DSC5211C QUANTITATIVE RISK MANAGEMENT

SESSION 2

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Introduction to Smoothing Forecasting Methods

Objectives

- Introduction to Forecasting methods
- Qualitative vs. Quantitative Forecasting
- Smoothing forecasting methods:
    - Simple Exponential Smoothing
    - Holt - Exponential Smoothing
    - Holt-Winters Exponential Smoothing
    - Qualitative Forecasting Methods

Main Forecasting Approaches

- **Qualitative Forecasting:** these are methods based on the opinion of experts in a given subject.

- **Quantitative Forecasting:** these are methods that are based on data and statistics for forecasting.

Quantitative Forecasting – Time Series

- What are we looking in a Time Series in order to forecast?

  - Trend – this represents a pattern of behaviour that is a function of time.

  - Seasonality – this represents a pattern of variation around the trend. Typically these lead to repetitions of patterns with the duration of one year.

  - Cycle – this represents a pattern of repetitions with a length of more than one year.

Measures of Variability

- Measures of Variability allow us to evaluate how well a forecasting model is able to approximate real data.

- Mean Absolute Percentage Error (MAPE) measures the accuracy of the fitted time series values. It expresses it as a percentage:

$$\text{MAPE}=\frac{\sum_{i=1}^{n}\left|\frac{x_i-\overline{x_i}}{x_i}\right|}{n}\times 100$$

x_i - this is the actual value

$\overline{x_i}$ - this is the estimated value

n – the number of observations

- Mean Absolute Deviation (MAD) measures the accuracy of the fitted time series values. It expresses it in the same units as the data:

$$\text{MAD}=\frac{\sum_{i=1}^{n}\left|x_i-\overline{x_i}\right|}{n}$$

- Mean Squared Deviation (MSD) is similar to the MAD but it is more sensitive to large forecasting errors.

$$\text{MSD}=\frac{\sum_{i=1}^{n}\left|x_i-\overline{x_i}\right|^2}{n}$$

Smoothing Forecasting Methods

- Assuming there is no trend or seasonality, we can use the most recent smoothed value m_t as forecast

$$f_{t+1} = m_t$$

- Simple moving average of recent points

e.g. $$m_t = 1/3\ Y_t + 1/3\ Y_{t-1} + 1/3\ Y_{t-2}$$

- Weighted moving average

e.g. $$m_t = 1/2\ Y_t + 1/3\ Y_{t-1} + 1/6\ Y_{t-2}$$

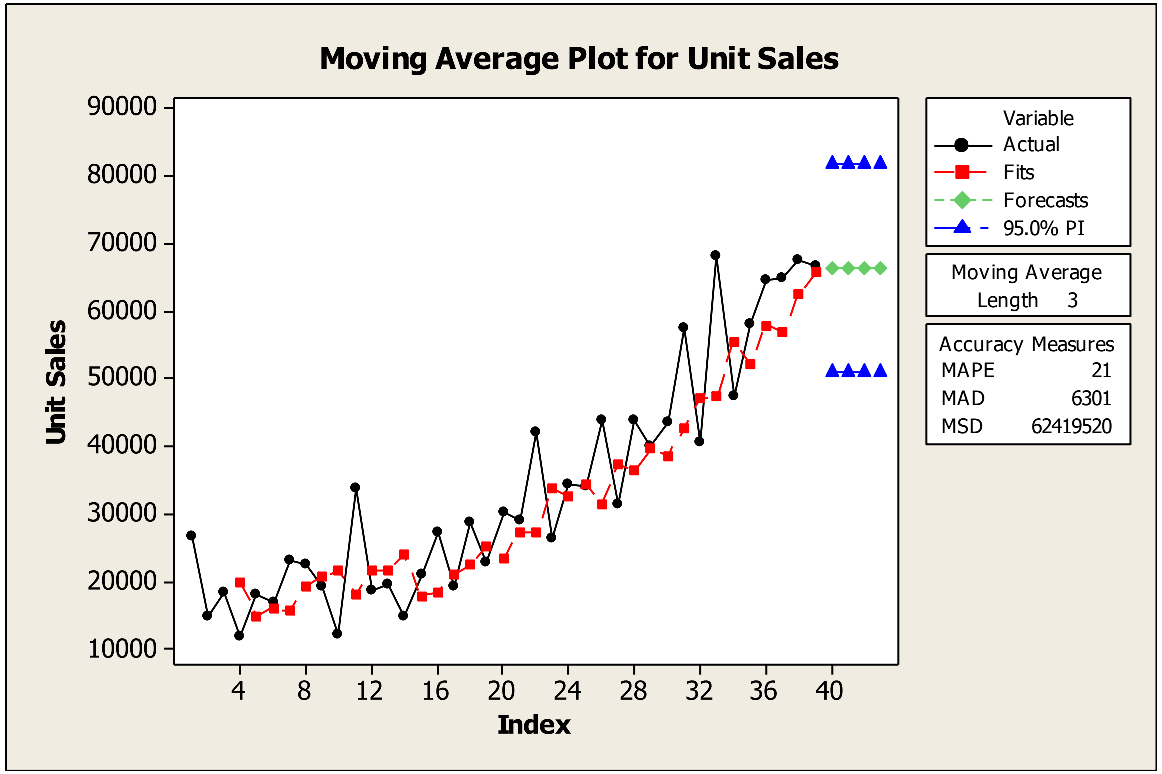
- Exponentially weighted moving average

e.g. $$m_t = 1/2\ Y_t + 1/4\ Y_{t-1} + 1/8\ Y_{t-2} + \ldots.$$

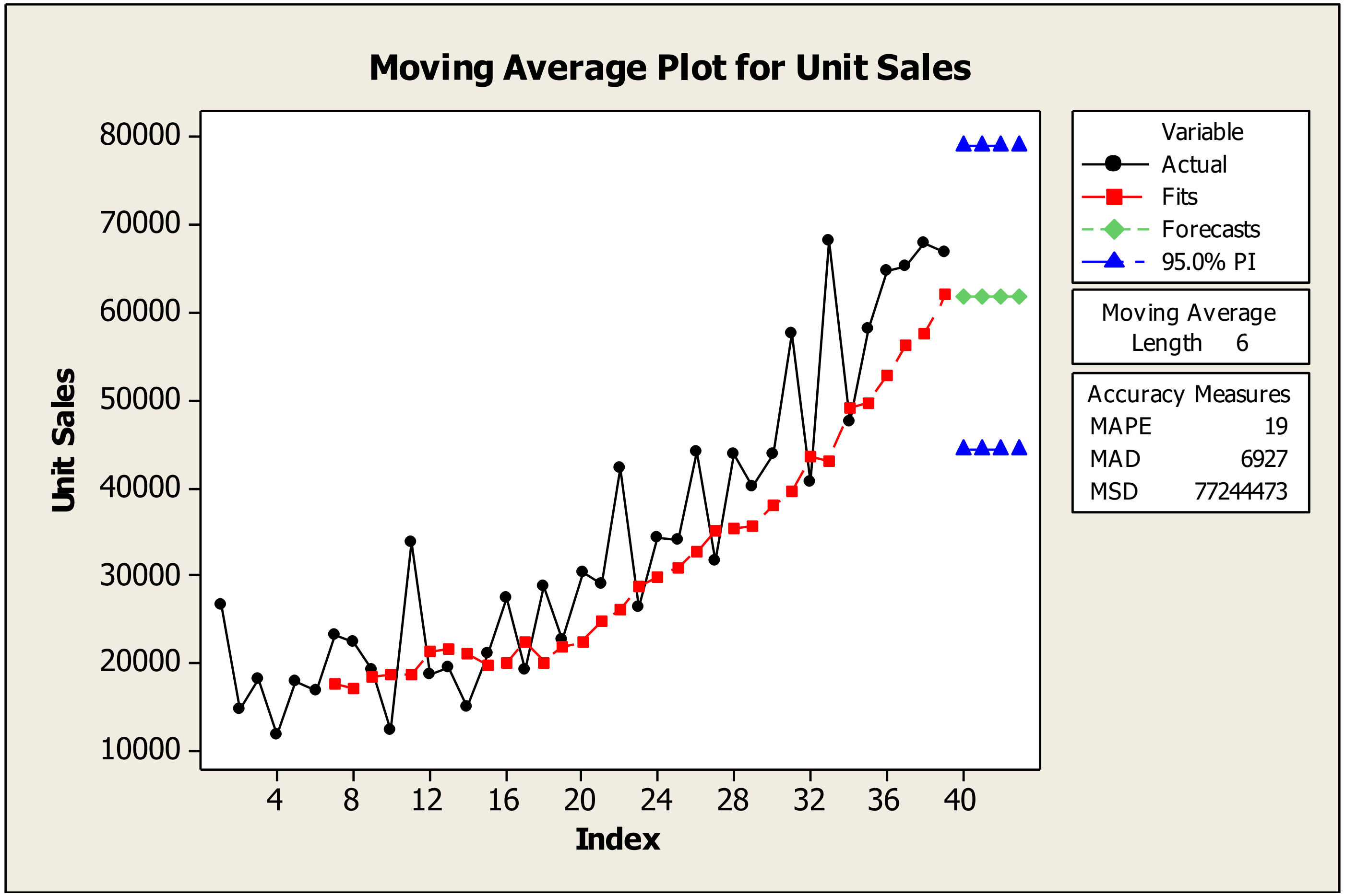
Moving Average: 3 weeks

Week	Unit Sales	m_t	Forecast
01-Jan-99	26,520		
08-Jan-99	14,660		
15-Jan-99	18,240	19806.7	
22-Jan-99	11,850	14916.7	19806.7
29-Jan-99	18,000	16030.0	14916.7
05-Feb-99	16,740	15530.0	16030.0
12-Feb-99	23,170	19303.3	15530.0
30-Jul-99	57,570	47130.0	42560.0
06-Aug-99	40,730	47343.3	47130.0
13-Aug-99	68,190	55496.7	47343.3
20-Aug-99	47,570	52163.3	55496.7
27-Aug-99	58,110	57956.7	52163.3
03-Sep-99	64,780	56820.0	57956.7
10-Sep-99	65,080	62656.7	56820.0
17-Sep-99	67,750	65870.0	62656.7
24-Sep-99	66,650	66493.3	65870.0
01-Oct-99			66493.3
08-Oct-99			66493.3
15-Oct-99			66493.3

Moving Average: Length 3



Moving Average: Length 6



Simple Exponential Smoothing

- More generally:

$$m_t = \alpha\ Y_t + \alpha\ (1-\alpha)\ Y_{t-1} + \alpha\ (1-\alpha)^2\ Y_{t-2} + \ldots\ldots$$
$$= \alpha\ Y_t + (1-\alpha)\ m_{t-1}$$

- Since we assume no trend or seasonality:

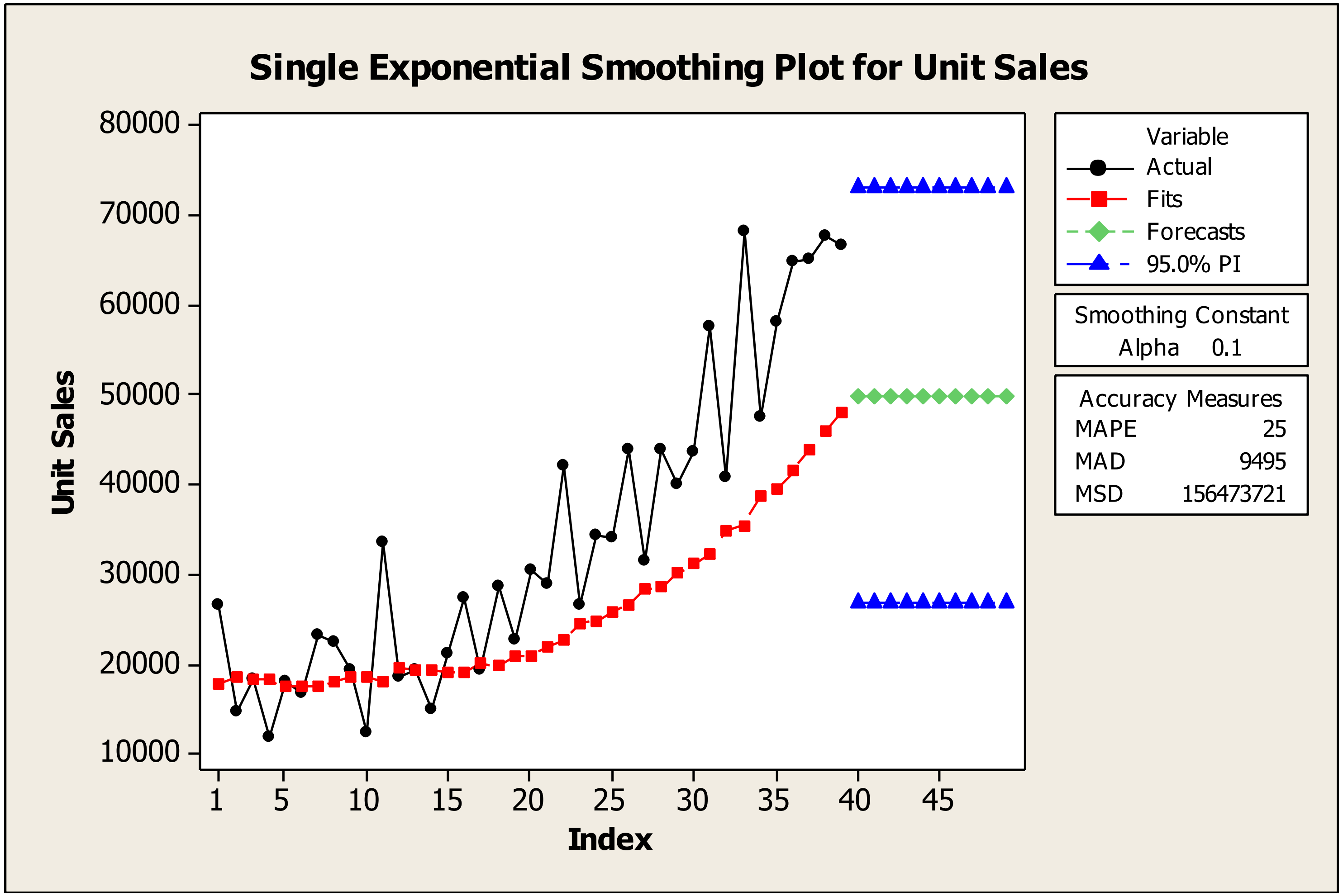
$$f_{t+1} = f_{t+2} = \ldots\ldots = f_{t+k} = m_t$$

- $m_1 = Y_1$
- $0 \leq \alpha \leq 1$

alpha: *0.1*

Week	Unit Sales	m_t	Forecast
01-Jan-99	26,520	26520.0	
08-Jan-99	14,660	25334.0	26520.0
15-Jan-99	18,240	24624.6	25334.0
22-Jan-99	11,850	23347.1	24624.6
29-Jan-99	18,000	22812.4	23347.1
05-Feb-99	16,740	22205.2	22812.4
30-Apr-99	28,690	22114.9	21384.3
07-May-99	22,670	22170.4	22114.9
14-May-99	30,330	22986.4	22170.4
21-May-99	28,970	23584.7	22986.4
28-May-99	42,150	25441.2	23584.7
04-Jun-99	26,440	25541.1	25441.2
11-Jun-99	34,230	26410.0	25541.1
18-Jun-99	33,980	27167.0	26410.0
25-Jun-99	43,980	28848.3	27167.0
02-Jul-99	31,550	29118.5	28848.3
09-Jul-99	43,860	30592.6	29118.5
16-Jul-99	40,090	31542.4	30592.6
23-Jul-99	43,730	32761.1	31542.4
30-Jul-99	57,570	35242.0	32761.1
06-Aug-99	40,730	35790.8	35242.0
13-Aug-99	68,190	39030.7	35790.8
20-Aug-99	47,570	39884.7	39030.7
27-Aug-99	58,110	41707.2	39884.7
03-Sep-99	64,780	44014.5	41707.2
10-Sep-99	65,080	46121.0	44014.5
17-Sep-99	67,750	48283.9	46121.0
24-Sep-99	66,650	50120.5	48283.9
01-Oct-99			50120.5
08-Oct-99			50120.5
15-Oct-99			50120.5
22-Oct-99			50120.5
29-Oct-99			50120.5
05-Nov-99			50120.5

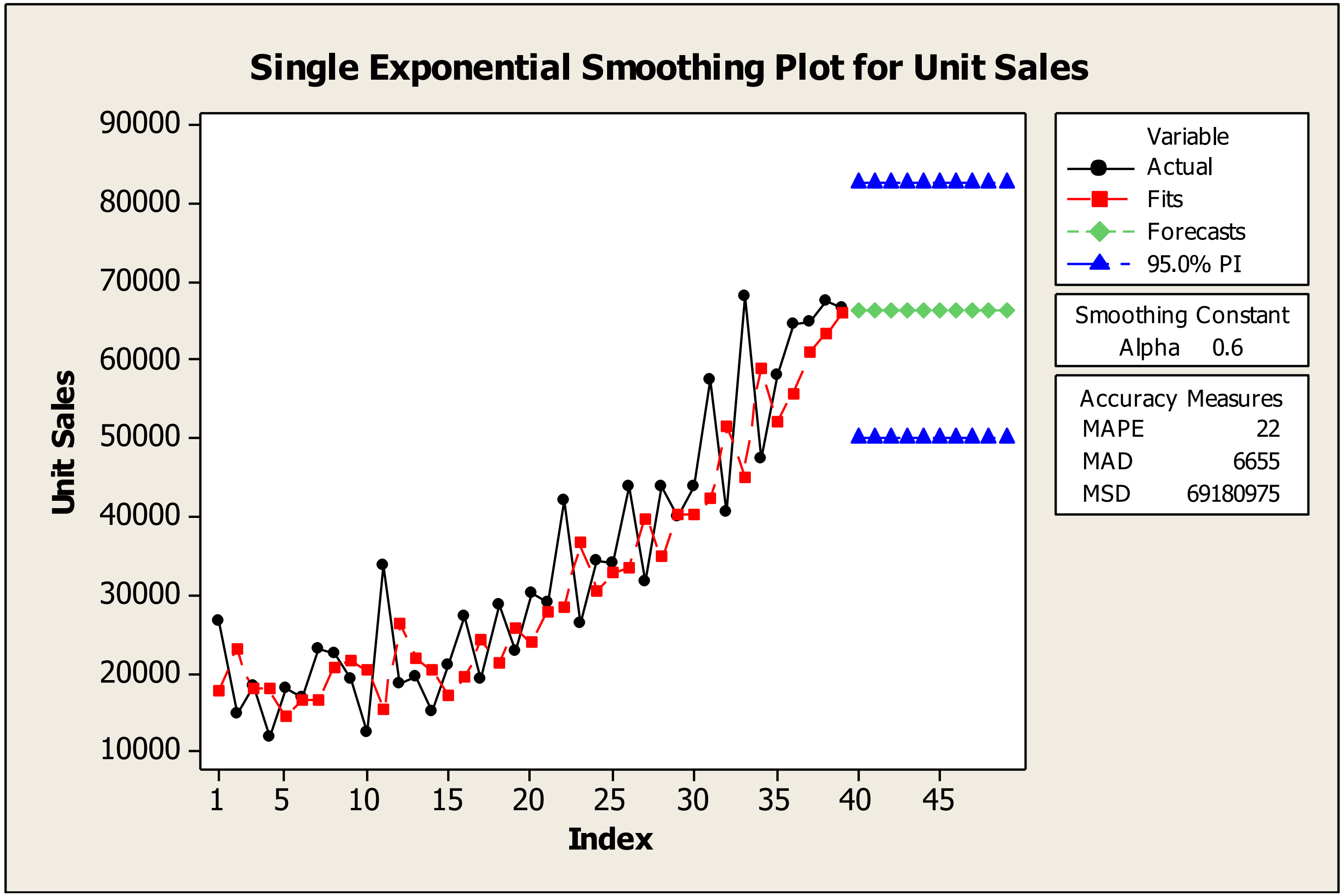
Simple Exponential Smoothing: Alpha 0.1



alpha: *0.6*

Week	Unit Sales	m_t	Forecast
01-Jan-99	26,520	26520.0	
08-Jan-99	14,660	19404.0	26520.0
15-Jan-99	18,240	18705.6	19404.0
22-Jan-99	11,850	14592.2	18705.6
29-Jan-99	18,000	16636.9	14592.2
05-Feb-99	16,740	16698.8	16636.9
30-Apr-99	28,690	25683.2	21173.0
07-May-99	22,670	23875.3	25683.2
14-May-99	30,330	27748.1	23875.3
21-May-99	28,970	28481.2	27748.1
28-May-99	42,150	36682.5	28481.2
04-Jun-99	26,440	30537.0	36682.5
11-Jun-99	34,230	32752.8	30537.0
18-Jun-99	33,980	33489.1	32752.8
25-Jun-99	43,980	39783.6	33489.1
02-Jul-99	31,550	34843.5	39783.6
09-Jul-99	43,860	40253.4	34843.5
16-Jul-99	40,090	40155.4	40253.4
23-Jul-99	43,730	42300.1	40155.4
30-Jul-99	57,570	51462.1	42300.1
06-Aug-99	40,730	45022.8	51462.1
13-Aug-99	68,190	58923.1	45022.8
20-Aug-99	47,570	52111.3	58923.1
27-Aug-99	58,110	55710.5	52111.3
03-Sep-99	64,780	61152.2	55710.5
10-Sep-99	65,080	63508.9	61152.2
17-Sep-99	67,750	66053.6	63508.9
24-Sep-99	66,650	66411.4	66053.6
01-Oct-99			66411.4
08-Oct-99			66411.4
15-Oct-99			66411.4
22-Oct-99			66411.4
29-Oct-99			66411.4
05-Nov-99			66411.4
12-Nov-99			66411.4
19-Nov-99			66411.4

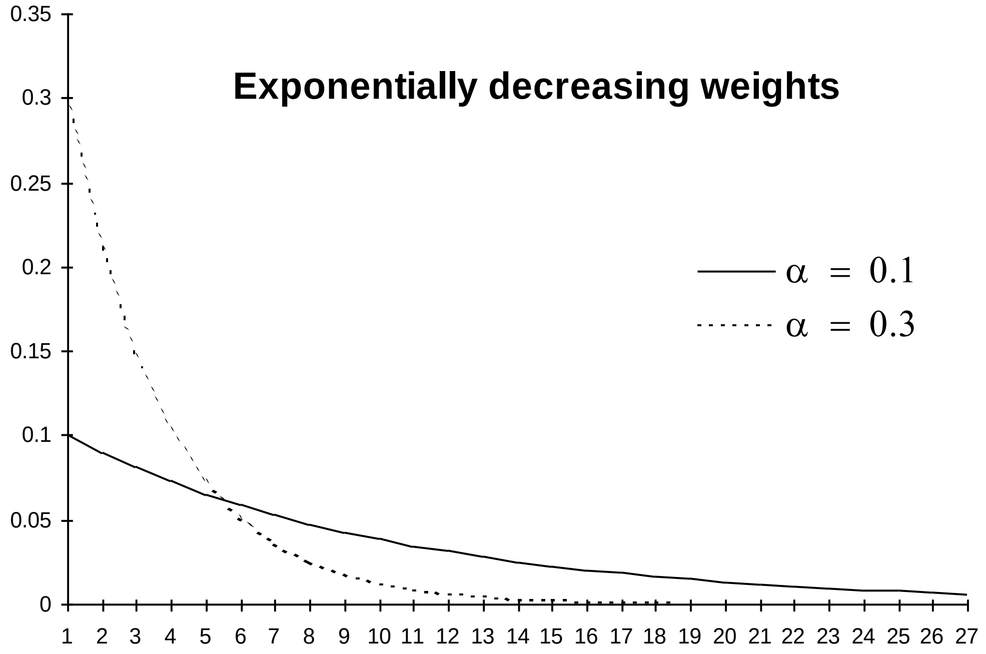
Simple Exponential Smoothing: Alpha 0.6



Choice of α

$$m_t = \alpha\ Y_t + (1-\alpha)\ m_{t-1}$$
$$m_t = \alpha\ Y_t + \alpha\ (1-\alpha)\ Y_{t-1} + \alpha\ (1-\alpha)^2\ Y_{t-2} + \ldots\ldots$$

$$f_{t+1} = m_t$$



Choice of α

- $0 \leq \alpha \leq 1$
  Usually 0.1 to 0.3

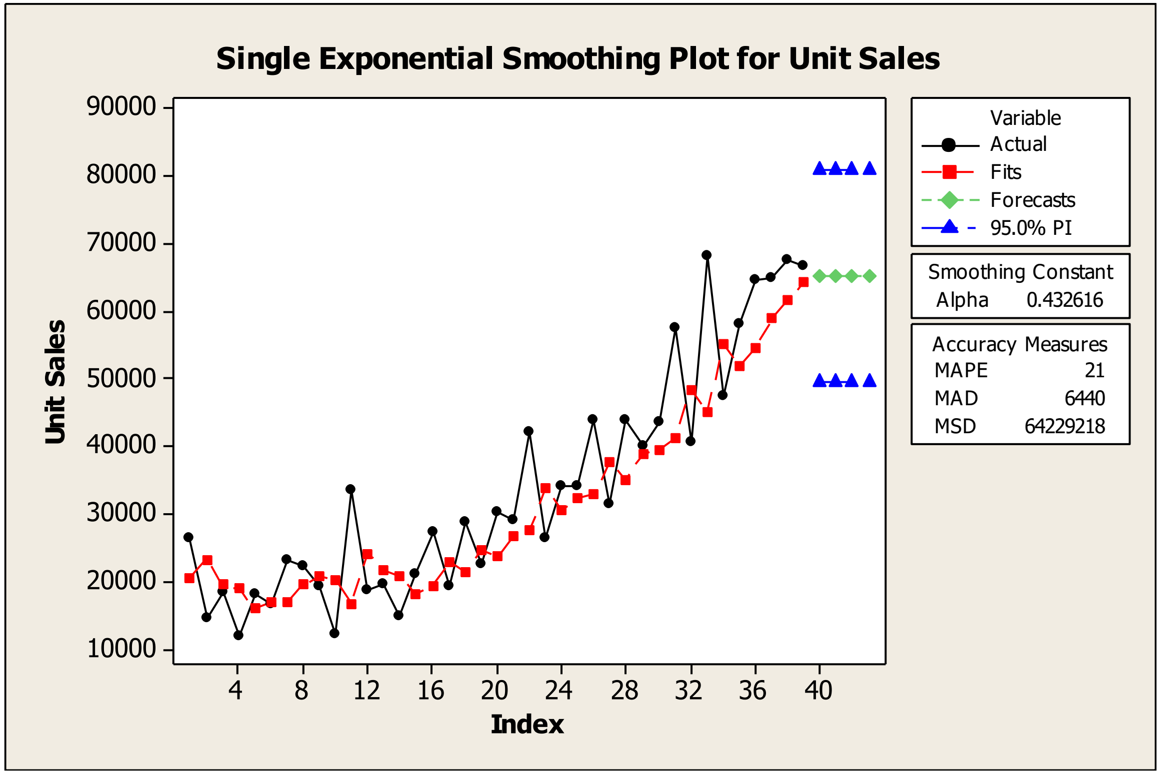
- Low α
  - Low weight to latest observation
  - Much smoothing
  - Slow response to structural change
  - Best for rapidly fluctuating, noisy series

- High α
  - Low weight to old observations
  - Little smoothing
  - Quick response to structural change
  - Best for slight random fluctuations

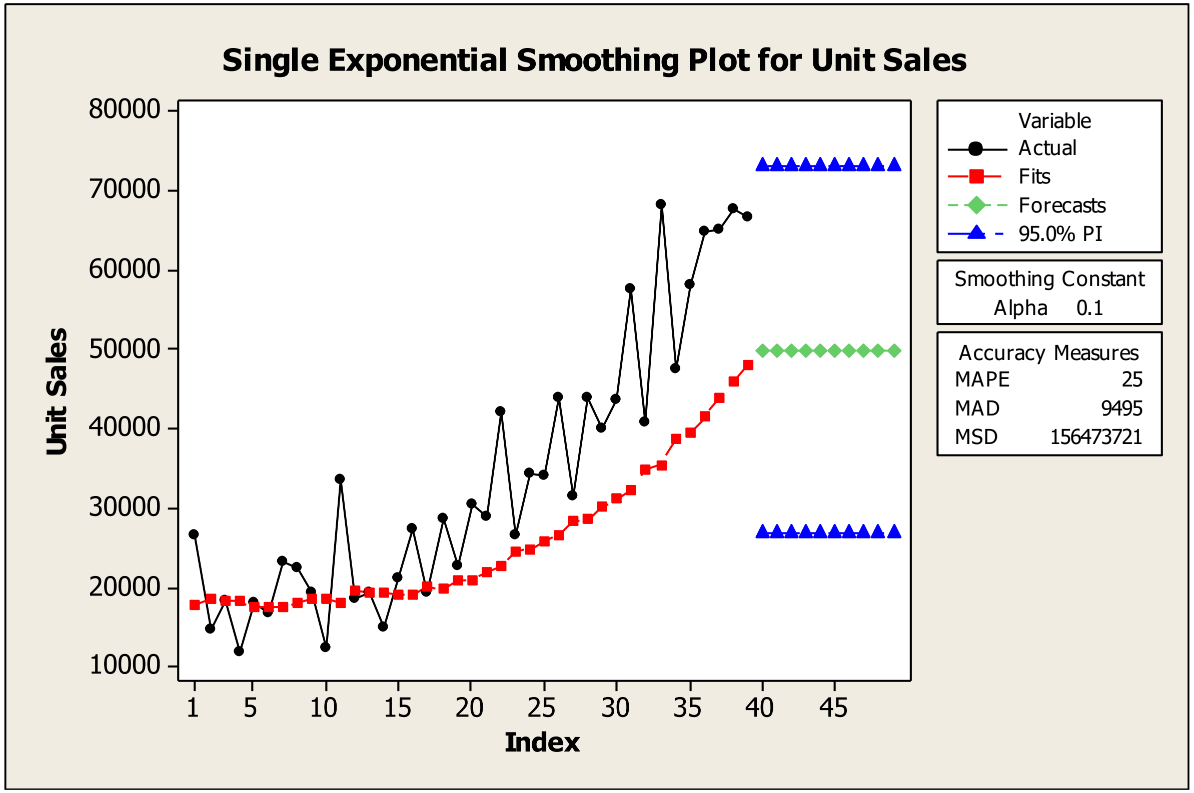
Choice of α

- Software allows α to be optimised as in regression:

$$\min \sum_{i=1}^{n} e_i^2$$



Simple Exponential Smoothing: Trending Series



- Simple exponential smoothing is not an appropriate forecasting technique for series with trend.

Holt - Exponential Smoothing

- Used on trending series
- Local linear trend assumed so forecasts given by:

$$f_{t+k} \quad = \quad (\text{Level})_t \;+\; k \times (\text{Growth})_t$$

- Holt's method takes into account both level and growth:

Level: $$m_t = \alpha\, Y_t \;+\; (1 - \alpha)(\, m_{t\text{-}1} + r_{t\text{-}1}\,)$$

Growth: $$r_t = \gamma\left(m_t - m_{t-1}\right) + (1 - \gamma) r_{t-1}$$

Forecast: $$f_{t+1} = m_t + \; r_t$$

k-period ahead forecast: $f_{t+k} = m_t + k \times r_t$

Holt - Exponential Smoothing

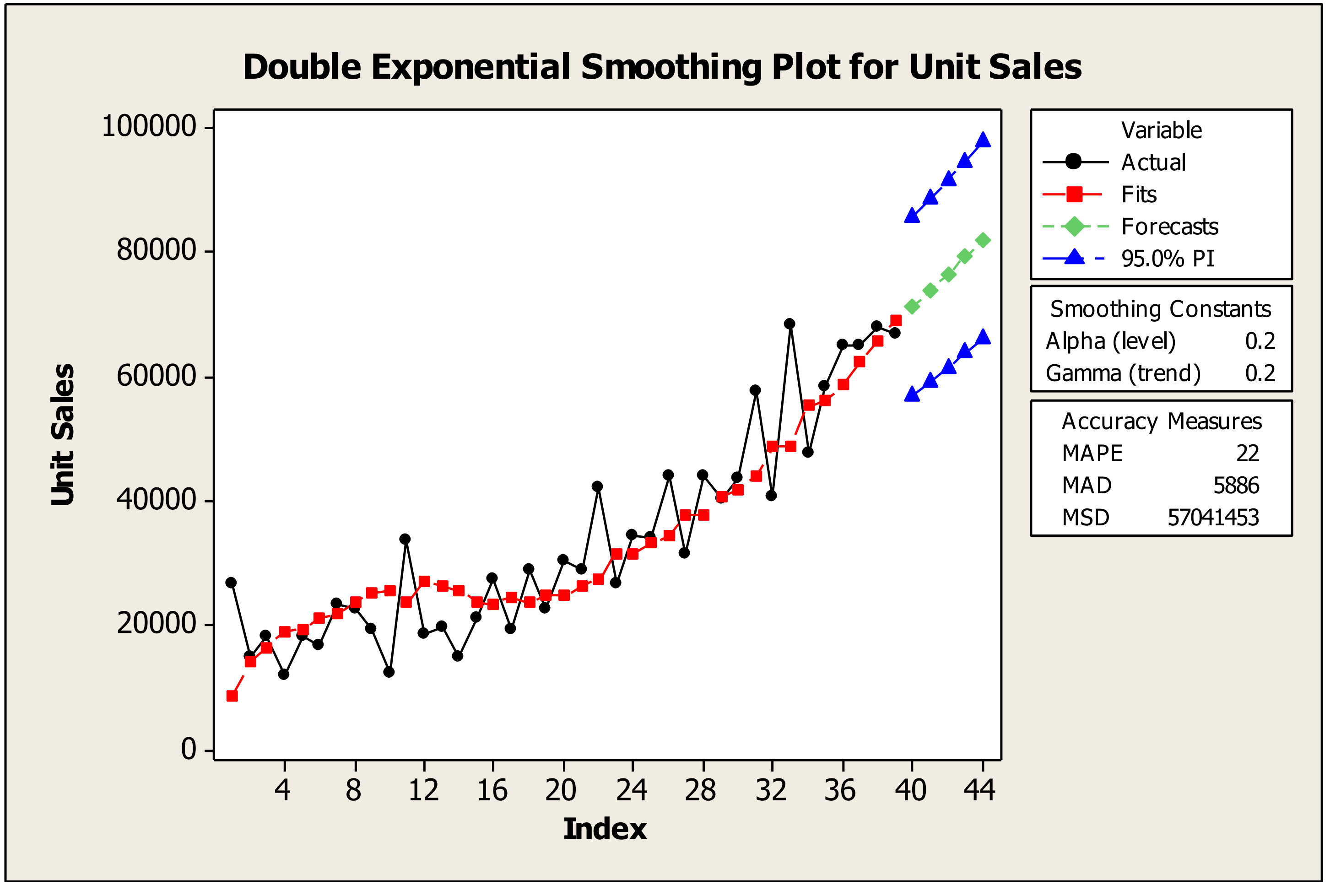
- Smoothing parameters: $0 \leq \alpha \leq 1$ $0 \leq \gamma \leq 1$

  - Slope evolves steadily: try high γ
  - Slope changes erratically: try low γ

  - Level evolves steadily: try high α
  - Level changes erratically: try low α

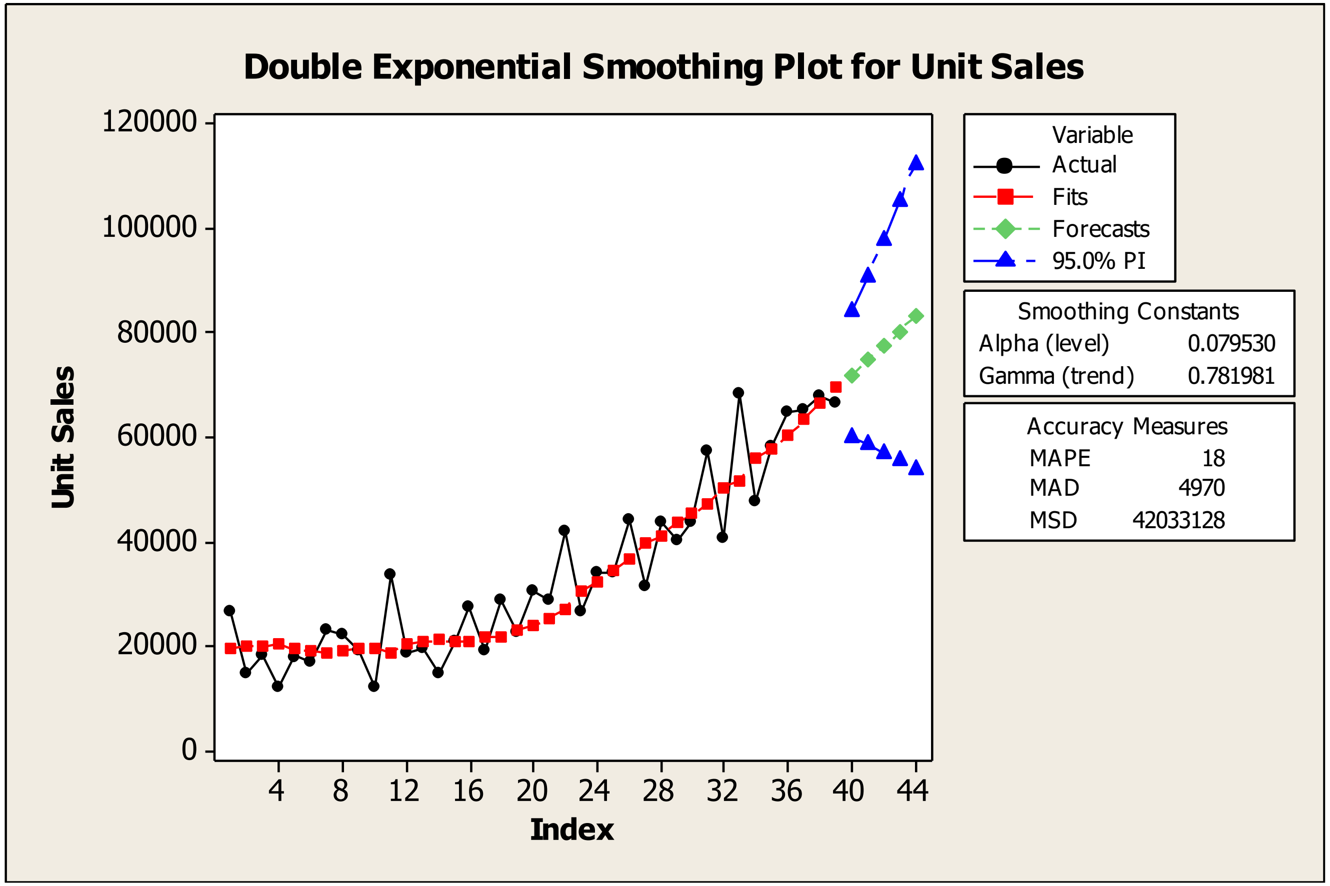
- It is not easy to subjectively select the parameters so, instead, we can optimise.

Holt - Exponential Smoothing



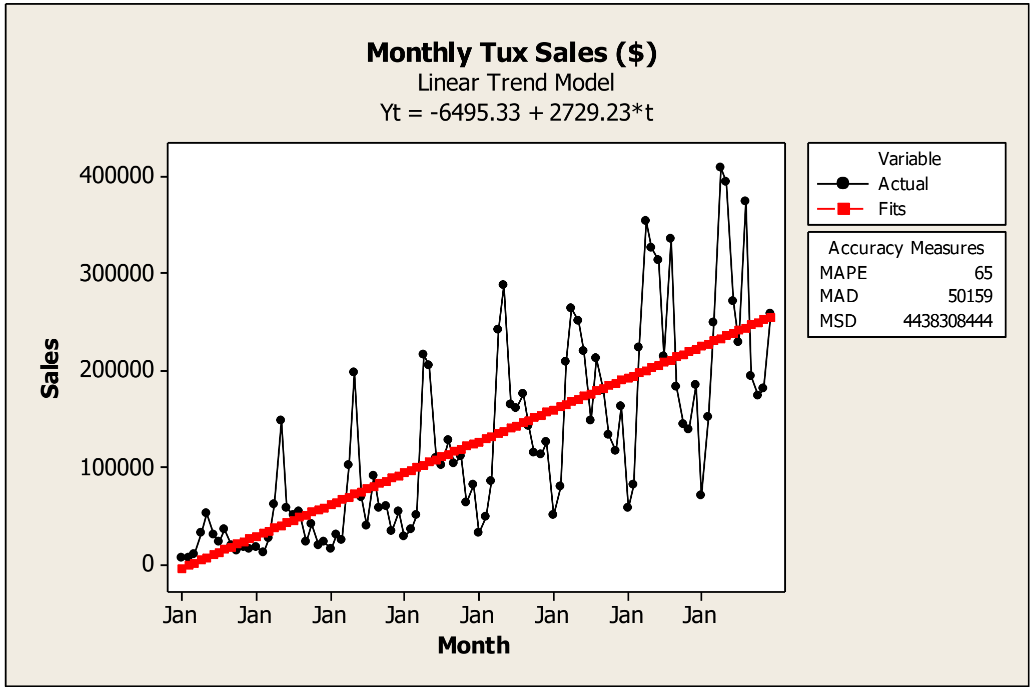
Holt - Exponential Smoothing

- Optimal Parameters



Holt-Winters Exponential Smoothing

- For series with trend and seasonality
- A straight line models the linear trend in monthly Tux rentals ($) but fails to model the seasonality



Holt-Winters Exponential Smoothing

- Holt-Winters method accounts for level, growth and **multiplicative seasonality**:

Level: $m_t = \alpha\, Y_t \,/\, S_{t\text{-}12} + (1 - \alpha)(\, m_{t\text{-}1} + r_{t\text{-}1}\,)$

Growth: $r_t = \gamma\left(m_t - m_{t-1}\right) + (1-\gamma)r_{t-1}$

Seasonality: $S_t = \delta\left(Y_t / m_t\right) + \left(1-\delta\right)S_{t-12}$

Forecast: $f_{t+1} = (\, m_t + r_t \,)\, S_{t\text{-}11}$

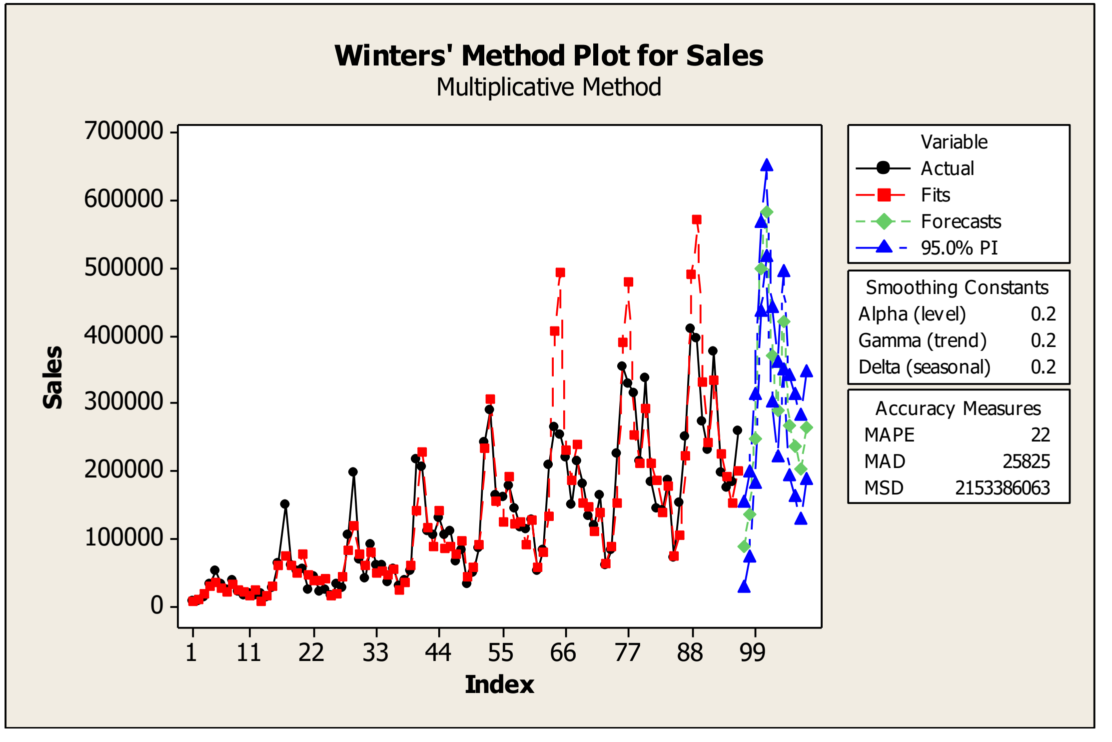
k-period ahead forecast: $f_{t+k} = (\, m_t + k \times r_t\,)\, S_{t+k\text{-}12}$

Holt-Winters Exponential Smoothing

- Smoothing parameters between 0 & 1

  Typically: $\alpha = \gamma = \delta = 0.2$

  If smoothly changing seasonality, could use high δ



Holt-Winters Exponential Smoothing

- Holt-Winters method accounts for level, growth and **additive seasonality**:

Level: $m_t = \alpha(Y_t - S_{t-12}) + (1-\alpha)(m_{t-1} + r_{t-1})$

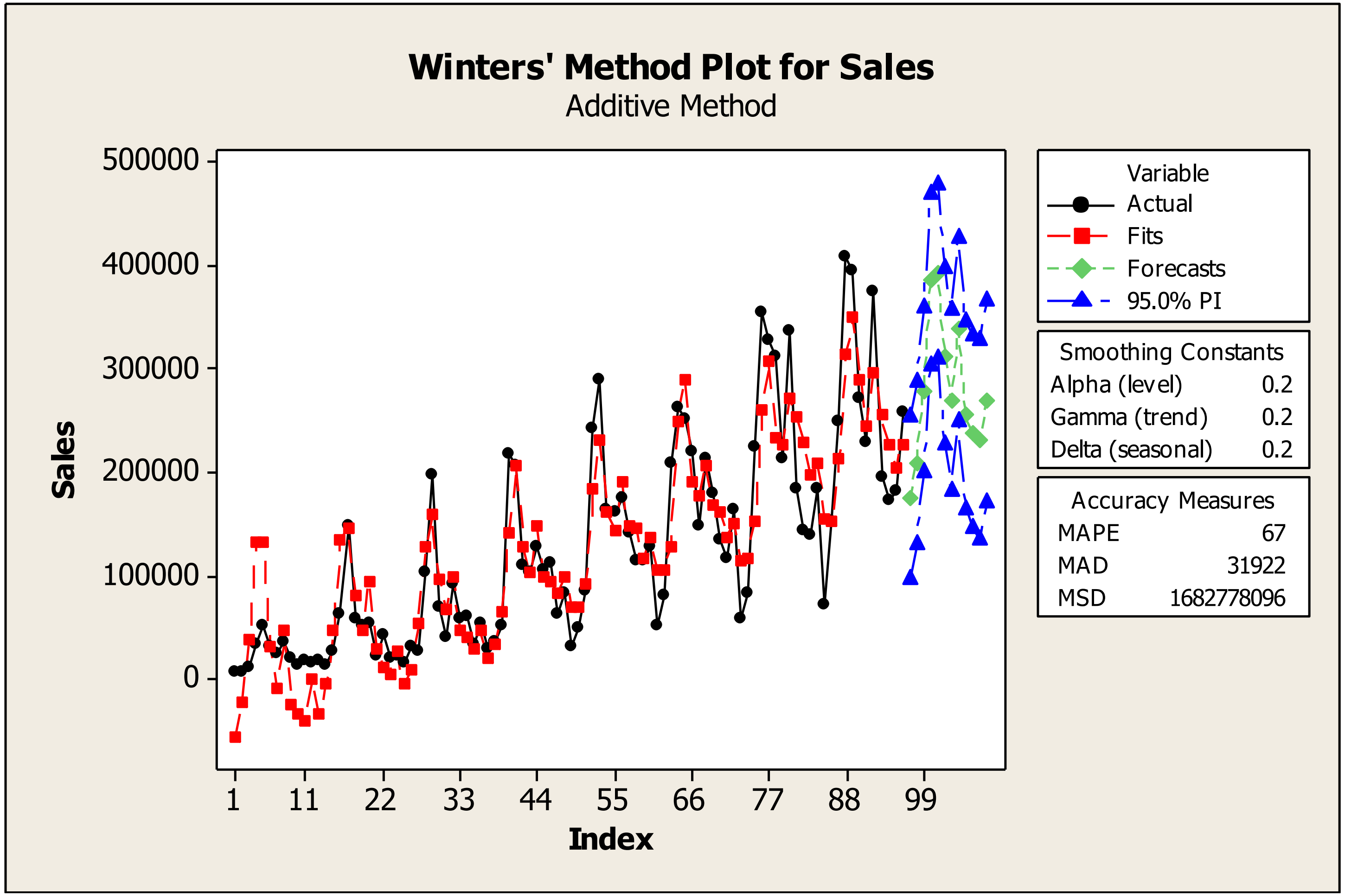
Growth: $r_t = \gamma(m_t - m_{t-1}) + (1-\gamma)r_{t-1}$

Seasonality: $S_t = \delta(Y_t - m_t) + (1-\delta)S_{t-12}$

Forecast: $f_{t+1} = m_t + r_t + S_{t-11}$

k-period ahead forecast: $f_{t+k} = m_t + k \times r_t + S_{t-k+12}$

Holt-Winters Exponential Smoothing



Exponential Smoothing – Advantages and Disadvantages

- Advantages
  - robustness
  - simplicity
  - minimal information storage
  - few observations required
  - low cost and can be made fully automatic
  - useful when a large number of forecasts are required
    e.g. inventory control

- Disadvantages
  - not reliable beyond short-term
  - methods attempting to model data generating process are more suitable if resources are available

Summary

- We have discussed the difference between qualitative and quantitative forecasting methods.
- We have looked at several measures of variability.
- We have analysed several smoothing forecasting methods.

“***As soon as you find the pattern, you break it. Otherwise it gets boring.*** “

John Lennon

Reading List

S. Makridakis, S.C. Wheelwright and R.J. Hyndman, 3rd Ed. Chapter 1, Chapter 2.1-2.2, Chapter 3.2

P.E. Gaynor and R.C. Kirkpatrick, Chapter 1

J.E. Hanke, D.W. Wichern and A.G. Reitsch, Chapter 1, Chapter 2