Time Series Analysis and Forecasting
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Workshop 4: Stationarity and ARIMA

(a) Dickey-Fuller Test

(i) Plotting correlogram

Correlogram is plotted to check if the time series is stationary. The autocorrelation graph is as shown,

Sample: 1960M01 2018M12
Included observations: 708

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.992	0.992	699.84	0.000
		2	0.980	-0.257	1384.0	0.000
		3	0.968	0.008	2051.6	0.000
		4	0.955	0.030	2703.3	0.000
		5	0.944	0.004	3339.9	0.000
		6	0.931	-0.037	3961.0	0.000
		7	0.919	0.027	4567.3	0.000
		8	0.909	0.068	5160.5	0.000
		9	0.901	0.111	5743.7	0.000
		10	0.893	-0.004	6318.5	0.000
		11	0.886	-0.055	6884.1	0.000
		12	0.876	-0.113	7438.2	0.000
		13	0.866	0.016	7980.0	0.000
		14	0.856	0.069	8510.8	0.000
		15	0.847	-0.008	9031.3	0.000
		16	0.838	0.019	9542.0	0.000
		17	0.830	0.021	10043.	0.000
		18	0.822	0.006	10535.	0.000
		19	0.814	0.010	11018.	0.000
		20	0.807	0.006	11494.	0.000
		21	0.800	-0.029	11963.	0.000
		22	0.792	-0.045	12423.	0.000
		23	0.783	-0.017	12873.	0.000
		24	0.774	0.002	13314.	0.000
		25	0.766	0.034	13745.	0.000
		26	0.758	0.031	14168.	0.000
		27	0.751	0.090	14585.	0.000
		28	0.746	0.012	14997.	0.000
		29	0.741	0.024	15404.	0.000
		30	0.738	0.000	15807.	0.000
		31	0.734	-0.002	16207.	0.000
		32	0.732	0.061	16605.	0.000

From the plot, we realize that the autocorrelation coefficients remain non-zero for many lags and do not die out quickly; hence it resembles characteristics similar to that of non-stationary series.

(ii) Dickey-Fuller test

```
Title:
 Augmented Dickey-Fuller Test

Test Results:
  PARAMETER:
    Lag Order: 19
  STATISTIC:
    Dickey-Fuller: -0.0962
  P VALUE:
    0.5857
```

Null Hypothesis: COPPER has a unit root
Exogenous: None
Lag Length: 1 (Automatic - based on SIC, maxlag=19)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.525351	0.4888
Test critical values:	1% level	-2.570670	
	5% level	-1.941606	
	10% level	-1.616176	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(COPPER)
Method: Least Squares
Date: 02/14/19 Time: 22:26
Sample (adjusted): 1985M03 2018M12
Included observations: 406 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
COPPER(-1)	-0.001635	0.003112	-0.525351	0.5996
D(COPPER(-1))	0.339270	0.046923	7.230366	0.0000

R-squared	0.113315	Mean dependent var	11.54389
Adjusted R-squared	0.111120	S.D. dependent var	305.6730
S.E. of regression	288.1898	Akaike info criterion	14.17003
Sum squared resid	33553558	Schwarz criterion	14.18977
Log likelihood	-2874.516	Hannan-Quinn criter.	14.17784
Durbin-Watson stat	1.975388		

The results from the Dickey-Fuller test is as shown above. The null hypothesis is to test whether COPPER series has unit root, meaning it is not stationary and imply random walk type behaviour. To check if the exchange rate series can be rejected, the t-statistic should be compared with Dickey-fuller distribution for unit root distribution. If t < DF or p-value is less than 0.05, then reject null hypotheses. In this case, since the result is opposite, we do not reject null hypothesis suggesting that series is indeed non-stationary.

(iii) Adding constant and trend in basic DF test

Dickey-Fuller test with constant and trend added is tested and the result is as shown,

```
Title:
 Augmented Dickey-Fuller Test

Test Results:
  PARAMETER:
    Lag Order: 19
  STATISTIC:
    Dickey-Fuller: -2.4675
  P VALUE:
    0.3804
```

Null Hypothesis: COPPER has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on SIC, maxlag=19)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.790857	0.2015
Test critical values:	1% level	-3.980823	
	5% level	-3.420930	
	10% level	-3.133194	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(COPPER)
Method: Least Squares
Date: 02/14/19 Time: 22:30
Sample (adjusted): 1985M03 2018M12
Included observations: 406 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
COPPER(-1)	-0.025480	0.009130	-2.790857	0.0055
D(COPPER(-1))	0.349569	0.046755	7.476613	0.0000
C	-91.23446	73.00562	-1.249691	0.2121
@TREND("1960M01")	0.393199	0.188703	2.083698	0.0378

R-squared	0.130186	Mean dependent var	11.54389
Adjusted R-squared	0.123695	S.D. dependent var	305.6730
S.E. of regression	286.1440	Akaike info criterion	14.16067
Sum squared resid	32915119	Schwarz criterion	14.20014
Log likelihood	-2870.616	Hannan-Quinn criter.	14.17629
F-statistic	20.05600	Durbin-Watson stat	1.985789
Prob(F-statistic)	0.000000		

From the result, we can see that the p-value is still high and t-statistic is not less than DF, therefore it does not give enough evidence to reject the null hypotheses, indicating that it's non-stationary data.

Since both above cases indicate Non Stationary in the dataset, we introduce stationary in the dataset by using the DIFF(dataset) function and then checking the Dickey Fuller test to verify if Non-Stationary is removed.

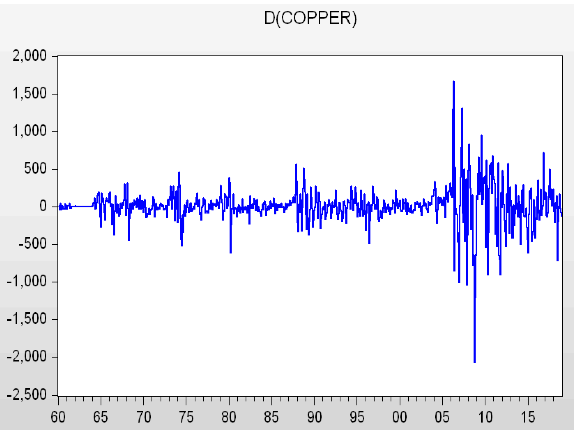
(iv) Augmented DF test including lagged values of dependent variables

We check the autocorrelation coefficients of the residuals to see if series is stationary.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.014	0.014	0.1372	0.711
		2	-0.030	-0.031	0.7918	0.673
		3	-0.021	-0.020	1.0975	0.778
		4	-0.028	-0.028	1.6539	0.799
		5	0.070	0.070	5.1854	0.394
		6	-0.052	-0.056	7.0883	0.313
		7	0.002	0.007	7.0918	0.419
		8	-0.169	-0.173	27.567	0.001
		9	-0.030	-0.022	28.218	0.001
		10	0.014	-0.006	28.349	0.002
		11	0.180	0.189	51.674	0.000
		12	0.071	0.053	55.268	0.000
		13	-0.104	-0.074	63.049	0.000
		14	-0.003	-0.016	63.054	0.000
		15	0.013	0.017	63.169	0.000
		16	0.005	-0.047	63.185	0.000
		17	0.052	0.059	65.147	0.000
		18	-0.021	-0.006	65.470	0.000
		19	-0.054	0.000	67.624	0.000
		20	-0.003	0.022	67.632	0.000
		21	0.078	0.062	72.074	0.000
		22	0.059	0.008	74.594	0.000
		23	0.054	0.051	76.734	0.000
		24	-0.050	-0.031	78.556	0.000
		25	-0.009	0.024	78.621	0.000
		26	-0.037	-0.068	79.653	0.000
		27	-0.024	-0.019	80.064	0.000
		28	-0.029	-0.055	80.699	0.000
		29	-0.041	-0.009	81.912	0.000
		30	0.036	0.065	82.863	0.000
		31	-0.081	-0.062	87.749	0.000
		32	0.016	-0.031	87.946	0.000
		33	0.020	0.002	88.244	0.000
		34	-0.015	-0.048	88.404	0.000
		35	0.045	0.043	89.927	0.000

From the residual ACF plot, we are able to see that the probability is high till lag 7 and then the probability drops below 0.05, indicating statistically significant values. This shows that there is autocorrelation and that the series is not stationary.

(v) Examining differences in series to check for stationarity



Sample: 1960M01 2018M12
Included observations: 707

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.333	0.333	78.954	0.000
		2	0.063	-0.055	81.743	0.000
		3	-0.020	-0.027	82.020	0.000
		4	-0.037	-0.021	82.987	0.000
		5	0.009	0.033	83.043	0.000
		6	-0.081	-0.107	87.696	0.000
		7	-0.098	-0.045	94.588	0.000
		8	-0.205	-0.176	124.70	0.000
		9	-0.090	0.037	130.57	0.000
		10	0.025	0.041	131.02	0.000
		11	0.166	0.160	150.89	0.000
		12	0.074	-0.058	154.87	0.000
		13	-0.082	-0.104	159.68	0.000
		14	-0.040	-0.004	160.86	0.000
		15	-0.012	-0.004	160.96	0.000
		16	-0.002	-0.046	160.96	0.000
		17	0.021	0.053	161.29	0.000
		18	-0.039	-0.037	162.37	0.000
		19	-0.066	0.001	165.51	0.000
		20	-0.007	0.026	165.54	0.000

```
Title:
 Augmented Dickey-Fuller Test

Test Results:
  PARAMETER:
    Lag Order: 19
  STATISTIC:
    Dickey-Fuller: -6.1267
  P VALUE:
    0.01
```

Null Hypothesis: D(COPPER) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=19)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-18.74024	0.0000
Test critical values:	1% level	-3.971104	
	5% level	-3.416195	
	10% level	-3.130392	

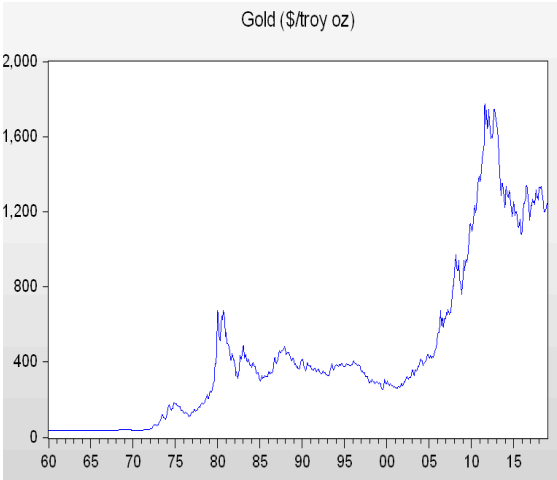
*MacKinnon (1996) one-sided p-values.

Differencing is performed on the time series variable, copper; and its line plot, correlogram and unit root test is analyzed to check if its first differences are stationary.

One can observe from its correlogram that autocorrelation coefficients die down to zero after the first lag indicating stationarity. DF unit root test also proves the same as the t-statistic is lower than the DF critical values. The p-value is also lower than 0.05 in this case, hence signifying evidence against the null hypothesis of having a unit root(non-stationarity).

(b) ARIMA Modeling

(i) Establishing Stationarity

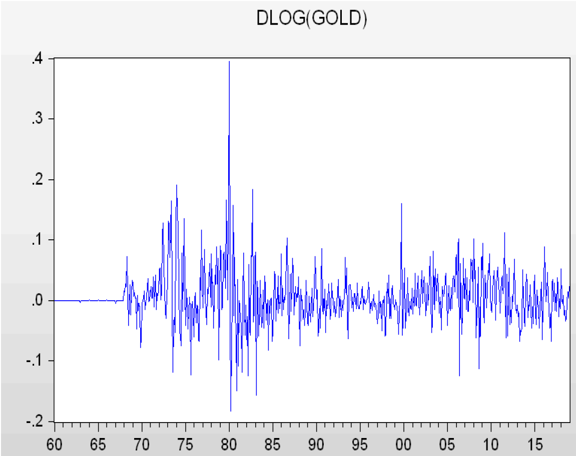


Sample: 1960M01 2018M12
Included observations: 708

Lag	AC	PAC	Q-Stat	Prob
1	0.995	0.995	704.08	0.000
2	0.990	-0.052	1401.6	0.000
3	0.985	0.023	2092.8	0.000
4	0.980	0.008	2778.1	0.000
5	0.975	-0.010	3457.1	0.000
6	0.969	-0.071	4129.2	0.000
7	0.963	-0.042	4793.6	0.000
8	0.956	-0.028	5450.1	0.000
9	0.950	-0.012	6098.5	0.000
10	0.943	0.008	6739.2	0.000
11	0.937	-0.005	7372.1	0.000
12	0.930	-0.043	7996.5	0.000
13	0.923	0.022	8612.9	0.000
14	0.916	-0.013	9221.0	0.000
15	0.909	-0.005	9820.9	0.000
16	0.902	-0.043	10412.	0.000
17	0.895	-0.009	10995.	0.000
18	0.888	0.017	11569.	0.000
19	0.880	-0.023	12134.	0.000
20	0.873	0.018	12692.	0.000

Correlogram of Gold time series is plotted and its autocorrelation coefficients are studied. Sine the coefficients decreases slowly without dying down quickly, the series is not stationary.

Differencing is done on the log return of the series and its correlogram is studied as shown,



Sample: 1960M01 2018M12
Included observations: 707

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.262	0.262	48.914	0.000
		2	-0.039	-0.116	49.995	0.000
		3	0.016	0.063	50.175	0.000
		4	0.062	0.039	52.931	0.000
		5	0.001	-0.025	52.932	0.000
		6	-0.047	-0.035	54.539	0.000
		7	0.099	0.129	61.525	0.000
		8	0.141	0.073	75.682	0.000
		9	0.101	0.067	82.974	0.000
		10	0.065	0.042	85.993	0.000
		11	0.150	0.133	102.16	0.000
		12	0.119	0.041	112.35	0.000
		13	-0.041	-0.065	113.56	0.000
		14	-0.096	-0.068	120.22	0.000
		15	0.019	0.032	120.48	0.000
		16	0.067	0.016	123.78	0.000
		17	-0.003	-0.028	123.79	0.000
		18	-0.052	-0.071	125.79	0.000
		19	-0.021	-0.054	126.11	0.000
		20	0.066	0.043	129.29	0.000

From the correlogram, its apparent that the coefficient pattern resemble that of a stationary series. Since stationarity is established, Box-Jenkins methodology can be applied using Moving Average with lag 1.

(ii)Identification

After removing the initial data, the correlogram is plotted again as shown,

Sample: 1985M01 2018M12
Included observations: 407

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.136	0.136	7.6014	0.006
		2	-0.085	-0.106	10.580	0.005
		3	-0.000	0.028	10.580	0.014
		4	0.047	0.035	11.506	0.021
		5	0.018	0.009	11.647	0.040
		6	0.005	0.009	11.657	0.070
		7	0.072	0.074	13.829	0.054
		8	0.024	0.001	14.060	0.080
		9	-0.013	-0.004	14.130	0.118
		10	0.009	0.013	14.163	0.166
		11	0.154	0.148	24.152	0.012
		12	0.030	-0.017	24.534	0.017
		13	-0.001	0.027	24.534	0.027
		14	0.035	0.027	25.056	0.034
		15	0.045	0.027	25.923	0.039
		16	0.042	0.034	26.664	0.045
		17	0.019	0.015	26.820	0.061
		18	0.057	0.038	28.209	0.059
		19	0.021	0.002	28.390	0.076
		20	0.081	0.089	31.181	0.053
		21	0.027	-0.002	31.496	0.066
		22	0.036	0.020	32.060	0.076
		23	0.037	0.025	32.656	0.087
		24	0.047	0.037	33.608	0.092
		25	0.009	-0.016	33.647	0.116

From the ACF and PACF, there are two significant spikes and then the coefficients die down with random spikes appearing in lag 7, 11 and 20. A number of ARMA processes could result in this pattern. The following reasonable candidate models are verified: ARMA(2,0), ARMA(0,2), ARMA(1,0), ARMA(0,1), and ARMA(1,1).

(iii) Estimation

ARMA(0,1) is tried out through linear regression to see if it's a reasonable candidate.

Sample: 1985M02 2018M12
Included observations: 407
Convergence achieved after 16 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.003484	0.002086	1.670293	0.0956
MA(1)	0.167650	0.040449	4.144712	0.0000
SIGMASQ	0.001196	6.34E-05	18.84932	0.0000

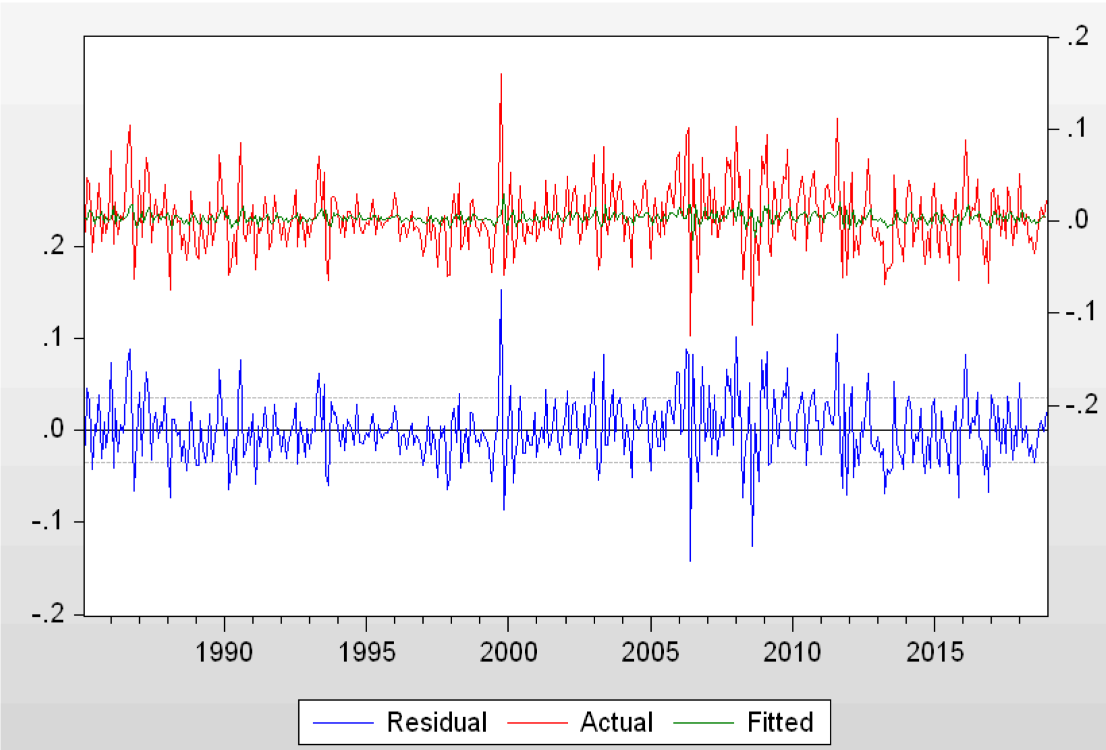
R-squared	0.022957	Mean dependent var	0.003484
Adjusted R-squared	0.018120	S.D. dependent var	0.035029
S.E. of regression	0.034710	Akaike info criterion	-3.876179
Sum squared resid	0.486726	Schwarz criterion	-3.846630
Log likelihood	791.8023	Hannan-Quinn criter.	-3.864485
F-statistic	4.746237	Durbin-Watson stat	2.026107
Prob(F-statistic)	0.009174		

Inverted MA Roots	-.17

Both its regression coefficients are statistically significant hence the model looks reasonable.

(iv) Testing

Consider the characteristics residuals by plotting it out as shown,



The residuals seem to be random in nature, having same variance. To verify its nature further, correlogram for residuals is plotted and studied,

Sample: 1985M01 2018M12
Included observations: 407
Q-statistic probabilities adjusted for 1 ARMA term

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.014	-0.014	0.0774	
		2	-0.083	-0.084	2.9417	0.086
		3	0.006	0.004	2.9572	0.228
		4	0.044	0.037	3.7588	0.289
		5	0.012	0.015	3.8229	0.431
		6	-0.009	-0.002	3.8558	0.570
		7	0.071	0.073	5.9516	0.429
		8	0.014	0.013	6.0305	0.536
		9	-0.013	-0.002	6.0984	0.636
		10	-0.014	-0.013	6.1854	0.721
		11	0.155	0.150	16.262	0.092
		12	0.006	0.005	16.277	0.131
		13	-0.007	0.020	16.296	0.178
		14	0.030	0.028	16.689	0.214
		15	0.034	0.025	17.181	0.247
		16	0.035	0.037	17.715	0.278
		17	0.004	0.014	17.723	0.340
		18	0.056	0.042	19.070	0.325
		19	-0.001	-0.006	19.071	0.387
		20	0.079	0.087	21.741	0.297

The ACF plot shows random spikes at certain lags and are independent proving that the time series is stationary.

Q-statistic figures are all above 0.05, hence they are not statistically significant. Other measures like RSS, SBC, AIC and HQ are noted to understand the quality of the fit of model.

RSS	0.487
SBC	-3.847
AIC	-3.88
HQ	-3.86

From the figures, we realize SBC, AIC and HQ being negative indicating reasonable fitness of the model.

To find an alternative model, other reasonable candidate models are tried out and their results are validated as shown,

Figure 1 : ARMA(2,0)

Sample: 1985M02 2018M12
Included observations: 407
Convergence achieved after 15 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.003473	0.001897	1.830899	0.0679
AR(1)	0.150628	0.039910	3.774163	0.0002
AR(2)	-0.105918	0.043571	-2.430929	0.0155
SIGMASQ	0.001188	6.36E-05	18.67461	0.0000

R-squared	0.029621	Mean dependent var	0.003484
Adjusted R-squared	0.022398	S.D. dependent var	0.035029
S.E. of regression	0.034634	Akaike info criterion	-3.878078
Sum squared resid	0.483406	Schwarz criterion	-3.838679
Log likelihood	793.1888	Hannan-Quinn criter.	-3.862486
F-statistic	4.100590	Durbin-Watson stat	1.992539
Prob(F-statistic)	0.006942		

Inverted AR Roots	.08+.32i	.08-.32i

Figure 2 : ARMA(1,1)

Sample: 1985M02 2018M12
Included observations: 407
Convergence achieved after 14 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.003484	0.002013	1.731338	0.0842
AR(1)	-0.330921	0.215195	-1.537772	0.1249
MA(1)	0.490279	0.203645	2.407521	0.0165
SIGMASQ	0.001190	6.33E-05	18.80114	0.0000

R-squared	0.027818	Mean dependent var	0.003484
Adjusted R-squared	0.020581	S.D. dependent var	0.035029
S.E. of regression	0.034666	Akaike info criterion	-3.876232
Sum squared resid	0.484304	Schwarz criterion	-3.836833
Log likelihood	792.8132	Hannan-Quinn criter.	-3.860640
F-statistic	3.843834	Durbin-Watson stat	2.011501
Prob(F-statistic)	0.009822		

Inverted AR Roots	-.33
Inverted MA Roots	-.49

Figure 4 : ARMA(1,0)

Sample: 1985M02 2018M12
Included observations: 407
Convergence achieved after 11 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.003486	0.002101	1.659647	0.0978
AR(1)	0.136010	0.039860	3.412218	0.0007
SIGMASQ	0.001201	6.43E-05	18.67665	0.0000

R-squared	0.018565	Mean dependent var	0.003484
Adjusted R-squared	0.013706	S.D. dependent var	0.035029
S.E. of regression	0.034788	Akaike info criterion	-3.871717
Sum squared resid	0.488914	Schwarz criterion	-3.842168
Log likelihood	790.8945	Hannan-Quinn criter.	-3.860024
F-statistic	3.820989	Durbin-Watson stat	1.969347
Prob(F-statistic)	0.022702		

Inverted AR Roots	.14

Figure 3: ARMA(0,2)

Sample: 1985M02 2018M12
Included observations: 407
Convergence achieved after 16 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.003479	0.001948	1.786143	0.0748
MA(1)	0.151899	0.040588	3.742491	0.0002
MA(2)	-0.079066	0.043340	-1.824306	0.0688
SIGMASQ	0.001188	6.36E-05	18.67115	0.0000

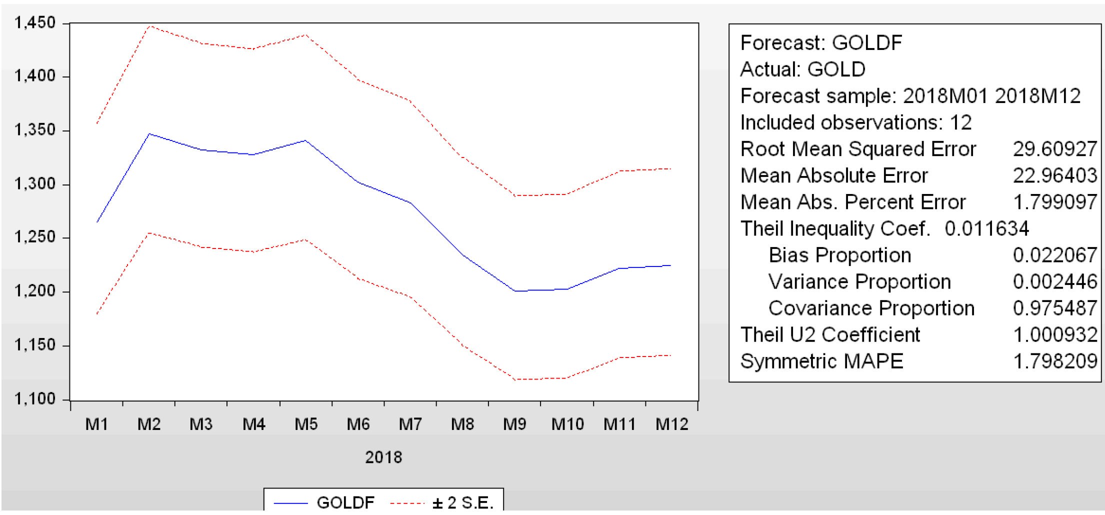
R-squared	0.029432	Mean dependent var	0.003484
Adjusted R-squared	0.022207	S.D. dependent var	0.035029
S.E. of regression	0.034637	Akaike info criterion	-3.877886
Sum squared resid	0.483500	Schwarz criterion	-3.838487
Log likelihood	793.1497	Hannan-Quinn criter.	-3.862294
F-statistic	4.073607	Durbin-Watson stat	1.996835
Prob(F-statistic)	0.007200		

Inverted MA Roots	.22	-.37

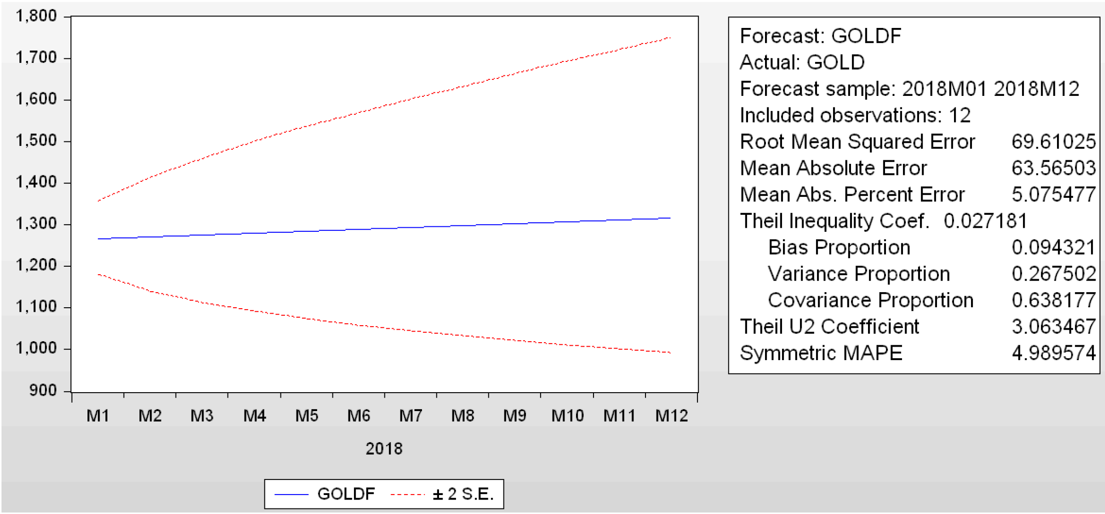
From the four different models, we are able to see that the ARMA(2,0) and ARMA(1,0) are more reasonable models comparable with ARMA(0,1) model since they have regression coefficients which are statistically significant.

(v) Forecasting

The last 12 observations are removed first before forecasting is done. After keying in the equation for the model ARMA(0,1), different forecasting methods are evaluated and the graphs are as shown,



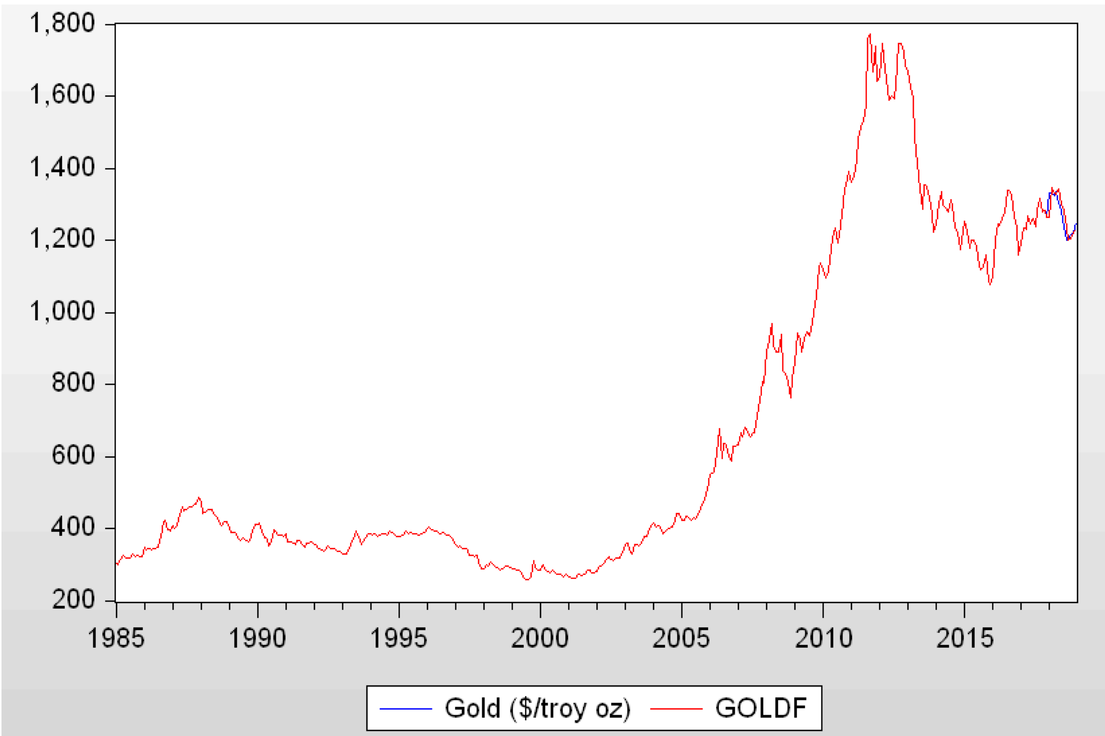
The graph above shows the forecast of gold using static method.



The graph above shows the forecast of gold using dynamic method.

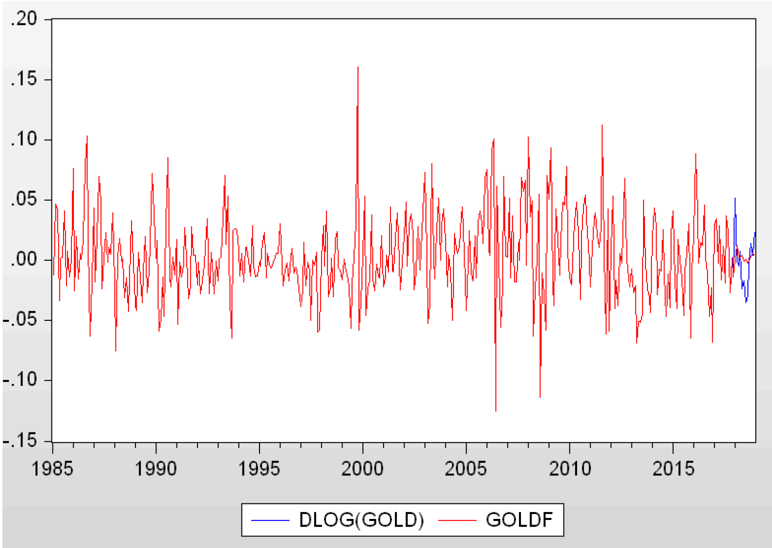
Dynamic produces a 1-step-ahead forecast, a 2-step-ahead forecast, a 3-step-ahead forecast, all the way till 12-step-ahead forecast for a 12 month forecast. By contrast, Static forecasting produces twelve 1 –step ahead forecasts.

Static forecast for gold are plotted to see the fitness of model.



One can observe that the forecast charts the actual time series values of gold, hence it's proves to be a good fit.

Static forecasting is also tried with DLOG(gold) and its fitness is validated.

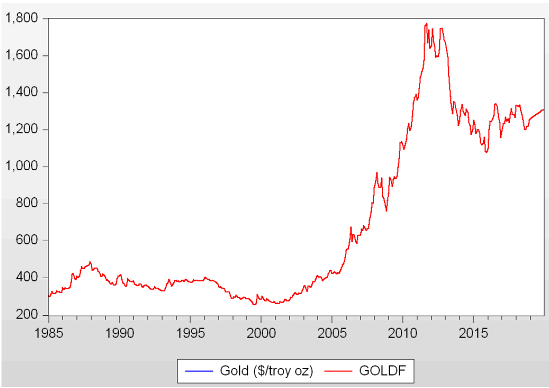


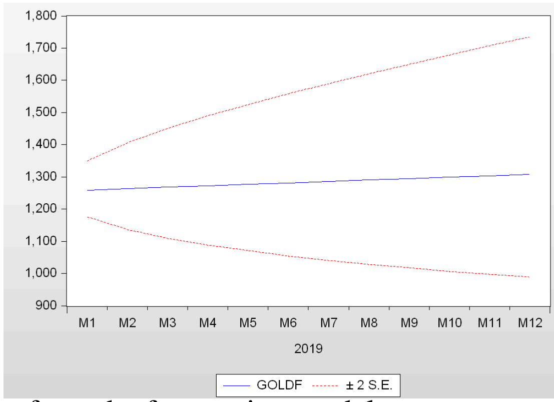
The forecast for DLOG(gold), GOLDF shows a upward trend around 2018 time period, indicating prices to go up over that year.

Do you expect gold prices to go up or down in the next 12 months? How about risk? What is the prediction interval ?

Prediction interval in this case is 12 months, from 2019 month1 to 2019 month12.

Forecasting for DLOG(gold) is done through dynamic forecasting and studied,





Prices of gold are expected to increase as seen from the forecasting model.

Risk of forecast in this case would be unpredictable decrease in gold price when it was expected to increase. An example would be buyinh GOLD commodity stocks assuming their level would appreciate over time, when in fact the stock prices decrease. This could cause them lose money invested.

Static forecasting makes use of actual values to make the next step forecast, which in this case in unavailable. In this scenario, forecast for the next 12 months are to be made based on past data. For this, dynamic forecasting is more appropriate as it previous forecasted values for prediction.