DSC5122C
Quantitative Risk Management
Group Project 3

Risk Aversion Project

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Introduction

Fruit has been recognized as a good source of nutrients that are vital for the health, especially vitamin C and vitamin A. Research has shown that a high daily intake of fruits could promotes health and potentially reduce the risk of chronic diseases[1]. However, there are many choices of fruits in the market with different flavors and prices. Our project aims to find a mix of fruits that satisfies requirements on the daily amount of nutrition with minimal cost.

Problem description

The nutrition information of four fruits are gathered along with a minimum required intake of each type of nutrition for an adult. The goal is to find the cheapest combination of fruits that will meet nutritional requirements by using 3 different approaches: risk neutral model, CVAR model and worst-case model. By adjusting model parameters, sensitivity analysis is conduced to determine how risk appetite and number of scenarios will impact the final result.

Mathematical model

Model Risk Neutral

$$Nutrition \in \{Calories, Protein, Fiber, Carbohydrate, Vitamin_A, Vitamin_C\}$$

$$s:\ scenario$$

$$f: fruit \in \{Apple, Lemon, Grape, Orange\}$$

$$\text{Minimize } cost = \sum_{f=1}^{4} price\_mean(f) * x(f)$$

s.t

$$\sum_{f=1}^{4} x(f) = 1$$

[1] Heiner Boeing, Angela Bechthold, Achim Bub, Sabine Ellinger, Dirk Haller, Anja Kroke, Eva Leschik-Bonnet, Manfred J. Müller, Helmut Oberritter, Matthias Schulze, Peter Stehle, Bernhard Watzl. Eur J Nutr. 2012 Sep; 51(6): 637–663. Published online 2012 Jun 9. doi: 10.1007/s00394-012-0380-y. PMCID: PMC3419346

$$\sum_{f=1}^{4}\sum_{s=1}^{100} Nutritions(s,f) * x(f)\ /card(s) \geq req(Nutritions)$$

$$\mathrm{x(f)} \geq 0$$

Model CVAR

$$\text{Minimize } cost = \sum_{f=1}^{4} price\_mean(f) * x(f)$$

s.t

$$\sum_{f=1}^{4} x(f) = 1$$

$$\sum_{f=1}^{4}\sum_{s=1}^{100} Nutritions(s,f) * x(f)\ /card(s) \geq req(Nutritions)$$

$$z(s) \geq \sum_{f=1}^{4} a(f,s) * x(f)\ -\ var$$

$$cvar = \frac{var + 1}{\big((1 - beta) * card(s)\big) * \sum_{s=1}^{100} z(s)}$$

$$\mathrm{x(f)} \geq 0$$

Model worst case

$$\text{Minimize } mu$$

s.t

$$\sum_{f=1}^{4} x(f) = 1$$

$$\sum_{f=1}^{4}\sum_{s=1}^{100} Nutritions(s,f) * x(f)\ /card(s) \geq req(Nutritions)$$

$$\mathrm{mu} \geq \sum_{f=1}^{4} a(f,s) * x(f)$$

$$x(f) \geq 0$$

Model Parameterization

1. f: set of fruits selected - Apple, Lemon, Grape, Orange
2. n: set of nutrients required - Calories, Protein, Fiber, Carbohydrate, Vitamin A, Vitamin C
3. s: number of scenarios
4. req(n): Daily requirement of each nutrient

Nutrient	min required
Energy (kcal)	46.35
Protein (g)	0.87
Dietary fiber (g)	3.23
Carbohydrate (g)	11.28
Vitamin A (mcg)	5.33
Vitamin C (mg)	34

Table 1 Minimum daily intake of nutrients required for an adult

5. price_mean(f) and price_SD(f)

Fruit	Mean price	Standard Deviation
Apple	0.55	0.112
Lemon	1.54	0.764
Grape	1.3	0.396
Orange	0.47	0.053

Table 2 Mean and SD of fruit prices

6. correlation(f,f): correlation matrix for fruit prices

	Apples	Lemon	Grapes	Oranges
Apples	1.000	0.784	0.052	0.346
Lemon	0.784	1.000	0.079	-0.061
Grapes	0.052	0.079	1.000	-0.383
Oranges	0.346	-0.061	-0.383	1.000

Table 3 correlation matrix for fruit prices

Correlation matrix is calculated based on historical fruits price from 2004-2014. (Source: U.S.

Bureau of Labor Statistics)

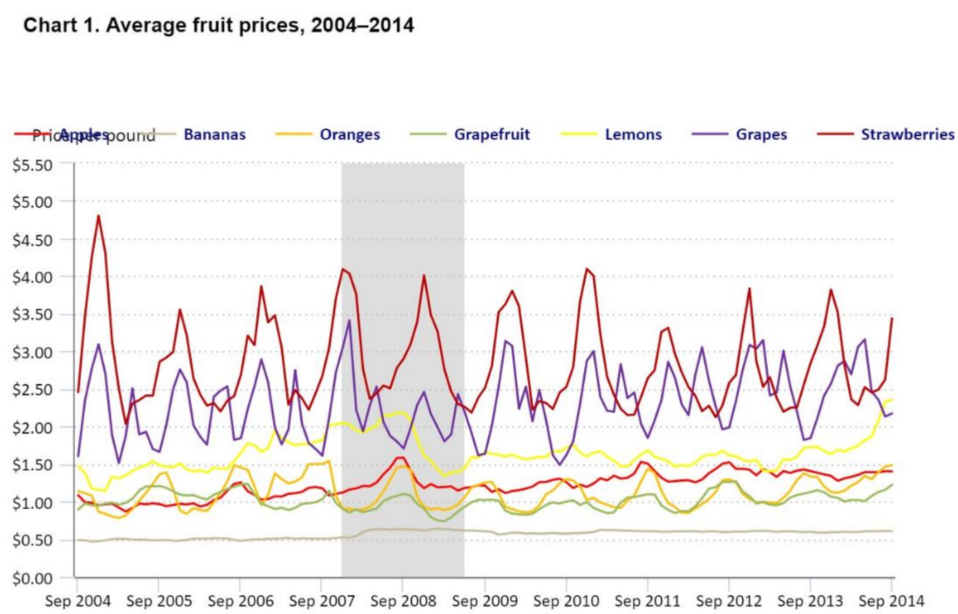

Figure 1 Average fruit prices from 2004 to 2014

7. char(*,n,f): fruit characteristics (pct)

	Apple. Mean	Apple.Variance	Lemon. Mean	Lemon.Variance	Grape.Mean	Grape.Variance	Orange. Mean	Orange. Variance
Energy (kcal)	54.39	3.63	20.00	0.00	70.24	4.79	40.75	2.85
Protein (g)	0.30	0.00	1.20	0.00	0.97	0.17	1.00	0.08
Dietary fibre (g)	2.27	0.05	4.70	0.00	3.43	0.09	2.50	0.22
Carbohydrate (g)	11.90	0.88	10.70	0.00	14.90	1.00	7.63	0.52
Vitamin A (mcg)	1.67	0.47	2.00	0.00	5.67	4.03	12.00	1.63
Vitamin C (mg)	5.33	0.47	77.00	0.00	4.00	2.94	51.67	5.79

Table 4 Fruit characteristics (pct)

Three different brands of each fruit except lemon are collected to compute the mean and variance of the nutrition composition.

8. create normal distribution of nutrients amount and fruit prices:
   cal(s,f) = normal(char('mean','Calories',f),sqrt(char('variance','Calories',f)));
   prot(s,f) = normal(char('mean','Protein',f),sqrt(char('variance','Protein',f)));
   fiber(s,f) = normal(char('mean','Fiber',f),sqrt(char('variance','Fiber',f)));
   carbo(s,f) = normal(char('mean','Carbohydrate',f),sqrt(char('variance','Carbohydrate',f)));
   va(s,f) = normal(char('mean','Vitamin_A',f),sqrt(char('variance','Vitamin_A',f)));
   vc(s,f) = normal(char('mean','Vitamin_C',f),sqrt(char('variance','Vitamin_C',f)));
   price(f,s) = normal(price_mean(f),price_SD(f));
9. variables: cost total cost per 100g
   x(f) fruit mix (per 100g)
   var value at Risk
   cvar conditional value at risk
   z(s) tail profit in scenario s
10. equations: equations defined for each nutrient requirement
    calc.. sum((f,s), cal(s,f)*x(f))/card(s) =g= req('Calories');
    protc.. sum((f,s), prot(s,f)*x(f))/card(s) =g= req('Protein');
    fiberc.. sum((f,s), fiber(s,f)*x(f))/card(s) =g= req('Fiber');
    carboc.. sum((f,s), carbo(s,f)*x(f))/card(s) =g= req('Carbohydrate');
    vac.. sum((f,s), va(s,f)*x(f))/card(s) =g= req('Vitamin_A');
    vcc.. sum((f,s), vc(s,f)*x(f))/card(s) =g= req('Vitamin_C');
11. mc: mixed constraint, the percentages of each fruit should add up to 1
12. total cost = sum(f, price_mean(f)*x(f))
13. tails(s): calculate the value of the tail
    z(s)=g=-sum(f,a(f,s)*x(f))-var
14. cvar_eq: calculates cvar of model
    cvar =e= var+1/((1-beta)*card(s))*sum(s,z(s))
15. objective is to minimizing total cost using linear programming

Analysis of the optimal policy

Risk-neutral policy

```
----    122 VARIABLE x.L  fruit mix per 100g

Apple  0.242,    Lemon  0.246,    Grape  0.255,    Orange 0.257


----    122 VARIABLE cost.L               =        0.998  total cost per 100g
```

The optimal policy under risk-neutral method is to intake 24.2% apple, 24.6% lemon, 25.5% grape and 25.7% orange. The objective cost is 0.998 on 100g

CV@R policy

```
----    133 VARIABLE x.L  fruit mix (per 100g)

Apple  0.153,    Lemon  0.210,    Grape  0.322,    Orange 0.315


----    133 VARIABLE cost.L               =        0.974  total cost per 100g
            VARIABLE var.L                =        1.772  value at Risk
            VARIABLE cvar.L               =        1.999  conditional value at
                                                          risk
```

The optimal policy under CV@R method is to intake 15.3% a% apple, 21.0% lemon, 32.2% grape and 31.5% orange. The objective cost is 0.970 on 100g. The V@R is 1.772 and the CV@R is 1.999.

Worst Case policy

```
----    128 VARIABLE x.L  fruit mix per 100g

Apple  0.053,    Lemon  0.169,    Grape  0.398,    Orange 0.381


----    128 VARIABLE cost.L               =        0.985  total cost per 100g
            VARIABLE mu.L                 =        2.900  the worst case cost
```

The optimal policy under worst case method is to intake 5.3% apple, 16.9% lemon, 39.8% grape and 38.1% orange. The worst case cost is 2.9 on 100g.

Sensitivity analysis

1. Scenario = 1000

	Risk Neutral	CVAR	CVAR	CVAR	CVAR	Worst Case
Scenario	1000	1000	1000	1000	1000	1000
Beta		0.9	0.95	0.99	0.999	
Mix						
Apple	24.20%	15.30%	15.50%	9.40%	5.30%	5.30%
Lemon	24.60%	21.00%	21.10%	18.60%	16.90%	16.90%
Grape	25.50%	32.20%	32.10%	36.70%	39.80%	39.80%
Orange	25.70%	31.50%	31.40%	35.40%	38.10%	38.10%
Cost	0.998	0.974	0.974	0.980	0.985	0.985
V@R		1.772	1.919	2.265	2.900	
CV@R		1.999	2.158	2.482	2.900	
Mu						2.900

2. Scenario = 5000

	Risk Neutral	CVAR	CVAR	CVAR	CVAR	Worst Case
Scenario	5000	5000	5000	5000	5000	5000
Beta		0.9	0.95	0.99	0.999	
Mix						
Apple	24.20%	17.10%	16.80%	19.00%	15.10%	3.50%
Lemon	24.90%	21.90%	21.80%	22.70%	21.10%	25.40%
Grape	25.60%	31.00%	31.20%	29.60%	32.50%	37.50%
Orange	25.40%	30.00%	30.20%	28.80%	31.30%	33.60%
Cost	1.001	0.975	0.976	0.973	0.978	1.056
V@R		1.744	1.992	2.340	2.733	
CV@R		2.035	2.210	2.526	2.826	
Mu						2.894

3. Scenario = 10,000

	Risk Neutral	CVAR	CVAR	CVAR	CVAR	Worst Case
Scenario	10000	10000	10000	10000	10000	10000
Beta		0.9	0.95	0.99	0.999	
Mix						
Apple	24.50%	18.60%	17.90%	18.40%	23.40%	23.90%
Lemon	25.10%	22.80%	22.50%	22.70%	24.80%	25.00%
Grape	25.40%	29.60%	30.20%	29.70%	25.90%	25.50%
Orange	25.00%	29.00%	29.50%	29.20%	25.90%	25.60%
Cost	1.002	0.974	0.975	0.974	0.969	0.969
V@R		1.749	1.962	2.375	2.916	
CV@R		2.030	2.212	2.552	2.999	
Mu						3.219

Findings from sensitivity analysis:

- Keeping the scenario still, with the increase of beta, V@R and CV@R will converge to worst case cost. And V@R is always less or equal than CV@R
- V@R and CV@R converge to worst case mu faster when the scenario is less, in other words, the distance between CV@R and worst mu with beta 0.999 under scenario 10,000 is larger than the distance between CV@R and worst mu with beta 0.999 under scenario 1,000.
- The larger the scenarios are, the higher the cost in Risk Neutral method and the higher the worst-case cost.
- The V@R and CV@R increase with the increase of beta.

Conclusions

This report analysed how to choose the fruit to meet the nutrition requirement and meanwhile minimizing the cost by applying stochastic programming. Three different approaches Risk Neutral, CVAR and Worst Case are used to model different risk appetite. The results of the analysis with different scenarios give different solutions for different degrees of uncertainty. If one has higher risk appetite, he could follow the model result when Beta=90%. Vice versa, for the one is more risk adverse, higher Beta or Worse Case might be a better choice for him. Last but not the least, the result also suggests that diversification of the fruit intake is important to meet different nutrition requirement rather than only eat one kind of fruit.

Reference

Nutrition Composition

https://focos.hpb.gov.sg/eservices/ENCF/foodsearch.aspx

Nutrition Requirement

http://www.mydailyintake.net/daily-intake-levels/

Price Information

https://redmart.lazada.sg/#home

Historical Price Information

https://www.bls.gov/opub/btn/volume-3/slicing-through-fruit-price-volatility.htm