Workshop 10: Worst-Case Analysis

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Task 2

In workshop 6 we have analyzed the feed mix problem. Download the file "feed_mix.gms" and adapt it to performance risk analysis over the feeding cost. The price variances and correlation matrix are in the gams file.

A) Include the code to generate correlated feed prices. Assume Barley as the "indexfeed".

```
----      90 PARAMETER a  stockastic returns

                   s1          s2          s3          s4          s5

barley         25.260      24.373      24.030      26.375      26.370
oats           27.028      26.463      26.697      28.316      28.417
sesame         39.968      39.396      38.639      41.499      40.815
grnd-meal      41.408      40.472      40.430      42.239      41.832

        +          s6          s7          s8          s9         s10

barley         25.287      23.379      24.924      25.149      26.137
oats           27.584      25.186      26.842      27.554      27.740
sesame         40.002      38.229      39.067      39.608      40.755
grnd-meal      40.840      39.279      40.483      41.454      41.732

        +         s11         s12         s13         s14         s15

barley         24.132      23.960      25.509      24.410      22.215
oats           26.042      26.126      27.617      26.191      24.588
sesame         38.666      38.279      40.232      38.932      36.204
grnd-meal      40.105      39.889      40.928      40.635      38.195
```

Figure 1 Correlated Feed Prices

The above graph shows the price for 4 kinds of commodities in 15 iterations.

[correction: sum((f,s), prot(s,f)*x(f))/card(s) =g= req(n)]

B) Modify the constraints to compute the CVAR and compute the optimal policy.

```
----     101 VARIABLE x.L  feed mix (pct)

barley     0.774,     grnd-meal 0.226


----     101 VARIABLE var.L                =      -25.827  value at Risk
             VARIABLE cvar.L               =      -25.827  conditional value at
                                                           risk
```

Figure 2 CVaR Result

As the graph shown above, the optimal policy is to spend 77.4% budget buying *barley* and 22.6% buying *grnd_meal*. The corresponding conditional value at risk is -25.827.

C) Modify the constraints to compute the Worst-Case analysis and compute the optimal policy.

```
----     97 VARIABLE x.L   feed mix  (pct)

barley     0.774,     grnd-meal 0.226


----     97 VARIABLE cost.L                    =        28.155  total cost per ton
            VARIABLE mu.L                      =        30.212  the cost
```

Figure 3 Worst-Case Analysis

We modified the constraint and set new variable *mu* greater or equal to two kinds of cost. The optimal policy stays the same with part(b). The objective min (max *mu*) in the Worst-Case analysis is 30.212.

D) Compare the results obtained from your analysis in B), C) and the risk-neutral policy (obtained from the original program).

```
----     69 VARIABLE x.L   feed mix  (pct)

barley     0.774,     grnd-meal 0.226


----     69 VARIABLE cost.L                    =        28.155  total cost per ton
```

Figure 4 Results Comparison

The optimal policy keeps the same while we modified the constraint. However, CVaR in part(b) is lower than the ones in the other two parts.

Eatra guidelines:

1. Relax the requirement constraint so that is not binding. req(n) =0.

We also change the requirements of protein from 21 to 30 and fats from 5 to 50. Then we increase the scenario from 100 to 1000. The final result is shown below, here we will use 5.3% to buy barley and the rest to buy grnd-meal.

```
----     90 VARIABLE x.L   feed mix  (pct)

barley     0.053,     grnd-meal 0.947


----     90 VARIABLE cost.L                    =        39.661  total cost per ton
            VARIABLE mu.L                      =        42.827  tail profit in scenar
                                                                io s
```

Figure 5 Results Comparison

From the result shown above, the tail profit in the scenario (mu) increases to 42.827.

2. Change the correlation matrix to have lower correlations between prices.

Here is the new correlation matrix we have.

Table correlation				
	barley	cats	sesame	grnd-meal
barley	1.0	0.2	0.2	0.2
cats	0.2	1.0	0.2	0.2
sesame	0.2	0.2	1.0	0.2
grnd-meal	0.2	0.2	0.2	1.0

```
----      90 VARIABLE x.L  feed mix (pct)

barley    0.774,    grnd-meal 0.226


----      90 VARIABLE cost.L                  =       28.155  total cost per ton
             VARIABLE mu.L                    =       29.931  tail profit in scenar
                                                              io s
```

```
----      101 VARIABLE x.L  feed mix (pct)

barley    0.774,    grnd-meal 0.226


----      101 VARIABLE var.L                  =      -26.092  value at Risk
              VARIABLE cvar.L                 =      -26.092  conditional value at
                                                              risk
```

Figure 6 Results Comparison (Worst case & CV@R)

After changing the correlation, CV@R sign also changes.

<u>3. Try different betas: 0.9, 0.95, 0.99, 0.999.</u>

```
----      101 VARIABLE x.L  feed mix (pct)

barley    0.774,    grnd-meal 0.226


----      101 VARIABLE var.L                  =      -25.827  value at Risk
              VARIABLE cvar.L                 =      -25.827  conditional value at
                                                              risk
```

```
----      101 VARIABLE x.L  feed mix (pct)

barley    0.774,    grnd-meal 0.226


----      101 VARIABLE var.L                  =      -25.827  value at Risk
              VARIABLE cvar.L                 =      -25.827  conditional value at
                                                              risk
```

Figure 7 Different Betas (0.95. 0.99, 0.999)

Result Comparison				
beta = 0.95	strategy: barley 0.774 grnd-meal 0.226			
	V@R	-25.827	CV@R	-25.827
beta = 0.99	strategy: barley 0.774 grnd-meal 0.226			
	V@R	-25.827	CV@R	-25.827
beta = 0.999	strategy: barley 0.774 grnd-meal 0.226			
	V@R	-25.827	CV@R	-25.827

The optimal solution and CV@R do not change with the change of different betas. The reason maybe that we reset the requirement and they are binding.

Task 1

You were asked to re-analyze the portfolio management problem by using worst-case analysis. What is the optimal policy? How does it compare with optimal policy derived for CV@R optimization? What is the respective expected profit and CV@aR?

```
----     79 VARIABLE x.L  fraction of portfolio invested in asset i

spy 0.494,    mcd 0.158,    qqq 0.276,    tlt 0.071


----     79 VARIABLE e_return.L              =        0.014  expected return of th
                                                             e portfolio
            VARIABLE mu.L                    =        0.055  cost
```

Figure 1 Results from Worst Case

The optimal policy is to invest 49.4% on spy, 15.8% on mcd, 27.6% on qqq and 7.1% on tlt, the worst case expected return is 5.5%

```
----     82 VARIABLE x.L  fraction of portfolio invested in asset i

spy  0.481,    aapl 0.073,    mcd  0.176,    qqq  0.268,    tlt  0.001


----     82 VARIABLE e_return.L              =        0.016  expected return of th
                                                             e portfolio
            VARIABLE var.L                   =  -3.66099E-4  value at Risk
            VARIABLE cvar.L                  =        0.005  conditional value at
                                                             risk
```

Figure 2 Results from CV@R

The optimal policy is to invest 48.1% on spy, 17.6% on mcd, 26.8% on qqq, 7.3% on aapl, and 0.1% on tlt. The expected return is 1.6%.