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CS-211

Discrete Mathematics

Syllabus

- sets and proposition
- mathematical Induction
- Principles of inclusion and exclusion
- Permutation & combination
- Relations
- Pigeon hole principle
- closure

} prilimitnary

- Graphs
- spanning trees.
- Prims and Kruskal algorithm
- Binary search tree (there but sorting notthere)

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Discrete Mathematics

Proposition and Predicates

⇒ Proposition ~~and~~ are statements used in mathematical logic, which either true or false but not both and we can definitely say whether a proposition is true/false

⇒ Lists of Topic

→ propositions and logical connections

→ Normal forms for well-formed formulae wff

→ Predicates

→ Rules of inferences for propositional calculus and predicate calculus

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⇒ True → T → 1

False → F → 0

<u>Examples</u>: 1. New Delhi is the capital of India (T)

∴ hence a proposition

2. The square of four is 16 (T) ∴ (")

3. The square of five is 27 (F) ∴ (")

4. Every college will have a computer by 2010 AD.

- (T/F confusion)

- If agree then true, if not agree false } ∴ hence a proposition

5. Mathematical logical is a difficult subject

∴ In the same way a previous 'd is a proposition.

6. Chennai is a beautiful city (∴ A proposition)

→ confusion but proposition

7. Bring me coffee! (Not a proposition)

8. No, thank you! (")

9. This statement is False. (Not A proposition)

Note: use capital letters to represent proposition

for e.g. P: The square is a polygon

Connectives (propositional connectives / logical conn.)

⇒ 5 basic connectives

(i) Negation (NOT)

(ii) conjunction (AND)

(iii) Disjunction (OR)

(iv) Implication (if ... then)

(v) If and only if

(i) Negation (NOT)

if P : proposition

~~No~~ NOT P : $\neg P$

P	$\neg P$
T	F
F	T

(ii) Conjunction (AND)

P ; Q : are two proposition :

~~AND~~ : $P \wedge Q$

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

(iii) Disjunction

Let P and Q are two proposition.

Disjunction denoted by $P \vee Q$

P	Q	$P \vee Q$
T	T	T
T	f	T
f	T	T
f	f	f

→ Also called inclusive OR

(iv)

Exercise → EXCLUSIVE OR

denote? truth table?

$\oplus$

P	Q	$P \oplus Q$
T	T	F
T	f	T
f	T	T
f	f	f

Example 1

P: This book is good

Q: This book is cheap

write the following sentences in symbolic form

a) This book is good and cheap ($P \wedge Q$)

b) This book is not good but cheap ($\neg P \wedge Q$)

c) This book is costly but good $\neg Q \wedge P$

d> This book is neither good not cheap $(\neg P \wedge \neg Q)$

e> This book is either good or cheap. $(P \vee Q)$

(iv) Implications (if then)

$P \Rightarrow Q$: denoted by

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Exersise

Example : Find the truth values of the following propositions

P : 2 integer Q : $\frac{1}{2}$ integer.

a> if 2 is not an integer then $\frac{1}{2}$ is an integer

$(\neg P \Rightarrow Q)$

b> if 2 is an integer, then $\frac{1}{2}$ is an integer

$(P \Rightarrow Q)$

(v) If and only if $(P \Leftrightarrow Q)$

Truth table

P	Q	$P \Leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

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Example

Translate the following sentences into propositional forms.

a) If it is not raining and I have the time then I will go to a movie.

Sol^n

P : if it is raining

Q : I've time

P: I'll go to a movie

$\neg P \wedge Q \Rightarrow R$

b) It is raining and I will not go to a movie.

P: It is raining

Q: I'll go to a movie

$P \wedge \neg Q$

c) It is not raining

d) It is not raining

Q.1 $\neg P \wedge Q \Leftrightarrow R$ (from previous example)

meaning:

i'll go to a movie iff it is not raining and i've time

Proof (LHS = RHS)

Q.2 $(Q \Rightarrow R) \wedge (R \Rightarrow Q) \longrightarrow Q \Leftrightarrow R$

Q3. $\underline{\neg (Q \vee R)}$

It is not the case that i'll go to a movie ~~and~~ or i've time

Q4 $R \Rightarrow \neg P \wedge Q$

~~itt~~ If it is not raining and i've time, then it is not raining

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Wellformed formulae (wff)

→ $P \wedge Q$ same as $Q \wedge P$

→ P and Q are propositional variables, symbols to represent proposition.

→ A wff is the propositional variables involving connectives

→ Recursive proposition:

e.g $0! = 1$ $n! = n \times (n-1)!$

wff using recursion:

Defn : A wff is defined recursively as

(i) If P is a propositional variable, then it is a wff

(ii) If α is wff then $\neg\alpha$ is a wff

(iii) If α and β are wff then $(\alpha \vee \beta)$, $(\alpha \wedge \beta)$, $(\alpha \Rightarrow \beta)$, and $\alpha \Leftrightarrow \beta$ are wffs

(iv) A string of symbols is a wff if and only if it is obtained by a finite no of applications of (i) – (iii)

Example: 1

(i) $\neg(P \wedge Q) \wedge (\neg Q \wedge R) \Rightarrow Q$ is a wff

(ii) $(\neg P \wedge Q) \Leftrightarrow Q$ is a wff

Truth Table for wff

Ex. 2

Obtain the truth table for

$$\alpha = \boxed{(P \vee Q) \wedge (P \Rightarrow Q)} \wedge (Q \Rightarrow P)$$

P	Q	$P \vee Q$	$P \Rightarrow Q$	$(P \vee Q) \wedge (P \Rightarrow Q)$	$(Q \Rightarrow P)$	α
T	T	T	T	T	T	T
T	F	T	F	F	T	F
F	T	T	T	T	F	F
F	F	F	T	F	T	F

∴ wff whose outcomes are always true are called **tautology**

∴ wff whose outcomes are always false are called **contradictions** or **absurdity**.

Dn: A **tautology** or is a universally true formula is a wff whose truth value is T for all the possible of ~~the~~ truth values to the propositional variation

e.g. $P \vee \neg P = T$

Example : 1. $P \vee \neg P$

2. $(P \wedge Q) \Rightarrow P$

3. $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$ is a tautology.

4. Show that

$$\alpha = (P \Rightarrow (Q \Rightarrow R)) \Rightarrow ((P \Rightarrow Q) \Rightarrow (P \Rightarrow R))$$ is a

tautology. $(\underbrace{P \Rightarrow A}_{X} \Rightarrow \underbrace{B \Rightarrow C}_{Y})$

P	Q	R	(A) $Q \Rightarrow R$	(B) $P \Rightarrow Q$	(C) $P \Rightarrow R$	(X) $P \Rightarrow A$	(Y) $B \Rightarrow C$	$X \Rightarrow Y$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	F	T
T	F	T	T	F	T	T	T	T
T	F	F	F	F	F	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	F	T	T	T	T	T
F	F	F	T	T	T	T	T	T

∴ hence a tautology

Contradiction: A contradiction/ absurdity is a wff whose truth value is F for all possible assignments of truth values to the propositional variables

e.g (i) $P \wedge \neg P$

(ii) $(P \wedge Q) \wedge \neg Q$

$\boxed{\text{Note:}}$ ⇒ α is a contradiction iff $\neg\alpha$ is a tautology

Equivalence of wff

D^n: Two wff α and β in propositional variables $P_1, P_2 \ldots P_n$ are equivalent (or Logically equivalent) if the formulae $\alpha \Leftrightarrow \beta$ is a tautology

when α and β are equivalent, we write $\alpha \equiv \beta$

but $(\alpha \Leftrightarrow \beta) \neq (\alpha \equiv \beta)$

a wff as defined previously

not a wff as $\equiv$ not defined previously

Example: show that

$$\underbrace{(P \Rightarrow (Q \vee R))}_{\alpha} \equiv \underbrace{(P \Rightarrow Q) \vee (P \Rightarrow R)}_{\beta}$$

L.H.S | RHS

P	Q	R	$Q \vee R$	$P \Rightarrow Q \vee R$	X: $(P \Rightarrow Q)$	Y: $(P \Rightarrow R)$	$X \vee Y$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

As. ~~RHS=~~ LHS $\equiv$ RHS

hence equivalent.

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Logical identifier.

⇒ Some equivalences are useful for deducing other equivalences called identifier

⇒ There are 12 logical identifier used to simplify wff as given below.

1. Idempotent Law

$P \vee P \equiv P \qquad P \wedge P \equiv P$

00	0 0	0
11	1 1	1

2. Commutative Law

$P \vee Q \equiv Q \vee P$

$P \wedge Q \equiv Q \wedge P$

3. Associative Law

$P \vee (Q \vee R) \equiv (P \vee Q) \vee R$

$P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$

4. Distributive Law

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

5. Absorption Law

$P \vee (P \wedge Q) \equiv P, \qquad P \wedge (P \vee Q) \equiv P$

6. Demorgan's Law

$\neg (P \vee Q) \equiv \neg P \wedge \neg Q$

$\neg (P \wedge Q) \equiv \neg P \vee \neg Q$

7. Double Negation

$P \equiv \neg (\neg P)$

8.

$P \vee \neg P \equiv T$

$P \wedge \neg P \equiv F$

9. $P \vee T \equiv T$, $P \wedge T \equiv P$, $P \vee F \equiv P$, $P \wedge F \equiv F$

10. $(P \Rightarrow Q) \wedge (P \Rightarrow \neg Q) \equiv \neg P$

11. contrapositive

$P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P$

12. $P \Rightarrow Q \equiv \neg P \vee Q$

$\Rightarrow$ If a wff β is a part of another formula α, and β is equivalent to β', then we can replace β by β' in α and the resulting wff is equivalent to α.

Example show that

$(P \wedge Q) \vee (P \wedge \neg Q) \equiv P$

Sol^n

LHS $= (P \wedge Q) \vee (P \wedge \neg Q)$

$= P \wedge (Q \vee \neg Q)$ (distributive law and absorption law)

$= P \wedge T$ (I 8)

$= P = $ RHS proved

Ex 2 Show that $(P \Rightarrow Q) \wedge (R \Rightarrow Q) \equiv (P \vee R) \Rightarrow Q$

LHS $= (P \Rightarrow Q) \wedge (R \Rightarrow Q)$ I.12.

$\equiv (\neg P \vee Q) \wedge (\neg R \vee Q)$ (commutative)

$\equiv (Q \vee \neg P) \wedge (Q \vee \neg R)$

$\equiv Q \vee (\neg P \wedge \neg R)$

$\equiv Q \vee \neg (P \vee R)$

$\equiv \neg (P \vee R) \vee Q \equiv (P \vee R) \Rightarrow Q \equiv$ RHS

Normal forms of wffs

⇒ A method of reducing a given formula to an equivalent form called a "normal form".

⇒ We also use "sum" for disjunction

"product" for conjunction &

literal P for variable

Defn: An elementary product is a product of literals. An Elementary sum is a sum of literals.

Ex. $P \wedge \neg Q$, $\neg P \wedge \neg Q$, $P \wedge Q$, $\neg P \wedge Q$ are elementary products

$P \vee \neg Q$, $P \vee \neg R$ are elementary sums

Defn A formula is in disjunctive normal form if it is a sum of elementary products.

Ex. $P \vee Q \vee R$ and $P \vee (\neg Q \wedge R)$

are disjunctive normal form.

$(P \wedge (P \vee Q)$ is not in disjunctive normal form.

⇒ How to obtain a disjunctive normal form of a given formula?

Algorithm

Step 1: Eliminate $\Rightarrow$ and $\Leftrightarrow$ using logical identifiers

e.g. $P \Rightarrow Q \equiv (\neg P \vee Q)$

Step2: Use De Morgan's law (I_6) to eliminate $\neg$ before sums or products. The resulting formula has $\neg$ only before the proportional variables, i.e. it involves sum, products and literals.

Step3: Apply distributive law (I_4) repeatedly to eliminate the product of sums. The resulting formula will be a sum of product of literals i.e. sums of elementary products

Ex: obtain a disjunctive normal form of
$P \vee (\neg P \Rightarrow (Q \vee (Q \Rightarrow \neg R)))$

Soln:
$P \vee (\neg P \Rightarrow (Q \vee (Q \Rightarrow \neg R)))$
$\equiv P \vee (\neg P \Rightarrow (Q \vee (\neg Q \vee \neg R)))$
$\equiv P \vee (P \vee (Q \vee (\neg Q \vee \neg R)))$
$\equiv (P \vee P \vee Q \vee \neg Q \vee \neg R \longrightarrow$ disjunctive normal form

Ex. $(P \wedge \neg(Q \wedge R)) \vee (P \Rightarrow Q)$
$(P \wedge \neg Q \vee \neg R) \vee (\neg P \vee Q)$

Ans: $(P \wedge \neg Q) \vee (P \wedge \neg R) \vee (\neg P \vee Q)$

$\Rightarrow$ For the same formula, we may get different disjunctive normal form.

$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R)$ and $P \wedge Q$ are disjunctive normal form of $P \wedge Q$

⇒ Principal disjunctive normal form or d on the sum of products cannonical form

Advantages of constructing the principal disjunctive normal form

(1) For a given formula its principal disjunctive normal form is unique

(2) Two formula are equivalent iff their principal disjunctive normal for coincide.

Def^n

A min-term in n propositional variable $P_1, P_2 \cdots P_n$ is $Q_1 \wedge Q_2 \wedge Q_3 \cdots \wedge Q_n$ where each Q_i is either P_i or $\neg P_i$

Ex. The min term in P_1 and P_2 are $P_1 \wedge P_2$, $\neg P_1 \wedge P_2$, $P_1 \wedge \neg P_2$, $\neg P_1 \wedge \neg P_2$.

The no. of min terms in n variables is 2^n.

Def^n A formula α is in principal disjunctive normal form if α is a sum of min terms

Algorithm to construct principal disjunctive norm. form

Step 1 Obtain a disjunctive normal form

Step 2 Drop the elementary products which are contradictions (such as $P \wedge \neg P$)

Step 3 If P_i and $\neg P_i$ are missing in an elementary product α_1 replace α by $(\alpha \wedge P_i) \vee (\alpha \wedge \neg P_i)$

Step 4 Repeat step 3 until all the elementary product are reduced to sum of minterm. use the idempotent laws to avoid the repetition of minterms

e.g. obtain the cannonical SOP forms of

$$\alpha = P \vee (\neg P \wedge \neg Q \wedge R)$$

$$\therefore P \equiv (P \wedge Q) \vee (P \wedge \neg Q)$$

$$\equiv (P \wedge Q \wedge R) \vee (P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R)$$
$$\vee (P \wedge Q \wedge \neg R)$$

$$\equiv ((P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R)) \vee ((P \wedge \neg Q \wedge R))$$
$$\vee (P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge R)$$

$\alpha =$

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DNF : Disjunctive Normal form

PDNF: Principal Disjunctive Normal form

Sum of product (Every term mint con

disjunction of conjunction.

Ex. Obtain the principal disj. NF of

$\alpha = (\neg P \vee \neg Q) \Rightarrow (\neg P \wedge R)$

Soln:

PDNF: • All the variables must be present in each term.

or perfect

• It is unique.

• DNF: can be one or many.

$\alpha = (\neg P \vee \neg Q) \Rightarrow (\neg P \wedge R)$ (∵ $P \Rightarrow$ $\neg P$

$(P \vee Q) \wedge T \equiv \neg(\neg P \vee \neg Q) \vee (\neg P \wedge R)$

$= (P \vee Q) \wedge (R \vee \neg R) \equiv \boxed{(P \vee Q)} \vee (\neg P \wedge R)$

$=$

$\equiv (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge R \wedge Q)$
$\vee (\neg P \wedge Q \wedge \neg Q)$

$\equiv (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R)$
$\vee (\neg P \wedge \neg Q \wedge R)$

$P \wedge Q$

$= (P \wedge Q) \wedge (R \wedge \neg R)$ $\quad P \wedge (R \wedge \neg R)$

∴ This is PDNF

⇒ Or disjunctions of conjunct

$(P \wedge Q)$

$(P \vee R) \wedge (P \vee \neg R)$

$(P \wedge Q \vee R)$

$P \wedge T$
$= P$

$P \wedge Q$

$= (P \wedge Q) \wedge (R \vee \neg R)$

$= (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R)$

Def^n: A minterm of the form $Q \wedge Q \wedge \cdots \wedge Q_n$ can be represented as $a_1, a_2 \ldots a_n$ where $a_i = 0$ if $Q_i = \neg P_i$ and $a_i = 1$ if $Q_i = P_i$. So the PDNF can be represented by a sum of binary string

e.g. $(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge R)$ can be represented as $111 \vee 110 \vee 001$

(If negation write 0 if no negation write 1.

e.g. for a given formula α, the truth table are given in Table find the PDNF

P	Q	R	α	
T	T	T	T	row1
T	T	F	F	
T	F	T	F	
T	F	F	T	row4
F	T	T	T	row5
F	T	F	F	
F	F	T	F	
F	F	F	T	row8.

$(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R)$
$\vee (\neg P \wedge \neg Q \wedge \neg R)$

$\therefore$ PDNF for this truth table.

for conjunctive normalform we will ~~print~~ find product of sum.

CNF is a dual of DNF

PCNF $\Rightarrow$ dual of PDNF (apply negation operator to find the

Conjunctive Normal form

⇒ A formula is in conjunctive normal form if it is a product of elementary sums (POS)

⇒ if α is in DNF then $\neg\alpha$ is in CNF

⇒ To construct the conjuctive normal form, we construct a DNF of ~~α~~ $\neg\alpha$ and use negation.

Def^n 1. A maxterm in n propositional variables $P_1, P_2 \ldots P_n$ is $Q_1 \vee Q_2 \vee \cdots Q_n$ where each Q_i is either P_i or $\neg P_i$

Def^n 2. A wff α is in principal CNF if α is product of maxterms.

eg. Find the PCNF for $\alpha = P \vee (Q \Rightarrow R)$

Sol^n: $\neg\alpha \equiv \neg(P \vee Q \Rightarrow R)$

$\equiv \neg(P \vee (\neg Q \vee R))$

$\equiv \neg P \wedge (\neg(\neg Q \vee R))$

$\equiv \neg P \wedge (Q \wedge \neg R)$

→ PCNF of $\neg\alpha$

$\boxed{\neg\neg\alpha = \alpha = (P \vee \neg Q \vee R)}$ for verification

Ex 1 Find PCNF $(P \Leftrightarrow Q)$

$$P \Leftrightarrow Q \equiv (P \Rightarrow Q) \wedge (Q \Rightarrow P)$$

$$\equiv \underbrace{(\neg P \vee Q) \wedge (\neg Q \vee P)}_{\text{PCNF}}$$

Ex 2 Find the boolean exp in CNF for the boolean fun^n of given by the following truth table

X	Y	Z	f (X, Y, Z)	
1	1	1	1	
1	1	0	1	
1	0	1	0	row 4 $(\neg X \vee Y \vee Z)$
1	0	0	1	
0	1	1	1	
0	1	0	0	row 7 $(X \vee Y \vee \neg Z)$
0	0	1	0	row 8 $(X \vee Y \vee Z)$
0	0	0		

$$f = (\neg X \vee Y \vee Z) \wedge (X \vee Y \vee \neg Z) \wedge (X \vee Y \vee Z)$$

POS or CNF

↓

product of sum.

Ex. 4 Express the following boolean expression in CNF in the variables present in the exp.

$\neg x \vee (y \wedge z)$

Solⁿ: $\neg x \vee y \wedge z$

$\equiv (\neg x \vee y) \wedge (\neg x \vee z)$

$\equiv (\neg x \vee y \vee (z \wedge \neg z)) \wedge (\neg x \vee z \vee (y \wedge \neg y))$

$\equiv (\neg x \vee y \vee z) \wedge (\neg x \vee y \vee \neg z) \wedge (\neg x \vee y \vee z) \wedge (\neg x \vee \neg y \vee z)$

$\equiv (\neg x \vee y \vee z) \wedge (\neg x \vee y \vee \neg z) \wedge (\neg x \vee y \vee z)$

CNF

Ex. 5. Find the CNF for the following formula.

$((p \wedge q) \vee (r \wedge s)) \vee (\neg q \wedge (p \vee t))$

CNF. $(p \vee r \vee \neg q) \wedge (p \vee r \vee p \vee t) \wedge (q \vee r \vee \neg q)$

$\wedge (q \vee r \vee p \vee t) \wedge$

$(p \vee s) \vee \neg q) \wedge (p \vee s \vee p \vee t) \wedge$

$(q \vee s \vee \neg q) \wedge (q \vee s \vee p \vee t)$

Solⁿ: $\overline{\alpha} = (p \wedge q) \vee (r \wedge s) \vee (\neg q \wedge p) \vee (\neg q \wedge t)$

CNF $\alpha = (\neg p \vee \neg q) \wedge (\neg r \vee \neg s) \wedge (q \vee \neg p) \wedge (q \vee \neg t)$

Rules of inference of propositional calculus

premises or hypothesis : if any proposition will have truth values. Premises are essentially tautologies

wff (: more than one wff

conclusion: from a no. of premises we find conclusion. it can be truth or false.

from premises / conclusion we find if wff is valid or not.

Tautology : The wff that always gives truth value. or a premise

$P \Rightarrow (P \vee Q)$ is a tautology or rules of inference

premise — to conclusion $\dfrac{P}{\therefore P \vee Q}$

Important rules of inferences

Implication form

R_{i1}: Addition :

$$\frac{P}{\therefore P \vee Q}$$

$P \Rightarrow (P \vee Q)$

$P \wedge Q \Rightarrow P$

$R_{i2} \Rightarrow$ conjunction

$$\frac{\begin{matrix} P \\ Q \end{matrix}}{\therefore P \wedge Q}$$

$R_{i3} \Rightarrow$ simplification.

$(P \wedge Q) \Rightarrow P$

$$\frac{P \wedge Q}{P}$$

R24 Modus ~~Pollens~~ ponens

$$\begin{array}{l} P \\ P \Rightarrow Q \\ \hline \therefore Q \end{array}$$

$(P \wedge (P \Rightarrow Q)) \Rightarrow Q$

$P \wedge (\neg P \vee Q)$

$\Rightarrow \underbrace{(P \wedge \neg P)} \vee (P \wedge Q)$

$\Rightarrow P \wedge Q \Rightarrow Q$

R25 Modus Tollens

$(\neg Q \wedge (P \Rightarrow Q)) \Rightarrow \neg P$

$$\begin{array}{l} \neg Q \\ P \Rightarrow Q \\ \hline \neg P \end{array}$$

R26 Disjunctive syllogism

$(\neg P \wedge (P \vee Q)) \Rightarrow Q$

$$\begin{array}{l} \neg P \\ P \vee Q \\ \hline Q \end{array}$$

R27 Hypothetical syllogism

$(P \Rightarrow Q) \wedge (Q \Rightarrow R)$

$\Rightarrow (P \Rightarrow R)$

$$\begin{array}{l} P \Rightarrow Q \\ Q \Rightarrow R \\ \hline P \Rightarrow R \end{array}$$

R28 constructive dilema.

$$\begin{array}{l} (P \Rightarrow Q) \wedge (R \Rightarrow S) \\ (P \vee R) \\ \hline Q \vee S \end{array}$$

R29 Disjunctive dilema.

$$\begin{array}{l} (P \Rightarrow Q) \wedge (R \Rightarrow S) \\ \neg Q \vee \neg S \\ \hline \neg P \vee \neg R \end{array}$$

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eg can we conclude S from the following premises

(i) $P \Rightarrow Q$

(ii) $P \Rightarrow R$

(iii) $\neg(Q \wedge R)$

(iv) $S \vee P$

(apply any rule any time but apply all rules to reach S)

These problems are NP hard.

Sol^n:

$\neg P \vee Q$

$\neg P \vee R$

$\neg Q \vee \neg R$

Resolution

$Q \vee \neg P$

$\neg Q \vee \neg R$

$\therefore \neg P \vee \neg R$

$\neg P \vee R$

$\therefore \neg P$

$P \vee S$

$\therefore S$

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Premises and conclusion

→ Premise, conclusion, argument

→ A premise is a statement in an argument that provides reasons can support for the conclusion

-es

→ An argument ~~conclusi~~ consists of one or many premises

→ A conclusion is a statement in an argument

→ A conclusion indicates the way of convincing answers to the receiver / listener

→ An argument has any no. of supporting claims, → Premises and one supported claim → conclusion

conclusion

→ Eg. we should not take spices on the meal because spices makes us sick

→ Premise

P Q

Q Conclusion

Validity and conclusion of the argument

① Sound Argument

② The conclusion should follow for the premises.

Valid argument the truth of the premises should make conclusion (likely to be true)

(ii) The premise should be acceptible, the premises should (likely to be) true

(1) valid

If (1) and (2) both satisfy then sound argum

How to identify premises and conclusion of the arguments

3 approaches

1. What does the author really want me to believe?

2. why should I believe it?

3. Utilising or understanding of conclusion and premise indicator.

Conclusion indication	premises Indicator
Therefore	Since
In conclusion	because
it follow that	for
x implies y	Given that
so etc.	since that.
	due to the fact

Example

A) I need to get a flue shot. If you want to decrease your chances of getting the flue then you need to get the shot. I can not afford to get sick this year.

(H) If P from Q $P \Rightarrow Q$

(1) P = " "

Q = " "

R = " "

S = " "

(1) $P \Rightarrow Q$

(2) R

(3) S

Ex B> If we bring our troops out of Afganisthan too quickly, the govt. there will collapse and (U.S can't suffer through the embarassment of the foreign relation's failure of the govt. collapse there.) so, we should not bring the troops out of too quickley.

P

Q

conclusion so, is there so, rest are premises

If then is also there so, premises

Conclusion : we should not bring the troops out of Afganisthan too quickly. (we can write own way not necessarily accurate words)

conclusion also and there are. extraneous word can ignore

c> The president, being human, is mortal as all human beings are mortal

premises → 2

conclusion → 1

Premises - 1 (P1) : All human beings are mortal

Premises - 2 (P2) : The president is a human being

conclusion : Therefore, the President is mortal.

(S) P1
P2
S.

1 (D): God doesn't exist because if he did there would be no suffering and evil ~~exist~~ in the world but obviously suffering and evil do ~~evil do~~ exist. Thus there is no god.

Conclusion: There is no god.

Deduction & Induction

⇒ A deductive argument is one where the truth of the conclusion is (claimed to be) guaranted by the truth of the premises.

Example: Mathematics is deductive

⇒ An inductive argument is one where the truth of the conclusion (claimed to be) more likely gives the truth of the premises.

Ex: Science is inductive
(as earlier it was believed sun rotates around the earth)

Fallacy

⇒ Bad arguments are called fallacies

⇒ There are many falacies of which many people think that they are good arguments (myth is a falacies)

Consistency of fallacies

→ A set of formulae $\alpha_1, \alpha_2 \cdots \alpha_n$ is said to be consistent if their conjuction has the truth value T for some assignment of the truth values to the atomic variable appearing in $\alpha_1, \alpha_2 \ldots \alpha_n$.

→ A set of formulae $\alpha_1, \alpha_2 \dots \alpha_m$ is inconsistent if their conjunction implies a contradiction or reduction or absurd or idirect method of proof.

Ex. Show the following system (set of premises or inference rule) is inconsistent

$P \Rightarrow Q$

$P \Rightarrow R$

$Q \Rightarrow \neg R$

$(\neg P \vee Q) \wedge (\neg P \vee R) \wedge (\neg Q \vee \neg R)$

Sol^n: (i) $P \Rightarrow Q$ (given)

(ii) P (given)

(iii) Q (conclude drawn from first two premises)

(iv) $P \Rightarrow R$ (given)

(v) R

(vi) $Q \cap R$ (conjunctive operator)

(vii) $Q \Rightarrow \neg R$

(viii) $\neg Q \vee \neg R$

($P \Rightarrow Q$ means $\neg P \vee Q$)

(ix) $\neg(Q \wedge R)$

(x) $(Q \wedge R) \wedge \neg(Q \wedge R)$ (This leads to contradiction) so, this set of rules are not consistent

(It is also called Indirect method of proof)

2. Using Indirect method

Assignment show that

$(R \Rightarrow \neg Q,\ R \vee S,\ S \Rightarrow \neg Q,\ P \Rightarrow Q) \Rightarrow \neg P$

(No hard and fast rules that we need to analyze from left to right we can do anyway)

(1) $P \Rightarrow Q$

(2) P

(3) Q

(4) $S \Rightarrow \neg Q$

(5) $\neg S$ (modus tollens rules) (from (4) we can infer (5))

(6) $R \vee S$ (disjunctive --)

(7) R

(8) $R \Rightarrow \neg Q$

(9) $\neg R \vee \neg Q$

(10) $\neg(R \wedge Q)$ S

(11) $R \wedge Q$

(12) $(R \wedge Q) \wedge \neg(R \wedge Q)$ (There is contradiction

Ex: Can we conclude S from the following premises?

(i) $P \Rightarrow Q$ S

(ii) $P \Rightarrow R$

(iii) $\neg(Q \wedge R)$

Ex.2: Derive (S) from the following premises using argument

(i) $P \Rightarrow Q$

(ii) $Q \Rightarrow \neg R$

(iii) $P \vee S$

(iv) R

$Q \Rightarrow \neg R$
$\neg Q \vee \neg R$
R
$\therefore \neg Q$ (modes tollens)

$\neg P \vee Q$
$\therefore \neg P$ (modus tollens)

$P \vee S$
$\therefore S$ (modus tollens)

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Valid Argument form

⇒ Many valid argument forms

⇒ And, the following 4 forms are more common:

1. Affirmimming the argument

if p then q

p therefore q

2. Denying ~~Designing~~ the consequent

If p then q

Not q Therefore not p

3. chain argument

if p then q

If q then r.

~~not~~ therefore if p then r

4. Disjunctive syllogism

Either p or q

Not p therefore q

If an argument whose form is identical to one of the valid argument form, — it must be a valid argument.

Example

Check the validity of the following argument

if Ram has completed BTech (CSE) or MBA then he is assured to get a good job. If Ram is assured of a good job, he is happy. Ram is not happy so, Ram has not completed MBA.

→ conclusion

Soln

P = "Ram has completed BTech (CSE)

Q = "Ram has completed MBA.

R = "Ram is assured of a good job"

S = "Ram is happy"

Given Premises are

(i) $P \vee Q \Rightarrow R$

(ii) $R \Rightarrow S$

(iii) $\neg S$

$\therefore$ conclusion : $\neg Q$

1. $P \vee Q \Rightarrow R$ (given)

2. $R \Rightarrow S$ given

3. $(P \vee Q) \Rightarrow S$ (transitive) $\neg (P \vee Q) \vee S$ T

$\neg S$ T

4. $\neg S$ (given)

5. $\neg (P \vee Q)$ — modus tollens

$$\frac{\neg Q \quad P \Rightarrow Q}{\neg P}$$ concludes

6. $\neg P \wedge \neg Q$

7. $\neg Q \wedge \neg P$ T

8. $\neg Q$ (simplification rule) $\frac{P \wedge Q}{P}$

Therefore, this argument is valid.

Q Test the validity of the following argument:

If milk is black then every cow is white then it has four legs. If every cow has four legs then buffalo is white and brisk. The milk is black.

Therefore, the buffalo is white.

Sol^n

P = "The milk is black"

Q = "Every cow is white"

R = "Every cow has 4 legs"

S = "Every buffalo is white"

T = "Every buffalo is brisk"

Given premises are:

(i) $P \Rightarrow Q$

(ii) $Q \Rightarrow R$

(iii) $R \Rightarrow S \wedge T$

(iv) T

$\therefore$ Conclusion: S

Sol^n

1. P
2. $P \Rightarrow Q$
3. Q Modus Ponens
4. $Q \Rightarrow R$
5. R
6. $R \Rightarrow S \wedge T$
7. $S \wedge T$ Modus Ponens
8. S

(As we are able to ~~app~~ conclude hence the argument is valid

Ex Show that Q is a valid inference from the premises

(i) $P \Rightarrow Q$
(ii) $P \vee Q$
(iii) $\neg Q$

Or are we able to conclude Q from this set of premises.

Sol^n

$$\begin{array}{l} P \Rightarrow Q \\ \neg P \vee Q \\ P \vee Q \\ \hline \therefore Q \end{array}$$

$$\begin{array}{l} P \vee Q \\ \neg Q \\ \hline \therefore P \\ \neg P \vee Q \\ \hline Q \end{array}$$

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PREDICATE CALCULUS

⇒ How predicate calculus is different from propositional calculus?

⇒ What are the alternative names for predicate calculus?

⇒ Propositional calculus deals with unanalysed propositions, (either true or false) related by connectives (and, or, if ... then)

⇒ Propositional calculus do not use quantifiers and relations

A generalization of

predicate calculus ⊃ propositional calculus.

First order logic
Elementary logic
Logic of quantifier
functional calculus
Restricted predicate calculus
Relational calculus
Theory of quantification.

also the name of predicate calculus

⇒ predicate :- A generalization of propositional calculus by adding relations and quantifier.

- developing languages of logic programming
- Eg prolog ('C doesn't include quantifier)

- for specifying the requirements of computer applications

- program correctness

→ consider two propositions:

"Ram is a student" &

"sam is a student"

⇒ We can replace two propositions by a single statement

"x is a student"

⇒ " is a student" is called a predicate

x is a variable

<u>predicate</u> : A part of a declarative ~~st~~ sentence describing the properties of an object or relation among object is called a predicate.

⇒ Description of predicates

Let P denotes the predicate → &

x is a variable denoting any object

$P(x)$

<u>Example</u> : "x is the father of y" is a sentence involving the predicate " is the father of"

Here, predicate describes the relation between two persons $F(x, y)$.

$2x + 3y = 4z$

$S(x, y, z)$

predicate & relation.

predicate is a more generalized form of the relation.

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Universe of discourse

Def^n for a declarative sentence involving a predicate, the universe is the set of all possible values which are assigned to variables

universe of $p(x)$: "x is a student" can be taken as the set of all human names

Universe of $E(x)$: "n is an even integer"

set of all integer.

(1) Universal and (2) Existential Quantifiers

(3) Nested Quantifier: (hybrid of universal & existential quantifiers

⇒ The phrase "for all" (denoted by $\forall$) is called the universal quantifiers

⇒ for all x, $x^2 = (-x)^2$ as $\forall x\, Q(x)$ where $Q(x)$ is $x^2 = (-x)^2$

⇒ "There exists" $\exists$

e.g There exists x s.t $x^2 = 5$

$\exists x\, R(x)$ where $R(x)$ is $x^2 = 5$

$f(x)$: $x^2 > x$ where x is a set of all real nos

$x = 0$ $0^2 > 0$ False

$x = 1$ $1^2 > 1$ False

$x = 2$ $2^2 > 2$ True

$\exists x\, F(x)$ is false

$\exists x\, F(x)$ is true

$\forall$ "for every", "for any", "for each" or "for arbitrary"

$\exists$ "for some" "for atleastonce"

How to use connectives

Write in predicate calculus form.

Examples : 1. All students are clever

2. Some students are not successful

3. Every clever student is successful

4. There are some successful students who are not clever

5. Some students are clever and successful.

Identify

→ Quantifiers

→ Universe of discourse

→ predicate

→ proposition.

b. Let $C(x)$ denotes "x is clever"

$S(x)$ denotes "x is successful"

1. $\forall x \; C(x)$

x is called Bound variable

Note: Variables are of two types -

(1) Bound Variable (associated with a relation)

(2) free variable (not " " " ")

2. $\exists x(\neg S(x))$

3. $\forall x(C(x) \Rightarrow S(x))$

4. $\exists x(S(x) \wedge \neg C(x))$

5. $\exists x(C(x) \wedge S(x))$

Wff of predicate calculus

Def^n A ~~wff~~ well formed formula of predicate calculus is a string of variables such as $x_1, x_2 \ldots x_n$ then connectives, parenthesis and quantifiers (for nesting variables & quantifiers) defined recursively by the following rule.

(i) $P(x_1, \ldots x_n)$ is a wff, where P is an predicate involving n variables $x_1, x_2 \ldots x_n$.

(ii) If α is a wff then $\neg\alpha$ is also a wff

(iii) If α ~~is a~~ and β are wff then $\alpha \wedge \beta$, $\alpha \vee \beta$, $\alpha \Rightarrow \beta$, $\alpha \Leftrightarrow \beta$ are also wffs.

(iv) If α is a wff and x is any variable then $\forall x(\alpha)$, $\exists x(\alpha)$ are wffs.

(v) A string is a wff iff it is obtained by a finite no. of application of rules (i) to (iv).

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Wff of predicate Calculus

→ $P(x_1, \dots x_n)$ T, θ

→ α $\neg\alpha$

→ $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$

→ $\forall$, $\exists$

Ex. 1 "Not every integer is even."

$E(x)$ means "x is even" (assume)

Universe: a set of integer.

$\alpha_1 = \neg\forall x\, E(x)$

"some integers are not even"

$\alpha_2 = \exists x\, \neg E(x)$

both are equivalent or $\alpha_1 \equiv \alpha_2$

Ex. 2 "Some integers are even and some are odd."

Let $O(x)$ means "x is odd".

$E(x)$ means "x is even

$$\exists x\, E(x) \wedge \exists x\, O(x)$$

Ex 3 "No integer is even"

$$\forall x\, \neg E(x)$$

Ex 4 "If an integer is not even then it's odd".

$$\forall x\, (\neg E(x) \rightarrow O(x))$$

Ex. 5. "2 is even"

$E(x) =$ "x is even"

$E(2)$

Now, U = set of numbers

$I(x) =$ "x is an integer"

$E(x) =$ "x is even"

$O(x) =$ "x is odd"

Ex. 1. "All integers are even"

$$\forall x \, (I(x) \rightarrow E(x))$$

Ex. 2 "Some integers are odd"

$$\exists x \, (I(x) \wedge O(x))$$

Ex. 3 "A number is even only if it is integer"

$$\forall x \, [E(x) \rightarrow I(x)]$$

Defn: Let α and β be two predicate formulae in variables $x_1 \ldots x_n$ and Let U be a universe of discourse for α & β, Then α & β are equivalent to each other over U if for every positive assignment of values to each variable in α & β, then resulting statements have the same truth values we can write $\alpha \equiv \beta$ over U.

Note: choosing U is very imp as choosing Large set will increase the complexity of our probleem.

Defn If a formulae of the form $\exists x\, P(x)$ or $\forall x\, P(x)$ occurs as part of the predicate formulae α, then such part is called x-~~based~~ bound part α_i and the occurance of x is called bound occurence of x. A occurence of x is free if it is not a bound occurence. A predicate variable in α is a free if its occurence is free in any part of α

Ex.

$$\alpha = (\exists x_1\, P(x_1, x_2)) \wedge (\forall x_2\, P(x_1, x_2))$$

bound variable.

x_2 is independent hence a free variable

Defn A predicate formula is valid if for all possible assignment of values from any universe of discourse to free variables, the resulting proposition have the truth value T. (predicate formula that are ~~vari~~ valid they are called tautologies.)

Defn A predicate formula is satisfiable if for some assignment of values to predicate variables the resulting proposition has the truth value T

Defn A Pf is unsatisfiable if for all possible assignment of values from U to predicate variables, the resulting proposition have the truth value F.

(predicate formula that are unsatisfiable are called contradiction)

Assignment

Ex. Let w_1 and w_2 be wff.

1. $\forall x\, P(x) : W_1$ $\qquad W_2 : \neg \exists x\, \neg P(x)$

2. $\forall x (P(x) \wedge Q(x)) \equiv (\forall x P(x) \wedge \forall x\, Q(x))$

Rules of inferences for predicate calculus

I_{13} Distributivity of $\exists$ over $\vee$,

$$\exists x\, (P(x) \vee Q(x)) \equiv \exists x\, P(x) \vee \exists x\, Q(x)$$

$\boxed{\text{Note:}}$ No need to proof they are trivial and they're truth value

$$\exists x\, (P \vee Q(x)) \equiv P \vee (\exists x\, Q(x))$$

means if a variable is independ of x then we can taken that in ~~common~~ common

I_{14} $\forall$ over $\wedge$

$$\forall x\, (P(x) \wedge Q(x)) \equiv \forall x\, P(x) \wedge \forall x\, Q(x)$$

$$\forall x\, (P \wedge Q(x)) \equiv P (\forall x\, Q(x))$$

Negation.

I_{15} $\neg(\exists x\, P(x)) \equiv \forall x\, \neg(P(x))$

I_{16} $\neg (\forall x\, P(x)) \equiv \exists x\, \neg(P(x))$

I_{17} $\exists x\, (P \wedge Q(x)) \equiv P \wedge (\exists x\, Q(x))$

I_{18} $\forall x\, (P \vee Q(x)) \equiv P \vee (\forall x\, Q(x))$

I_{19} $\forall x\, P(x) \Rightarrow \exists x\, P(x)$ $\quad$ ($\because$ U: set of integer)

(P(x): x is even)

I_{20} $\forall x P(x) \vee \forall x\, Q(x) \Rightarrow \forall x\, (P(x) \vee Q(x))$

C I_{21} $\exists x(P(x) \wedge Q(x)) \Rightarrow \exists x P(x) \wedge \exists x Q(x)$

(including rules for propositional calculus.)

Special rules (related to universal instantiation)

RI 1 Universal Instantiation

$$\frac{\forall x\, P(x)}{\therefore P(c)}$$

where c is some element of the variable
$c \in x$.

RI 2 Existential Instantiation

$$\frac{\exists x\, P(x)}{\therefore P(c)}$$

I

RI 3 Universal generalization

$$\frac{P(c) \text{ for an arbitrary } c}{\forall x\, P(x)}$$

condⁿ: x shouldn't be free in any of the given premises.

RI 4 Existential generalization

$$\frac{P(c) \text{ for some element } c}{\therefore \exists x\, P(x)}$$

~~where~~ p is some element of the variable.

where p is some statement of the variable

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Predicate Calculus

- wff or atomic formula.
- Equivalence
- validity
- Satisfiability and unsatisfiability.

Example 1: Discuss the validity of the following argument.

→ All graduates are educated

→ Ram is a graduate

→ Therefore, Ram is educated.

Sol^n: Let $G(x)$ denote "x is a graduate"

Let $E(x)$ denote "x is educated"

Let R denote "Ram"

So the premises are

(i) $\forall x(G(x) \Rightarrow E(x))$

(ii) $G(R)$

Conclusion $E(R)$ i.e., Ram is educated.

$\forall x(G(x) \Rightarrow E(x))$ — By using Premise(i)

$G(R) \Rightarrow E(R) \longrightarrow$ Universal instantiation (RI_{13})

// Because universal set is common i.e person

$G(R) \longrightarrow$ By using a premise (ii)

$E(R) \longrightarrow$ modus ponens (RI_{14})

The conclusion is valid.

Ex.2

Discuss the validity of the following argument.

All graduates can read and write.

Ram can read and write.

Therefore, Ram is a graduate.

Sol^n:

Let $R(x)$ denote "x can read."

Let $W(x)$ denote "x can write"

Let $G(x)$ denote "x is a graduate"

Let R' denote Ram

So the premises are

(i) $\cancel{\forall x (R(x) \wedge W(x))}$ $\forall x (G(x) \rightarrow R(x) \wedge W(x))$

(ii) $R(R') \wedge W(R')$

(iii) conclusion $G(R)$ i.e Ram is a graduate

$$\forall x (G(x) \rightarrow R(x) \wedge W(x))$$

$$\downarrow$$

$$G(R') \rightarrow (R(R') \wedge W(R'))$$ (universal instantaneous)

$$\downarrow$$

$$R(R') \wedge W(R')$$

We can't derieve $G(R)$ from.

$\Rightarrow$ it is not a tautology (Invalid)

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Assignment 1 / Ex. Discuss the validity of the following argument.

→ All educated ~~peo~~ persons are well behaved

→ Ram is educated

→ No well-behaved persons is quarrel-some

→ Therefore, Ram is not quarrelsome.

Step 1: ~~Soln~~ U = set of all educated persons

P(x) = " x is well-behaved"

y = Ram

Q (x) = " x is Quarrelsome"

Step2 : Write the premises.

(i) $\forall x \; P(x)$

(ii) y U a particular element of the universe of discourse

(iii) $\forall x \; (P(x) \Rightarrow \neg Q(x))$

$\neg Q(y)$

Step3: Check whether the above argument is valid/invalid.

⇒ what are the rules of inference needed for checking the validity of the argument.

$\forall x \; P(x)$

$\exists y \; P(y)$

$\exists y \; (P(y) \to \neg Q(y))$

$P(y)$

$\neg Q(y)$ modus ponens proved valid.

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Principle of Induction

The process of ~~recovering~~ reasoning from general observation to specific truth is called induction

The following ~~prop~~ properties applies to the set of natural number and principle of induction.

property 1: Zero is a natural number.

property 2: The successor of any natural number is also a natural number.

prop 3: Zero is not the successor of a natural number

p. 4: No two natural number have the same successor.

p5: Let a property p(n) be defined for every natural number n. If (i) p(n) is true and (ii) p(successor of n) is true whenever p(n) is true, then p(n) is true for all n.

A proof by ~~induction~~ complete enumeration of all possible combination is called a perfect induction.

Eg proof by truth table.

Method of proof by induction

There are 3 steps:

Step 1: Prove $p(n)$ for $n = 0/1$. This is called the basis proof for the basis.

BASIS

Step 2: Assume the result / properties for $p(n)$. This is called the induction hypothesis.

HYPOTHESIS

the induction hypothesis.

Step 3: prove $p(n+1)$ using the induction hypothesis.

FINAL

Ex. Prove that

$1 + 3 + 5 + \dots + r = n^2$ for all $n > 0$

where r is an odd integer and n is the number of terms in the sum.

proof:

a) Basis step:

for $n = 1$ LHS = 1 and R.H.S $= 1^2 = 1$

Hence the result is true for $n = 1$

$\boxed{\text{Note}}$ (in basis step we can also take $n = 2, 3 \dots$ to proof / verify)

b) Induction hypothesis Step:

$1 + 3 + 5 + \dots r = n^2$

A $(r = 2n - 1)$

$1 + 3 + 5 + \dots + (2n-1) = n^2$

LHS $= 1 + 3 + 5 + \dots +$

(c) we have to prove.

$1+3+5+ \cdots + r + (r+2) = (n+1)^2$

$LHS = (1+3+5+ \cdots r) + (r+1)$

$\Rightarrow n^2 + r+2 = n^2+2n-1+2 = n^2+2n+1$

$= (n+1)^2 = R.H.S$

$\therefore$ replace r by $(2n-1)$

proved

Ex.2 Prove the following theorem by induction,

$1+2+3+ \cdots n = n(n+1)/2$

ba