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## Motivation and project overview

Japan equity market is one of the biggest financial markets in the world and largest among Asia market. The Nikkei 225 or more commonly known as Nikkei Index is a price-weighted index composed of Japan's top 225 blue-chip companies traded on the Tokyo Stock Exchange (TSE), and is often used to identify opportunities and risks in the market. To understand how Japan equity market will perform in 2019, we conduct six months forecast for Nikkei Index. Adjusted Closing Value is chosen as the index monthly performance.

In the later part of the discussion, we will further study the relationship of Hang Seng Index, Nikkei and S&P 500. Any of the index is the main driver and leading the other two? Is Japan highly affected by US or Hong Kong market?

## Data description

For time series analysis, we collected 20 years (1999.1-2019.1) of **Nikkei 225, Hang Seng Index**, and **S&P 500 Stock Index** data on monthly frequency (a total of 240 observations) from Yahoo Finance, which can be downloaded <a href="here">here</a>. The **Adjusted Closing Value** is selected for our analysis.

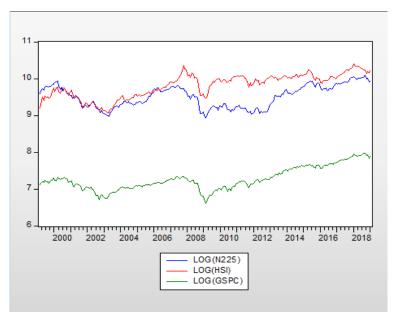


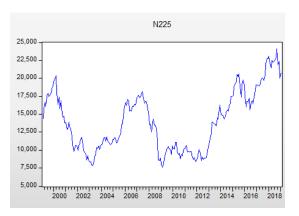
Figure 1 Monthly trend of Hang Seng Index , S&P 500 Stock Index and Nikkei 225 from 1999 Jan to 2019 Jan

# Theoretical explanation for the forecasting model

#### Model Identification

The forecast series must be stationary so that Auto Regressive Integrated Moving Average can be performed. Thus, it's necessary for us to check the stationarity first before the model building.

We can see that Nikkei time series has instances of both positive and negative trend. Overall, it is very volatile, which tells us that we will have to transform the data for the Box-Jenkins Methodology to predict with better accuracy.



Exogenous: None Lag Length: 0 (Fixed)	has a unit root		
		t-Statistic	Prob.*
Augmented Dickey-Ful	ller test statistic	0.245269	0.7566
Augmented Dickey-Ful Test critical values:	ller test statistic 1% level	0.245269 -2.574674	0.7566
			0.7566

Augmented Dickey-Fuller Unit Root Test on N225

Figure 2 Monthly trend of Nikkei Index from 1999 Jan to 2019 Jan

Figure 3 Augmented Dickey-Fuller Unit Root Test on N225

From the result above, the p value of DF test is 0.76, higher than 0.05 threshold, thus we cannot reject the null hypothesis that N225 has a unit root and we can conclude that Nikkei 225 is not stationary.

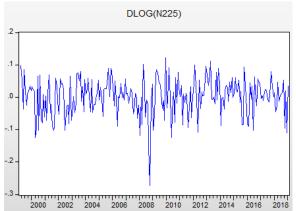


Figure 4 Monthly trend of DLOG(N225) from 1999 Jan to 2019 Jan

Null Hypothesis: DLOG(N225) has a unit root Exogenous: None Lag Length: 0 (Fixed)								
		t-Statistic	Prob.*					
Augmented Dickey-Fu	Augmented Dickey-Fuller test statistic							
Test critical values:	1% level	-2.574714						
	5% level	-1.942164						
	10% level	-1.615810						

Figure 5 Augmented Dickey-Fuller Unit Root Test on DLOG(N225)

After DLOG transformation, the series graph looks stationary. Dickey-Fuller Test is further conducted to verify the stationarity. The t-Statistics is now significant and null hypothesis can be rejected, thus DLOG(N225) is stationary.

# Estimation of the model parameters

Correlogram of DLOG(N225)									
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob			
		1	0.125	0.125	3.7877	0.052			
ւիլ	ווןו	2	0.055	0.040	4.5262	0.104			
ı <b>þ</b> i		3	0.081	0.071	6.1279	0.106			
1)1	1 1	4	0.016	-0.004	6.1887	0.185			
1 1	1(1	5	-0.001	-0.009	6.1891	0.288			
' <b>[</b> ] '	'[] '	6		-0.080	7.5332	0.274			
1)1	ונןי	7	0.022	0.041	7.6587	0.364			
1)1		8	0.010	0.010	7.6839	0.465			
1 <b>þ</b> 1	וון י	9	0.046	0.055	8.2152	0.513			
1 <b>j</b> j i		10	0.028	0.012	8.4124	0.589			
1 1	'  '	11	0.021	0.011	8.5270	0.665			
1 1	'  '	12		-0.015	8.5348	0.742			
1 1			-0.006		8.5452	0.806			
	'  '		-0.016		8.6089	0.855			
1 1	1  1	15	0.011	0.024	8.6383	0.896			
	'  '	16	-0.017		8.7094	0.925			
' <b>[</b> ] '	'[[ '	17		-0.052	9.5547	0.921			
'  '	' '		-0.011		9.5850	0.945			
1 <b>j</b> i	יון י	19	0.057	0.065	10.433	0.941			
1 <b>j</b> j i	'  '	20	0.028	0.020	10.636	0.955			
' <b>[</b> '	'[  '		-0.028		10.840	0.966			
' <b> </b>  '		22	0.010	0.002	10.865	0.977			
' '	' '		-0.005		10.871	0.984			
'  '	']'		-0.011		10.906	0.990			
' <b> </b>	' '		-0.024		11.063	0.993			
'[[ '	'[[ '		-0.041		11.524	0.994			
']'	']'	27	0.011	0.020	11.560	0.996			
'[[ '	'[[ '	28		-0.028	11.764	0.997			
۱ <b>۱</b> ۱	'   '	29	0.033	0.040	12.071	0.998			
']'	']'	30	0.016	0.006	12.139	0.998			
' <b>[</b> ] '	'[[ '	31		-0.055	12.791	0.998			
:]:	<u> </u>	32	0.007	0.011	12.803	0.999			
' <b>[</b> ] '	'[]'		-0.056		13.693	0.999			
'] '	']'		-0.018		13.786	0.999			
'[]'	'¶'		-0.054		14.618	0.999			
10 1		36	-0.064	-0.036	15.783	0.999			

Figure 6 Correlogram of DLOG(N225)

As shown in the correlogram of DLOG(N225), the PACF and ACF function have a significant spike at period one and then cut-off. Therefore, we start our model by using AR (1).

## Analysis of the model results

#### ARIMA models

Dependent Variable: DLOG(N225)

Method: ARMA Generalized Least Squares (BFGS) Date: 02/19/19 Time: 13:52

Sample: 1999M03 2018M01 Included observations: 227

Convergence achieved after 3 iterations

Coefficient covariance computed using outer product of gradients

d.f. adjustment for standard errors & covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.	
AR(1)	0.151876	0.065767	2.309285	0.0218	
R-squared	0.021405	Mean depen	0.002092		
Adjusted R-squared	0.021405	S.D. depend	0.056392		
S.E. of regression	0.055786	Akaike info c	riterion	-2.930101	
Sum squared resid	0.703321	Schwarz crite	erion	-2.915013	
Log likelihood	333.5665	Hannan-Quir	Hannan-Quinn criter.		
Durbin-Watson stat	1.997658				
Inverted AR Roots	.15				

Figure 7 Summary of AR (1)

Dependent Variable: DLOG(N225)

Method: ARMA Generalized Least Squares (BFGS)

Date: 02/19/19 Time: 13:48 Sample: 1999M03 2018M01 Included observations: 227

Convergence achieved after 15 iterations

Coefficient covariance computed using outer product of gradients d.f. adjustment for standard errors & covariance

Coefficient	Std. Error	t-Statistic	Prob.
0.554610	0.314500 1.763465		0.0792
-0.418790	0.343108	0.2235	
0.024058	Mean depen	0.002092	
0.019720	S.D. depend	ent var	0.056392
0.055834	Akaike info o	riterion	-2.923967
0.701415	Schwarz crit	erion	-2.893792
333.8703	Hannan-Qui	nn criter.	-2.911791
1.975590			
.55			
.42			
	0.554610 -0.418790 0.024058 0.019720 0.055834 0.701415 333.8703 1.975590	0.554610 0.314500 -0.418790 0.343108 0.024058 Mean depen 0.019720 S.D. depend 0.055834 Akaike info c 0.701415 Schwarz crit 333.8703 Hannan-Qui	0.554610 0.314500 1.763465 -0.418790 0.343108 -1.220579  0.024058 Mean dependent var 0.019720 S.D. dependent var 0.055834 Akalke info criterion 0.701415 Schwarz criterion 333.8703 Hannan-Quinn criter. 1.975590

Figure 9Summary of AR (1) MA (1)

Dependent Variable: DLOG(N225)

Method: ARMA Generalized Least Squares (BFGS)
Date: 02/19/19 Time: 13:59

Sample: 1999M03 2018M01 Included observations: 227 Estimation settings: tol= 0.00010 Initial Values: C(1)=0.14097, C(2)=0.02383 Convergence achieved after 5 iterations

Coefficient covariance computed using outer product of gradients d.f. adjustment for standard errors & covariance

Coefficient	Std. Error	t-Statistic	Prob.
0.148378	0.066646	2.226361	0.0270
0.024198	0.066657	0.363023	0.7169
0.021970	Mean depen	0.002092	
0.017623	S.D. depende	ent var	0.056392
0.055893	Akaike info c	riterion	-2.921862
0.702916	Schwarz crite	erion	-2.891686
333.6313	Hannan-Quir	nn criter.	-2.909686
1.995073			
.25	10		
	0.148378 0.024198 0.021970 0.017623 0.055893 0.702916 333.6313 1.995073	0.148378	0.148378

Figure 8 Summary of AR (1) AR (2)

Dependent Variable: DLOG(N225)

Method: ARMA Generalized Least Squares (BFGS)

Date: 02/19/19 Time: 13:56 Sample: 1999M03 2018M01 Included observations: 227

Convergence achieved after 16 iterations

Coefficient covariance computed using outer product of gradients

d.f. adjustment for standard errors & covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.126335	0.062845	2.010260	0.0456
AR(3)	-0.384964	0.274644	-1.401683	0.1624
MA(3)	0.516890	0.257832	2.004756	0.0462
R-squared	0.035909	Mean depen	dent var	0.002092
Adjusted R-squared	0.027301	S.D. depend	ent var	0.056392
S.E. of regression	0.055617	Akaike info	riterion	-2.927017
Sum squared resid	0.692898	Schwarz crit	erion	-2.881753
Log likelihood	335.2164	Hannan-Qui	nn criter.	-2.908752
Durbin-Watson stat	1.947664			
Inverted AR Roots	.41+.63i	.4163i	69	
Inverted MA Roots	.40+.70i	.4070i	80	

Figure 10 Summary of AR (1) AR (3) MA (3)

Among all the models above, AR (1) model has the lowest AIC, Schwarz criterion and Hannan-Quinn criter. The coefficient of AR (1) is also significant at level 0.05. As with regression, residuals should have zero mean, fixed variance, no autocorrelation, we further conduct the residual test. Here, we cannot use DW when lag is an explanatory variable.

### Residual test

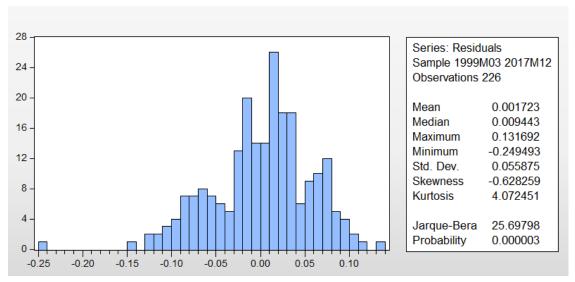
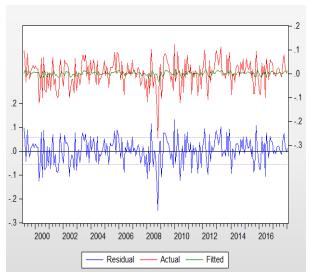


Figure 11 Normality test



Date: 02/19/19 Time: 14:00 Sample: 1999M02 2018M01 Included observations: 227 Q-statistic probabilities adjusted for 1 ARMA term

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
1 1	1 1	1	-0.006	-0.006	0.0092	
1 1	1 1	2	0.008	0.008	0.0231	0.879
ı 🛅		3	0.092	0.093	2.0044	0.367
1 1 1	1)1	4	0.010	0.011	2.0256	0.567
1 1	1 1	5	0.007	0.006	2.0364	0.729
10 1	III	6	-0.078	-0.087	3.4493	0.631
(1)	1 1	7	0.027	0.024	3.6256	0.727
1 1	1 1	8	0.004	0.005	3.6301	0.821
1 📗		9	0.042	0.058	4.0496	0.853
1   1   1	1 1	10	0.010	0.007	4.0748	0.906
1 1	1  1	11	0.020	0.019	4.1699	0.939
1   1	1 1	12		-0.009	4.1875	0.964
1 1	1 1	13	0.002	0.003	4.1886	0.980
1 1	1 1	14	-0.010	-0.015	4.2121	0.989
(1)	1 11	15	0.020	0.028	4.3124	0.993
1111	1 1 1		-0.011		4.3420	0.996
10	'[[ '	17	-0.059	-0.055	5.1942	0.995
1 1 1	1 11	18	-0.010	-0.019	5.2182	0.997
1 🔟	III	19	0.068	0.072	6.3660	0.994
1   1   1	'  '	20	0.021	0.030	6.4737	0.997
14 1	'4 '	21	-0.051	-0.045	7.1334	0.996
1 11	1 1	22	0.030	0.011	7.3560	0.997
1 1	1 1	23			7.3782	0.998
1   1	1 11	24	0.006	0.013	7.3874	0.999

Figure 12 AR (1) Residual Plot

Figure 13 : AR (1) Residual Correlogram

To verify the assumptions of residual, we did a residual diagnosis in EViews. From figure 12 and figure 13, we can conclude that the residual of our model satisfies the three assumptions:

- normally distributed
- independent (no autocorrelation)
- same variance (no heteroscedasticity)

#### **Forecast**

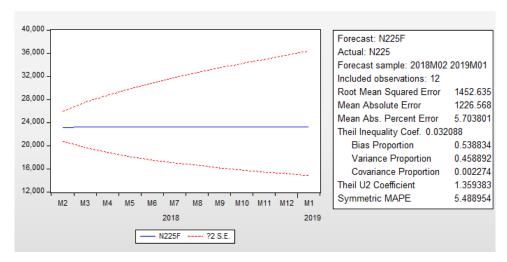


Figure 14 In-sample forecast for one year(dynamic)

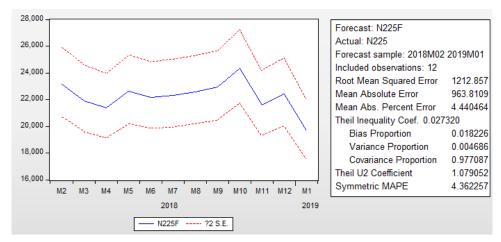


Figure 15 In-sample forecast for one year(static)

To evaluate our static forecast result, we can have a look at the index. The Theil Inequality Coefficient is 2.7%, which indicates it's a very good fit. For Bias Proportion, 1.8% tells us that the forecast for the mean are very close to the actual values. For the Variance Proportion, 0.4% tells us the variance of the forecasts is almost perfect. For Covariance Proportion, 97.7% tells us that a big proportion of forecasting errors are unsystematic. Overall, this is a very good model.

#### Mixed ARIMA model

To further improve the model, Japan Unemployment Rate (Aged 15 and Over), Consumer Purchase Index and the Long-Term Government Bond interest rate are included in our Mixed

Arima Model, we also run multiple regressions to select the best model to do the forecast.

Dependent Variable: DLOG(N225)

Method: ARMA Generalized Least Squares (BFGS)

Date: 02/19/19 Time: 15:23 Sample: 1999M03 2018M01 Included observations: 227

Convergence achieved after 3 iterations

Coefficient covariance computed using outer product of gradients

d.f. adjustment for standard errors & covariance

Dependent Variable: DLOG(N225) Method: ARMA Generalized Least Squares (BFGS) Date: 02/19/19 Time: 15:26 Sample: 1999M03 2018M01 Included observations: 227 Convergence achieved after 3 iterations

Coefficient covariance computed using outer product of gradients

u.i.	aujustinentioi	Stariuaru	ciiois	& covariance	

Variable	Coefficient	Std. Error	t-Statistic	Prob.					
Valiable	Cocincient	Old. Elliol	Cotatione	1100.	Variable	Coefficient	Std. Error	t-Statistic	Prob.
C DLOG(UER) DLOG(CPI) DLOG(LTGB+0.3)	0.003272 0.051407 1.019345 0.125705	0.004239 0.120166 1.213828 0.028965	0.771870 0.427797 0.839777 4.339931	0.4410 0.6692 0.4019 0.0000	DLOG(UER) DLOG(CPI) DLOG(LTGB+0.3)	0.043387 1.031767 0.124479	0.119536 1.212787 0.028881	0.362963 0.850741 4.310149	0.7170 0.3958 0.0000
AR(1)	0.152039	0.066941	2.271248	0.0241	AR(1)	0.154849	0.066733	2.320416	0.0212
R-squared Adjusted R-squared S.E. of regression Sum squared resid Log likelihood F-statistic Prob(F-statistic)	0.101984 0.085803 0.053919 0.645409 343.3194 6.302875 0.000080	Mean dependent var S.D. dependent var Akaike info criterion Schwarz criterion Hannan-Quinn criter.		0.002092 0.056392 -2.980788 -2.905348 -2.950347 1.990627	R-squared Adjusted R-squared S.E. of regression Sum squared resid Log likelihood Durbin-Watson stat	0.099581 0.087467 0.053870 0.647136 343.0157 1.990216	Mean depend S.D. depende Akaike info cri Schwarz criter Hannan-Quin	nt var terion tion	0.002092 0.056392 -2.986922 -2.926571 -2.962569
Inverted AR Roots	.15				Inverted AR Roots	.15			

Dependent Variable: DLOG(N225)

Method: ARMA Generalized Least Squares (BFGS)

Date: 02/19/19 Time: 15:27 Sample: 1999M03 2018M01 Included observations: 227

Convergence achieved after 4 iterations
Coefficient covariance computed using outer product of gradients

d.f. adjustment for standard errors & covariance

Dependent Variable: DLOG(N225)

Method: ARMA Generalized Least Squares (BFGS)

Date: 02/19/19 Time: 15:28 Sample: 1999M03 2018M01 Included observations: 227

Convergence achieved after 4 iterations

Coefficient covariance computed using outer product of gradients

d.f. adjustment for standard errors & covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.	Variable	Coefficient	Std. Error	t-Statistic	Prob.
DLOG(UER) DLOG(LTGB+0.3) AR(1)	0.036608 0.123842 0.158252	0.119105 0.028842 0.066287	0.307360 4.293767 2.387378	0.7589 0.0000 0.0178	DLOG(LTGB+0.3) AR(1)	0.123782 0.159456	0.028782 0.066107	4.300724 2.412094	0.0000 0.0167
R-squared Adjusted R-squared S.E. of regression Sum squared resid Log likelihood Durbin-Watson stat	0.096646 0.088580 0.053837 0.649246 342.6458 1.991587	S.D. depende Akaike info cri Schwarz criter	0.066287         2.387378         0.0178           Mean dependent var S.D. dependent var Akaike info criterion Schwarz criterion Hannan-Quinn criter.         0.002092 -2.992474 -2.992474 -2.947210 -2.974209		R-squared Adjusted R-squared S.E. of regression Sum squared resid Log likelihood Durbin-Watson stat	0.096266 0.092249 0.053728 0.649519 342.5978 1.989215	Mean depend S.D. depende Akaike info cri Schwarz criter Hannan-Quin	nt var terion rion	0.002092 0.056392 -3.000862 -2.970686 -2.988686
Inverted AR Roots	.16				Inverted AR Roots	.16			

Figure 16: Mixed Arima Model

Among all the models above, the fourth model which consists of AR (1) and DLOG (LTGB+0.3) has the lowest AIC, Schwarz criterion and Hannan-Quinn criter. The coefficient of AR (1) and DLOG (LTGB+0.3) are all significant at level 0.05. As with regression, residuals should have zero mean, fixed variance, no autocorrelation, we further conduct the residual test. Here, we cannot use DW when lag is an explanatory variable.

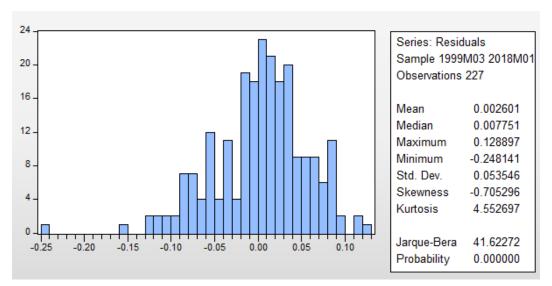


Figure 17 Residual Plot Histogram

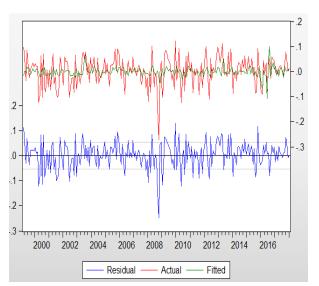


Figure 18 Residual Correlogram

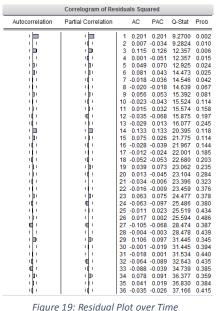


Figure 19: Residual Plot over Time

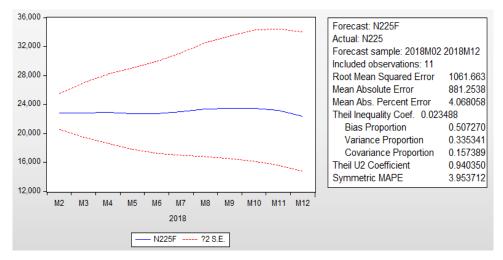


Figure 20: In-sample dynamic forecast

From the in-sample dynamic forecast, we can see that there is a slightly upward trend from M6 to M9, followed by a downward trend from M10 to M12.

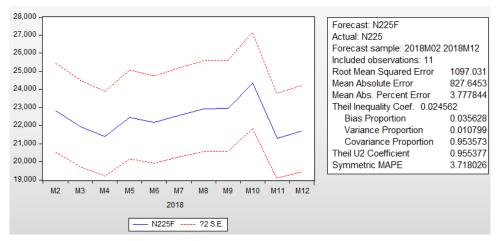


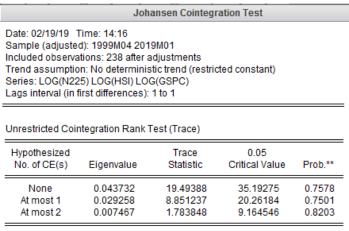
Figure 21 In-sample static forecast

Vector Autoregression Estimates	
Vector Autoregression Estimates	
Date: 03/05/19 Time: 14:20	
Sample (adjusted): 1999M04 2019M01	
Included observations: 238 after adjustments	
Standard errors in ( ) & t-statistics in [ ]	

	DLOG(HSI)	DLOG(N225)	DLOG(GSPC)
DLOG(HSI(-1))	0.115910 (0.09330) [1.24236]	0.098460 (0.08247) [1.19382]	0.094858 (0.06262) [1.51472]
DLOG(N225(-1))	0.078961 (0.09686) [ 0.81517]	0.045906 (0.08563) [ 0.53612]	0.083354 (0.06502) [1.28203]
DLOG(GSPC(-1))	-0.144239 (0.14688) [-0.98201]	0.032805 (0.12984) [ 0.25265]	-0.096427 (0.09859) [-0.97805]
С	0.003787 (0.00409) [ 0.92692]	0.000580 (0.00361) [0.16066]	0.002907 (0.00274) [1.06003]
R-squared	0.012942	0.026251	0.026805
Adj. R-squared Sum sq. resids	0.000288 0.923792	0.013767 0.721884	0.014328 0.416207
S.E. equation	0.062832	0.055543	0.042174
F-statistic	1.022727	2.102753	2.148346
Log likelihood	322.9258	352.2739	417.8050
Akaike AIC	-2.680049	-2.926672	-3.477353
Schwarz SC	-2.621691 0.003939	-2.868314 0.001140	-3.418996 0.003122
Mean dependent S.D. dependent	0.062841	0.055929	0.003122

Figure 22 Vector Autoregression Estimates

VAR Model with lag 1 is estimated as above. All three index have no significant impact on each other. There is no major influencer in the model.



Trace test indicates no cointegration at the 0.05 level

Figure 23 Cointegration test

Cointegration test shows that none of the hypothesis can be rejected at significance level 0.05. No cointegration equation can be establish among Nikkei, S&P500 and Heng Seng Index. Thus, Vector Error Correction Models (VECM) cannot be applied in this case.

## Out-of-sample forecasts for at least five periods.

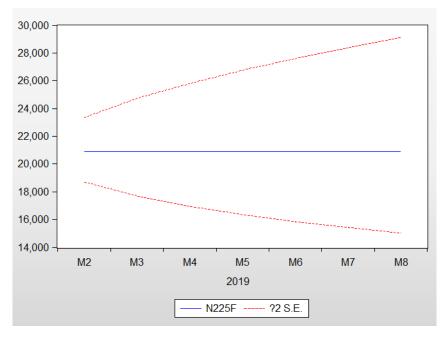


Figure 24 Out-of-sample forecasts(dynamic)

<sup>\*</sup> denotes rejection of the hypothesis at the 0.05 level

<sup>\*\*</sup>MacKinnon-Haug-Michelis (1999) p-values

AR(1) is selected to perform out of sample forecast. The out of sample forecast result shows a flat trend for the following 6 months. This suggest that Nikkei will keep fluctuate with 21,000 level.

The standard error widens up over the time. The suggests that the ARIMA model can compete reasonably well in short-term prediction, but not in the long run.

### Conclusions

Stock price prediction is an important topic in finance and economics which has attracted the interest of researchers over the years to develop better predictive models. This paper presents extensive process of building ARIMA model for Nikkei price prediction. The results obtained with best ARIMA model demonstrated that the simpler model perform better. Even though the prediction result doesn't have high degree of accuracy, time series model is still good to predict the overall trend of the stock price movement.

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