Assignment Week 4

Exponential Distribution

Definition

According to [Wikipedia](#):

> The exponential distribution [. . .] is the probability distribution that describes the time between
> events in a Poisson process, i.e. a process in which events occur continuously and independently
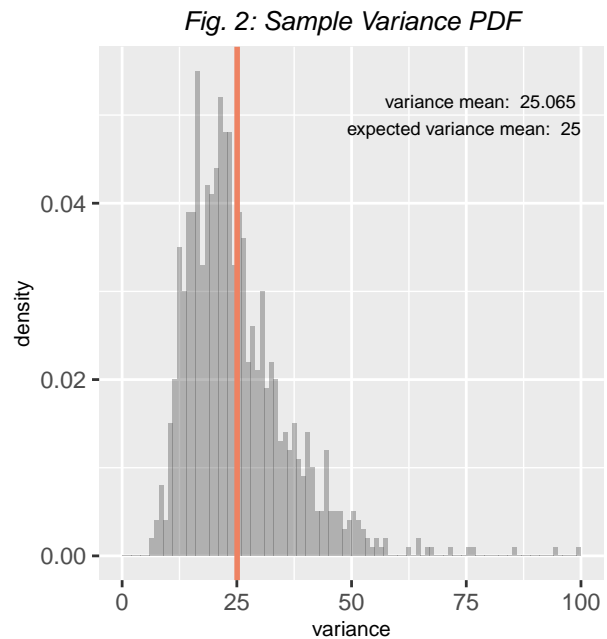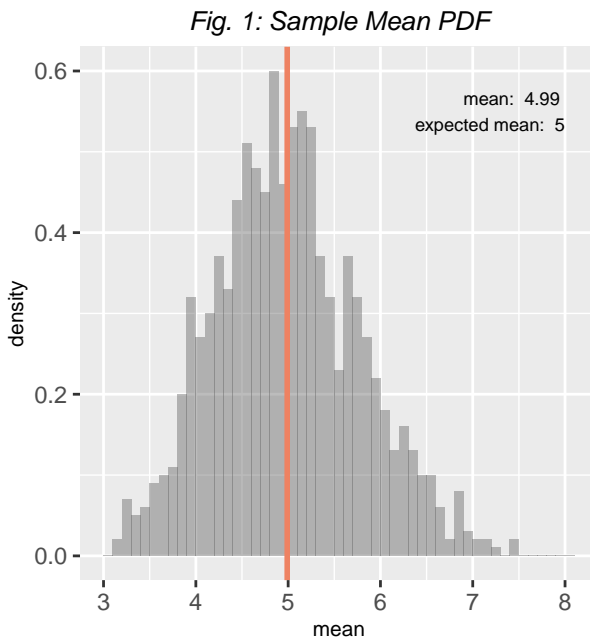> at a constant **average rate** λ.

- Its mean is $1/\lambda$
- Its standard deviation is also $1/\lambda$

In our example, we will use $\lambda=\mathbf{0.2}$.

Samples mean & variance

Samples mean & sample variance are **consistent estimators** of the populations mean & variance: they
converge to the correct value as the number of samples increases.

Let's study the distribution of 1000 averages of 40 exponentials. Fig.1 and Fig.2 show the distribution of
sample mean & sample variance, plus their mean and theoretical values:

Fig. 1: Sample Mean PDF

mean: 4.99
expected mean: 5

Fig. 2: Sample Variance PDF

variance mean: 25.065
expected variance mean: 25

We clearly see that **their mean is close to the value they estimate**.

Central Limit Theorem

Definition

The **Central Limit Theorem** states that, according to the Law of Large Numbers:

> The **sample mean distribution** of iid variables (mean $= \mu$, sd $= \sigma$) will become **normal**, or nearly normal, as the sample size n increases. It will be approximated by:
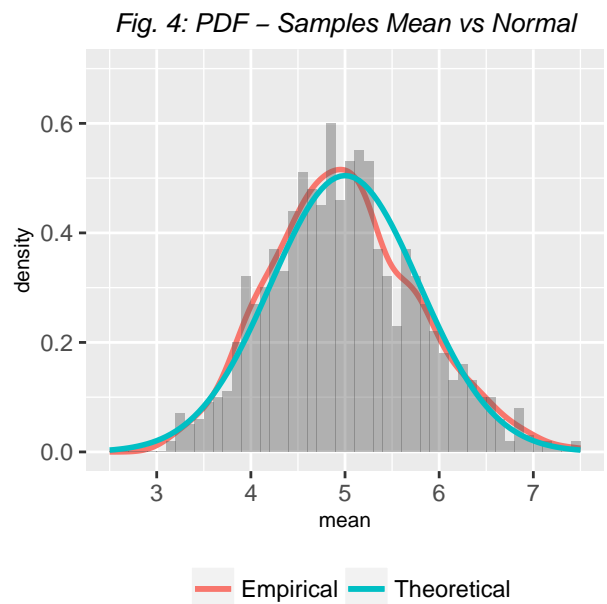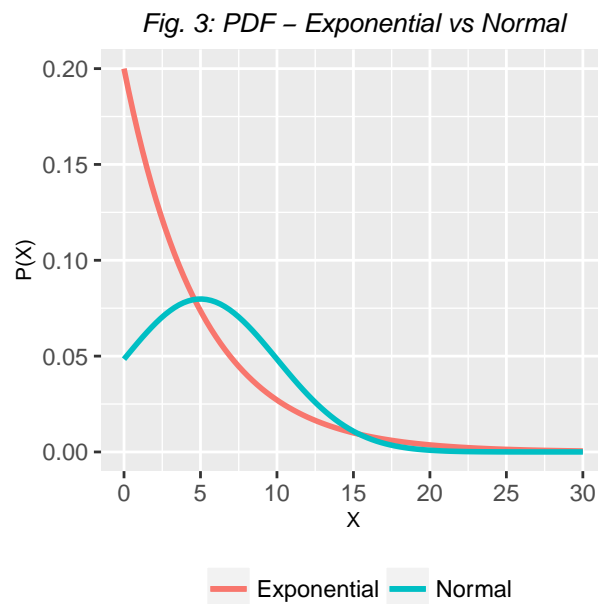>
> $\bar{X} \sim N\ (\mu, \sigma^2/n)$

An important point is that the CLT works **even if the original distribution is not normal**.

Example

Fig. 3 shows the PDF for our distribution (λ=0.2) vs a Normal with the same mean and standard deviation.

Fig.4 shows the empirical distribution of samples mean *(cf. Fig.1)* VS the CLT predicted one:

- $Est = 1/\lambda = 5$
- $SE_{Est} = 1/(\lambda \times \sqrt{n}) = 0.625$

We clearly see that:

- our exponential distribution is **not even close to being normal**
- **the empirical distribution is very close to being normal**, as predicted by the CLT