Workshop 9: V@R and CV@R Optimization

ZHANG AO (A0176595L)
WANG XINRUI (A0186103M)
LI LIPING (A0186040M)

Task 1 – Probabilistic Constraints

A) What is the optimal policy and the V@R for a level of significance (probability of the loss exceeding the V@R) of 1%, 5%, 10%, in a given month?

For confidence level of 1%, Z = 2.58, optimal policy is as below:

```
----     53 VARIABLE x.L  fraction of portfolio invested in asset i

aapl 0.197,    mcd  0.051,    qqq  0.455,    spy  0.296


----     53 VARIABLE e_return.L           =        0.020  expected return of th
                                                         e portfolio
            VARIABLE variance.L           =  3.261921E-4  variance of portfolio
            VARIABLE var.L                =       -0.047  value at risk
```

For confidence level of 5%, Z = 1.96

```
----     53 VARIABLE x.L  fraction of portfolio invested in asset i

aapl 0.197,    mcd  0.051,    qqq  0.455,    spy  0.296


----     53 VARIABLE e_return.L           =        0.020  expected return of th
                                                         e portfolio
            VARIABLE variance.L           =  3.261921E-4  variance of portfolio
            VARIABLE var.L                =       -0.035  value at risk
```

For confidence level of 10%: Z = 1.64

```
----     53 VARIABLE x.L  fraction of portfolio invested in asset i

aapl 0.197,    mcd  0.051,    qqq  0.455,    spy  0.296


----     53 VARIABLE e_return.L           =        0.020  expected return of th
                                                         e portfolio
            VARIABLE variance.L           =  3.261921E-4  variance of portfolio
            VARIABLE var.L                =       -0.030  value at risk
```

B) What is the different between the risk-averse and the risk-neutral optimal policies?

As shown in the results above, the allocations are the same at different confidence level. The only thing that changes is the VAR. At lower confidence level (5%), the VaR is 0.035, this is the risk-neutral policy. While at confidence level 10%, the VaR is 0.030, and this is the risk-averse policy.

C) What is the impact of the target on the monthly return on the V@R and on the optimal policy?

```
----     53 VARIABLE x.L  fraction of portfolio invested in asset i

aapl 1.000


----     53 VARIABLE e_return.L           =        0.029  expected return of th
                                                         e portfolio
            VARIABLE variance.L           =        0.002  variance of portfolio
            VARIABLE var.L                =       -0.096  value at risk
```

When we change the monthly return from 0.02 to 0.03, the policy changes drastically as shown in the figure above. The portfolio only consists one asset aapl, and the VaR is very high at 0.096.

Task 2 – CV@R Optimizations

A) What is the optimal policy for beta of 0.9, 0.95 and 0.99? What is the CV@R, V@R, and expect return of the optimal policies?

@0.90

```
----     82 VARIABLE x.L  fraction of portfolio invested in asset i

spy  0.481,    aapl 0.073,    mcd  0.176,    qqq  0.268,    tlt  0.001


----     82 VARIABLE e_return.L           =        0.016  expected return of th
                                                         e portfolio
            VARIABLE var.L                =  -3.66099E-4  value at Risk
            VARIABLE cvar.L               =        0.005  conditional value at
                                                         risk
```

@ 0.95

```
----     82 VARIABLE x.L  fraction of portfolio invested in asset i

spy  0.491,    aapl 0.066,    mcd  0.176,    qqq  0.257,    tlt  0.009


----     82 VARIABLE e_return.L           =        0.016  expected return of th
                                                         e portfolio
            VARIABLE var.L                =        0.004  value at Risk
            VARIABLE cvar.L               =        0.009  conditional value at
                                                         risk
```

@0.99

```
----     82 VARIABLE x.L  fraction of portfolio invested in asset i

spy  0.501,    aapl 0.057,    mcd  0.193,    qqq  0.233,    tlt  0.016


----     82 VARIABLE e_return.L           =        0.015  expected return of th
                                                          e portfolio
           VARIABLE var.L                 =        0.012  value at Risk
           VARIABLE cvar.L                =        0.016  conditional value at
                                                          risk
```

B) How does the number of scenarios influence the previous solution?

@ beta = 0.95, s = 50

```
----     82 VARIABLE x.L  fraction of portfolio invested in asset i

spy  0.542,    aapl 0.054,    mcd  0.208,    qqq  0.173,    tlt  0.024


----     82 VARIABLE e_return.L           =        0.015  expected return of th
                                                          e portfolio
           VARIABLE var.L                 =        0.002  value at Risk
           VARIABLE cvar.L                =        0.002  conditional value at
                                                          risk
```

@ beta = 0.95, s = 100

```
----     82 VARIABLE x.L  fraction of portfolio invested in asset i

spy  0.581,    aapl 0.128,    mcd  0.093,    qqq  0.164,    tlt  0.034


----     82 VARIABLE e_return.L           =        0.016  expected return of th
                                                          e portfolio
           VARIABLE var.L                 =        0.008  value at Risk
           VARIABLE cvar.L                =        0.010  conditional value at
                                                          risk
```

@ beta = 0.95, s = 500

```
----     82 VARIABLE x.L  fraction of portfolio invested in asset i

spy  0.522,    aapl 0.090,    mcd  0.154,    qqq  0.220,    tlt  0.015


----     82 VARIABLE e_return.L           =        0.016  expected return of th
                                                          e portfolio
           VARIABLE var.L                 =        0.004  value at Risk
           VARIABLE cvar.L                =        0.009  conditional value at
                                                          risk
```

@ beta = 0.95, s = 1000

```
----     82 VARIABLE x.L  fraction of portfolio invested in asset i

spy  0.506,    aapl 0.075,    mcd  0.159,    qqq  0.212,    tlt  0.048


----     82 VARIABLE e_return.L           =        0.015  expected return of th
                                                         e portfolio
            VARIABLE var.L                =        0.004  value at Risk
            VARIABLE cvar.L               =        0.009  conditional value at
                                                         risk
```

@ beta = 0.95, s = 10000

```
----     82 VARIABLE x.L  fraction of portfolio invested in asset i

spy  0.491,    aapl 0.066,    mcd  0.176,    qqq  0.257,    tlt  0.009


----     82 VARIABLE e_return.L           =        0.016  expected return of th
                                                         e portfolio
            VARIABLE var.L                =        0.004  value at Risk
            VARIABLE cvar.L               =        0.009  conditional value at
                                                         risk
```

The VaR and CVaR are quite sensitive to the number of scenarios when it is not high (lower than 500). When s is high, VaR and CVaR are not changing with the number of scenarios.

C) You believe that the historical correlations between the security returns and the SPY are not representative. Choose a correlation matrix that, in your view better represents the future behaviour of the stock returns and re-address question A).

We choose the correlation matrix between aapl and other securities for our model, since the correlation is relatively high.

AAPL @ beta = 0.90

```
----     82 VARIABLE x.L  fraction of portfolio invested in asset i

spy 0.536,    mcd 0.178,    qqq 0.283,    tlt 0.003


----     82 VARIABLE e_return.L           =        0.015  expected return of th
                                                         e portfolio
            VARIABLE var.L                =        0.007  value at Risk
            VARIABLE cvar.L               =        0.015  conditional value at
                                                         risk
```

AAPL @ beta = 0.95

```
----     82 VARIABLE x.L  fraction of portfolio invested in asset i

spy 0.552,    mcd 0.177,    qqq 0.253,    tlt 0.017


----     82 VARIABLE e_return.L           =        0.015  expected return of th
                                                         e portfolio
            VARIABLE var.L                =        0.014  value at Risk
            VARIABLE cvar.L               =        0.021  conditional value at
                                                         risk
```

AAPL @ beta = 0.99

```
----     82 VARIABLE x.L  fraction of portfolio invested in asset i

spy 0.592,    mcd 0.187,    qqq 0.205,    tlt 0.016


----     82 VARIABLE e_return.L           =        0.014  expected return of th
                                                         e portfolio
            VARIABLE var.L                =        0.025  value at Risk
            VARIABLE cvar.L               =        0.031  conditional value at
                                                         risk
```

As beta increases, VaR and CVaR also increase.