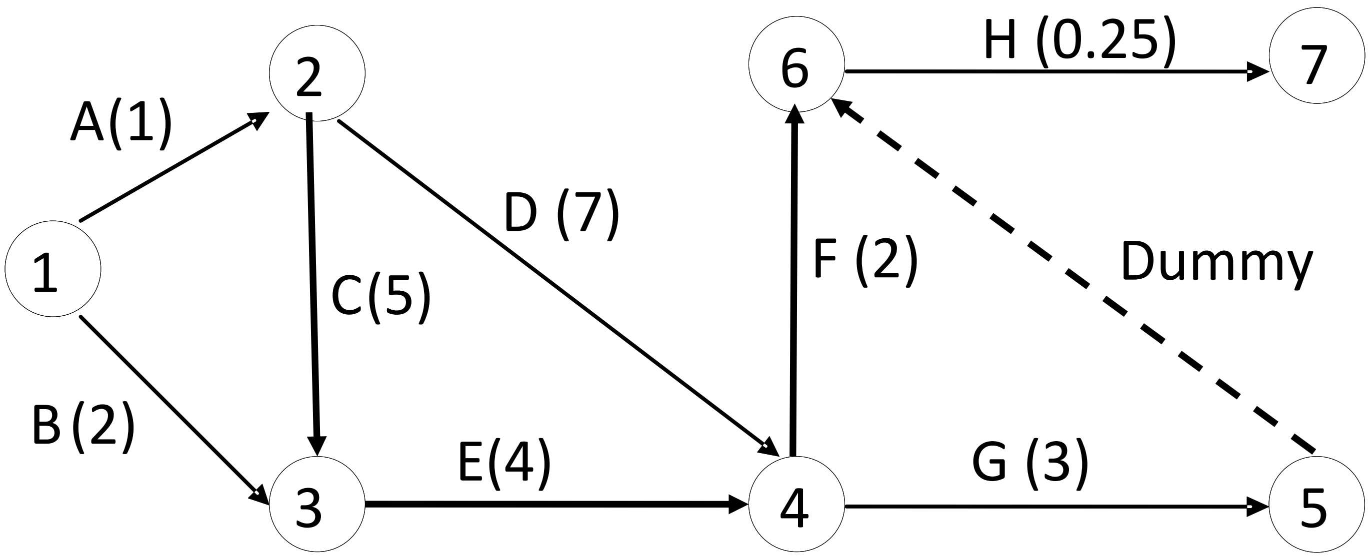
Drawing a Project Network - II

- An activity should not be represented by more than one arc in the network.

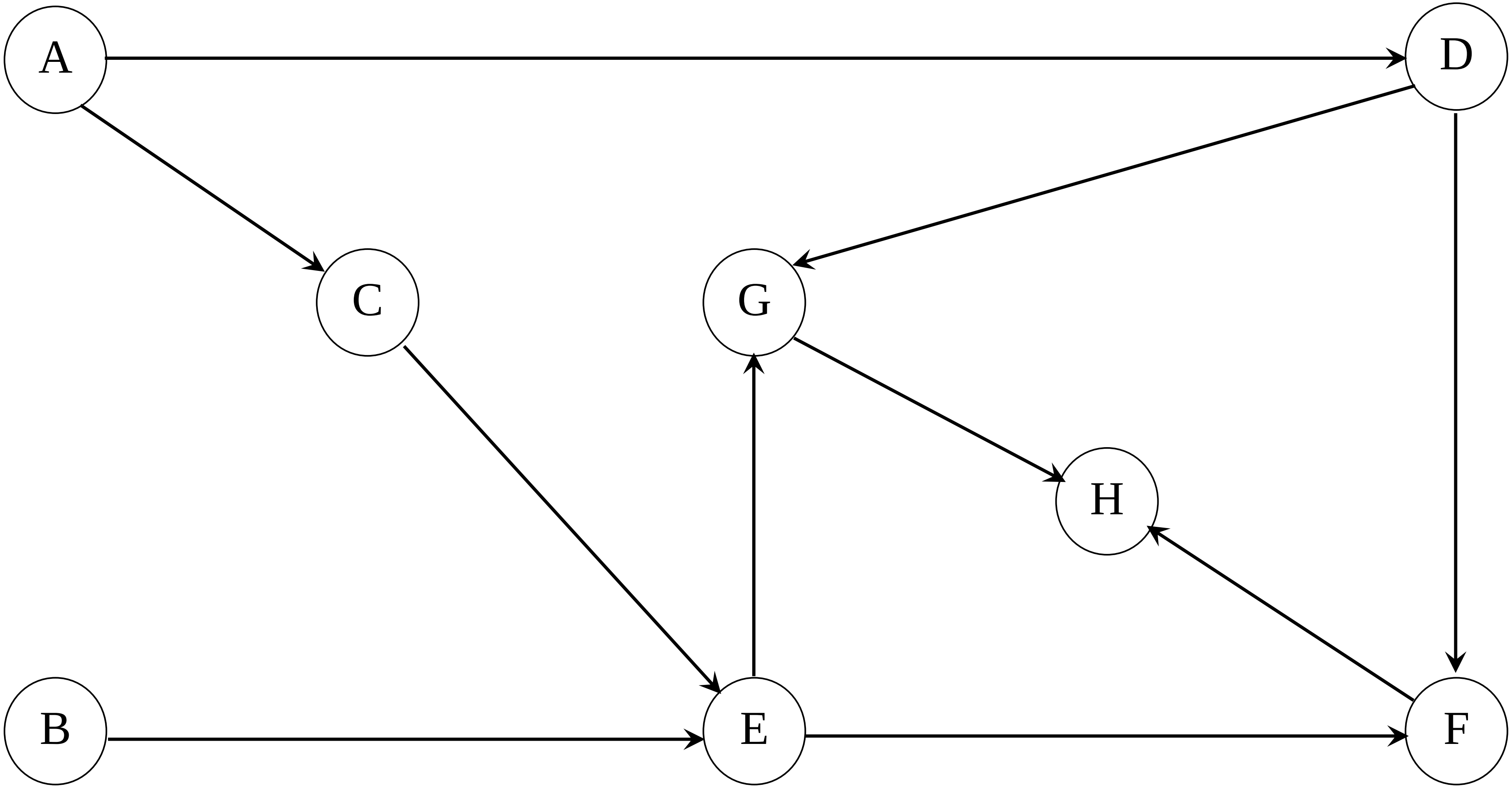
- Two nodes can be connected by at most one arc.

- Note: to avoid violating the last two rules it is sometimes necessary to use a **dummy activity** that takes zero time.

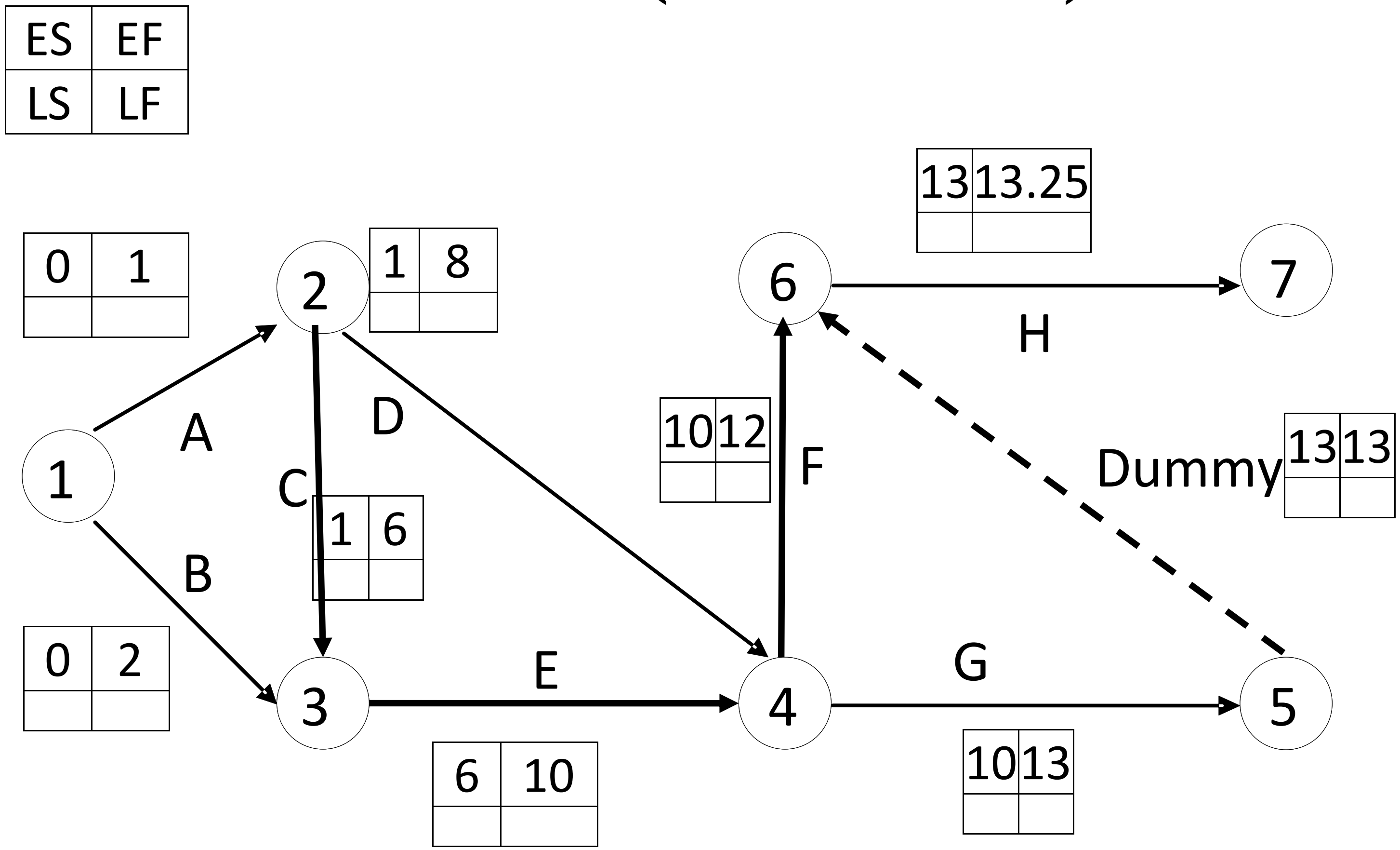
Draw the Project Diagram – Activity on Arc



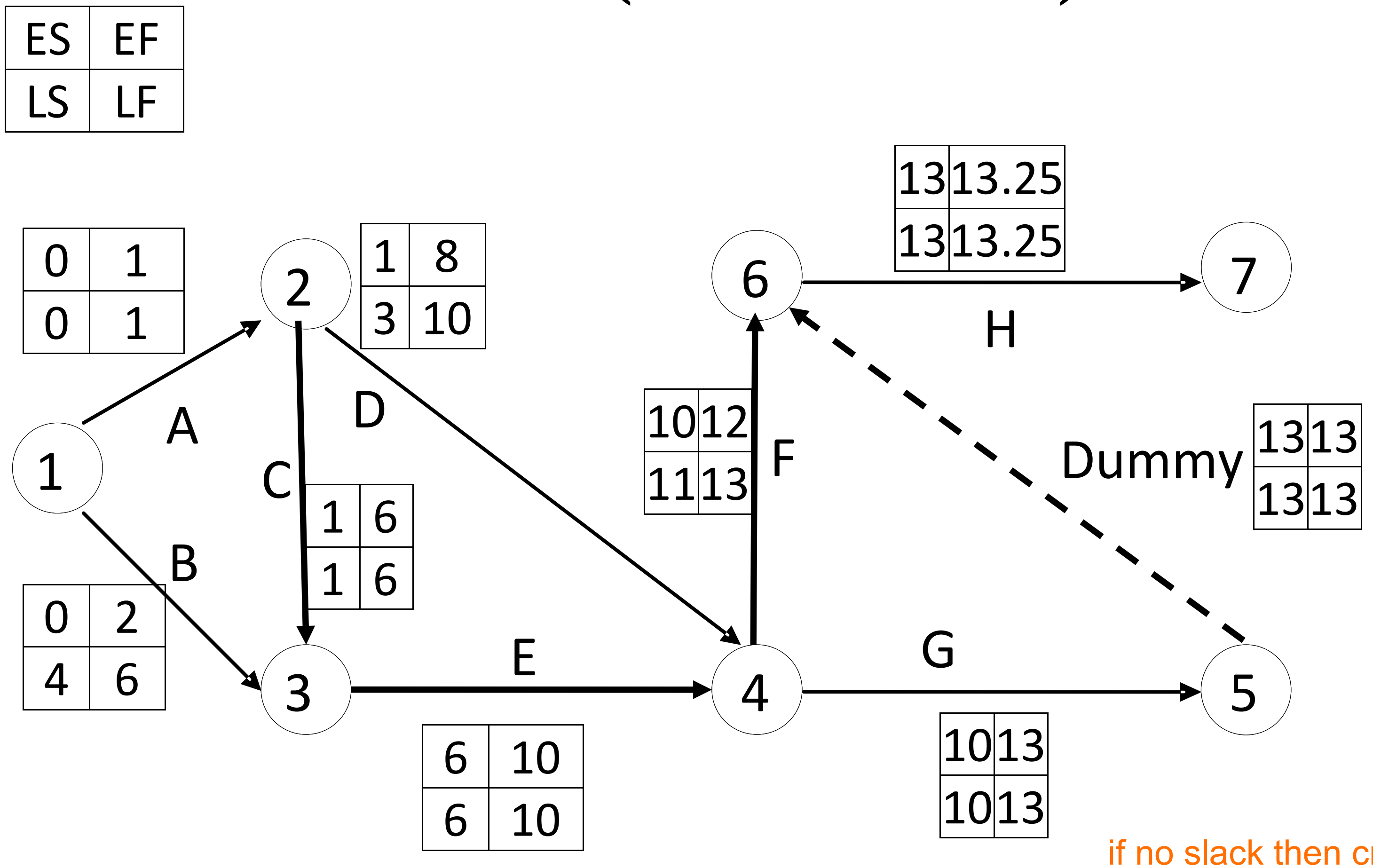
Draw the Project Diagram – Activity on Node



CPM – Early Start and Finish Times (Forward Pass)



CPM – Latest Start and Finish Times (Backward Pass)



if no slack then critical activity

CPM – Activity Slack Times

Slack = LS-ES = LF - EF

Activity	ES	EF	LS	LF	Slack
A - Define topic and main goals	0	1	0	1	0
B - Budgeting	0	2	4	6	4
C - Find Speakers	1	6	1	6	0
D - Find Local Partners	1	8	3	10	2
E - Find Sponsors	6	10	6	10	0
F - Advertising	10	12	11	13	1
G - Handle Registrations	10	13	10	13	0
H - Coordinate Event	13	13.25	13	13.25	0

- Critical Path: A, C, E, G, H
- Activities with zero slack.

PERT – Uncertain Activity Times

the time that

Activity	Pred.	Optimistic (a)	Most likely (m)	Pessimistic (b)
A - Define topic and main goals	--	0	1	1
B - Budgeting	--	1	2	4
C - Find Speakers	A	3	5	6
D - Find Local Partners	A	1	7	8
E - Find Sponsors	B, C	3	4	6
F - Advertising	D, E	1	2	4
G - Handle Registrations	D, E	1	3	3
H - Coordinate Event	F, G	0.2	0.25	0.3

PERT – Beta Distribution

- The time of each activity follows a Beta Distribution with

$$E = \frac{a + 4m + b}{6}$$

the average, 6 comes from 1+4+1

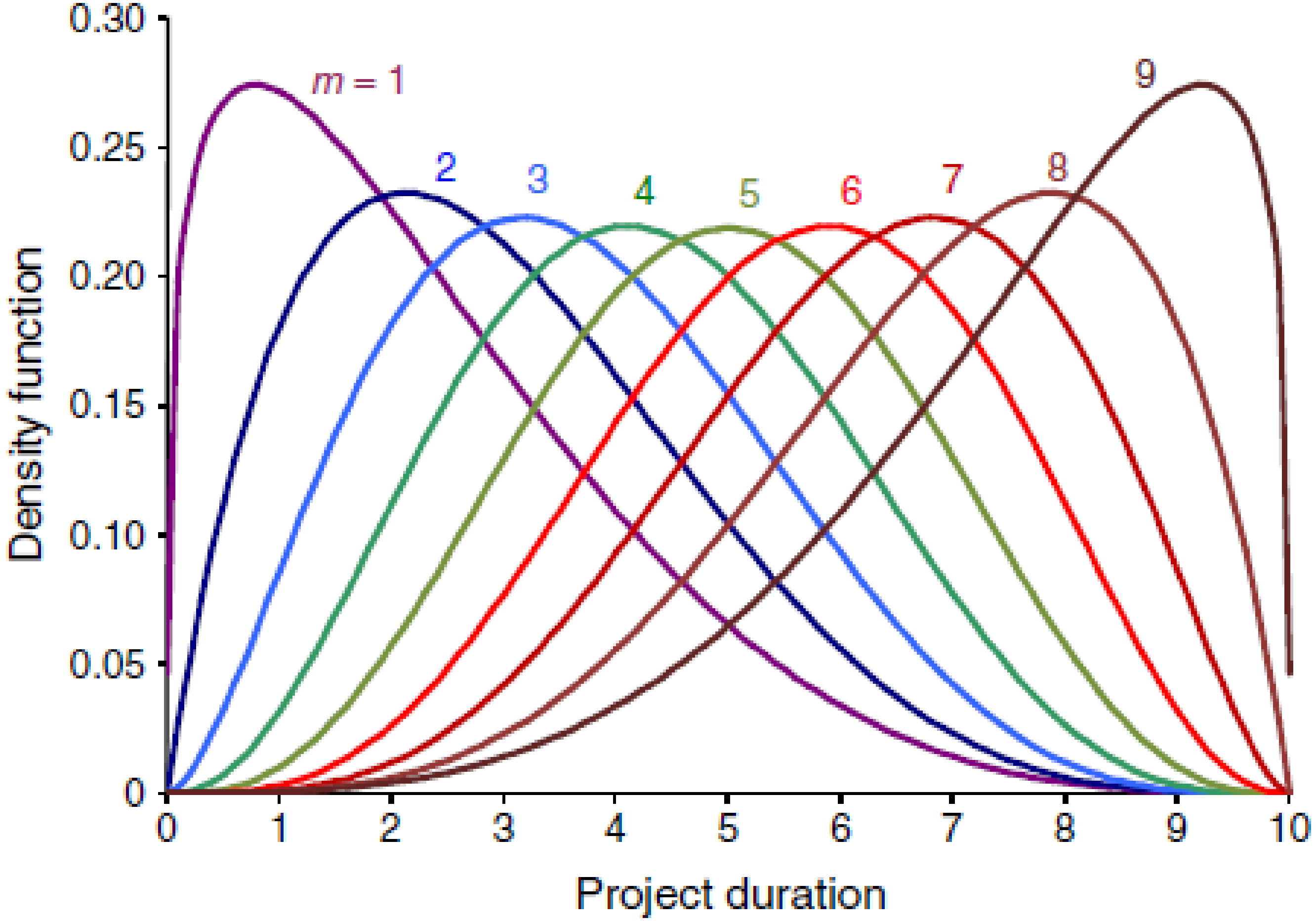
$$\sigma^2 = \left(\frac{b - a}{6}\right)^2$$

the variance

E – Expected activity time

σ^2- Variance of the activity time

PERT-Beta Function for *m* = 1…9, *a* = 0 and *b* = 10



just show the shape change with the change of m

PERT – Expected Times and Variances per Activity

Activity	Pred.	*a*	*m*	*b*	*E*	*Var.*
A - Define topic and main goals	--	0	1	1	0.833	0.028
B - Budgeting	--	1	2	4	2.167	0.25
C - Find Speakers	A	3	5	6	4.833	0.25
D - Find Local Partners	A	1	7	8	6.167	1.361
E - Find Sponsors	B, C	3	4	6	4.167	0.25
F - Advertising	D, E	1	2	4	2.167	0.25
G - Handle Registrations	D, E	1	3	3	2.667	0.111
H - Coordinate Event	F, G	0.2	0.25	0.3	0.25	0.0003

calculate by the formula
last page

Expected Time and Variance for the Critical Path

- Critical Path: A, C, E, G, H

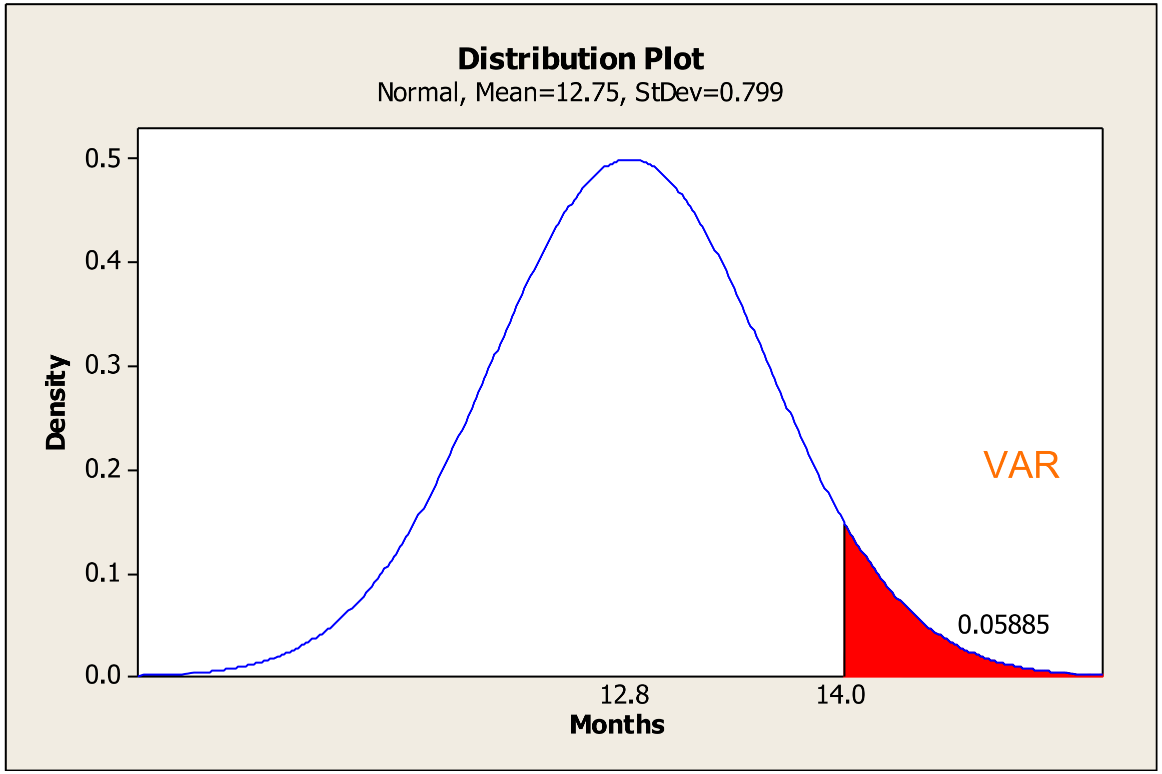
- A path of activities follows (approx.) a Normal Distribution (the larger the number of activities in the path the better the approximation.)

E (critical path) = 0.833 + 4.833 + 4.167 + 2.667 + 0.25 = 12.75 months

Var (critical path) = 0.028 + 0.25 + 0.25 + 0.11 + 0.0003 = 0.639 months^2

Probabilistic Network Analysis

- Calculate the probability that the project is completed within 14 months.



Probabilistic Network Analysis

$$Z = \frac{\text{Target} - E}{\sigma}$$

Assumption: normal distribution

$$Z = \frac{14 - 12.75}{\sqrt{0.639}} = 1.56$$

From which it follows, from the standard normal distribution, that

$$P(X \geq 14) = 0.0588$$
$$P(X \leq 14) = 0.9411$$

A Linear Program Formulation of the PERT

- Let the duration of activity *j* in scenario *s*, following a Pert-Beta distribution, with parameters *a*, *m*, *b*, be such that:

$$d_{js} \sim Beta(Pert\_\alpha, Pert\_\beta)$$

$$Pert\_\alpha = \frac{2(b + 4m - 5a)}{3(b-a)}\left[1 + 4 \times \frac{(m-a)(b-m)}{(b-a)^2}\right]$$

$$Pert\_\beta = \frac{2(5b - 4m - 5a)}{3(b-a)}\left[1 + 4 \times \frac{(m-a)(b-m)}{(b-a)^2}\right]$$

A Linear Program for PERT – Connections & Variables

- Let c_{ij} be a parameter representing a connection from predecessor activity i to activity j.

- $c_{ij} = 1$ if i is a predecessor of j

- $c_{ij} = 0$ if i is NOT a predecessor of j.

- Let x_{js} be a variable representing the finishing time of activity j in scenario s.

- Let y_s represent the finishing time of an End of Project Activity in scenario s.

A Linear Program: Connections & Variables

- Total duration in scenario *s*.

$$\min_{y_s, x_{js}} \; y_s$$ minimize the time of all activities

Subject to

$x_{js} \geq d_{js} + x_{is}$, for all pair i, j such that $c_{ij} = 1$

$y_s \geq x_{js}$, the last activity will be binding for all j

$x_{js} \geq 0$, for all j.

A Linear Program: Worst-Case Duration

$$\min_{y, x_{js}} \; y$$ the cost function

Subject to

$x_{js} \geq d_{js} + x_{is}$, for all i, j such that $c_{ij} = 1$**, and *s***

$y \geq x_{js}$, for all j **and *s***

the longest possible time to finish the project, the worst case

$x_{js} \geq 0$, for all j **and *s*.**

A Linear Program to Minimize V@R and CV@R.

$$\min_{z,x,u} \; u + \frac{\sum_s z_s}{(1-\beta)S}$$

CVAR

Subject to

$$z_s \geq c\left(x_s\right) - u$$

tails

for all s

$$x_s \in X_s, z_s \geq 0$$

for all s.

Extending the PERT Model to Include V@R and CV@R

$$\min_{z, x_{js}, y_s, u} \quad u + \frac{\sum_s z_s}{(1-\beta)S}$$

Subject to

$z_s \geq y_s - u$, for all s

$z_s \geq 0$, for all s

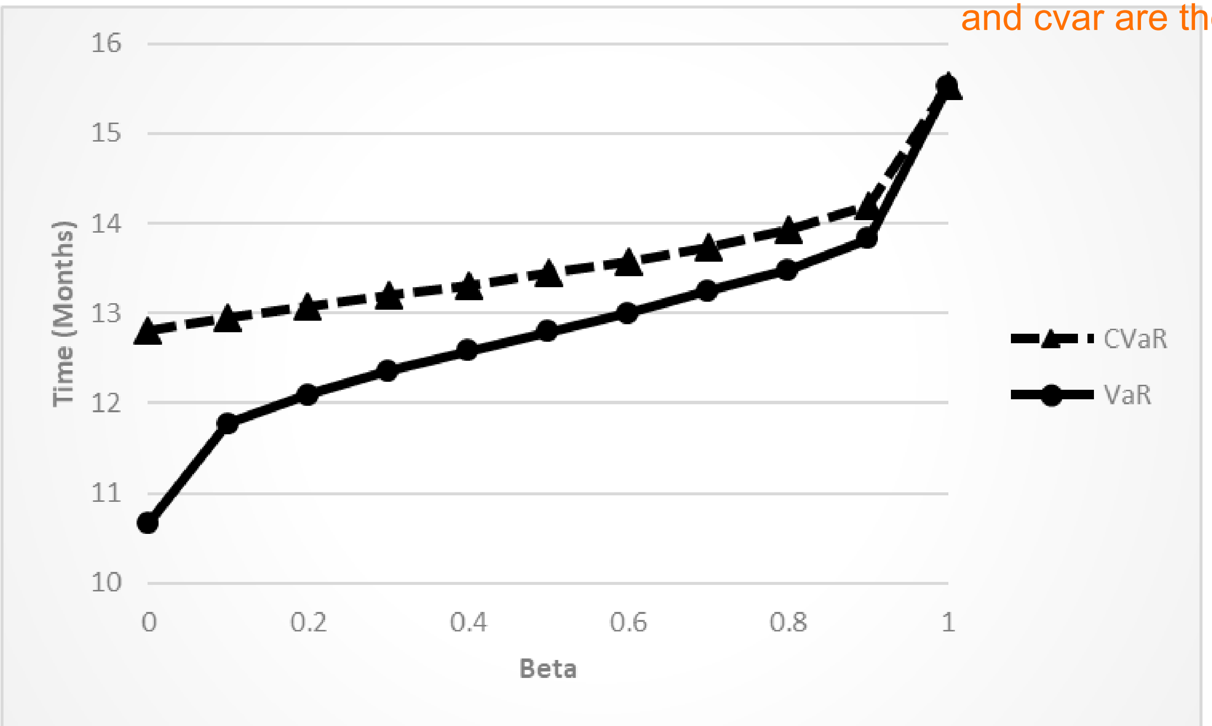
$x_{js} \geq d_{js} + x_{is}$, for all i, j such that $c_{ij} = 1$, and s

$y_s \geq x_{js}$, for all j and s

$x_{js} \geq 0$, for all j and s.

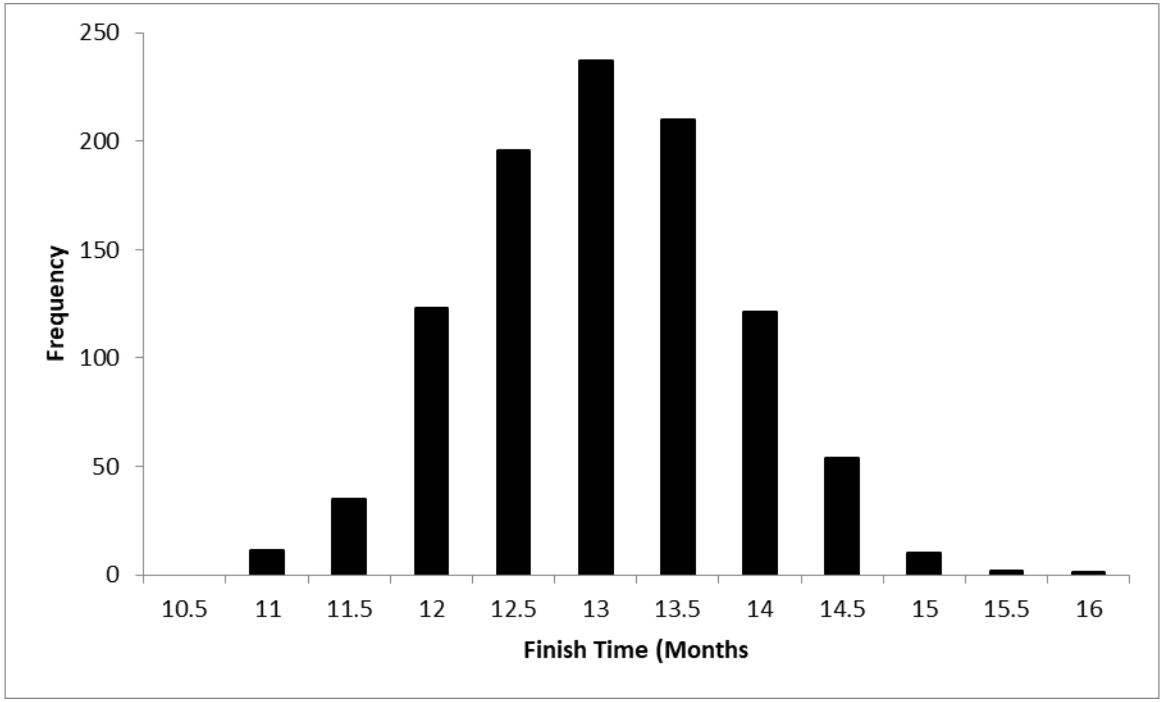
Example: PERT - CV@R

under the worst case the var and cvar are the same

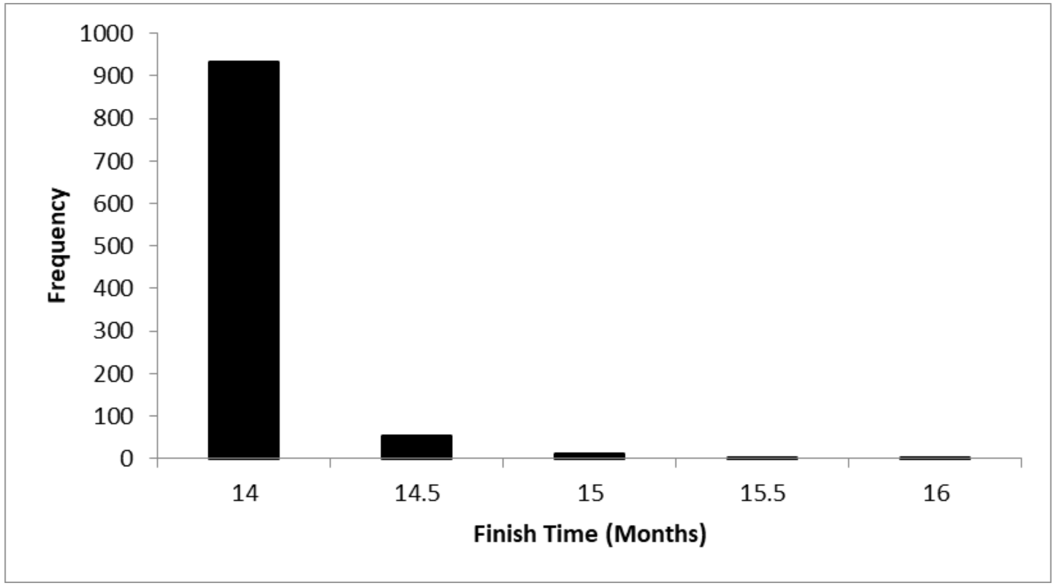


when the beta is 0, the tails has (the number of scenarios) points

Example: PERT – Risk Neutral – Finish Time



Example: PERT – Beta 0.9 – Finish Time



- Notice that this distribution is just the tail (10%) of the Risk-neutral one, as I have no decision variables to minimize risk.

Summary

- We have discussed the CPM and applied and discussed its advantages and limitations.
- We have analyzed the PERT which allows us to incorporate uncertainty and risk analysis in project scheduling.
- We have proposed a Linear Programming representation of the PERT problem to include CV@R.
    - We can better observe the distribution properties of the finish time.
    - We can analyse the finish time ***without assuming it is normal distributed***.

Final Project

- It is an open-book, computer-based, in-class project.

- You will be allocated a lecture theatre in BIZ 1: level 2 for your project.

- You will be given a data set (and a problem) to analyse:
    - The problem is randomly allocated to you.
    - *All data sets used in the final project were already given to you during the course.*
    - You need to submit all your files (PDF of the power point presentation, computer code, excel files) as a ZIP file in IVLE.

- You are asked to perform a risk analysis on the data you are attributed.
    - Time Series
    - CV@R optimization.

Final Project - Preparation

- Plan your presentation:
  - Some slides can be prepared at home, as they are standard
  - Have a template ready, with the intended titles of each slide.

- Make sure that the software you intend to use is working.
  - Have a template ready for the possible projects.

- Analyse the different data sets used in the course. Bring you analysis with you.

- Do not take risks during the 3 hours of the in-class project.

- Be innovative: prepare at home to innovate in your analysis! (There are points awarded to innovative solutions.)