DSC5211C QUANTITATIVE RISK MANAGEMENT

Workshop 1

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Models for Risk Analysis - Generating Distributions

Objectives for Workshop 1

- Introduction to the Concept of Uncertainty
- Practice in the Generation of Basic Statistic Distributions
    - Binomial
    - Normal
    - Lognormal
    - Poisson
- Sensitivity Analysis

Readings: Vose (Ch. 6-8).

1. Uncertainty and Variability

- Variability is the effect of chance and is a function of the system
- Variability: modelled with distributions.
- Uncertainty is a subjective construct that represents the lack of knowledge about the parameters in the system.
- Uncertainty: modelled with sensitivity analysis.

2. Binomial Probability Distribution

Four properties of a Binomial experiment:

- The experiment consists of a sequence of n identical trials
- Two outcomes, success and failure, are possible on each trial
- The probability of a success, denoted by p, does not change from trial to trial
- The trials are independent

Binomial Distribution: Applications

- The number of heads achieved when tossing a fair coin
- The number of people absent from a class
- The number of people possessing a particular characteristic in a group of people
- The number of defective items in a sample

Binomial Probability Function

- Our interest is in the number of successes occurring in the n trials

$$f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{(n-x)}$$

- In which:

  $f(x)$ = the probability of x successes in n trials

  n = the number of trials

  p = the probability of success on any one trial

Binomial Probability Function

$\frac{n!}{x!(n-x)!}$: Number of experimental outcomes providing exactly x successes in n trials

$p^x(1-p)^{(n-x)}$: Probability of a particular sequence of trial outcomes with x successes in n trials

- *Excel: = BINOMDIST(x,n,p,0)*

Exercise 1: Simulate the Number of Defectives

- ***Open the Excel spreadsheet Exercise 1.***
- Let the probability of a defective be equal to p = 5%.

- Generate 100 different Batches of articles. Each batch has 10 articles. Each article can be defective, i.e., assume value 1, with probability *p*

  - For each article do: := If(rand()<p, 1, 0)

a) Fill in the table with the appropriate formulas by copying the formulas in row 6 to the other cells (B7:L105).

Exercise 1: Simulate the Number of Defectives

Simulating the Number of Defectives

Probability of a Defective			p=	0.5							
Batches	Art 1	Art 2	Art 3	Art 4	Art 5	Art 6	Art 7	Art 8	Art 9	Art 10	Total
1	1	0	0	0	1	0	1	1	1	1	6
2											
3											
4											
5											
6											
7											

Exercise 1: Binomial Distribution Mean and Standard Deviation

b) Verify that, in your sample, for the total number of defectives per batch, the mean and the SD approximate:
   - Mean: np Standard Deviation: $\sqrt{np(1-p)}$

c) Press F9. This should generate new random numbers.
   i. What can you say about the mean and the standard deviation?
   ii. Collect 10 different means for the total number of defectives and compute their average and standard deviation. How do they compare with the formulas in b)?

d) Play with the probability of defective. What can you observe?

Exercise 1: Binomial Distribution

- Example of a Solution for the Mean and Standard Deviation

Total	
Average Number Defectives	0.57
Standard Deviation	0.82
Max	3
Min	0

Batch 1	
Average Number Defectives	0
Standard Deviation	0
Max	0
Min	0

3. The Normal Distribution

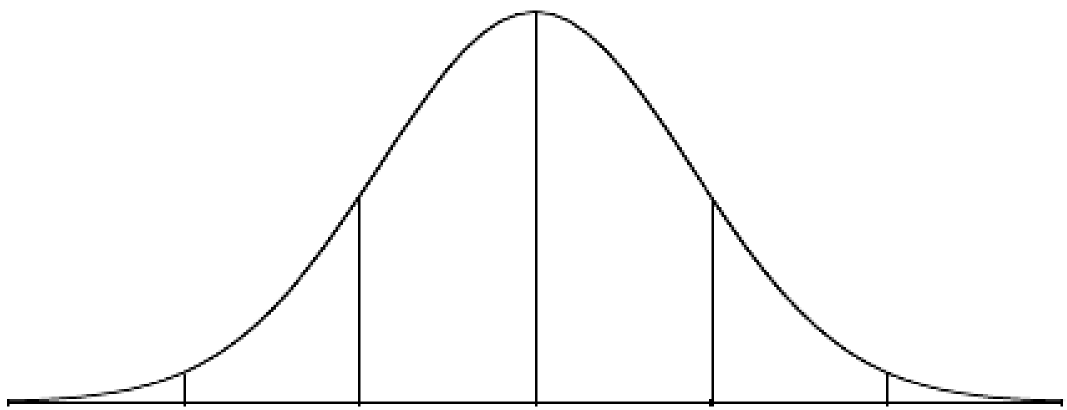
- The NORMAL(Gaussian) distribution ("The Bell Curve") is continuous.

- Defined by just two parameters N(μ, σ):
  - μ: mean (location)
  - σ: standard deviation (spread)

- Properties of Normal distribution:
  - symmetrical bell-shaped curve
  - mean = mode = median

3. Normal Distribution: Standardising Values

- Values for the normal distribution are available in tables – Only for the STANDARDISED normal distribution N(0,1)
- We need to standardise our value in order to use the table



135	140	145	150	155	160	165	$= x$	$\sim N(150, 5)$
-15	-10	-5	0	5	10	15	$= x - \mu$	$\sim N(0, 5)$
- 3	- 2	-1	0	1	2	3	$= (x - \mu) / \sigma$	$\sim N(0, 1)$

$$\mathbf{z = (x - \mu) / \sigma}$$

3. The Normal Distribution: Excel Functions

a) *NORMDIST*(*x*, *mean*, *stdev*, *true*)

Gives the area below x: $P(X < x)$

b) NORMSINV(RAND())

Gives the x of the standard normal distribution, with mean zero and stardard deviation 1, for which P(X<x)=RAND()

4. Modelling the Evolution of Prices
The Log-Normal Distribution

- Most financial models of stock prices assume that the stock's price follows a **log-normal** distribution.

- The same is true for some commodities such as oil and gas prices.

- Exchange rates also tend to follow this type of distribution.

- Log-normal distribution: the logarithm of the stock's price at any time is a normally distributed random variable:

$$\ln(P_t) \sim N\left(Mean, SD\right)$$

$$Mean = Ln(P_0) + \left(u - \frac{\sigma^2}{2}\right).t$$

$$SD = \sigma\sqrt{t}$$

- $P_0 =$ Price of the stock at time zero (this is known)
- t: any future time
- $P_t =$ Price of the stock at time t (it is a random variable and its value is not known until time t)
- Z: a standard normal random variable (having mean 0 and standard deviation 1)
- u : mean percentage growth of a stock expressed as a decimal
- σ : standard deviation of the growth of the stock expressed as a decimal

1. Log-Normal Distribution from Time zero to Time t:

$$P_t = P_0 . \exp\left[\left(u - \frac{\sigma^2}{2}\right).t + \sigma.Z.\sqrt{t}\right]$$

2. Log-Normal Distribution from Time t-1 to Time t:

$$P_t = P_{t-1} . \exp\left[\left(u - \frac{\sigma^2}{2}\right).t + \sigma.Z.\sqrt{t}\right]$$

In this case t refers to the proportion of one time period in the year. Example: 1/52 is a t for one week.

Exercise 2: Modelling the Price of a Stock

- ***Open the Excel spreadsheet Exercise 2.***

MODELLING THE PRICE OF A STOCK: McDonald's				Average growth	0.070	a year
				St. Dev	0.220	a year
				Time Period	0.019	(one week)
Weeks	Z	Price				
0		181.55				
1	1.69	191.31				

Step 1: In cells F1:F2 we enter the parameters for the mean percentage growth and for the standard deviation of the growth of the stock:

F1: = 0.07 F2: = 0.22

Exercise 2 (Cont)
Step 2: In cells F3 we enter the proportion of time of the week in a year:

F3:=1/52

Step 3: In cell B6 we enter the random number Z a standard normal random variable (having mean zero and standard deviation 1):

B6: =NORMSINV(RAND())

Step 4: Enter the initial price as a parameter in cell C5:

C5 := 181.55

Exercise 2 (Cont)

Step 5: Enter the formula to compute the final price in cell C6:

$$P_t = P_{t-1}.\exp\left[\left(u - \frac{\sigma^2}{2}\right).1/52 + \sigma.Z.\sqrt{(1/52)}\right]$$

C6:=C5*EXP((F1-(F2^2)/2)*F3+F2*B6*SQRT(F3))

Step 6: Copy the formulas for the random numbers and prices to cells B7:C57

Exercise 2: Sensitivity Analysis

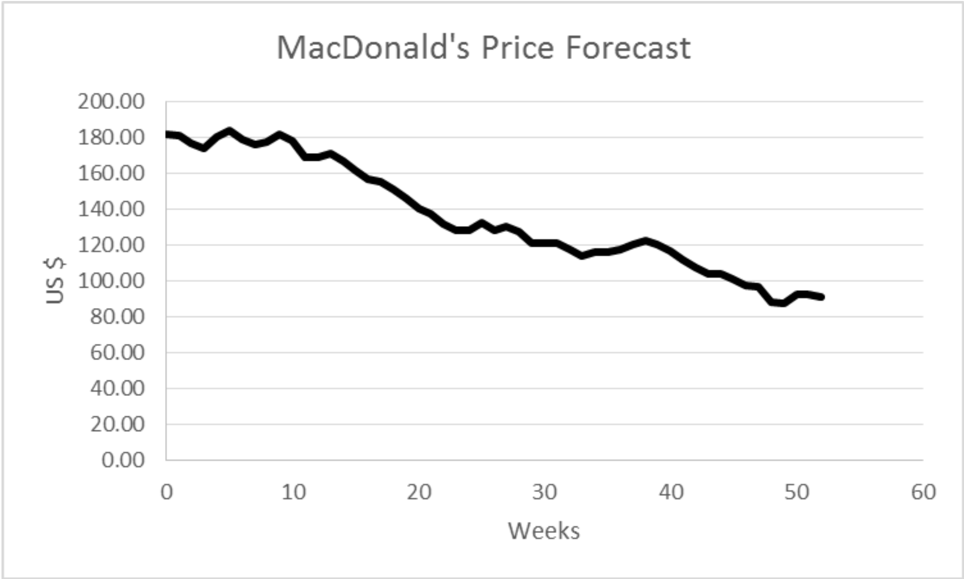
A) Build a Graph to observe the evolution of Prices. Press F9 to generate new real numbers. What can you observe?

B) By pressing F9, collect 10 observations for the price in week 52.

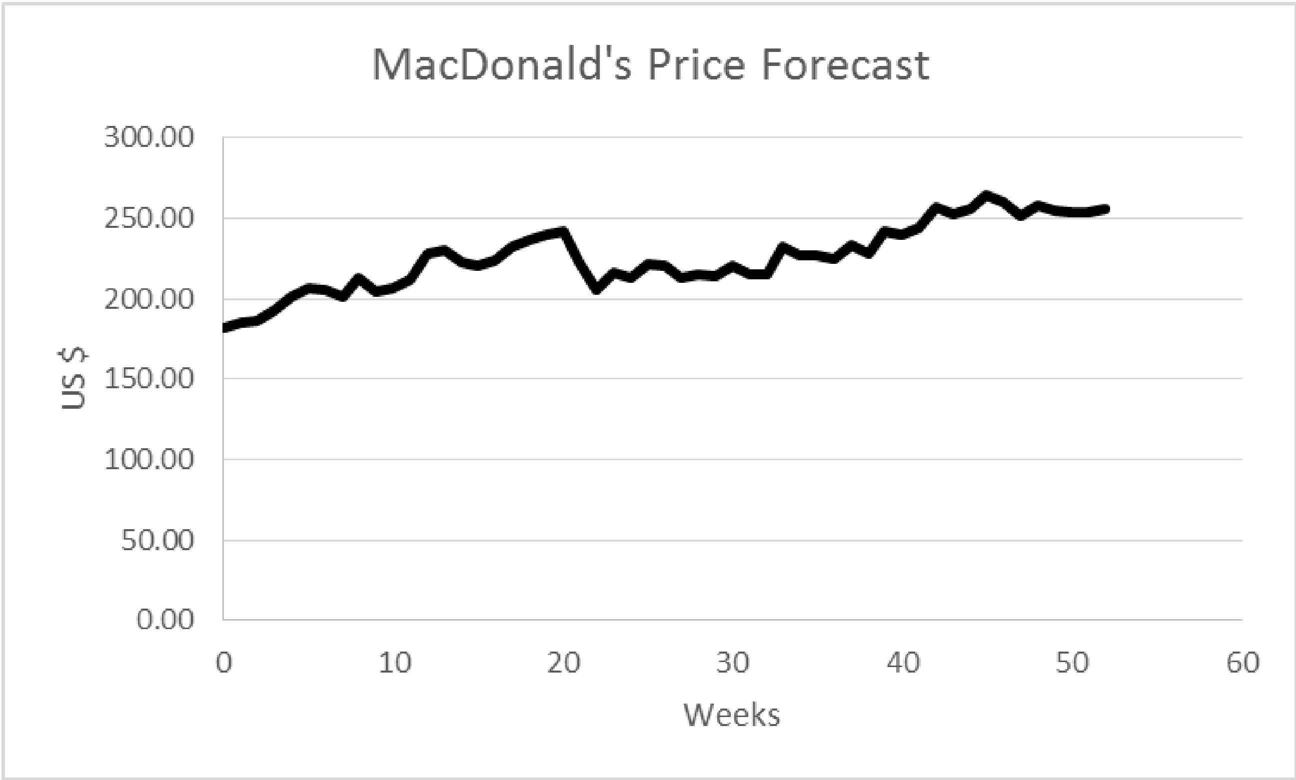
   i. What was the average price? What were the minimum and maximum prices?

C) Suggest different scenarios for growth and standard deviation. Redo A)-B).

Exercise 2: Example of a Forecast for the Stock Price



Exercise 2: A Second example …



Conclusions:

1. Variability is a result of the uncertainty inherent to a given system.

2. Uncertainty reflects the subjective nature of some of the parameters chosen by the analyst. This issue needs to be addressed by sensitivity analysis.

Extra-Work
Poisson Probability Distribution

5. Poisson Probability Distribution

- A Poisson distributed random variable is often useful in estimating the number of occurrences over a specified interval of time or space
- It is a discrete random variable that may assume an infinite sequence of values ($x = 0, 1, 2, \ldots$).
- The probability of an occurrence is the same for any two intervals of equal length
- The occurrence or nonoccurrence in any interval is independent of the occurrence or nonoccurrence in any other interval

5. Examples of a Poisson distributed random variable

- the number of knotholes in 24 linear feet of pine board
- the number of vehicles arriving at a toll booth in one hour
- the number of goals per match is a football game

5. Poisson Probability Function

$$f(x) = \frac{\mu^x e^{-\mu}}{x!}$$

- In which:

  $f(x)$ = probability of x occurrences in an interval

  μ = mean number of occurrences in an interval

  e = 2.71828

- Excel: *=POISSON(x, μ, 0)*

Exercise 3: Simulate the Number of Accidents

A recent article in a national newspaper reported that the number of fatal accidents on farms has averaged at 5 per year in the last 10 years.

Develop a model for analysing the ***monthly*** number of accidents per farm. This model should:

- ***Open the Excel spreadsheet Exercise 3.***

a) Simulate the number of accidents per month in 100 different farms.
b) Use the Poisson distribution to compute the accident probability
c) The average number of accidents peer month is equal to 5/12.

Tip: use =VLOOKUP to generate the number of accidents per farm for each month.

Exercise 3: View of the Model

Simulating the Number of Accidents using Poisson Dist.

Mean Number of Accidents (year)		U=	8	per year
Mean Number of Accidents (year)		U=	0.66667	per month

Poisson Distribution for the Monthly number of Accidents

N.	CDF	Lookup
	0	0
0	0.513417119	1
1	0.855695198	2
2	0.969787892	3
3	0.995141823	4
4	0.999367479	5
5	0.999930899	6
6	0.999993502	7
7	0.999999464	8
8	0.999999961	9
9	0.999999997	10

Note: in order to work with Vlookup we work with the second part of the table

Averages

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year
Average Number Defectives	0.78	0.62	0.62	0.63	0.54	0.72	0.77	0.76	0.53	0.69	0.60	0.61	7.87
Standard Deviation	0.76	0.80	0.77	0.76	0.64	0.86	0.86	0.98	0.70	0.84	0.72	0.73	2.41
Max	2	4	3	3	3	4	3	4	3	4	3	3	15
Min	0	0	0	0	0	0	0	0	0	0	0	0	2

Farm	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year
1	2	0	0	1	0	0	0	1	0	0	1	1	6
2	0	1	0	0	1	0	1	2	1	1	0	1	8
3	0	0	0	0	1	2	0	1	0	1	0	0	5

Exercise 3: Poisson Distribution

A) Verify that, in your sample the mean and the SD approximate:

Mean: μ Standard Deviation: $\sqrt{\mu}$

B) By pressing F9, collect 10 observations for the average number of accidents per year. Comment on the distribution of the average number of accidents per year. Comment also on the distribution of the average number of accidents per month.

C) Do a sensitivity analysis for the values of μ.