MDL-PAR

- Minimum Description Length Piecewise Autoregressive Predictor
  - sequentialize multidimensional signals
    - adaptive filter order and support shape

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Adaptive Sequential Prediction of Multidimensional Signals With Applications to Lossless Image Coding

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Linear Prediction

$$\chi^K(x_0) \;=\; (x_1, x_2, \ldots, x_K)$$

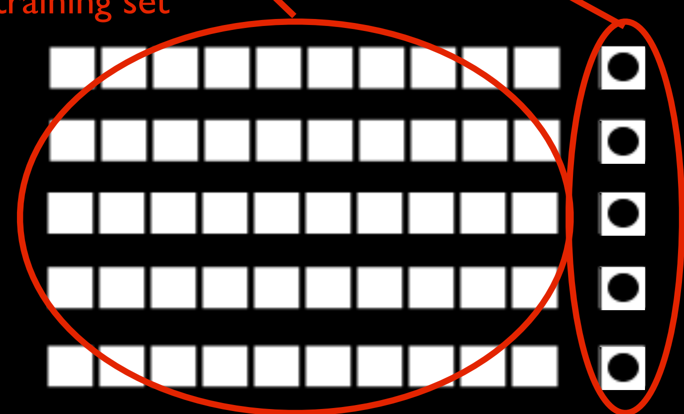
$$\mathbf{a} = \arg\min_{\alpha \in \Re^K} E_{\chi^K(x) \in S} \left\| x - \chi^K(x)\alpha^T \right\|_\ell$$

filter coefficients

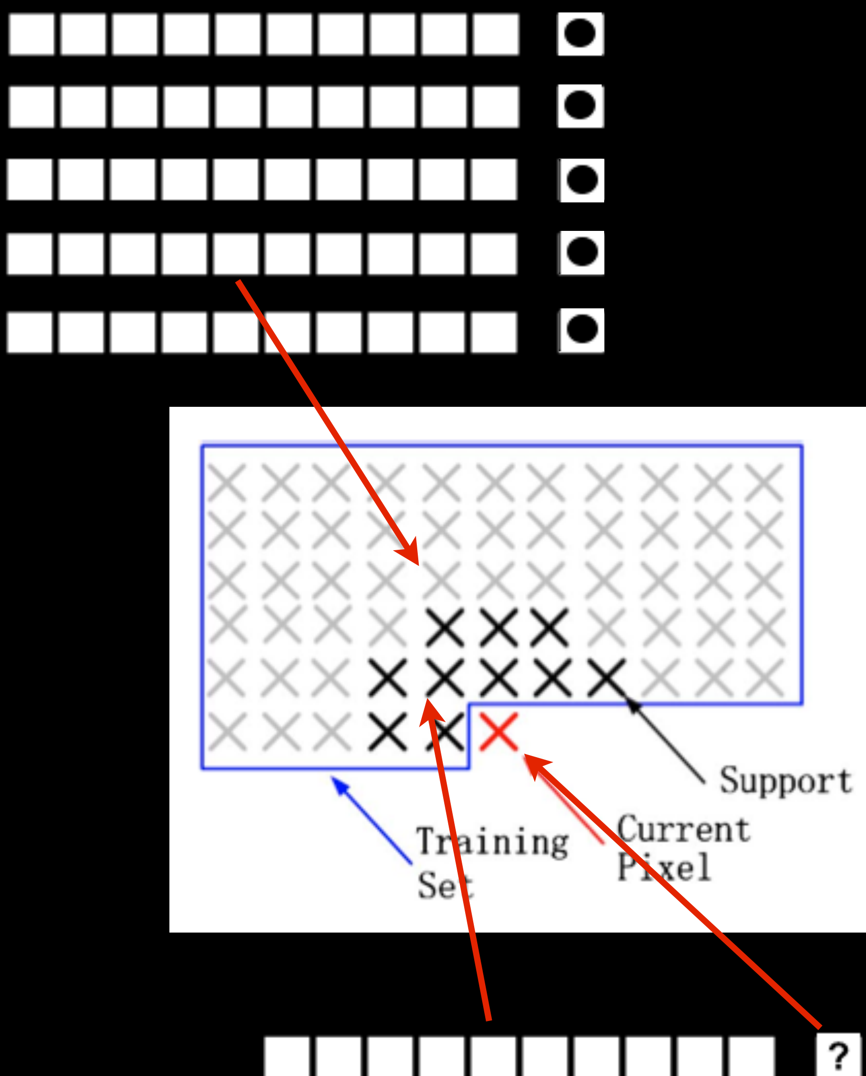
$$\mathbf{a} = (a_1, a_2, \ldots, a_K)$$

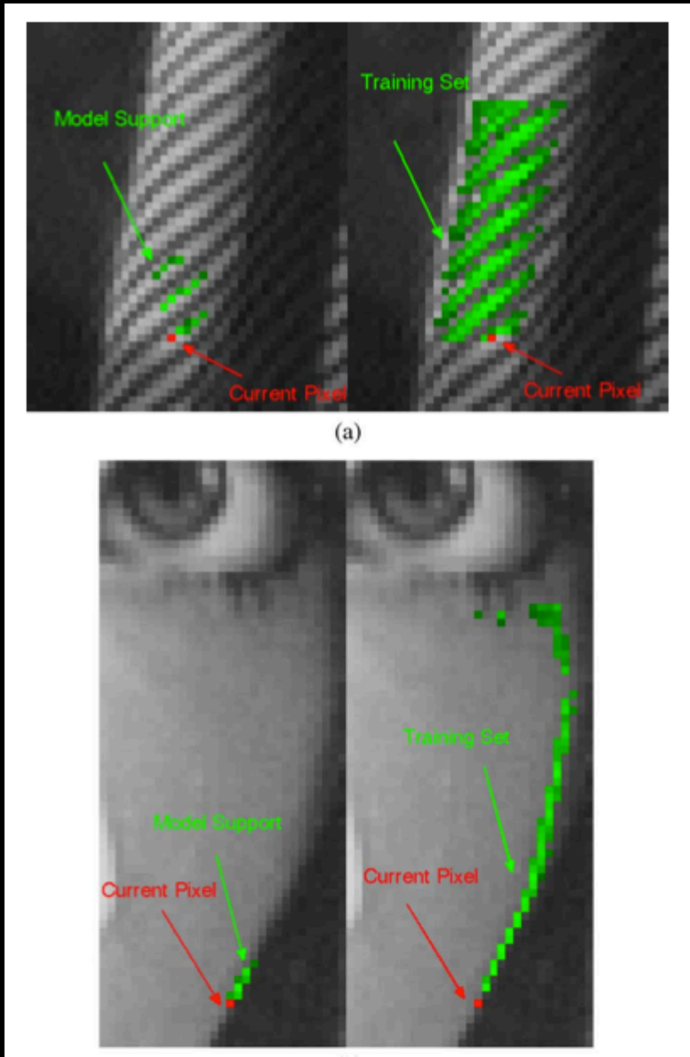
prediction value

$$\hat{x}_0 = \chi^K(x_0)\mathbf{a}^T$$



2D Linear Prediction

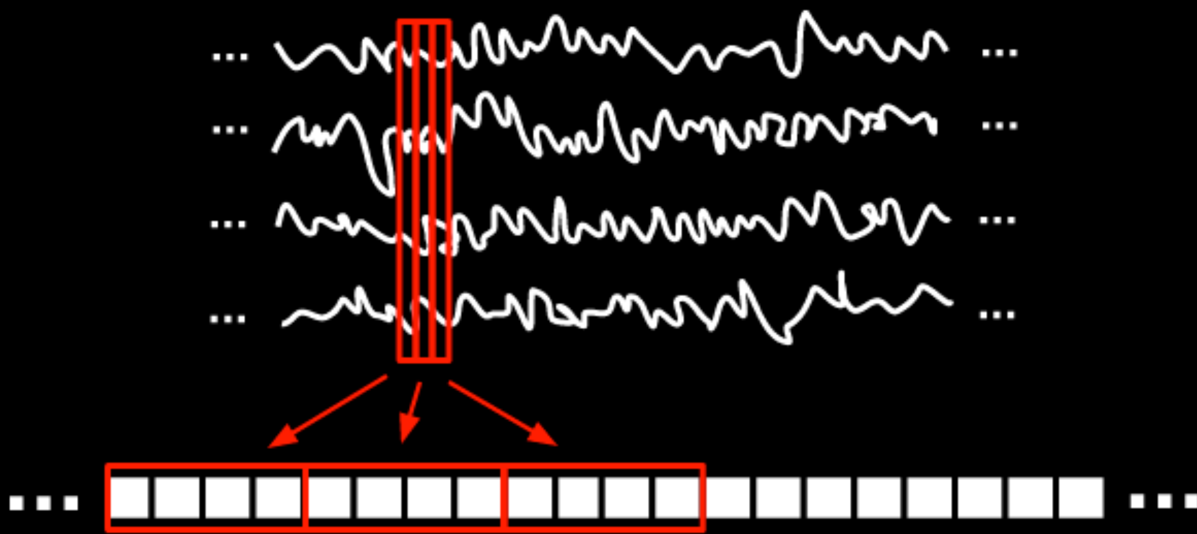




Wu et al., 2011

Data Conditioning

Serialization



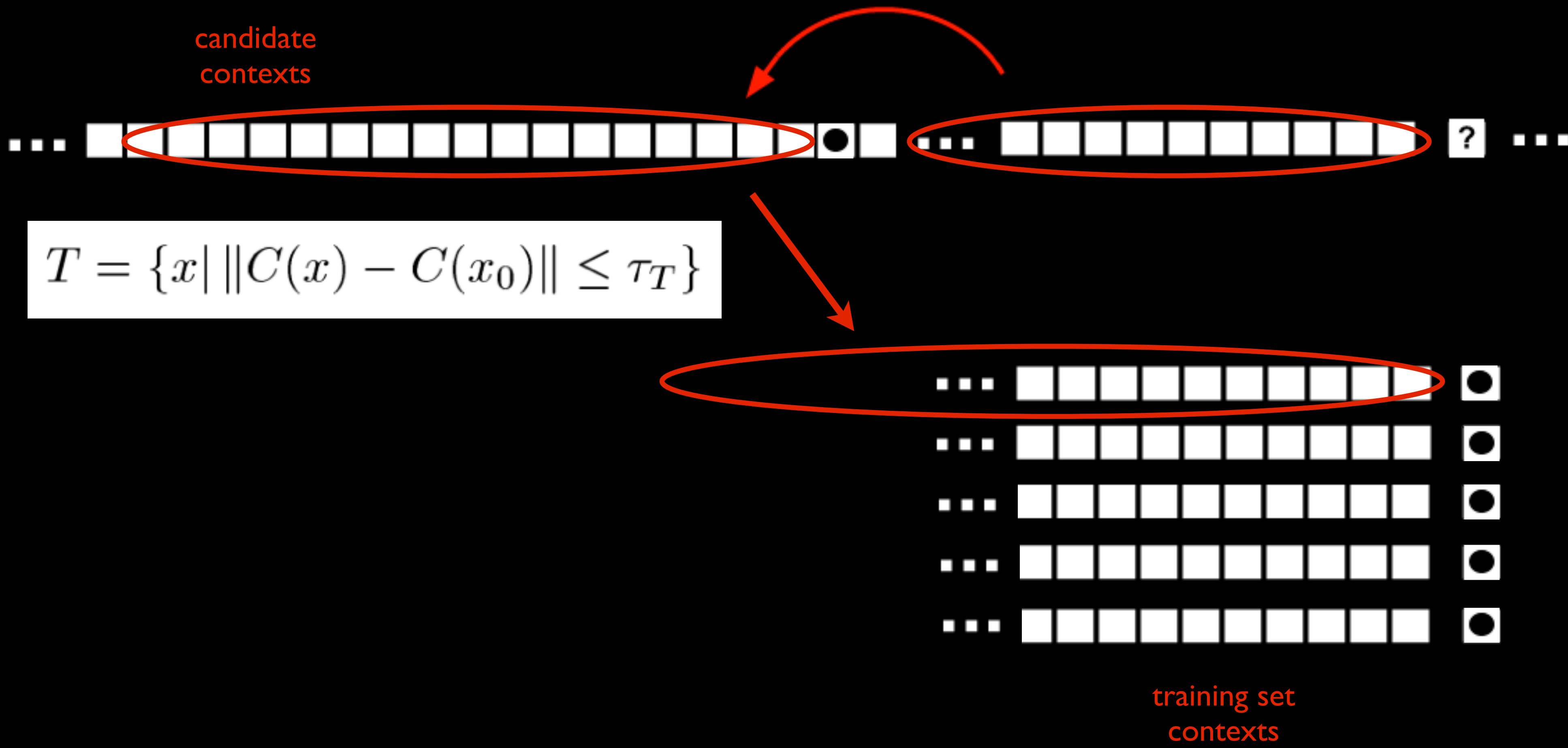
Dynamic Range Compression

(-32768 — 32767) —> (0 — 255)

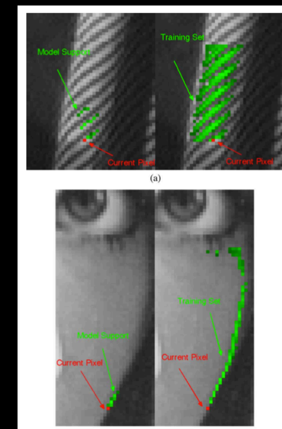
Context Selection*

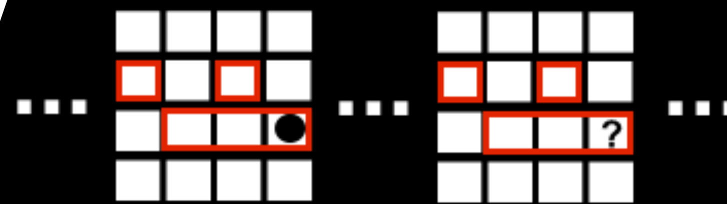
$$C(x) = (c_1(x), c_2(x), \ldots, c_M(x))$$

$$T = \{x | \, \|C(x) - C(x_0)\| \leq \tau_T\}$$

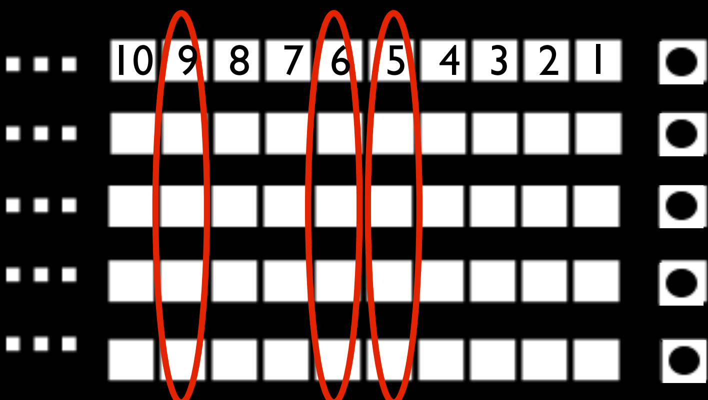


Training Set Selection





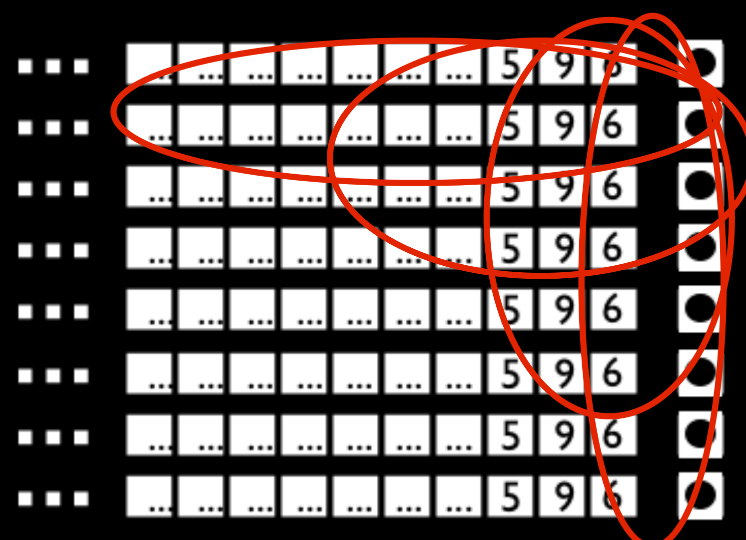
$$\hat{\rho}_{0,m} = \frac{|T| \sum_{x \in T} c_m(x) x - \sum_{x \in T} x \sum_{x \in T} c_m(x)}{\sqrt{|T| \sum_{x \in T} x^2 - \left(\sum_{x \in T} x\right)^2} \sqrt{|T| \sum_{x \in T} c_m(x)^2 - \left(\sum_{x \in T} c_m(x)\right)^2}}$$



$$S_k = \{x \,|\, \|\chi^k(x) - \chi^k(x_0)\| \leq \tau_S, x \in S_{k-1}\} \qquad k > 1$$
$$S_1 = T$$

$$S_1 \supseteq S_2 \supseteq \cdots \supseteq S_k \supseteq \cdots$$

nested training sets



$$\chi^K(x_0) = \{x_1, x_2, \cdots, x_K \,|\, |\rho_{0,1}| \geq |\rho_{0,2}| \geq \cdots \geq |\rho_{0,K}|\}$$

nested support shapes

Prediction*/Compression

$$\mathbf{a} = \arg\min_{\alpha \in \Re^K} E_{\chi^K(x) \in S} \left\| x - \chi^K(x)\alpha^T \right\|_\ell$$

MDL objective function

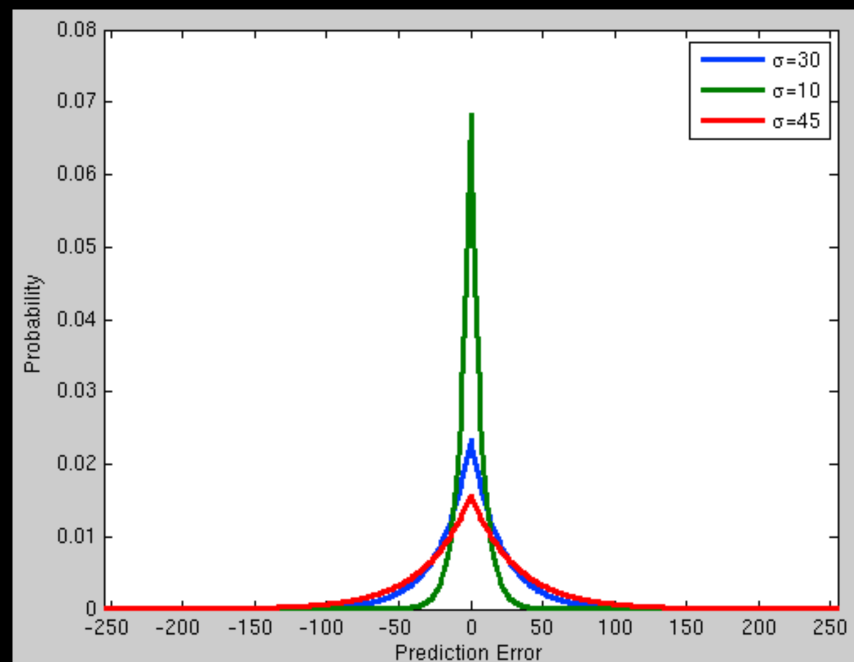
$$L(\chi^k, S) = H(\chi^k, S) + \frac{k}{2|S|} \log |S|$$

"compression code length"

"compression model size"

$$H(\chi^k, S_k) = - \sum_{-A \le d \le A} P(d) \log P(d)$$

$$P(d) = \begin{cases} 1 - e^{-\frac{1}{\sqrt{2}\sigma}}, & d = 0 \\ \frac{1}{2}\left(e^{-\frac{|d|-0.5}{\sigma/\sqrt{2}}} - e^{-\frac{|d|+0.5}{\sigma/\sqrt{2}}}\right), & 0 < |d| < A \\ \frac{1}{2} e^{-\frac{|d|-0.5}{\sigma/\sqrt{2}}}, & |d| = A. \end{cases}$$



$$\hat{x}_0 = \chi^K(x_0)\mathbf{a}^T$$

prediction error

$$x_0 - \hat{x}_0 = d$$

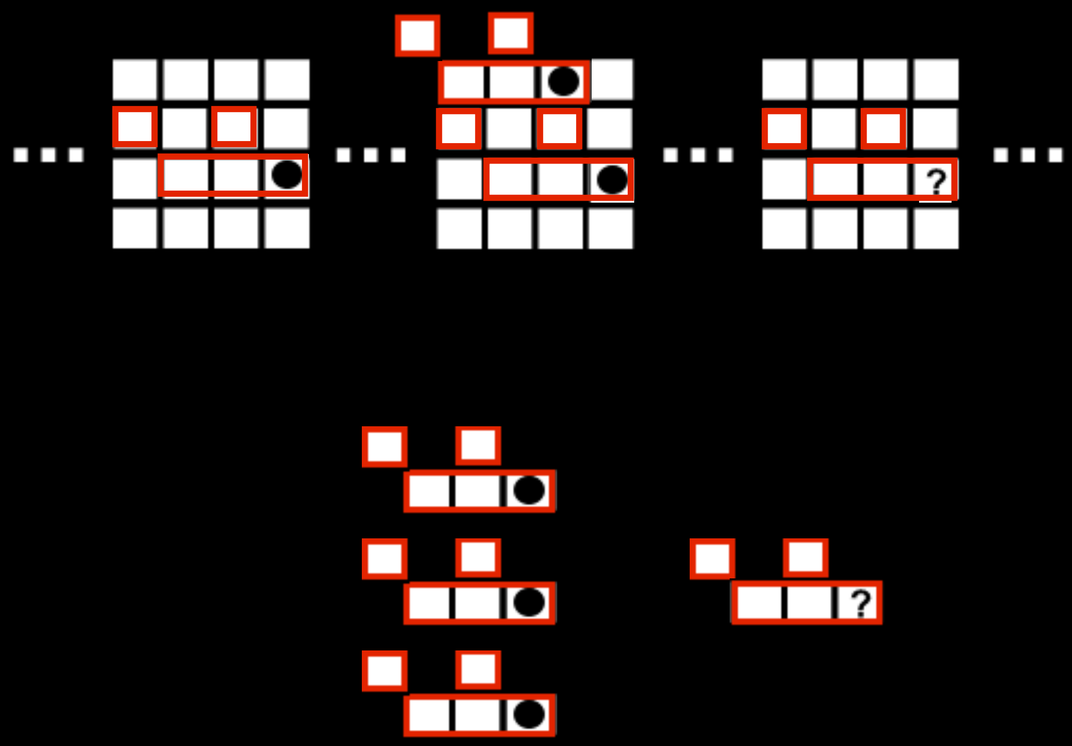
Minimum Description Length (MDL) Principle

- “The minimum description length (MDL) principle is a formalization of Occam's Razor in which the best hypothesis for a given set of data is the one that leads to the best compression of the data.” (Wikipedia)
- "[Occam's Razor] is often summarized as 'other things being equal, a simpler explanation is better than a more complex one.' […] The razor asserts that one should proceed to simpler theories until simplicity can be traded for greater explanatory power." (Wikipedia)

$$L(\chi^k, S) = H(\chi^k, S) + \frac{k}{2|S|} \log |S|$$

Wu et al., 2011

When your context is smaller than your “frame”...



Why haven't I set the context to a larger value?

$$\lim_{dim \to \infty} \frac{dist_{max} - dist_{min}}{dist_{min}} \to 0$$

$$\lim_{k \to \infty} \frac{Variance[d(p,q)]}{Expected[d(p,q)]} = 0$$

(In a naive or poorly constructed space!)

(And the algorithm gets sloooower...)

Performance

- Context selection [O(n) for a single scan in naive case]
  - scanning
  - distance calculation
  - sorting
- filter coefficient calculation [large matrix multiplication = size of training set, k]
- probability distribution construction [dynamic range / number of symbols]
- disk I/O [online vs batch writing]
- memory management [allocation and deallocation, stack vs heap, preallocation]

(Bottlenecks, and how close together are they?)

Algorithm Parameters

$$C(x) = (c_1(x), c_2(x), \ldots, c_M(x))$$

context size

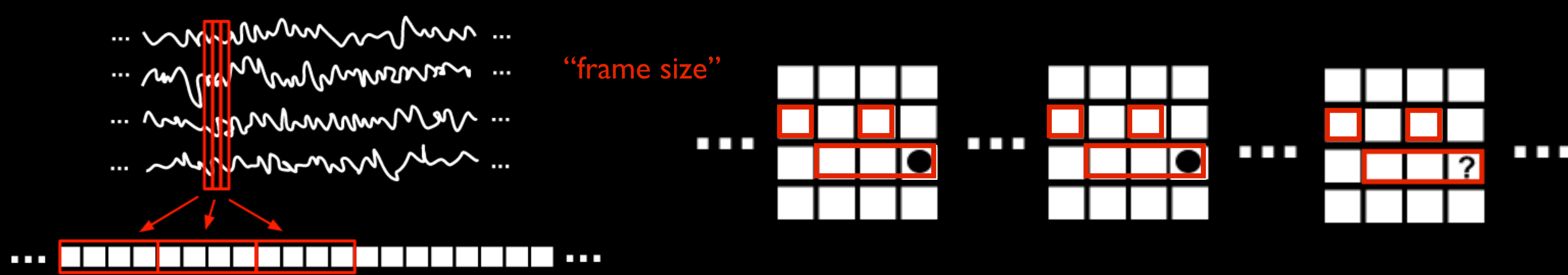
$$T = \{x \mid \|C(x) - C(x_0)\| \le \tau_T\}$$

candidate context similarity threshold

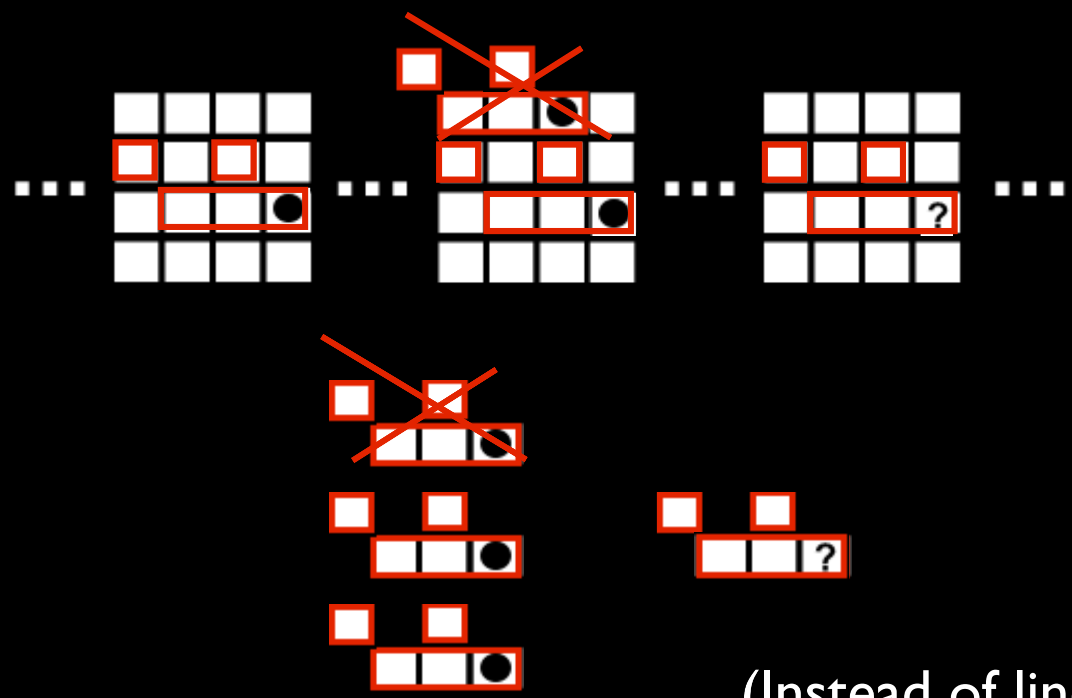
$$S_k = \{x \mid \|\chi^k(x) - \chi^k(x_0)\| \le \tau_S, x \in S_{k-1}\} \qquad k > 1$$
$$S_1 = T$$

candidate training set support threshold

Serialization

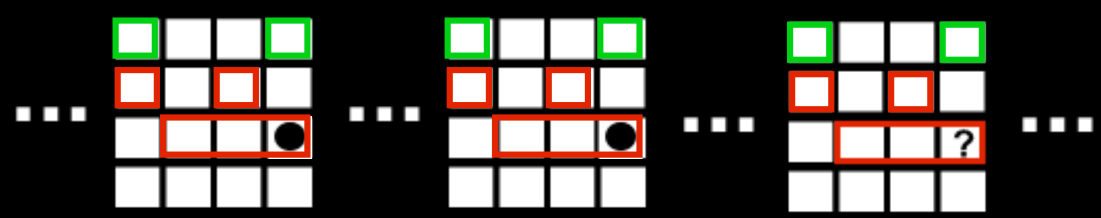


“Frame locking” (“frame size” as a new parameter)...



(Instead of linear scan, only search for candidate contexts at multiples of the frame size.)

But if context is still too small...



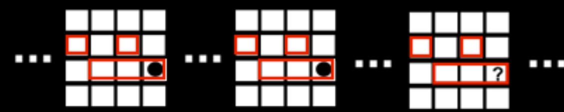
Moving forward...

- Key issues:
  - noisy context selection in high dimensions (probably)
  - performance

- Possible solution:
  - “frame locked” approximate nearest neighbor searching”
    - Better context matching
    - Faster context matching
    - Bigger contexts?

possible 50-100x speedup with better results

“frame-locked” approximate nearest neighbor searching



- a “context repository” for each
  - (predictionSymbolIndex *mod* frameSize)
- context repository is a red-black tree indexed on contexts collapsed to 1D (cf. chinese remainder theorem)
  - O(log n) vs O(n) [e.g. log2 (1,000,000) ~= 20]
- (it already works with a single context repository -- massive speedup, with slightly degraded compression)