

ADAPTIVE DECORRELATION USING GAIN-MODULATING INTERNEURONS

UT Austin

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Statistics of natural environments



La Jolla, CA

Statistics of natural environments

Dynamic range



La Jolla, CA

Statistics of natural environments

Dynamic range



La Jolla, CA



???

Statistics of natural environments

Dynamic range



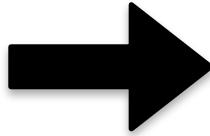
La Jolla, CA

Redundancy



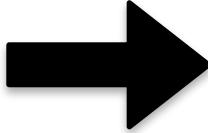
New York, NY

Efficient coding hypothesis



Attneave 1954; Barlow 1961; Laughlin 1981; Atick 1992; van Hateren 1997; Simoncell & Olshausen 2001; ...

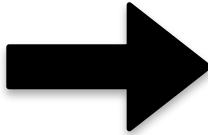
Efficient coding hypothesis



Neurons encode **maximum information** about the environment using limited resources

Attneave 1954; Barlow 1961; Laughlin 1981; Atick 1992; van Hateren 1997; Simoncell & Olshausen 2001; ...

Efficient coding hypothesis

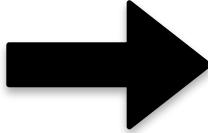


Neurons encode **maximum information** about the environment using limited resources

- Single neurons are adapted to use their entire **dynamic range**

Attneave 1954; Barlow 1961; Laughlin 1981; Atick 1992; van Hateren 1997; Simoncelli & Olshausen 2001; ...

Efficient coding hypothesis



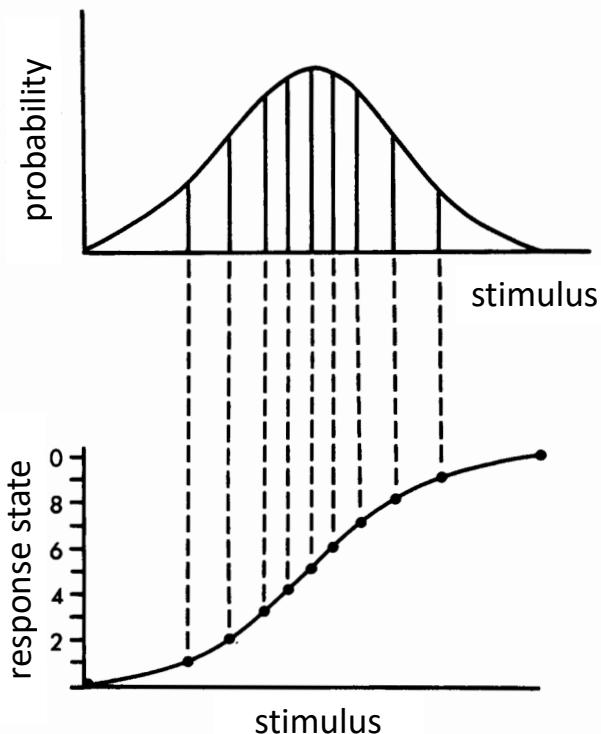
Neurons encode **maximum information** about the environment using limited resources

- Single neurons are adapted to use their entire **dynamic range**
- Neuron populations are adapted to **reduce redundancy**

Attneave 1954; Barlow 1961; Laughlin 1981; Atick 1992; van Hateren 1997; Simoncelli & Olshausen 2001; ...

Efficient coding in a neuron

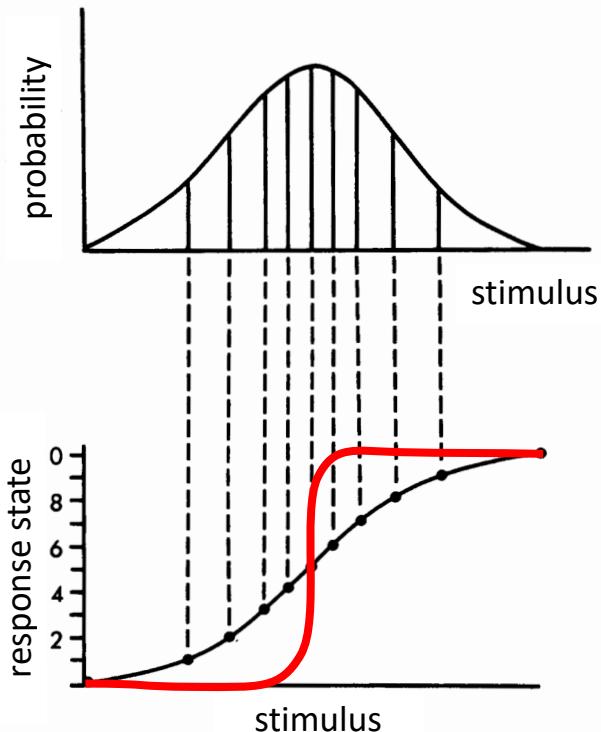
Theory



Laughlin 1981

Efficient coding in a neuron

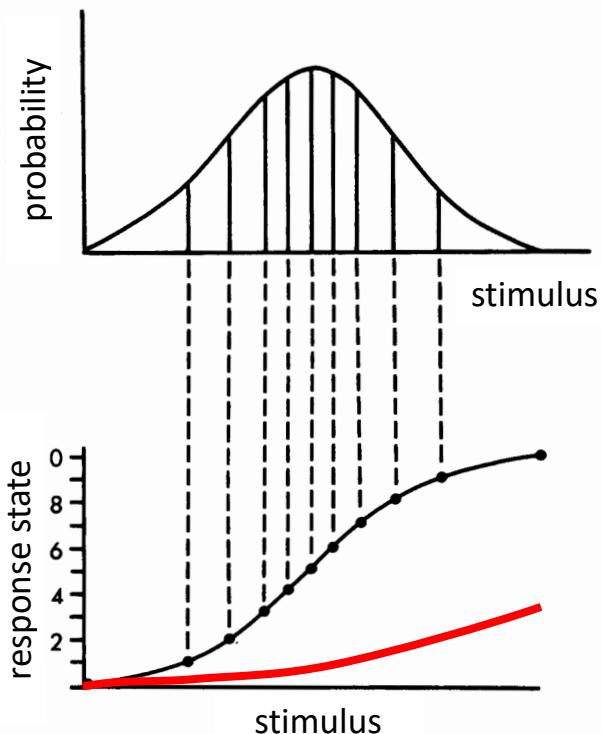
Theory



Laughlin 1981

Efficient coding in a neuron

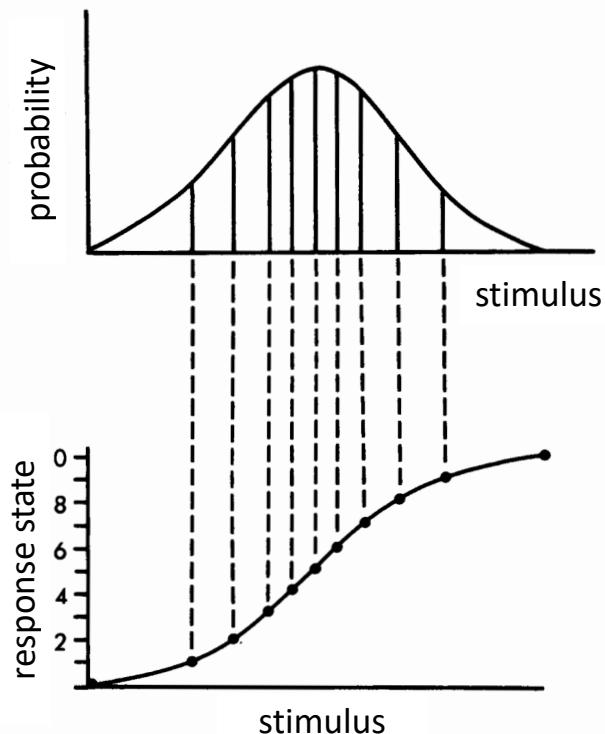
Theory



Laughlin 1981

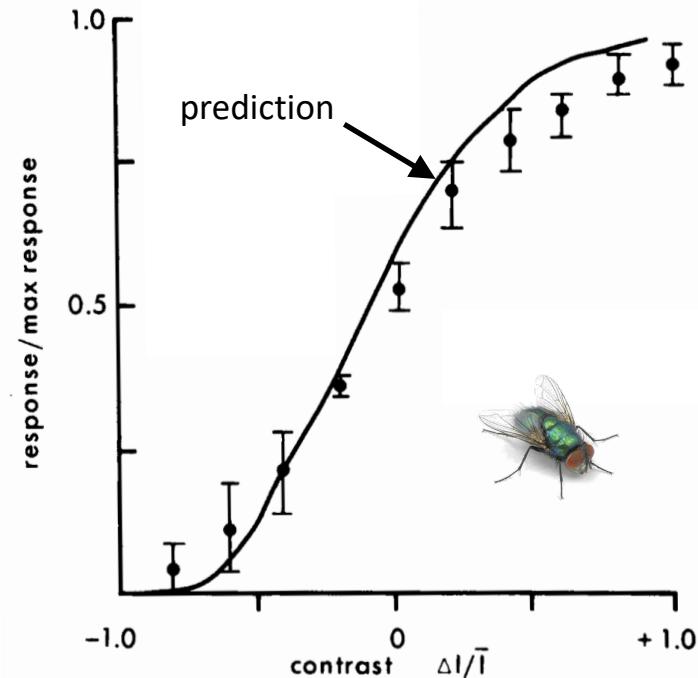
Efficient coding in a neuron

Theory



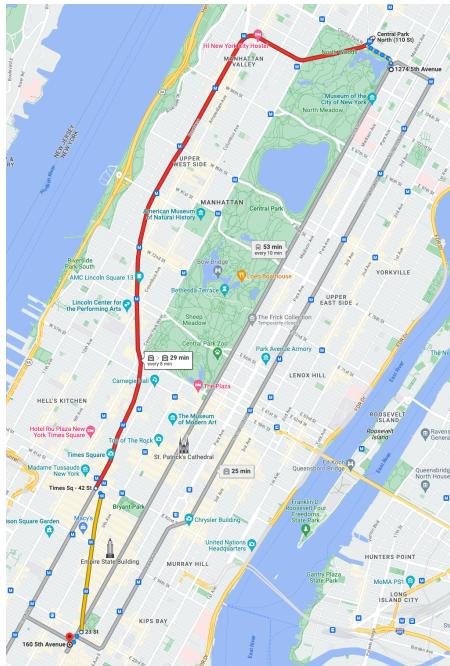
Laughlin 1981

Experiment



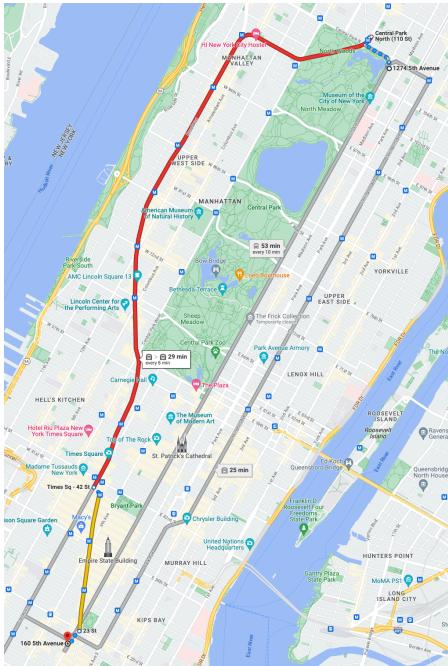
Environments are dynamic!

My commute



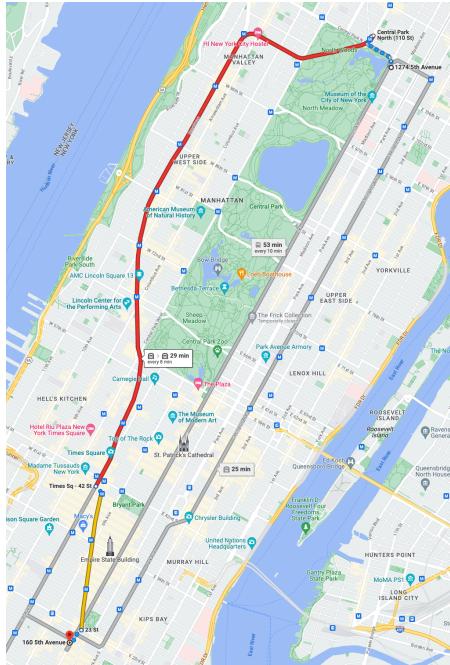
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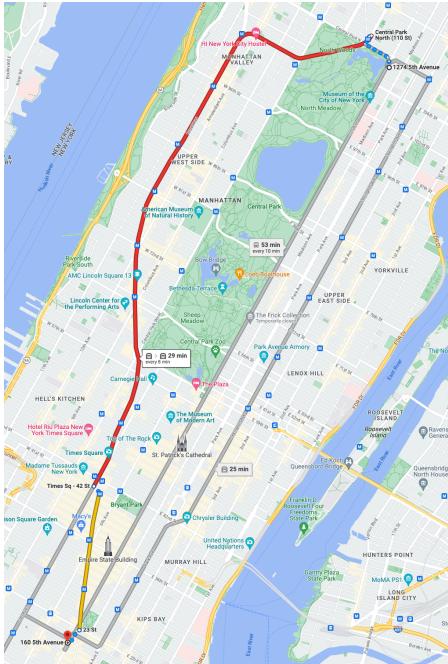
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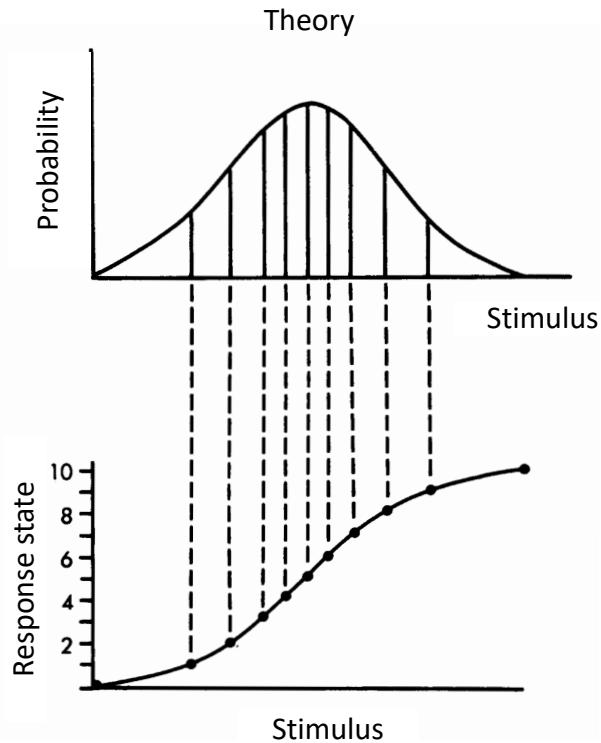


Environments are dynamic!

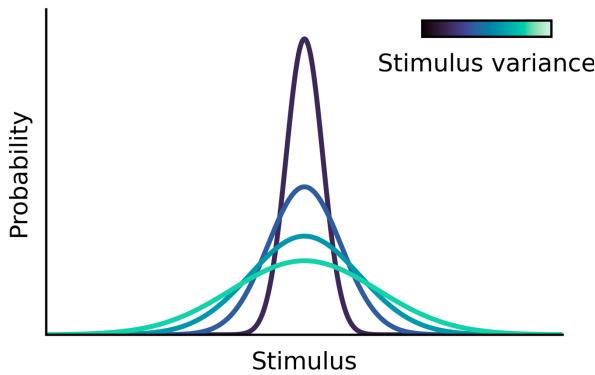
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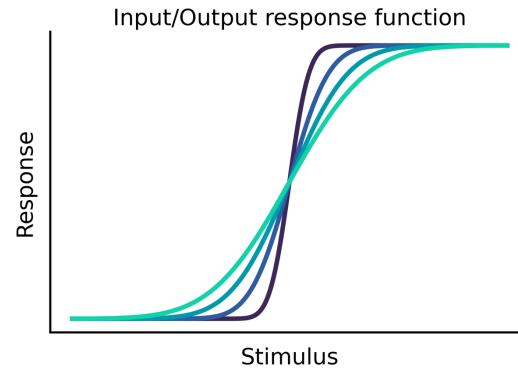
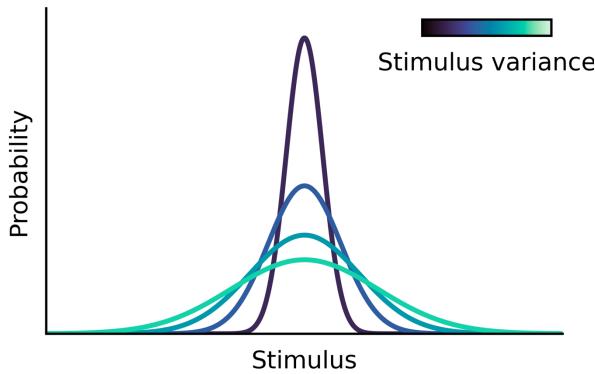
Adaptive efficient coding in single neurons



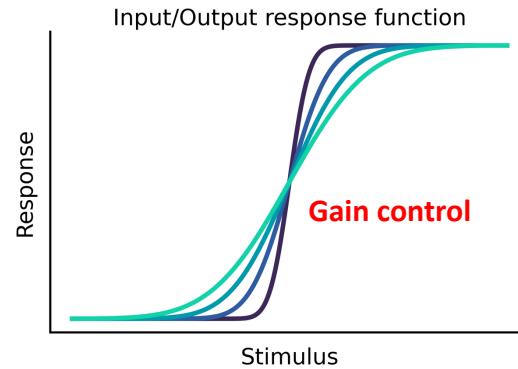
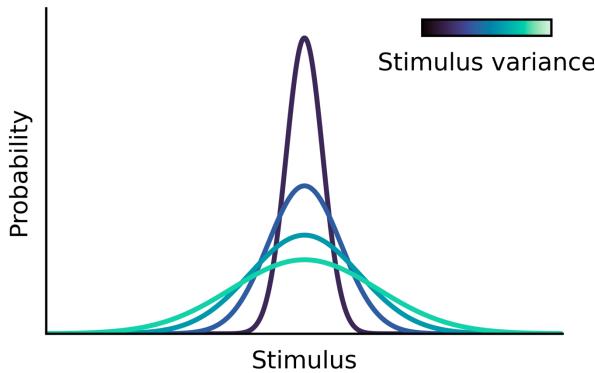
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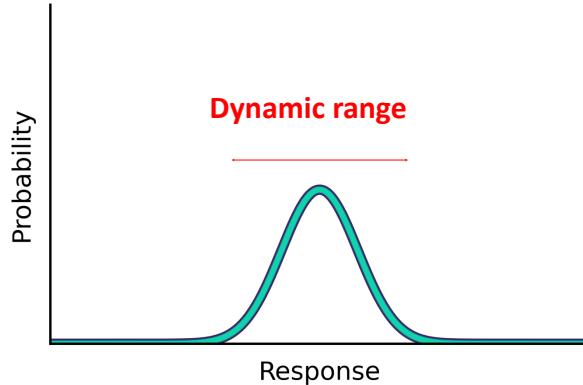
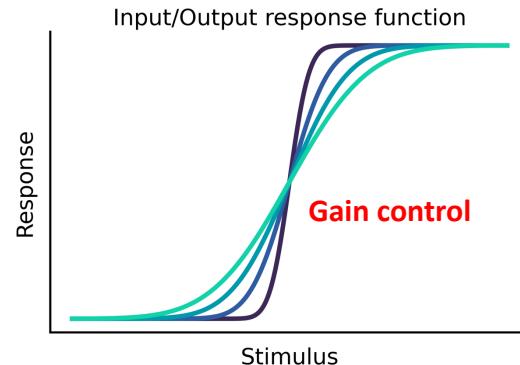
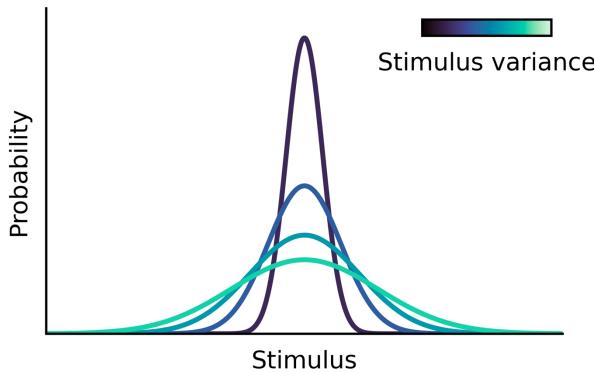
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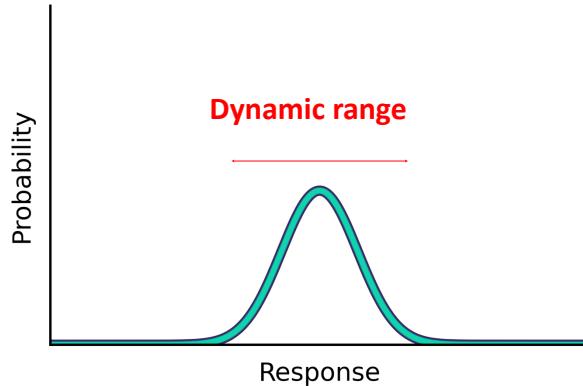
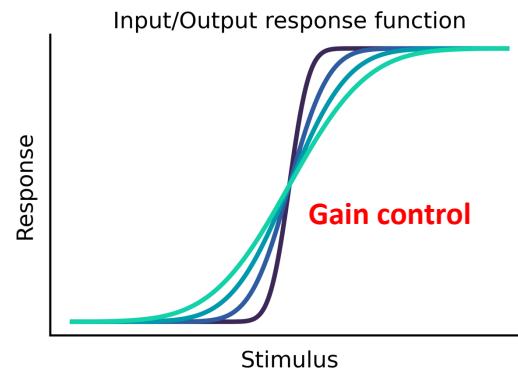
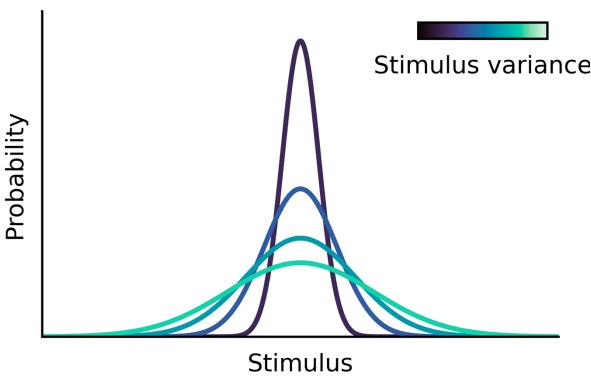
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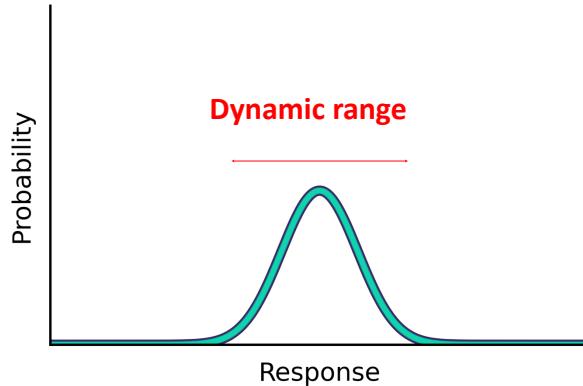
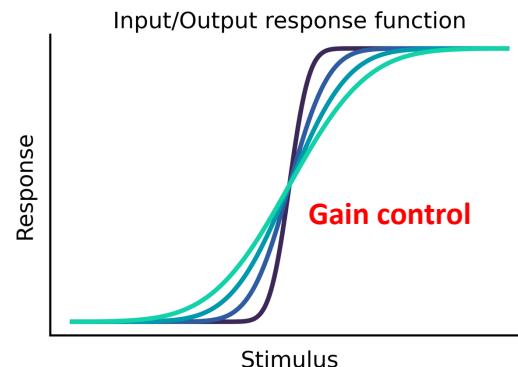
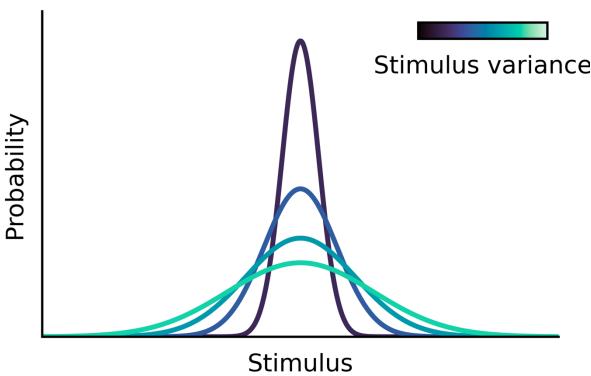
Adaptive efficient coding in single neurons



Reported in:

- Songbird auditory forebrain: [Nagel & Doupe, 2006]
- Fly vision: [Brenner et al. 2000; Fairhall et al., 2001]
- Salamander retina [Chander & Chichilnisky 2001, Baccus & Meister 2002]
- Cat LGN [Mante et al. 2005]
- & more

Adaptive efficient coding in single neurons

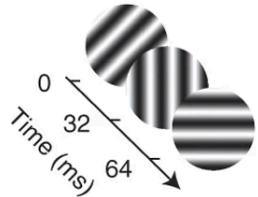


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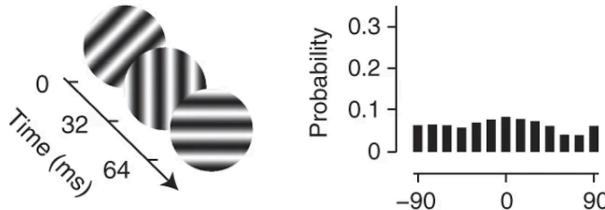
Fast & reversible!
~50ms

Adaptive decorrelation in a neural population

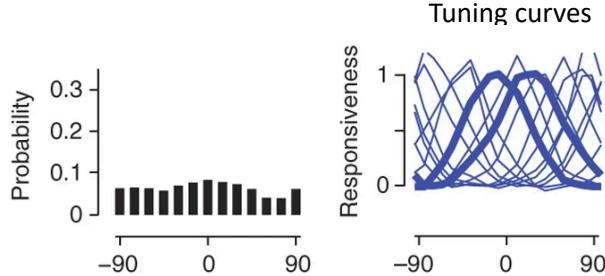
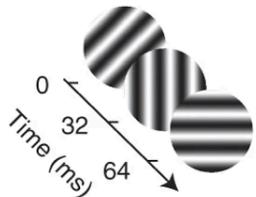


©Warren Photographic

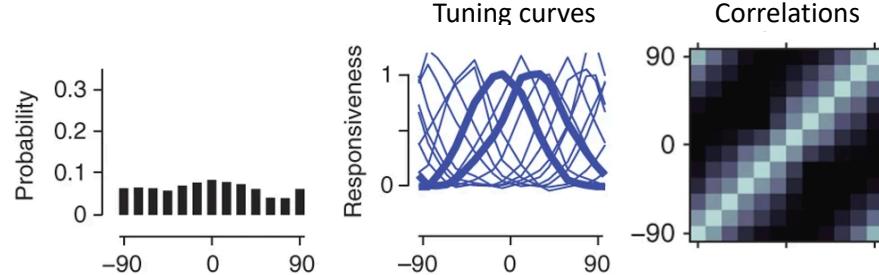
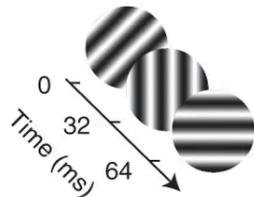
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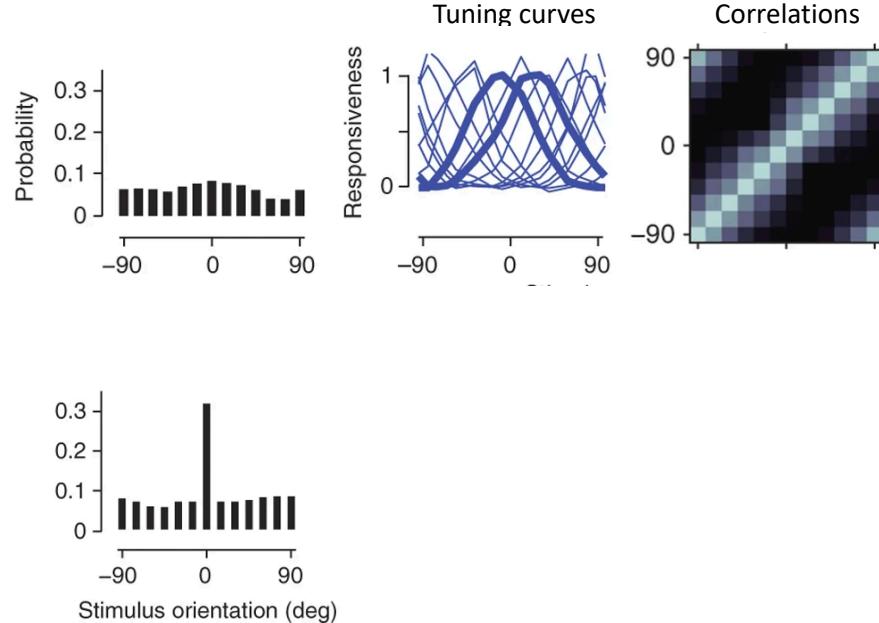
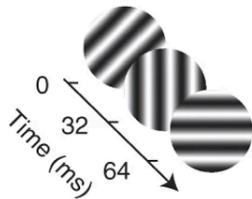
Adaptive decorrelation in a neural population



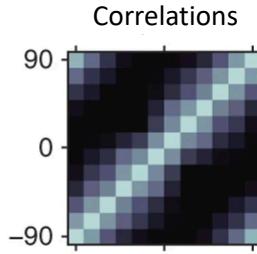
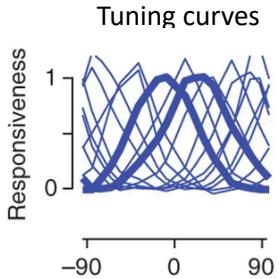
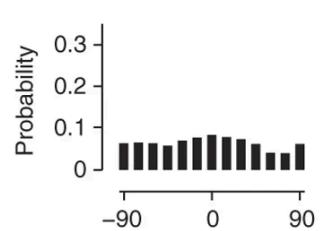
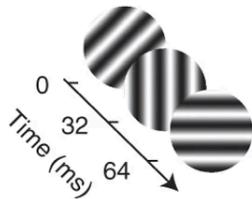
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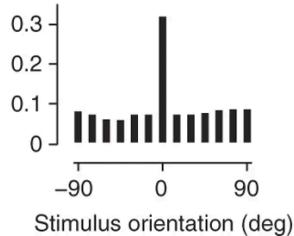
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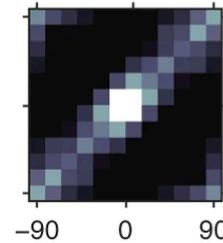
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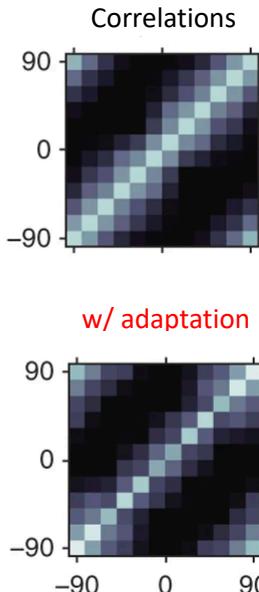
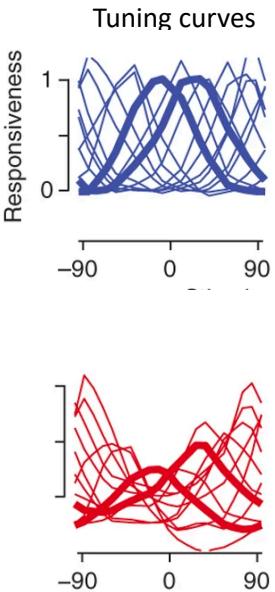
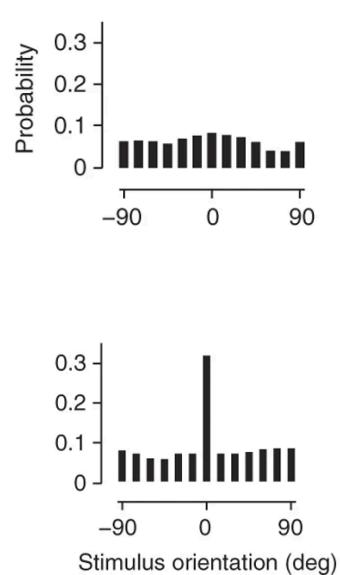
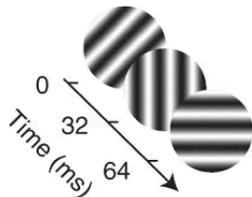
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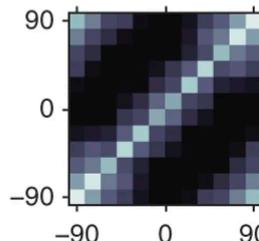
w/o adaptation



Adaptive decorrelation in a neural population

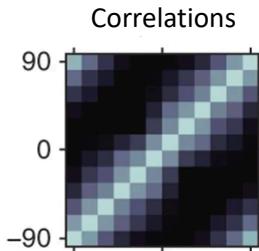
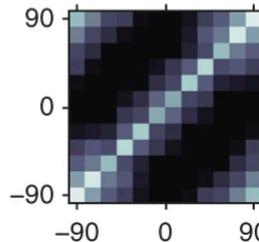
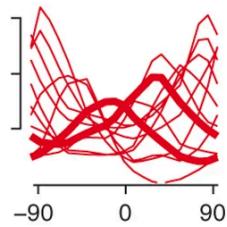
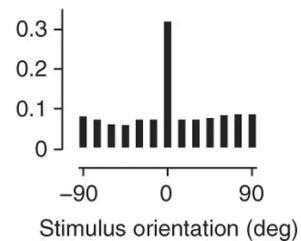
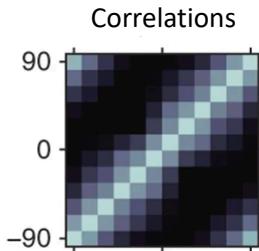
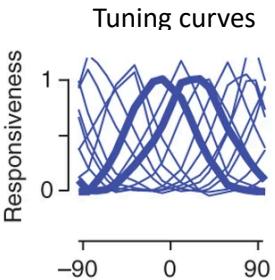
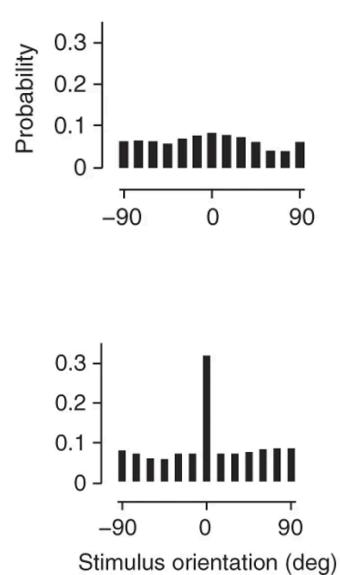
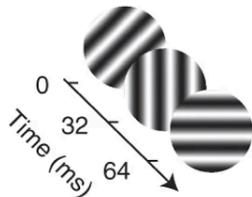


w/ adaptation



w/o adaptation

Adaptive decorrelation in a neural population

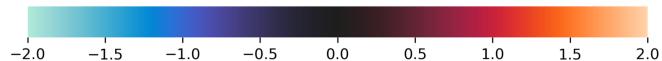
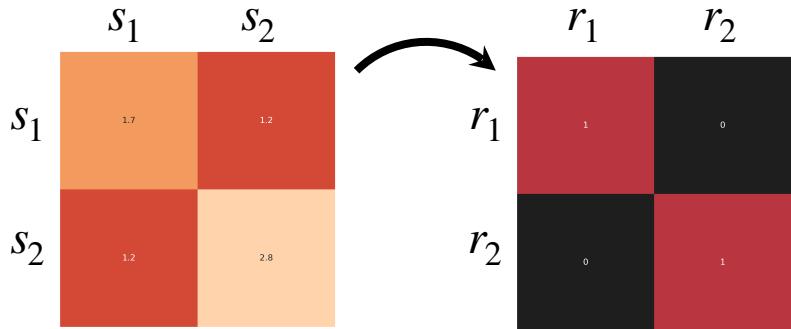


Benucci et al. 2013

timescale $\sim 1.5\text{s} / 50$ stimuli

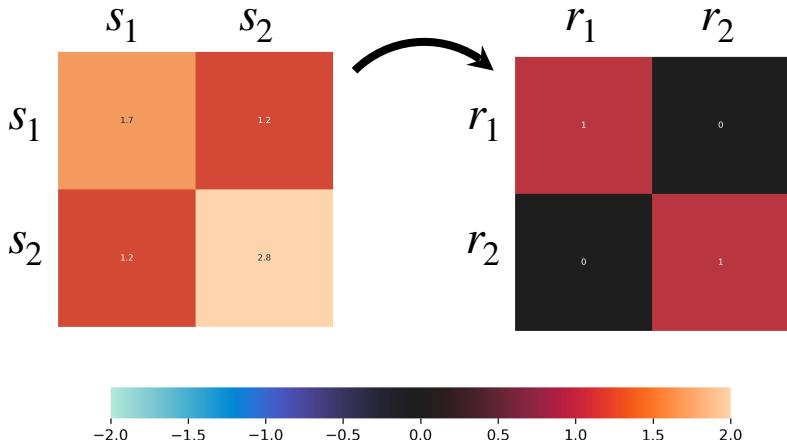
Whitening: normalization + decorrelation

Covariance matrix perspective

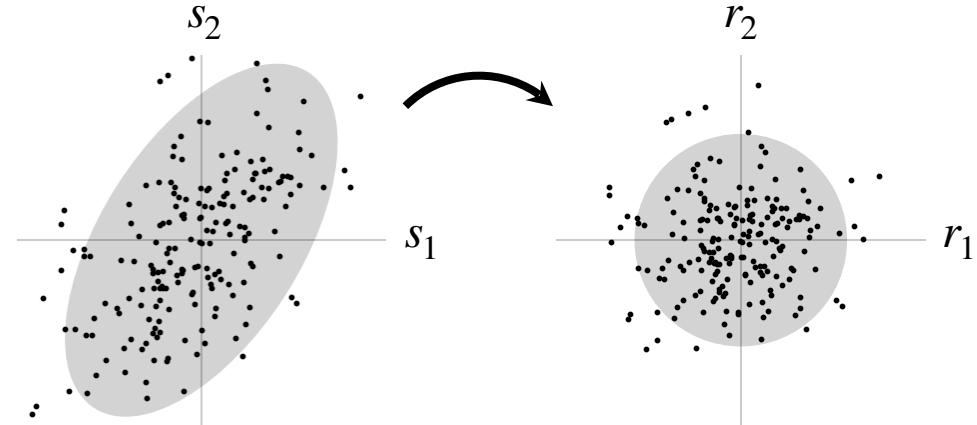


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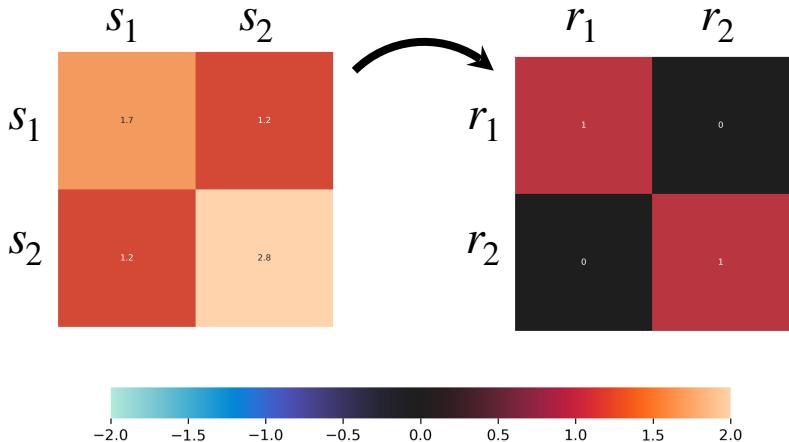


Geometric perspective

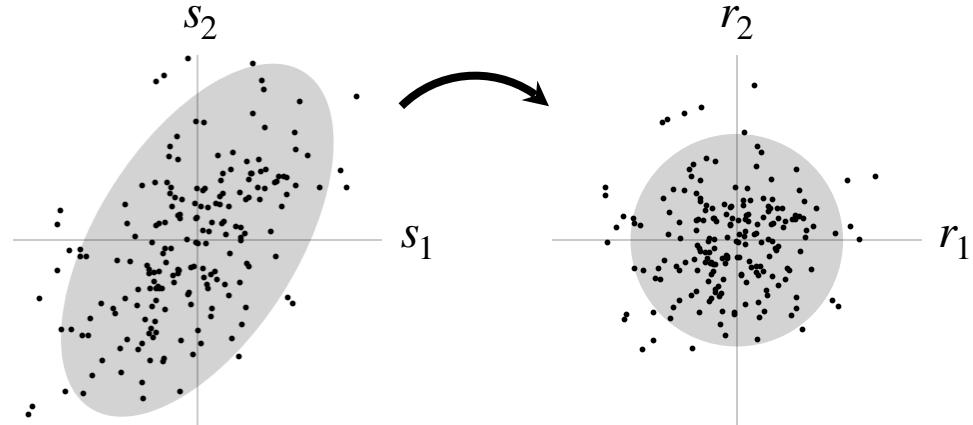


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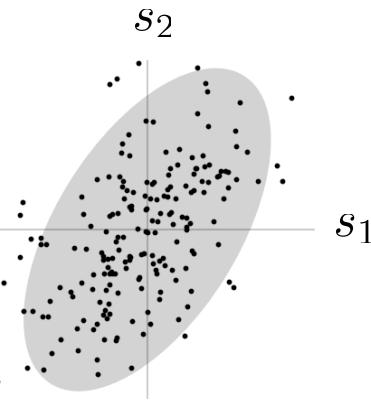
Fundamental to:

- Signal processing (e.g. ICA)
- Machine learning (unsupervised feature learning, self-supervised learning)
- **Neural computation?**
 - Cat V1 [Muller et al. 1999; Benucci et al. 2013]
 - Salamander retina [Pitkow & Meister 2012]
 - Zebrafish olfactory bulb [Wiechart et al. 2010; Wanner & Friedrich 2020]
 - Mouse olfactory bulb [Giridhar et al. 2011; Gschwend et al. 2015]

Gain modulation in neural populations

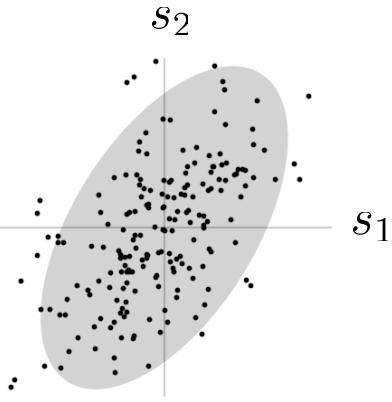
Gain modulation in neural populations

Stimulus distribution

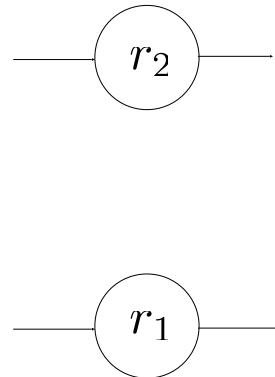


Gain modulation in neural populations

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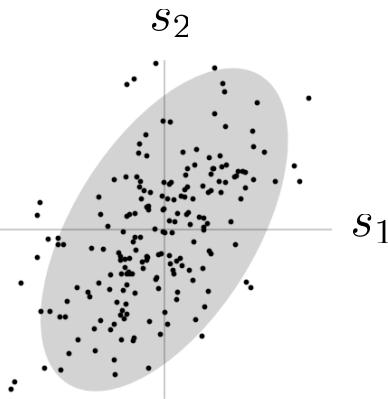


Single neuron gain adaptation

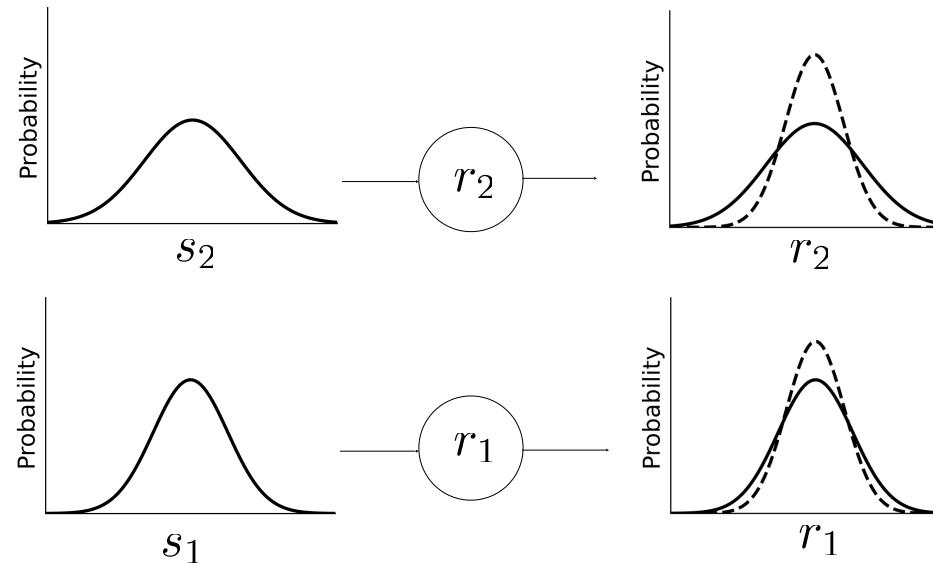


Gain modulation in neural populations

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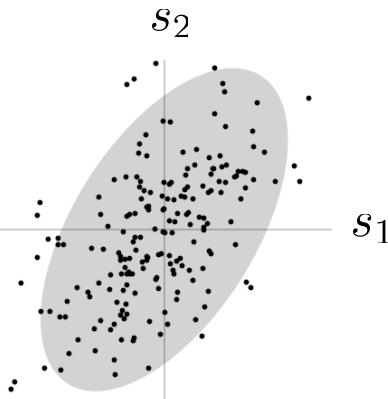


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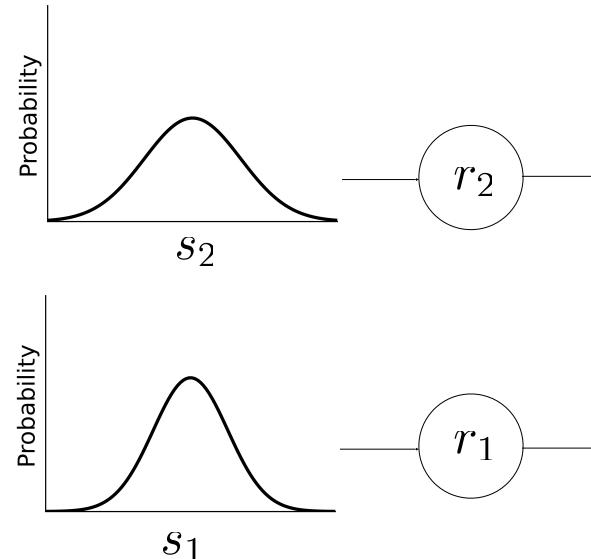


Gain modulation in neural populations

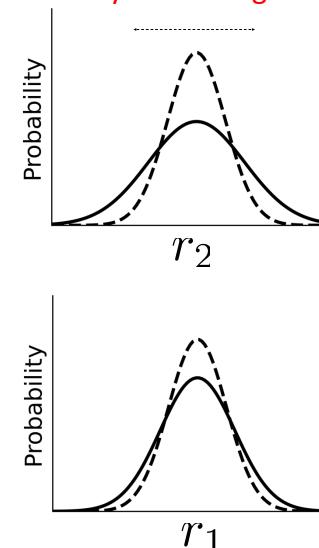
Stimulus distribution



Single neuron gain adaptation

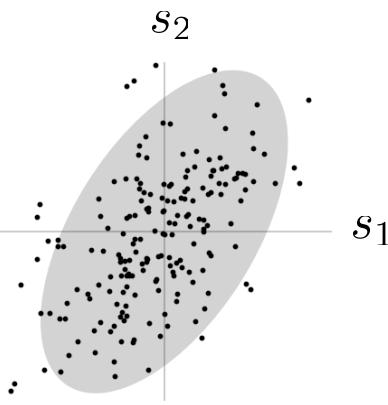


Dynamic range

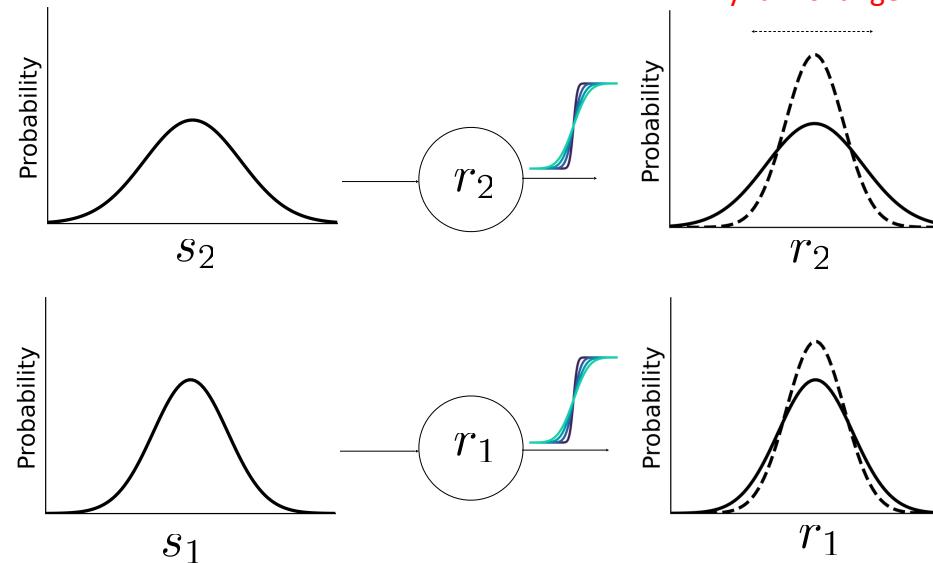


Gain modulation in neural populations

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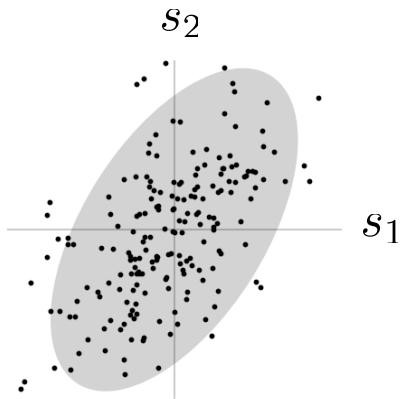
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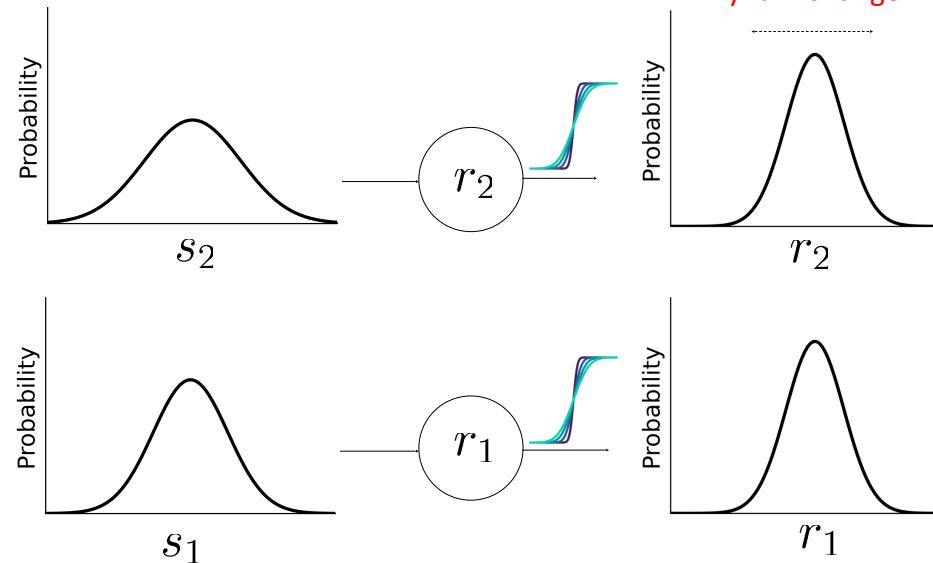
Dynamic range

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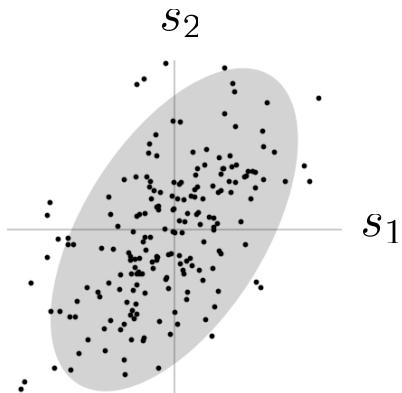


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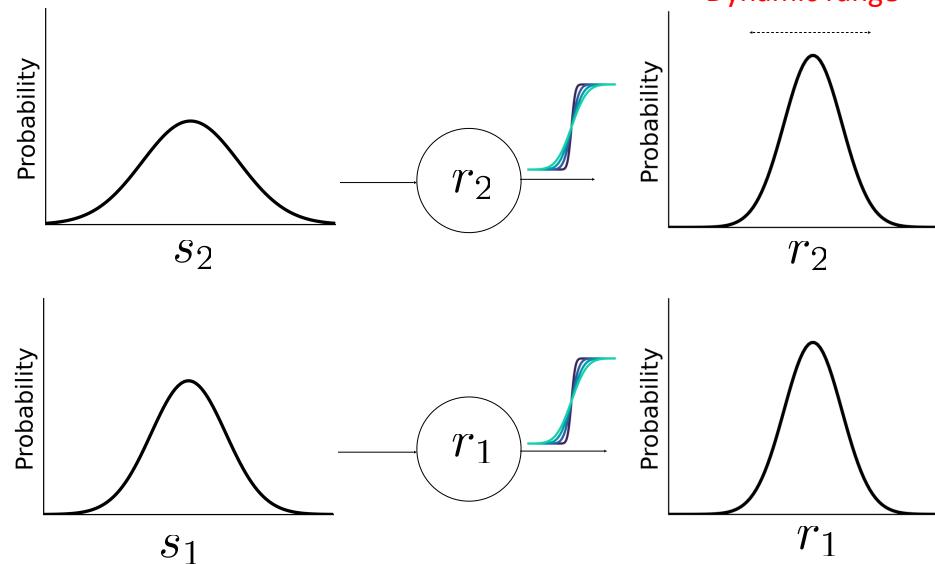


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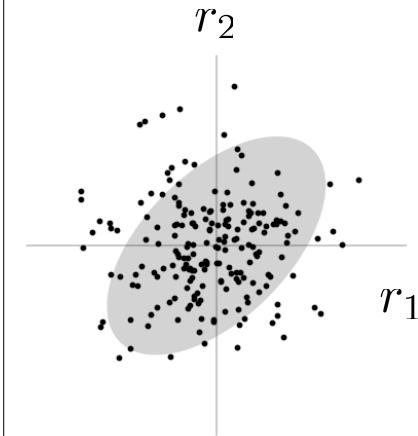
Stimulus distribution



Single neuron gain adaptation



Response distribution



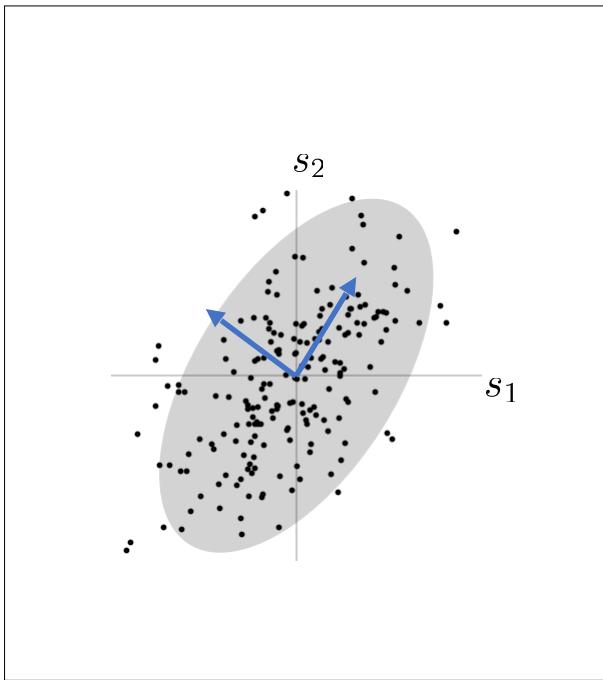
Correlations remain!

Existing adaptive neural network models

Wick et al. 2010; Pehlevan et al. 2015; Chapochnikov et al. 2023; ...

Existing adaptive neural network models

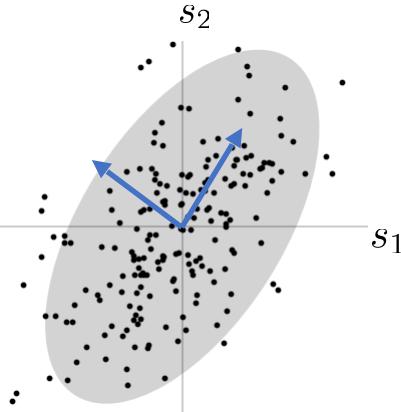
Stimulus distribution



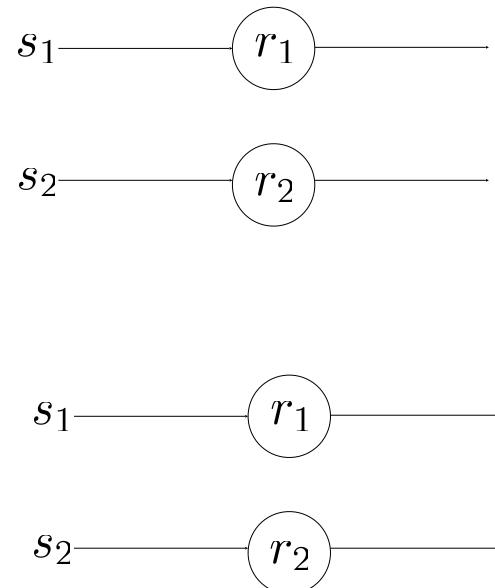
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Existing adaptive neural network models

Stimulus distribution



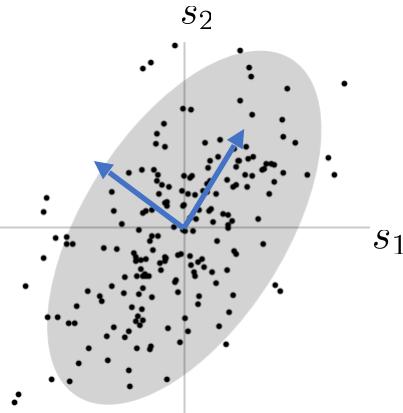
Models



Wick et al. 2010; Pehlevan et al. 2015; Chapochnikov et al. 2023; ...

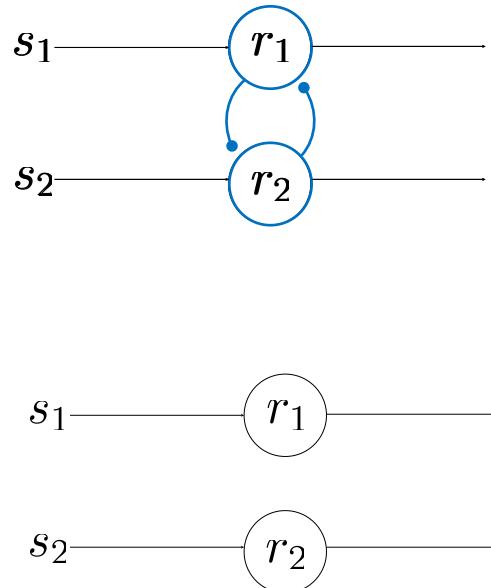
Existing adaptive neural network models

Stimulus distribution



Models

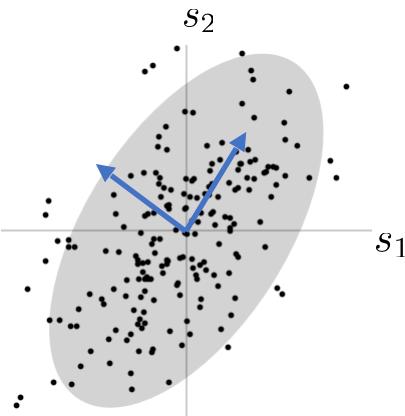
Direct connections



Wick et al. 2010; Pehlevan et al. 2015; Chapochnikov et al. 2023; ...

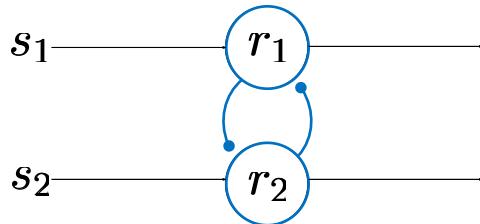
Existing adaptive neural network models

Stimulus distribution

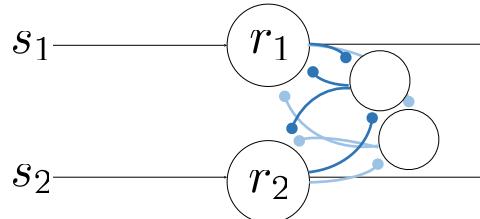


Models

Direct connections



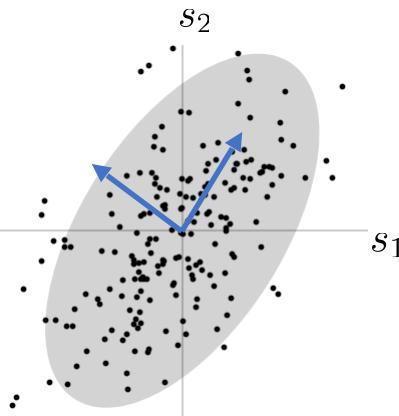
Auxiliary interneurons



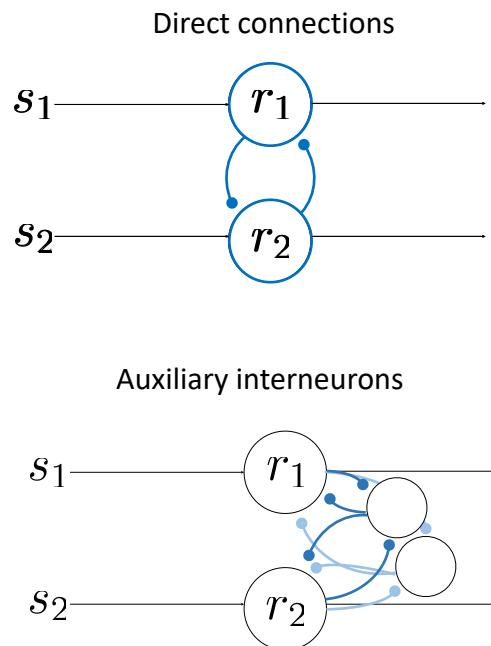
Wick et al. 2010; Pehlevan et al. 2015; Chapochnikov et al. 2023; ...

Existing adaptive neural network models

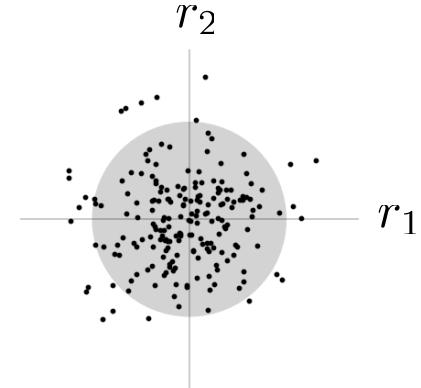
Stimulus distribution



Models



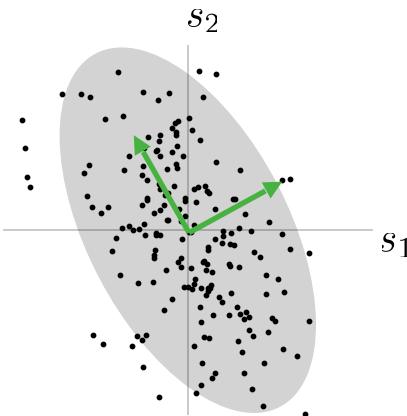
Response distribution



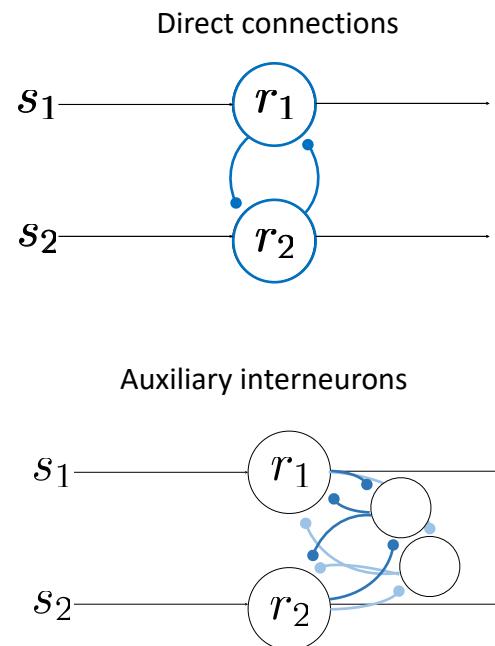
Wick et al. 2010; Pehlevan et al. 2015; Chapochnikov et al. 2023; ...

Existing adaptive neural network models

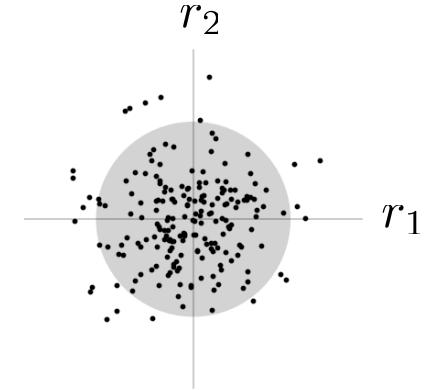
Stimulus distribution



Models



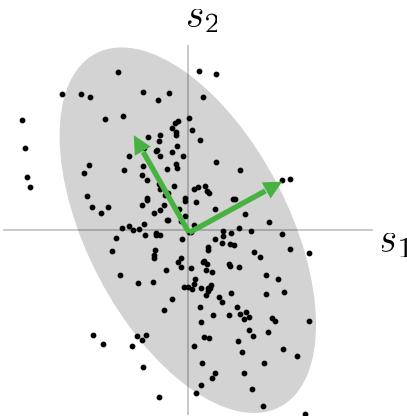
Response distribution



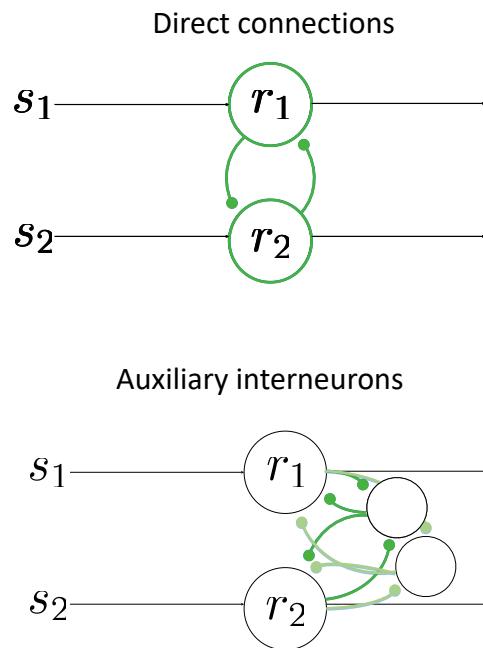
Wick et al. 2010; Pehlevan et al. 2015; Chapochnikov et al. 2023; ...

Existing adaptive neural network models

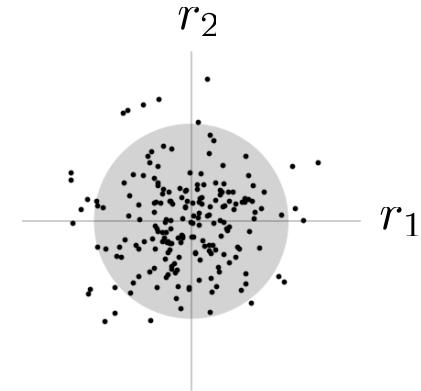
Stimulus distribution



Models



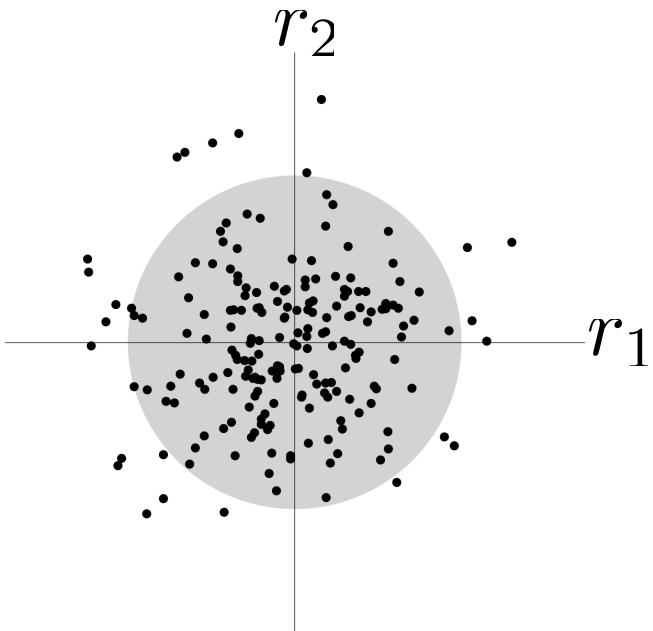
Response distribution



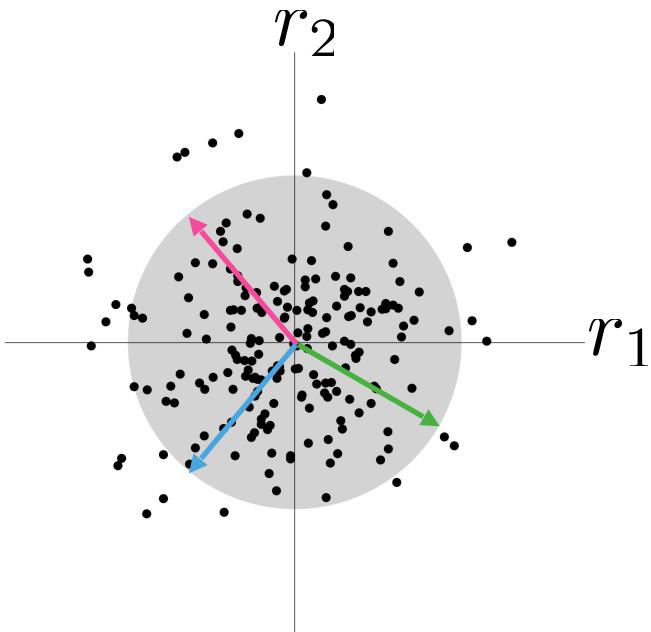
Wick et al. 2010; Pehlevan et al. 2015; Chapochnikov et al. 2023; ...

Q: Can neural circuits **decorrelate** their responses using **gain modulation**?

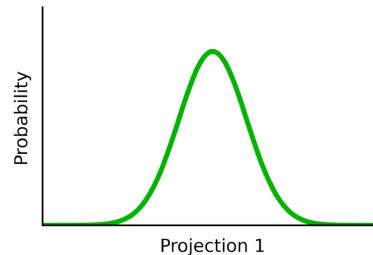
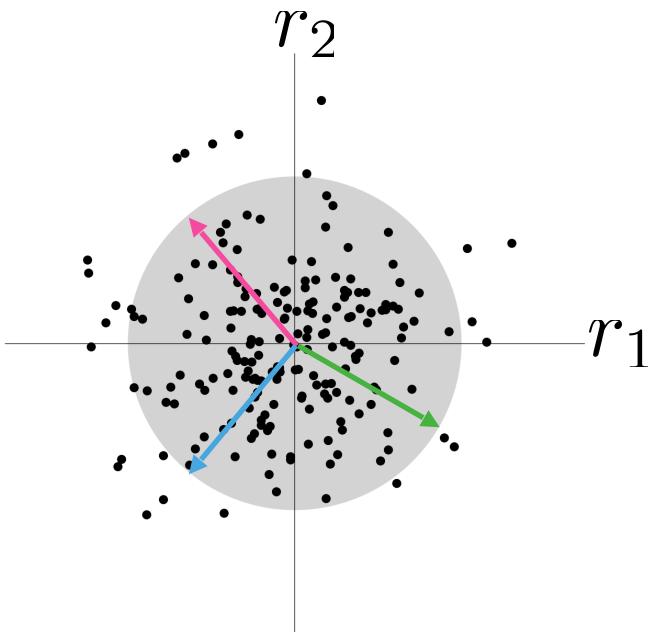
Geometric intuition



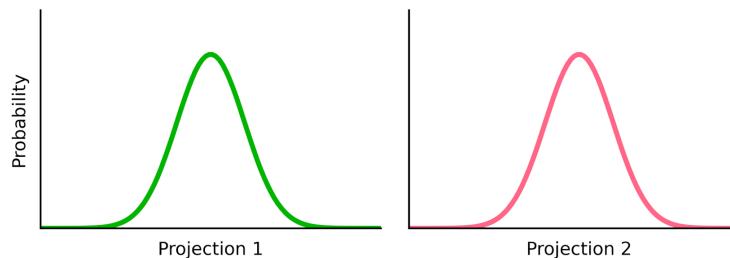
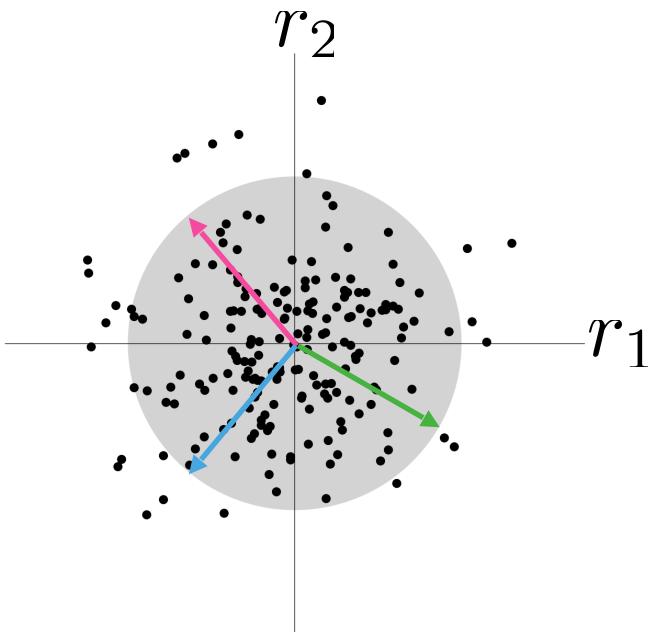
Geometric intuition



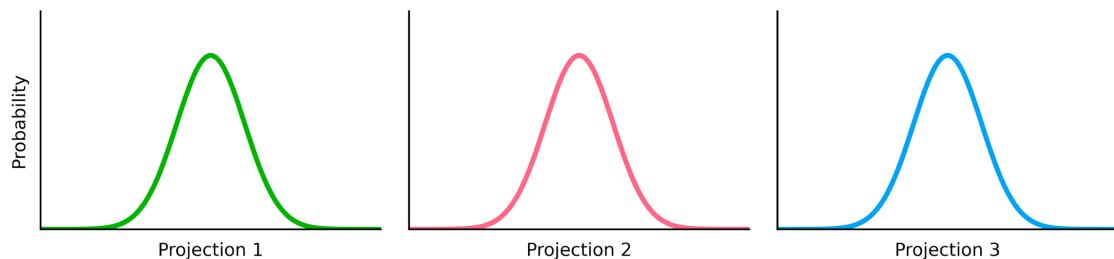
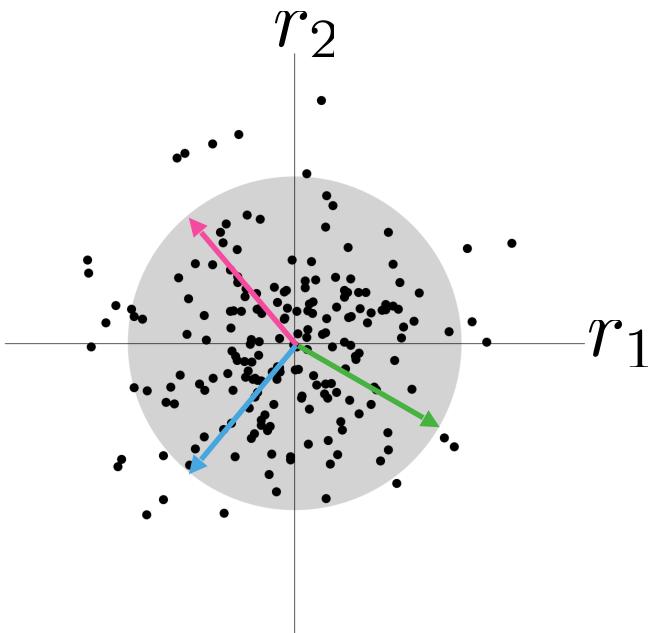
Geometric intuition



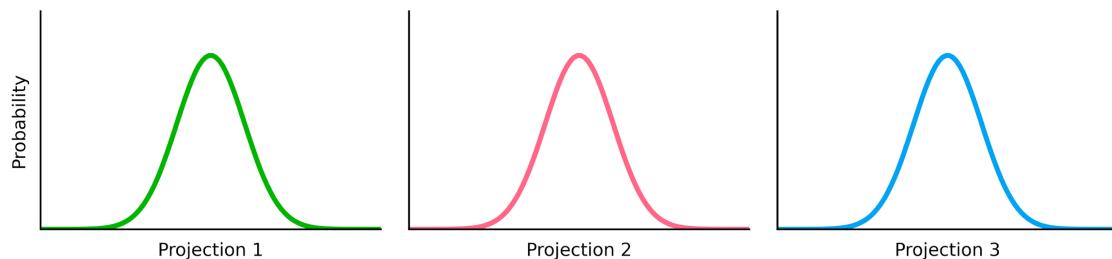
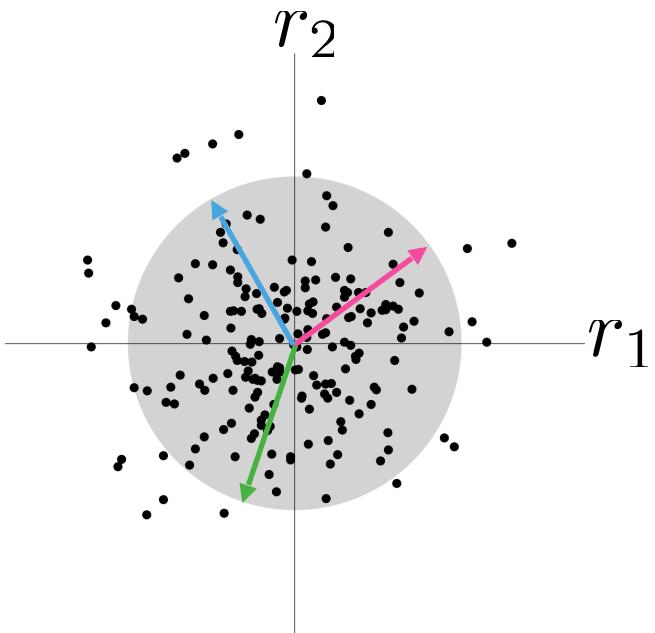
Geometric intuition



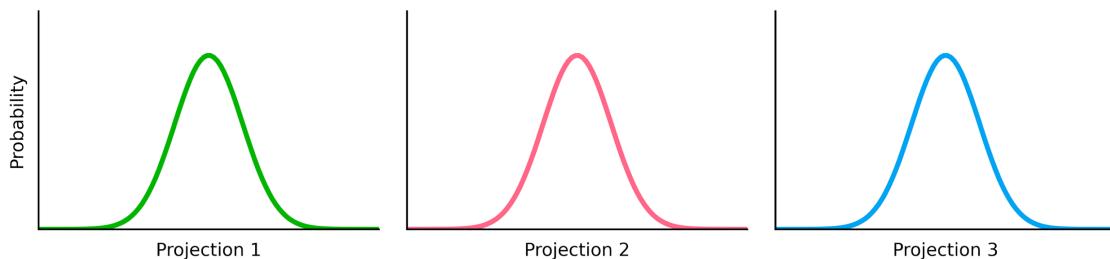
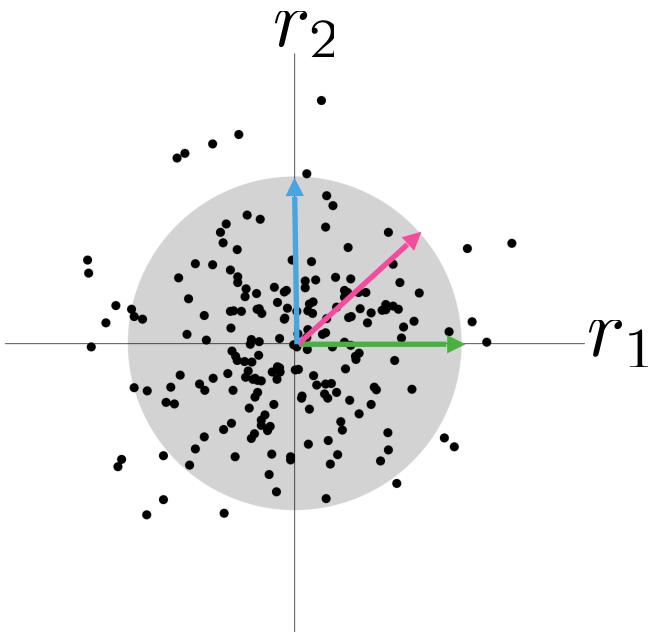
Geometric intuition



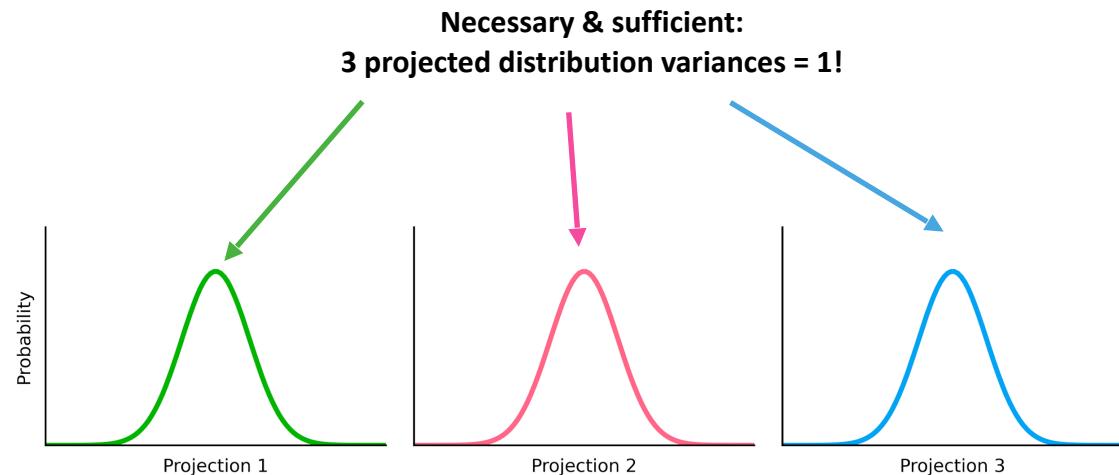
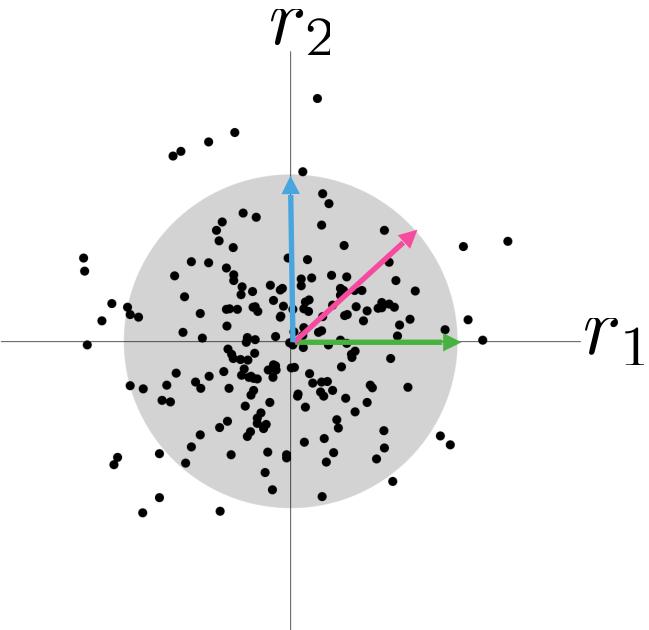
Geometric intuition



Geometric intuition



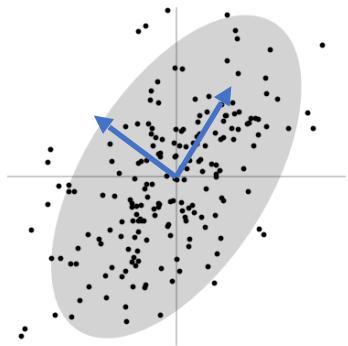
Geometric intuition



Novel matrix factorization

Novel matrix factorization

Traditional approaches (PCA)

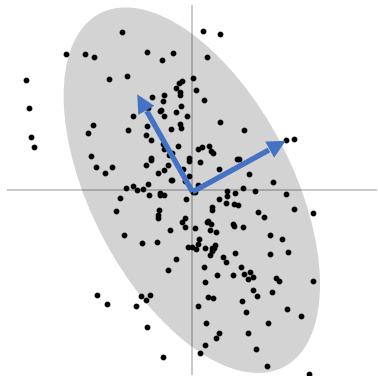


$$\begin{matrix} \text{C}^{1/2} \\ \text{V} \\ \Lambda^{1/2} \\ \text{V}^\top \end{matrix} = \begin{matrix} \text{C}^{1/2} \\ \text{V} \\ \Lambda^{1/2} \\ \text{V}^\top \end{matrix}$$

Principal axes must be **relearned** for different input densities.

Novel matrix factorization

Traditional approaches (PCA)

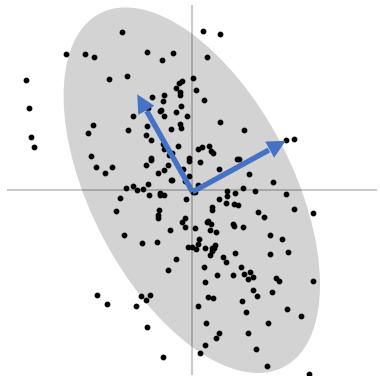


$$\begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix} = \begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix} \begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix} \begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix} \begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix}$$
$$\mathbf{C}^{1/2} \quad \mathbf{V} \quad \mathbf{\Lambda}^{1/2} \quad \mathbf{V}^T$$

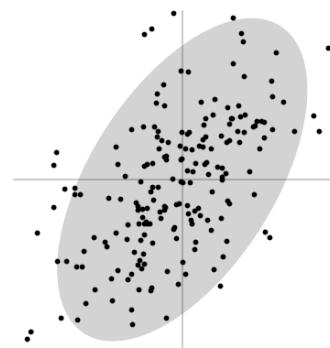
Principal axes must be **relearned**
for different input densities.

Novel matrix factorization

Traditional approaches (PCA)



Our approach

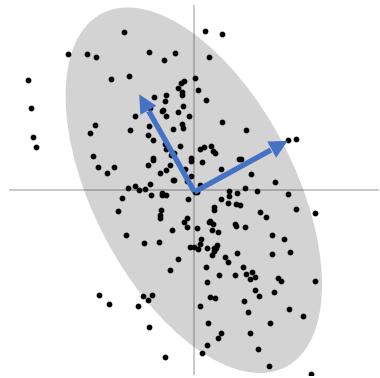


$$\begin{matrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{matrix} = \begin{matrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{matrix} \quad \begin{matrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{matrix} \quad \begin{matrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{matrix} \quad \begin{matrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{matrix}$$
$$\mathbf{C}^{1/2} \quad \mathbf{V} \quad \mathbf{\Lambda}^{1/2} \quad \mathbf{V}^T$$

Principal axes must be **relearned**
for different input densities.

Novel matrix factorization

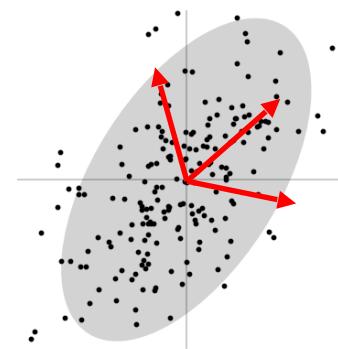
Traditional approaches (PCA)



$$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}^{1/2} = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}^{1/2} \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}^T$$

Principal axes must be **relearned** for different input densities.

Our approach

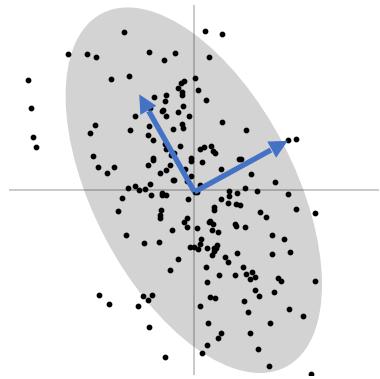


$$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}^{1/2} = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} g_1 & & & \\ & g_2 & & \\ & & g_3 & \\ & & & \cdot \end{bmatrix} \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}^T$$

$K = \frac{N(N+1)}{2}$ **arbitrary axes remain fixed for all densities!**

Novel matrix factorization

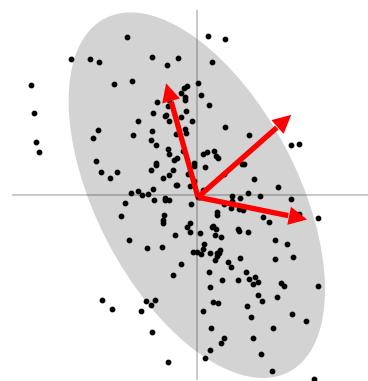
Traditional approaches (PCA)



$$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}^{1/2} = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}^{1/2} \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}^T$$

Principal axes must be **relearned** for different input densities.

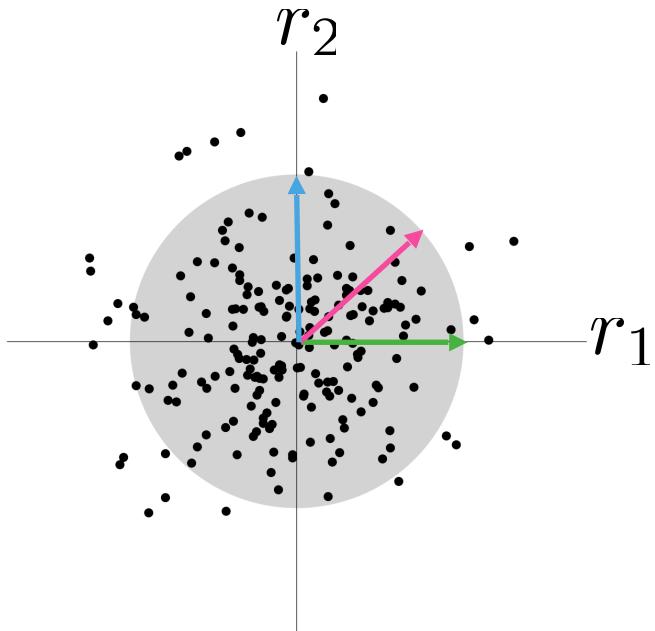
Our approach



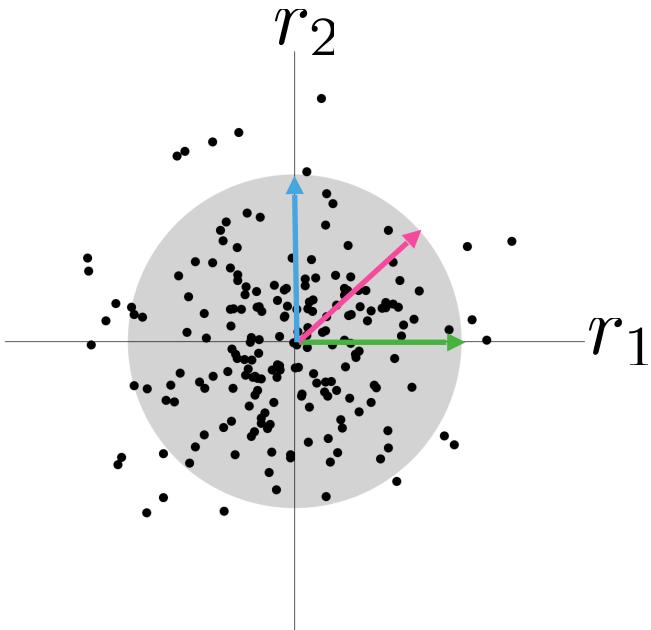
$$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}^{1/2} = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} g_1 & \cdot & \cdot \\ \cdot & g_2 & \cdot \\ \cdot & \cdot & g_3 \end{bmatrix} \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}^T$$

$K = \frac{N(N+1)}{2}$ **arbitrary axes remain fixed for all densities!**

Must be overcomplete



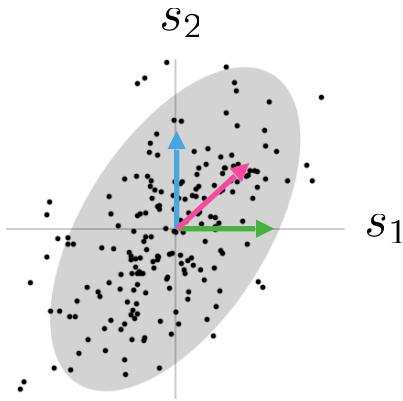
Must be overcomplete



# Primary neurons	# 1D Projections
2	3
3	6
10	55
100	5K

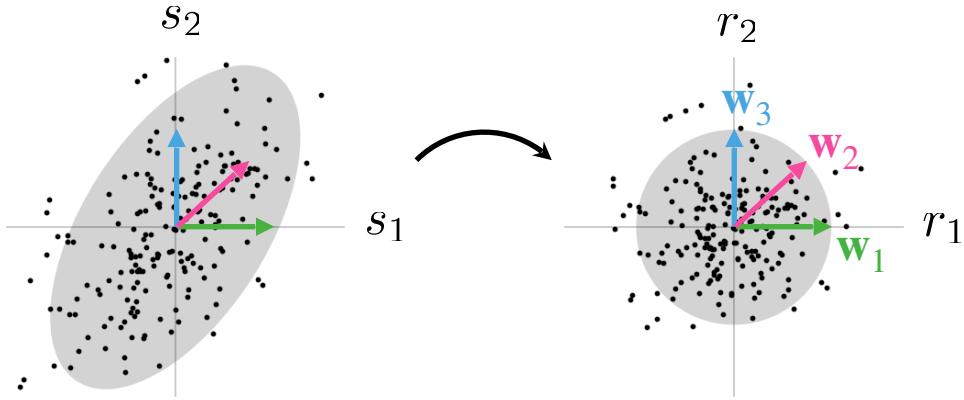
Adaptive whitening via gain modulation

Adaptation objective



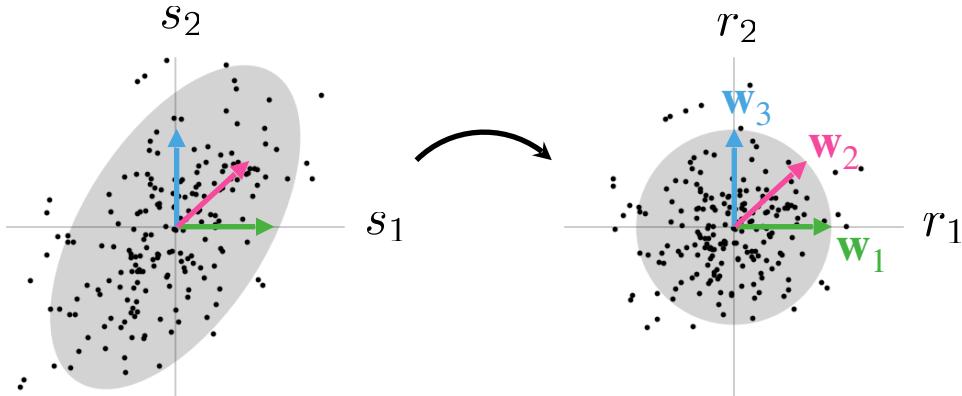
Adaptive whitening via gain modulation

Adaptation objective



Adaptive whitening via gain modulation

Adaptation objective

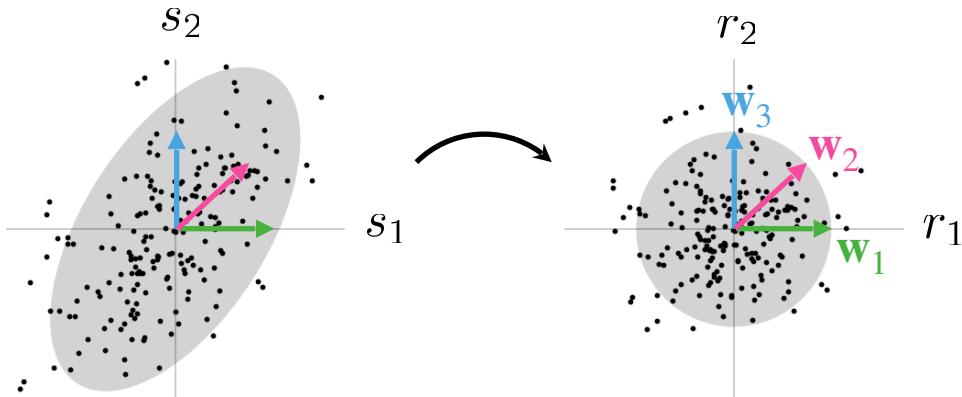


$$\max_{\mathbf{g}} \min_{\mathbf{r}_t} \langle \ell(\mathbf{g}, \mathbf{s}_t, \mathbf{r}_t) \rangle_t$$

$$\ell(\mathbf{g}, \mathbf{s}, \mathbf{r}) = \|\mathbf{r} - \mathbf{s}\|^2 + \sum_{i=1}^K g_i \left\{ (\mathbf{w}_i^\top \mathbf{r})^2 - 1 \right\}$$

Adaptive whitening via gain modulation

Adaptation objective



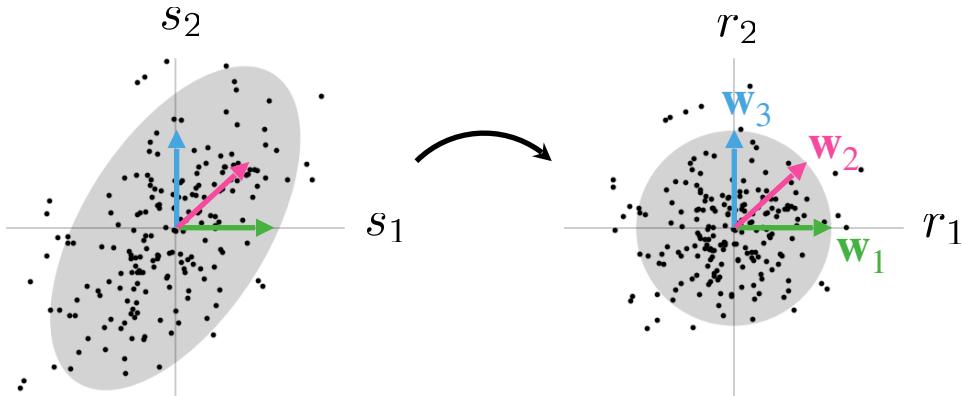
$$\max_{\mathbf{g}} \min_{\mathbf{r}_t} \langle \ell(\mathbf{g}, \mathbf{s}_t, \mathbf{r}_t) \rangle_t$$

$$\ell(\mathbf{g}, \mathbf{s}, \mathbf{r}) = \|\mathbf{r} - \mathbf{s}\|^2 + \sum_{i=1}^K g_i \left\{ (\mathbf{w}_i^\top \mathbf{r})^2 - 1 \right\}$$

match response
to stimuli

Adaptive whitening via gain modulation

Adaptation objective



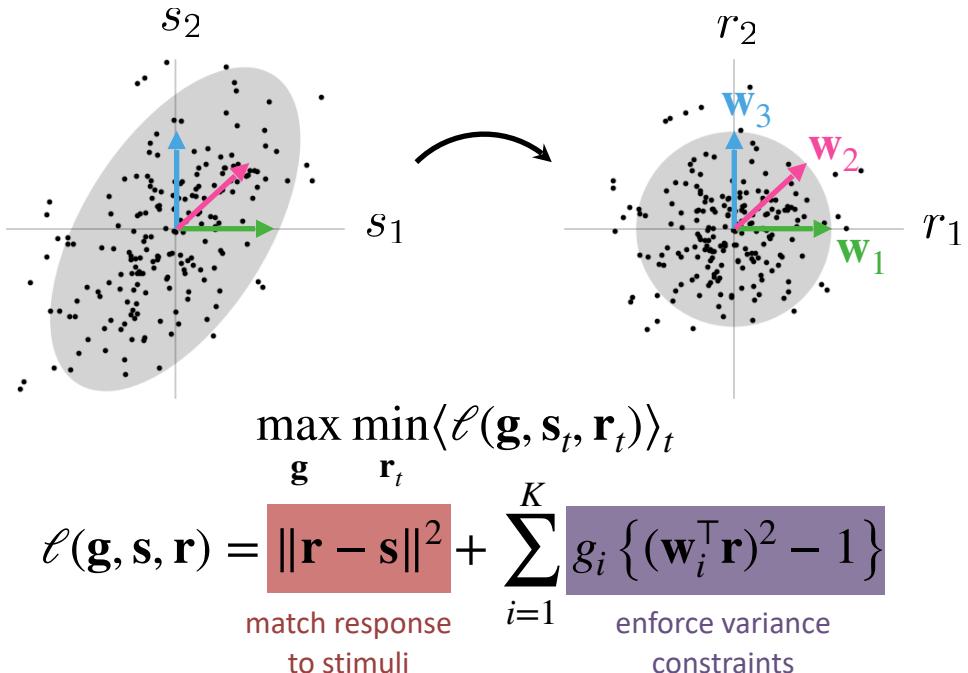
$$\max_{\mathbf{g}} \min_{\mathbf{r}_t} \langle \ell(\mathbf{g}, \mathbf{s}_t, \mathbf{r}_t) \rangle_t$$

$$\ell(\mathbf{g}, \mathbf{s}, \mathbf{r}) = \|\mathbf{r} - \mathbf{s}\|^2 + \sum_{i=1}^K g_i \left\{ (\mathbf{w}_i^\top \mathbf{r})^2 - 1 \right\}$$

match response to stimuli enforce variance constraints

Adaptive whitening via gain modulation

Adaptation objective

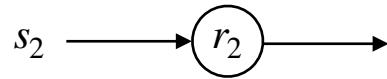
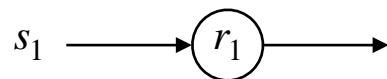


Adaptation algorithm

Algorithm 1: Adaptive whitening via gain modulation

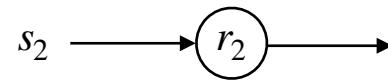
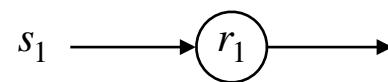
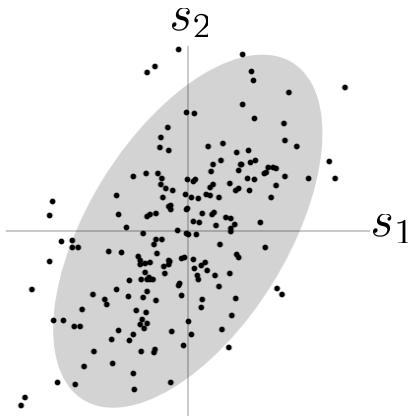
```
1: Input: Centered inputs  $\mathbf{s}_1, \mathbf{s}_2, \dots \in \mathbb{R}^N$ 
2: Initialize:  $\mathbf{W} \in \mathbb{R}^{N \times K}; \mathbf{g} \in \mathbb{R}^K; \eta, \gamma > 0$ 
3: for  $t = 1, 2, \dots$  do
4:    $\mathbf{r}_t \leftarrow 0$ 
5:   while not converged do
6:      $\mathbf{z}_t \leftarrow \mathbf{W}^\top \mathbf{r}_t$ 
7:      $\mathbf{r}_t \leftarrow \mathbf{r}_t + \gamma \{ \mathbf{s}_t - \mathbf{W}(\mathbf{g} \circ \mathbf{z}_t) - \mathbf{r}_t \}$ 
8:   end while
9:    $\mathbf{g} \leftarrow \mathbf{g} + \eta (\mathbf{z}_t^{\circ 2} - 1)$ 
10: end for
```

Adaptive neural circuit w/ **gain-modulating interneurons**



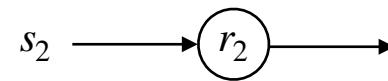
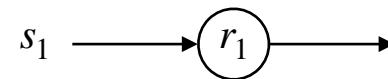
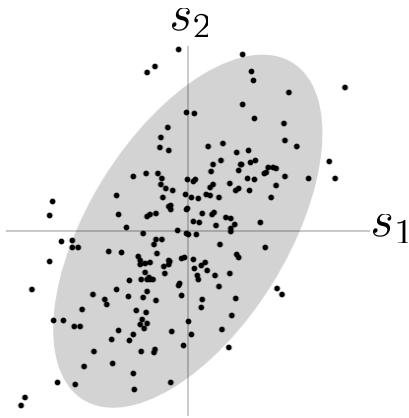
Adaptive neural circuit w/ gain-modulating interneurons

Stimulus distribution

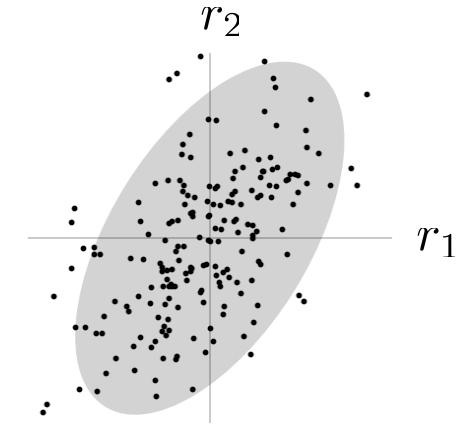


Adaptive neural circuit w/ gain-modulating interneurons

Stimulus distribution

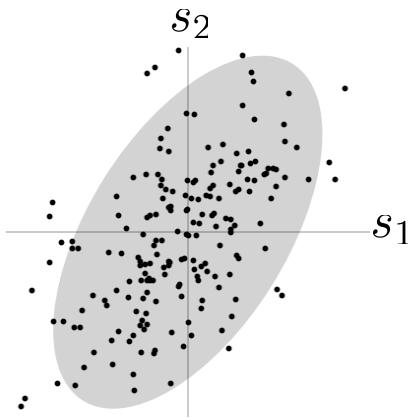


Response distribution

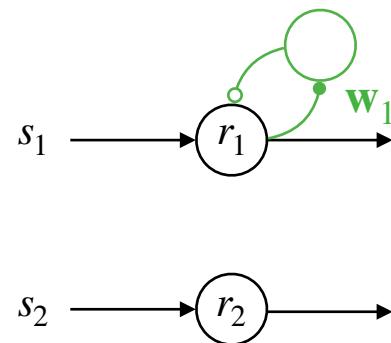


Adaptive neural circuit w/ gain-modulating interneurons

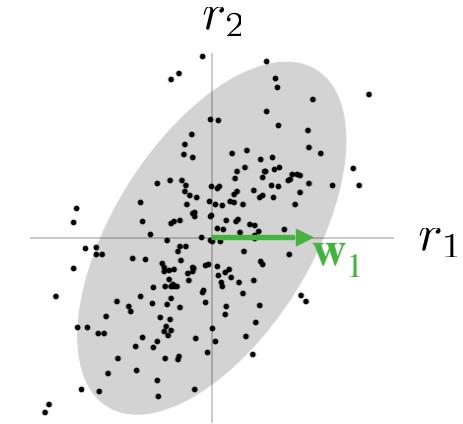
Stimulus distribution



Gain-modulating interneurons

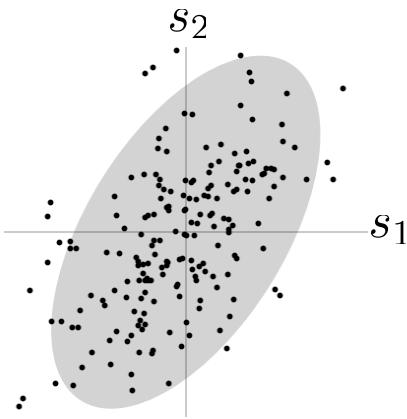


Response distribution

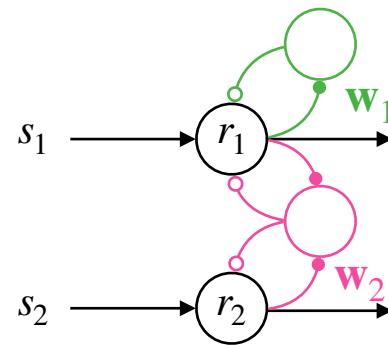


Adaptive neural circuit w/ gain-modulating interneurons

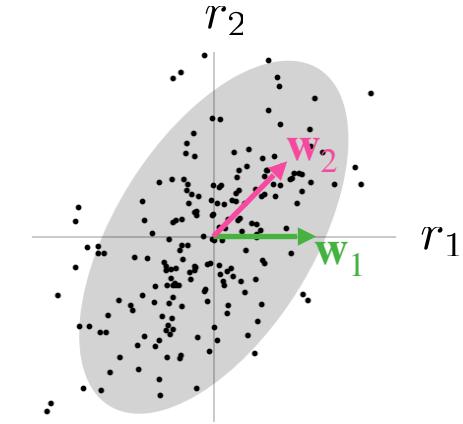
Stimulus distribution



Gain-modulating interneurons

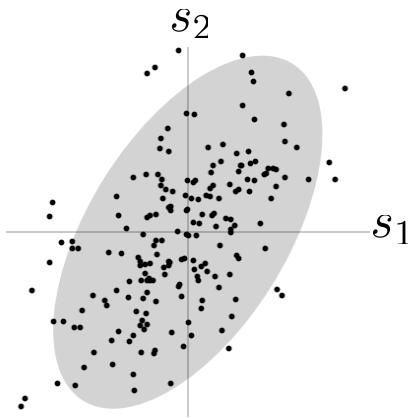


Response distribution

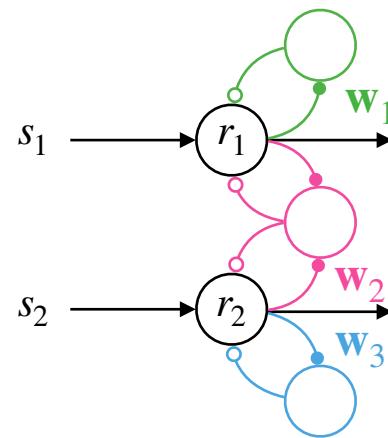


Adaptive neural circuit w/ gain-modulating interneurons

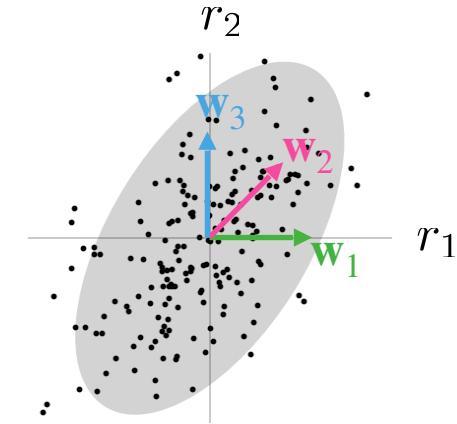
Stimulus distribution



Gain-modulating interneurons

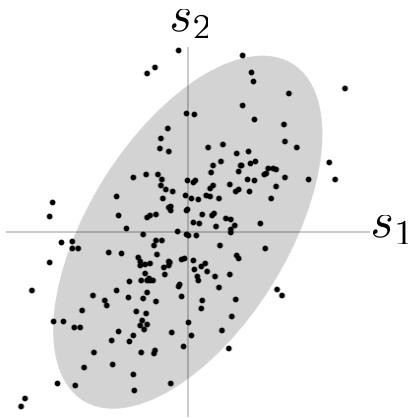


Response distribution

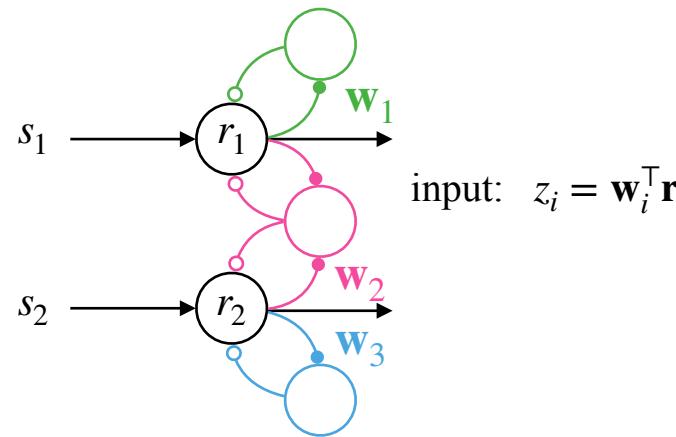


Adaptive neural circuit w/ gain-modulating interneurons

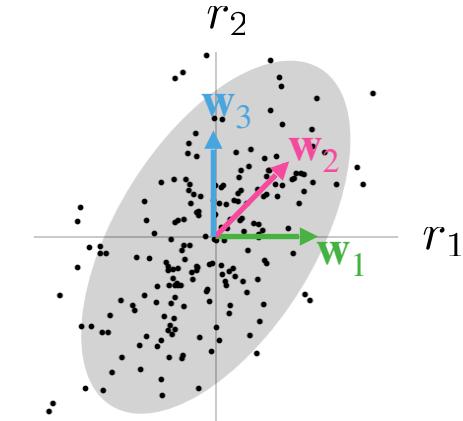
Stimulus distribution



Gain-modulating interneurons

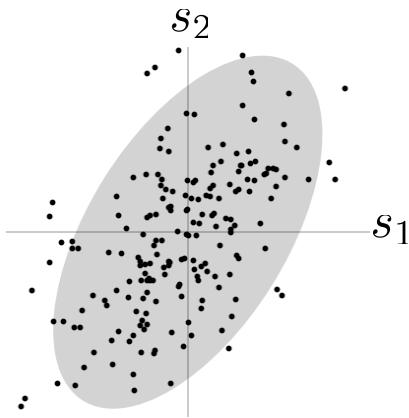


Response distribution

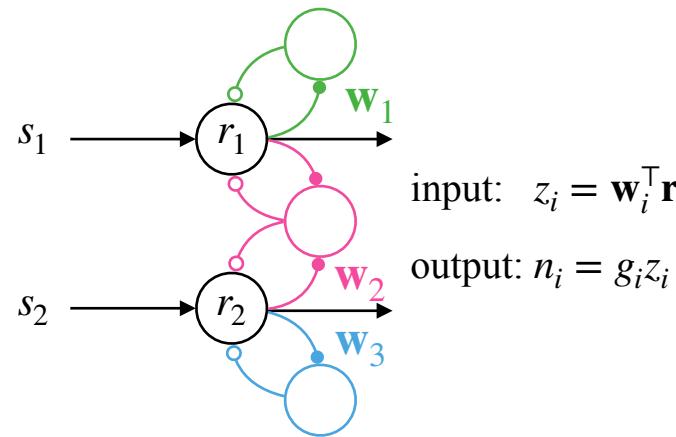


Adaptive neural circuit w/ gain-modulating interneurons

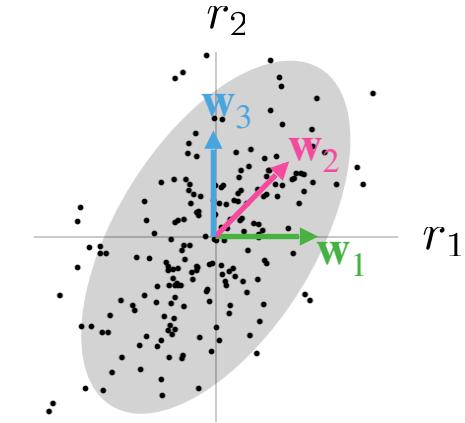
Stimulus distribution



Gain-modulating interneurons

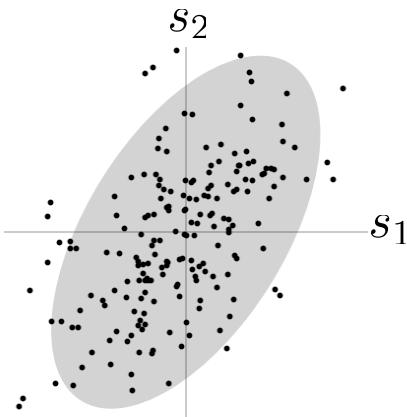


Response distribution

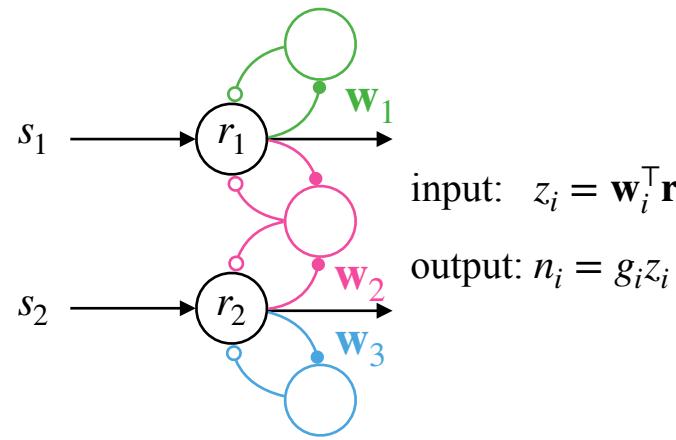


Adaptive neural circuit w/ gain-modulating interneurons

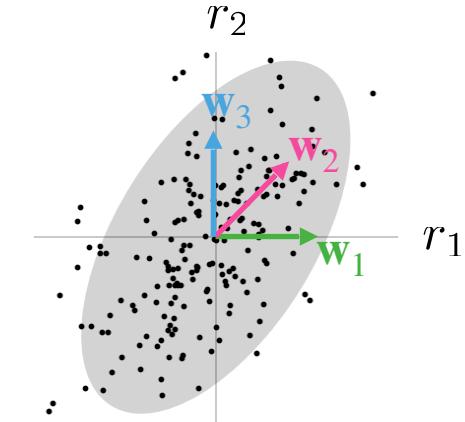
Stimulus distribution



Gain-modulating interneurons



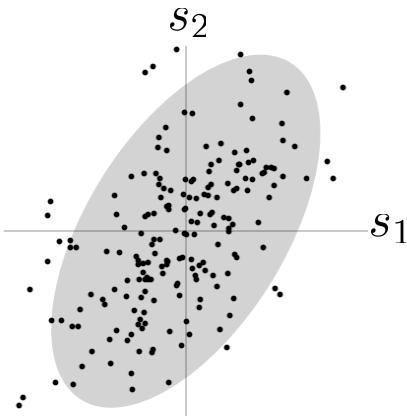
Response distribution



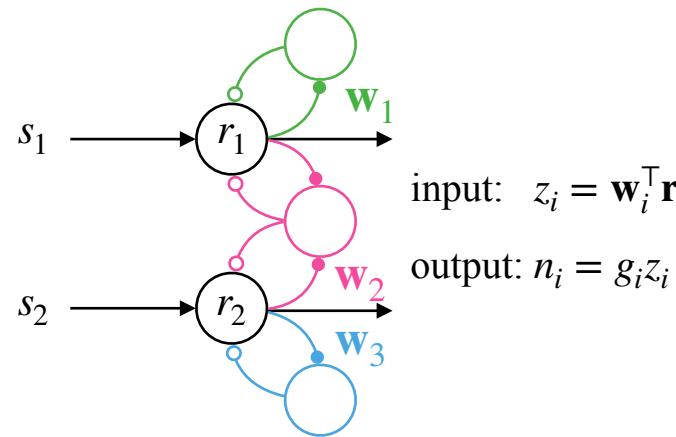
fast neural dynamics: $\frac{d\mathbf{r}}{dt} = \mathbf{s} - \mathbf{r} - \mathbf{W}\mathbf{n}$

Adaptive neural circuit w/ gain-modulating interneurons

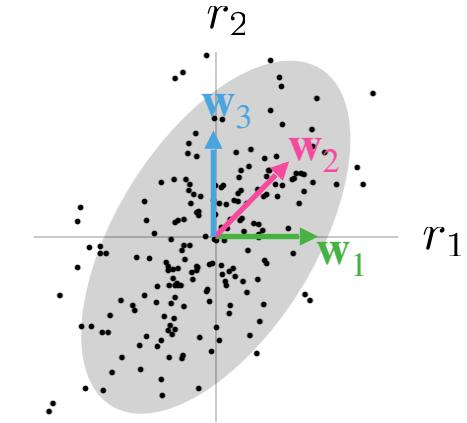
Stimulus distribution



Gain-modulating interneurons



Response distribution

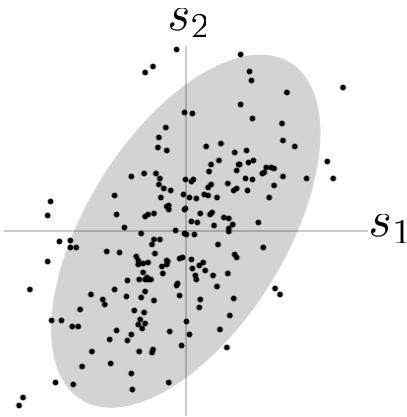


$$\text{fast neural dynamics: } \frac{d\mathbf{r}}{dt} = \mathbf{s} - \mathbf{r} - \mathbf{W}\mathbf{n}$$

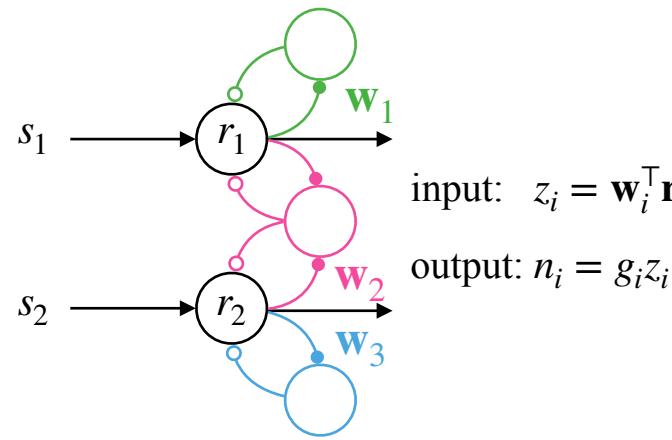
$$\text{slow gain updates: } \Delta g_i = \eta_g(\text{var}(z_i) - 1)$$

Adaptive neural circuit w/ gain-modulating interneurons

Stimulus distribution



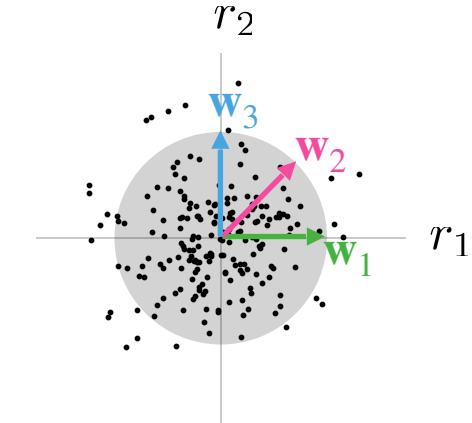
Gain-modulating interneurons



$$\text{fast neural dynamics: } \frac{d\mathbf{r}}{dt} = \mathbf{s} - \mathbf{r} - \mathbf{W}\mathbf{n}$$

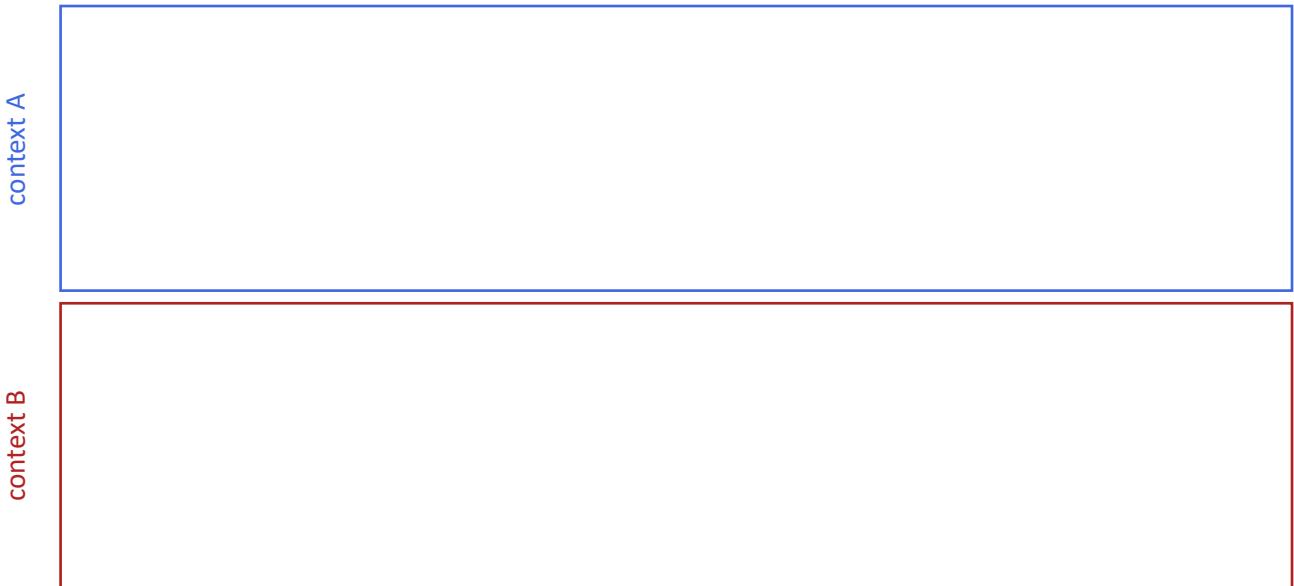
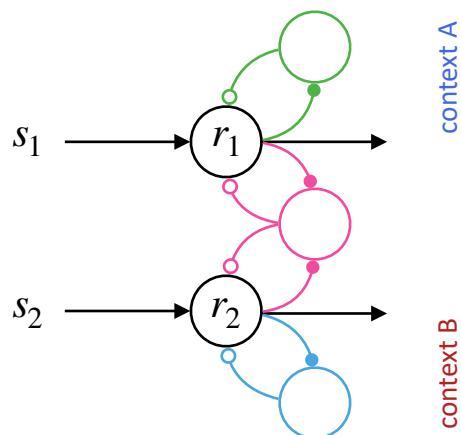
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Response distribution

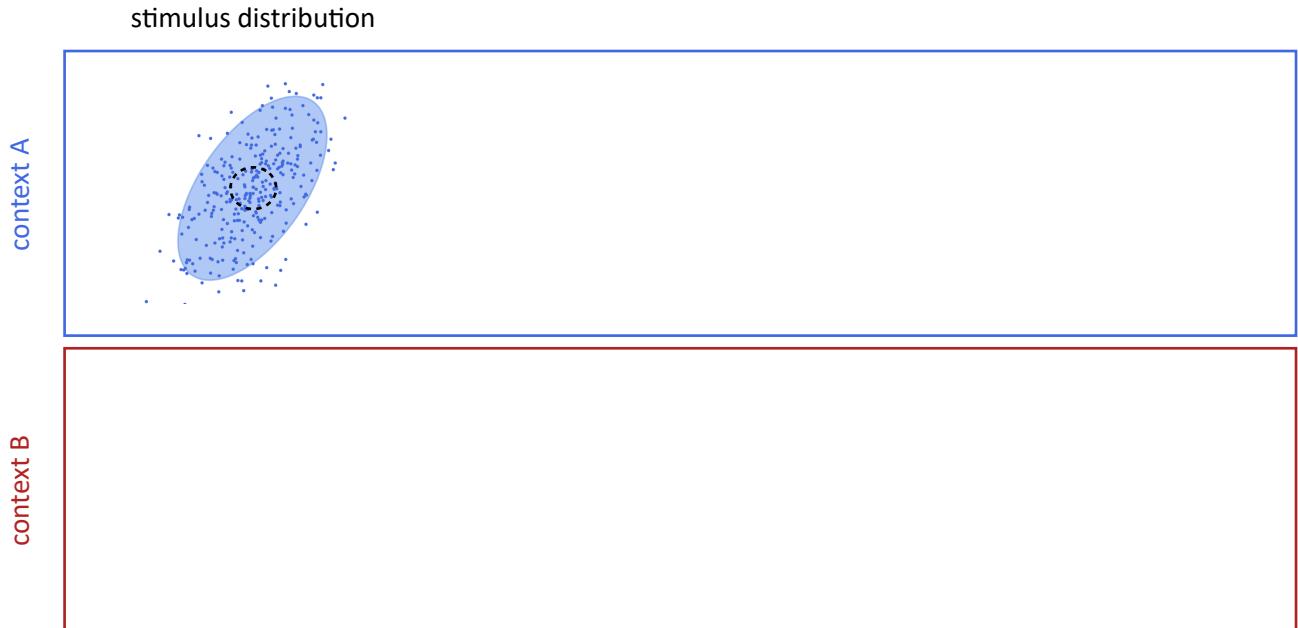
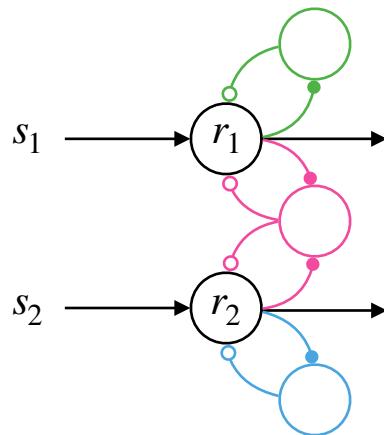


Correlations removed!

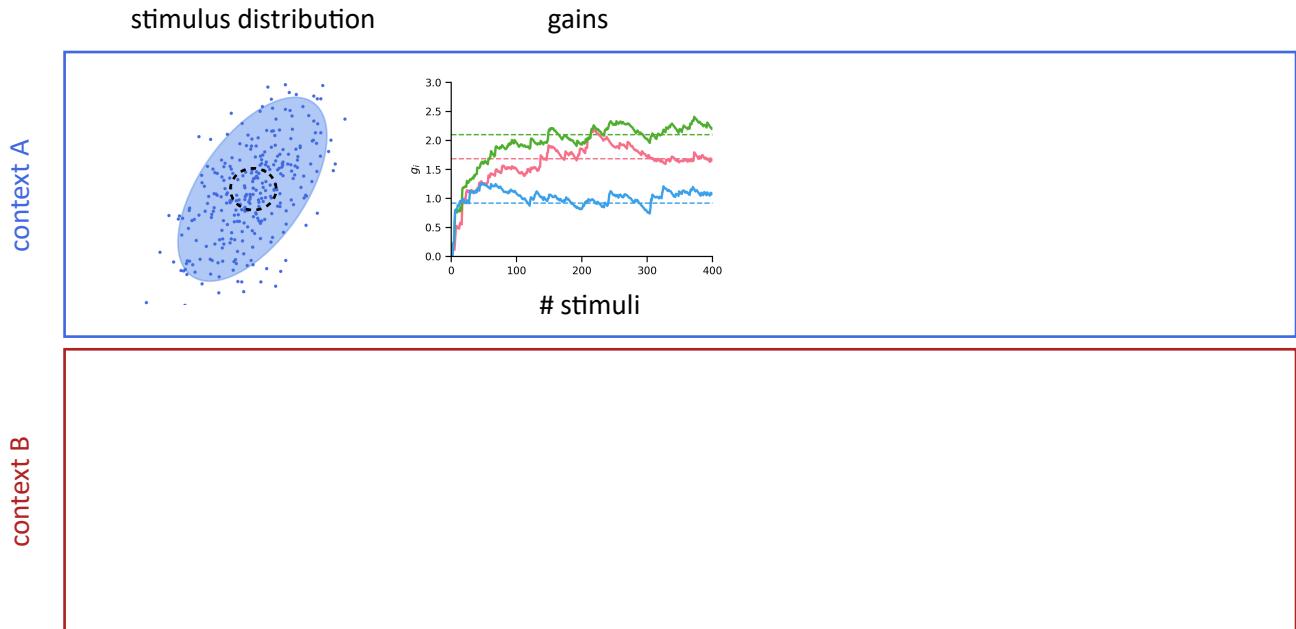
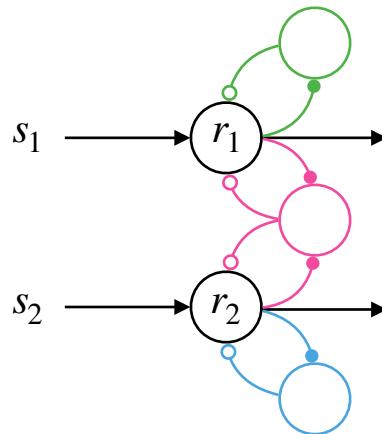
Numerical simulations



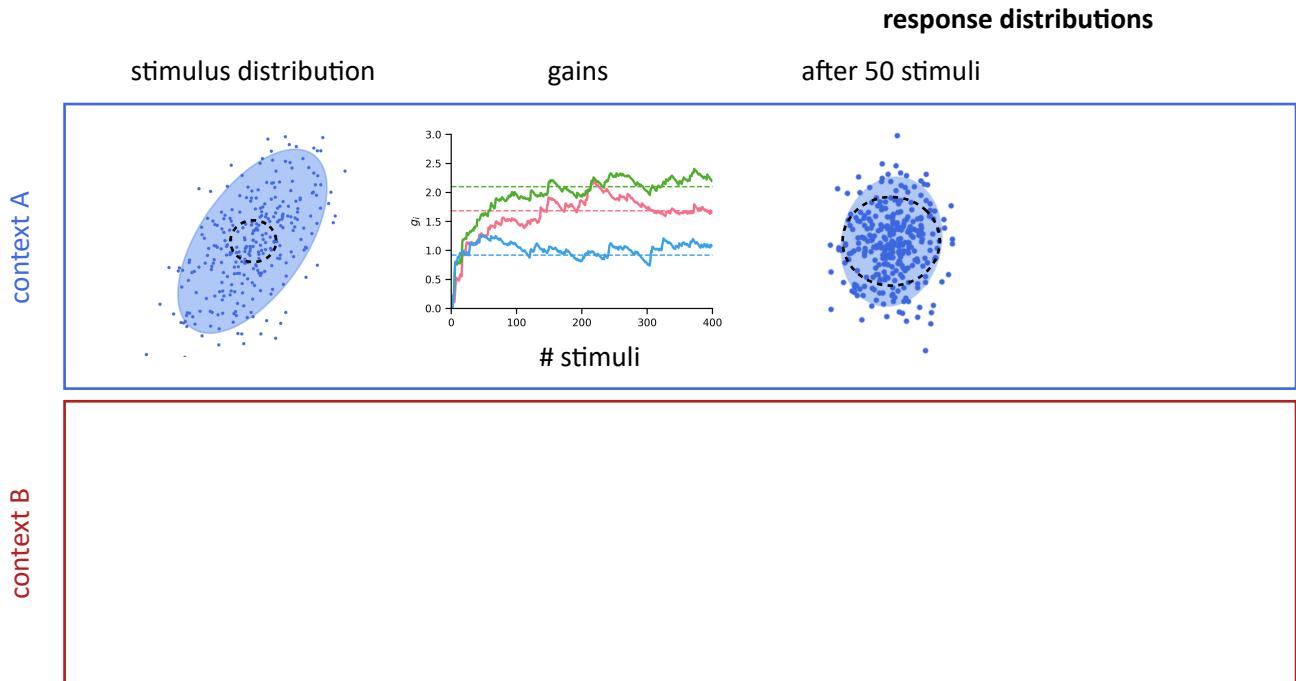
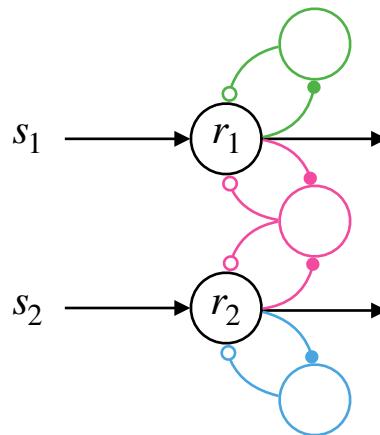
Numerical simulations



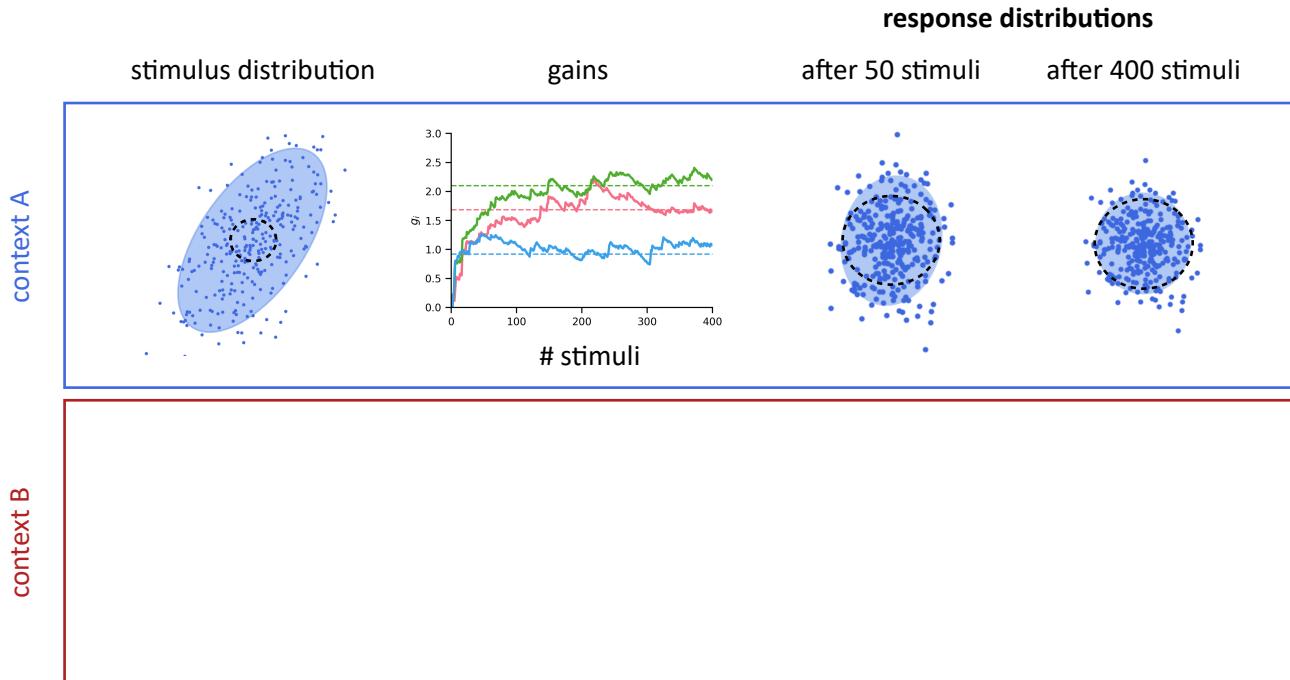
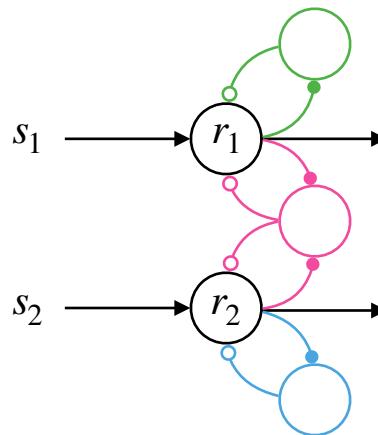
Numerical simulations



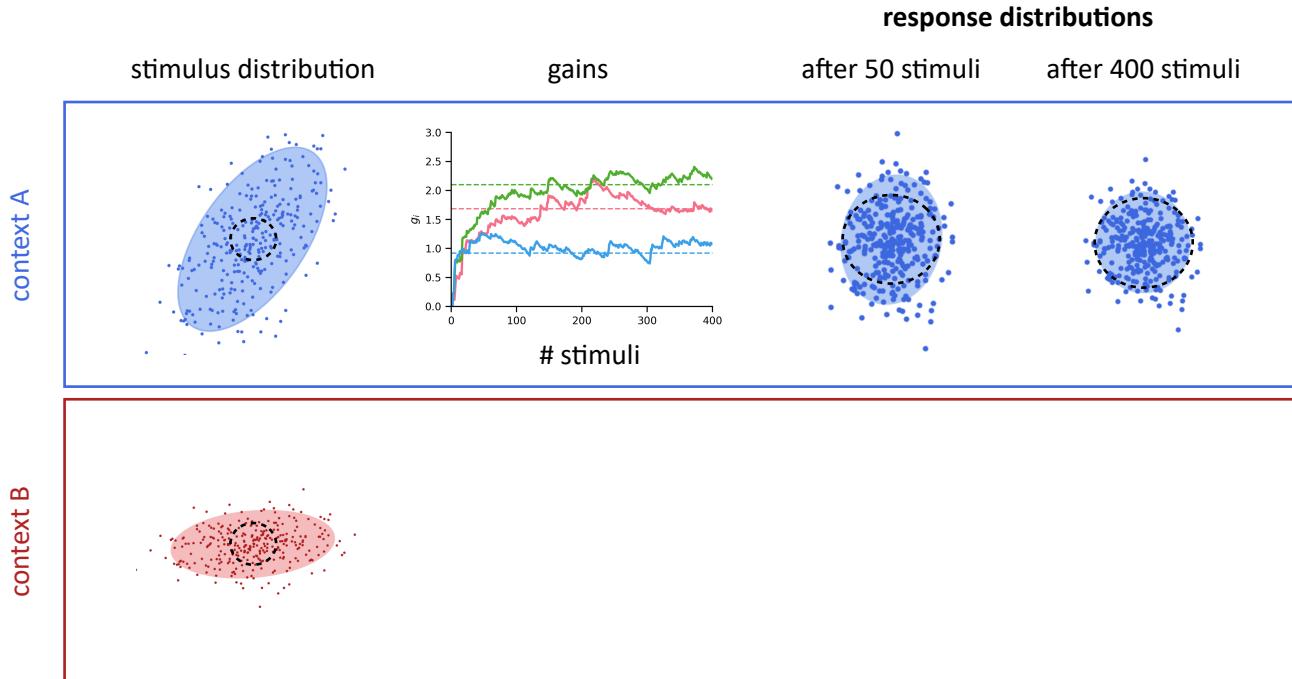
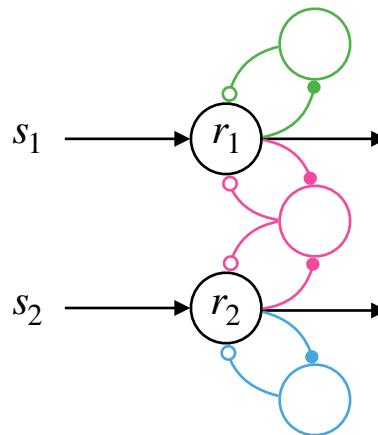
Numerical simulations



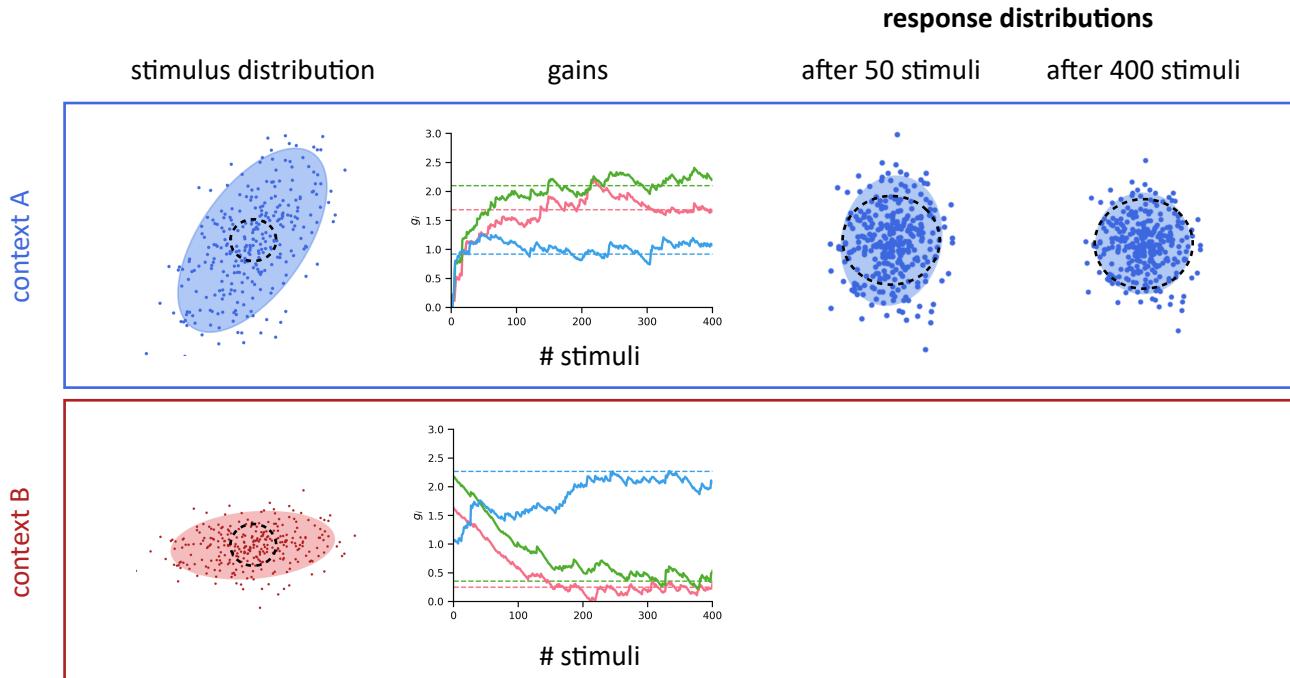
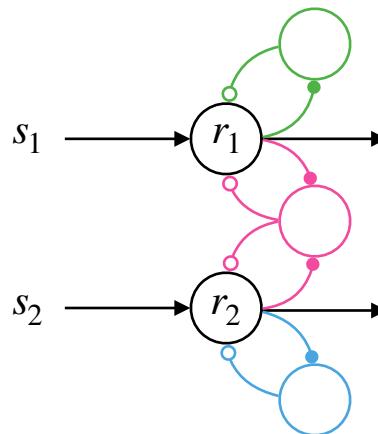
Numerical simulations



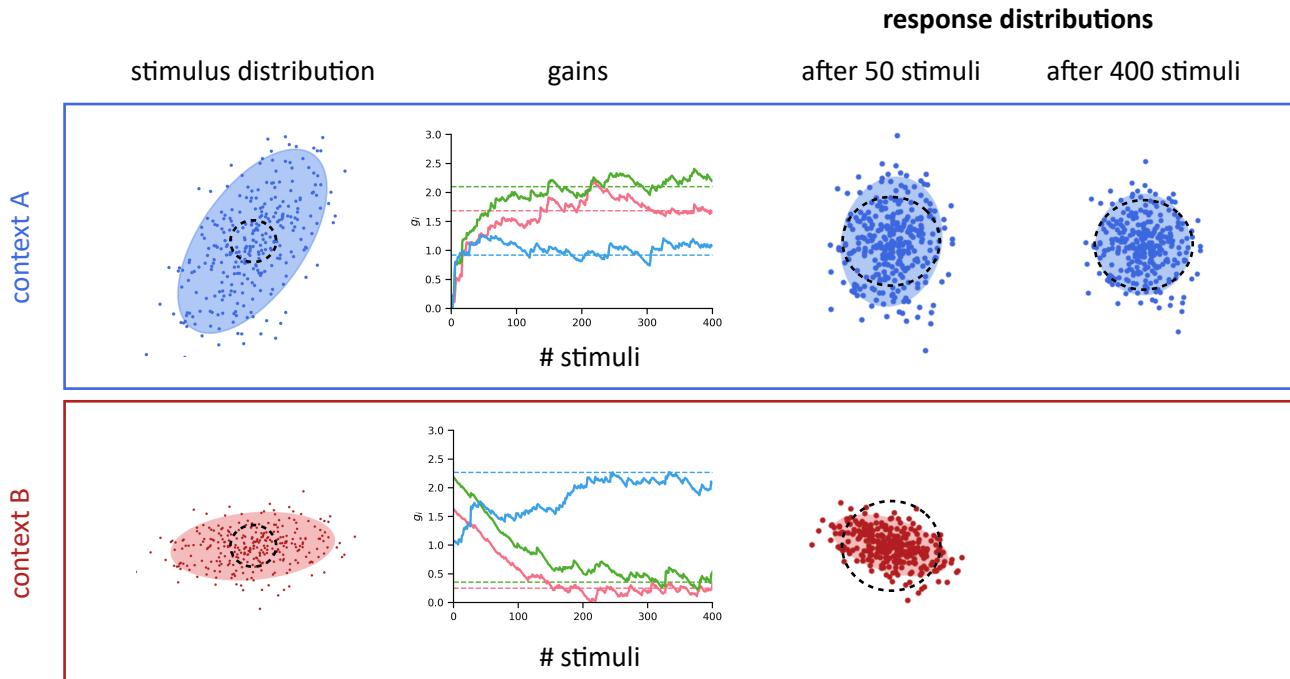
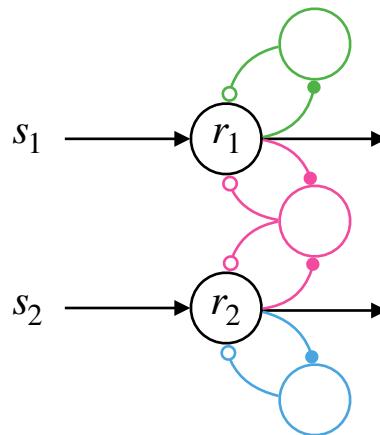
Numerical simulations



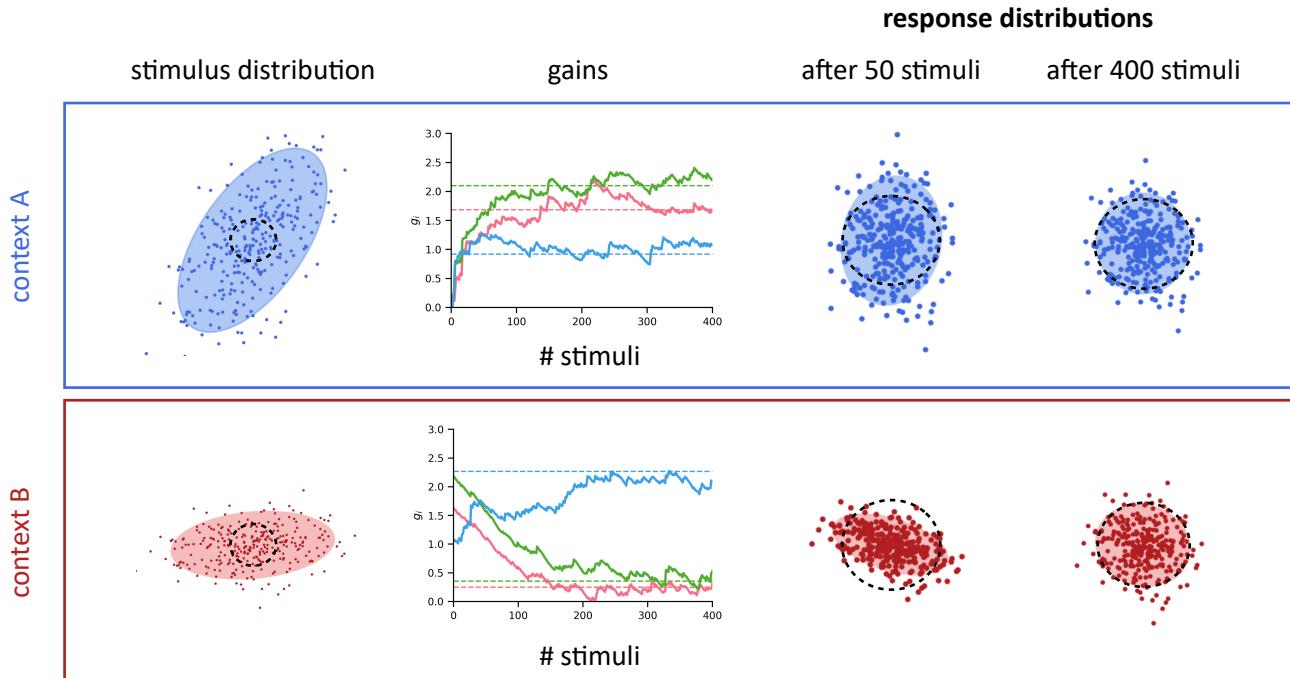
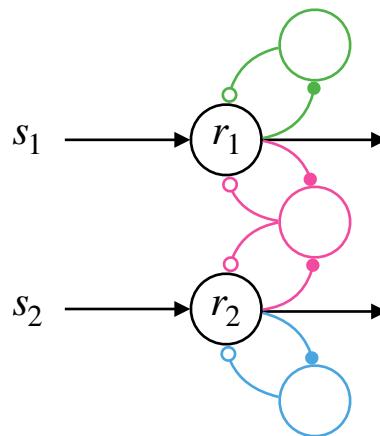
Numerical simulations



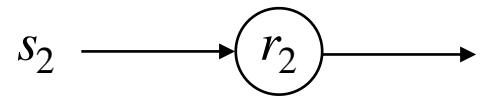
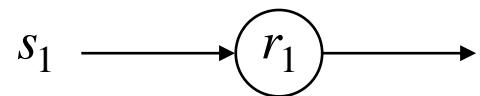
Numerical simulations



Numerical simulations

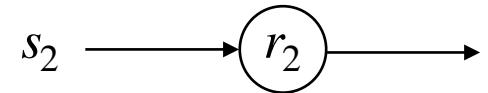
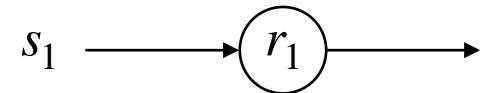


Summary



Summary

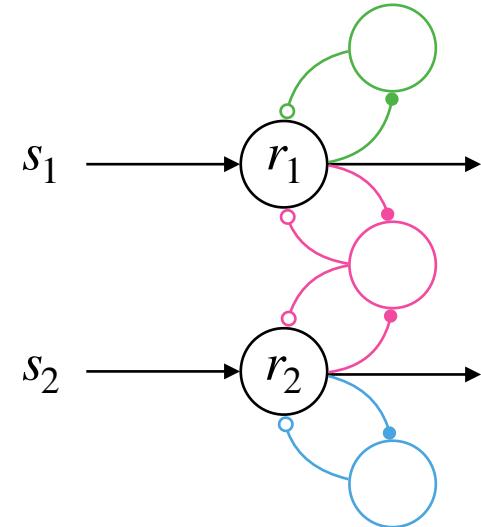
Q: Can neural circuits **decorrelate** their responses using **gain modulation**?



Summary

Q: Can neural circuits **decorrelate** their responses using **gain modulation**?

A: Yes. Using **gain-modulating interneurons** and a novel mathematical perspective.

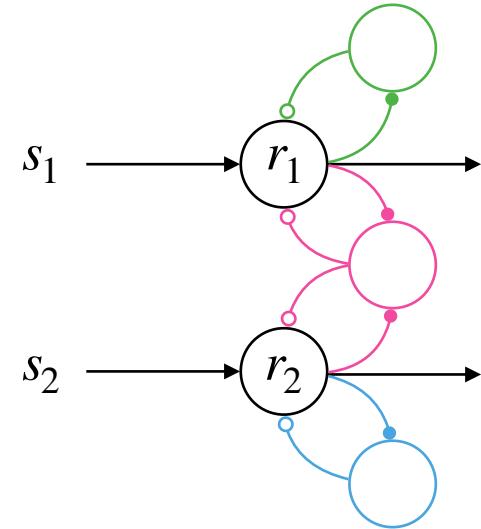


Summary

Q: Can neural circuits **decorrelate** their responses using **gain modulation**?

A: Yes. Using **gain-modulating interneurons** and a novel mathematical perspective.

Prediction: Local interneurons modulate their gains in response to changes in their **input variance**

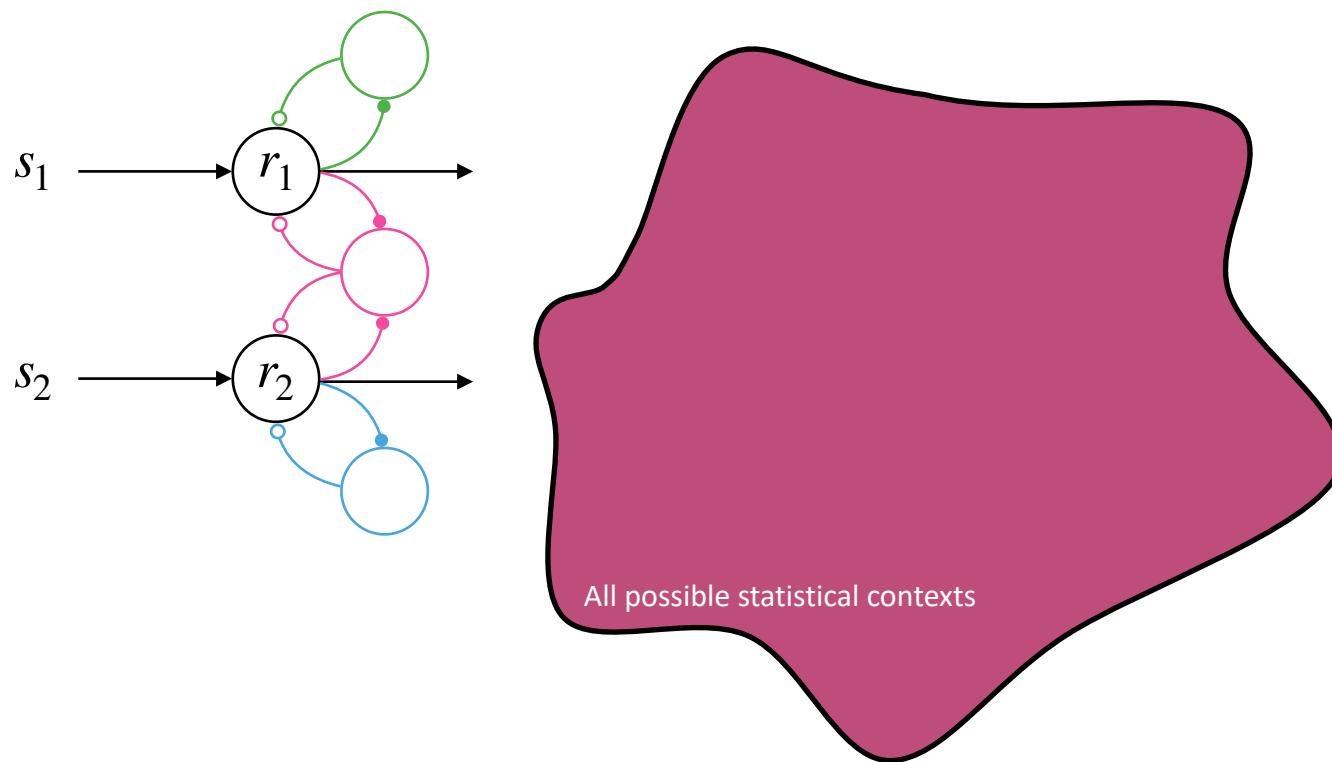


But wait...

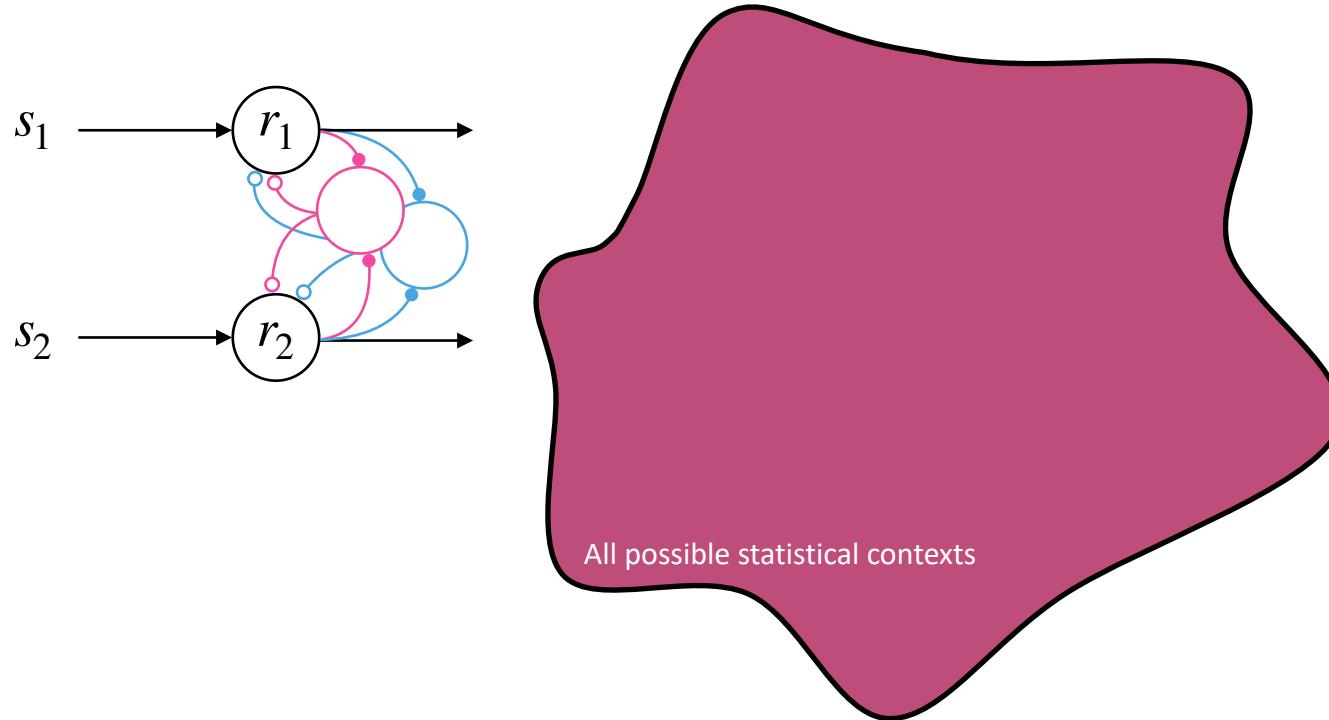
But wait...

# primary neurons	# interneurons
2	3
3	6
10	55
100	5K

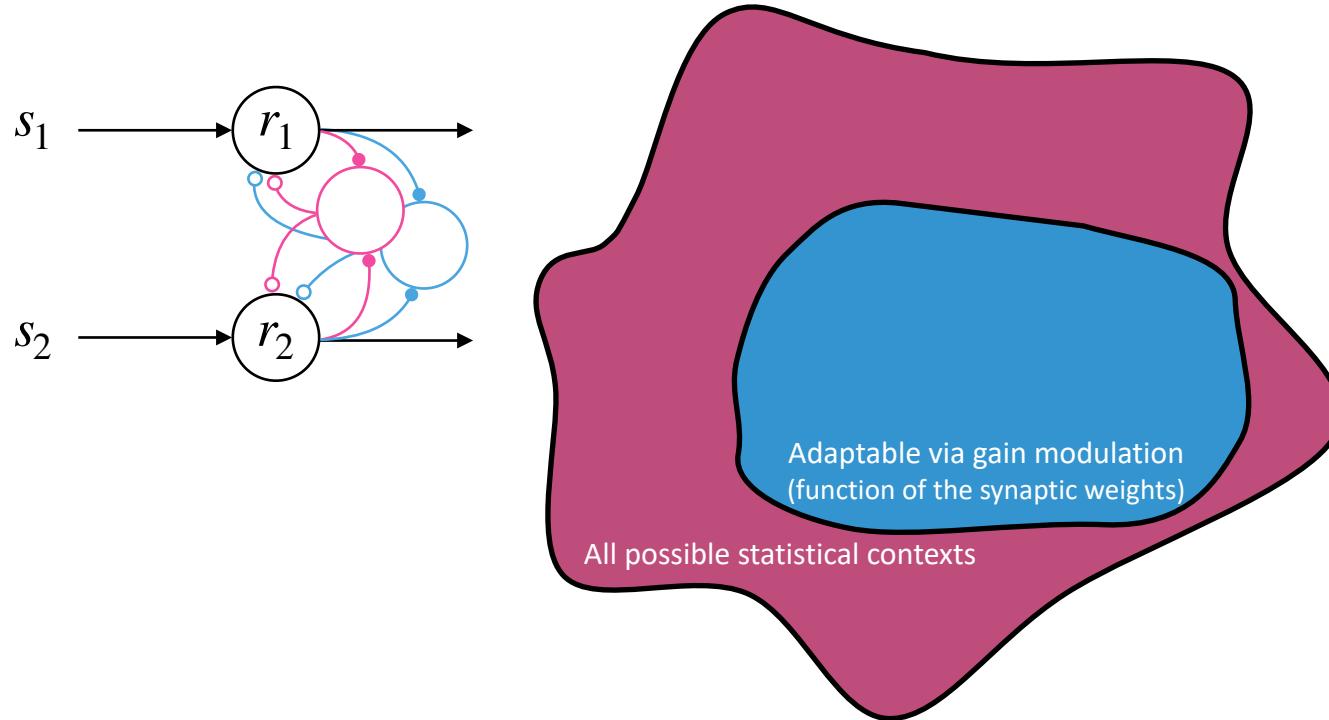
Multi-timescale model intuition



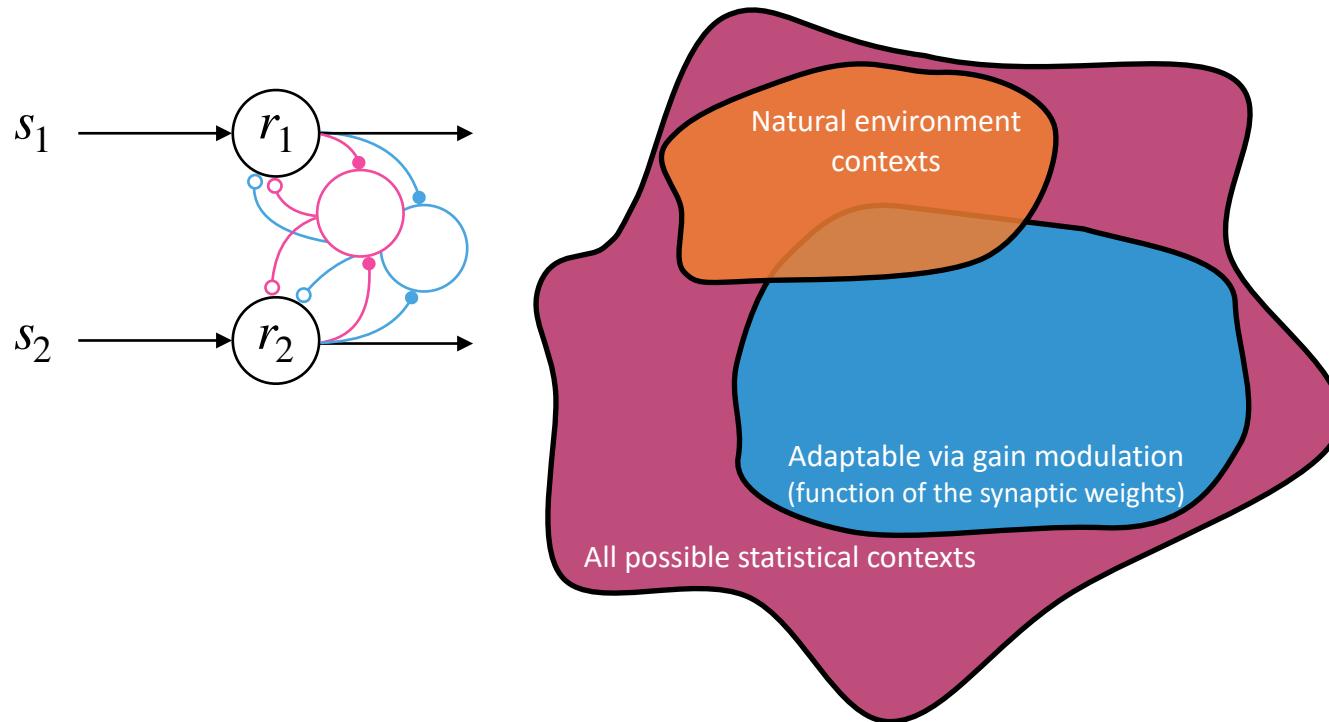
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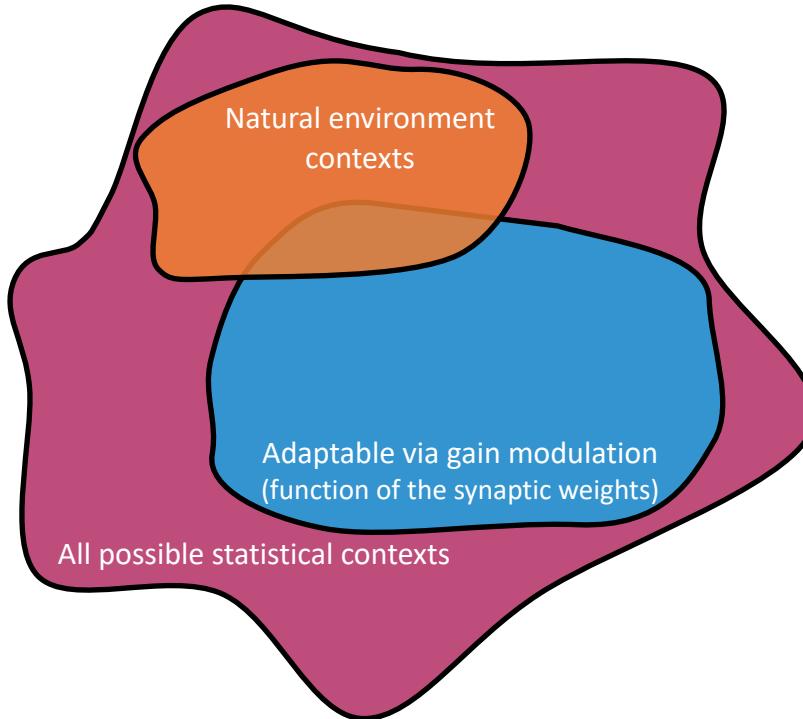
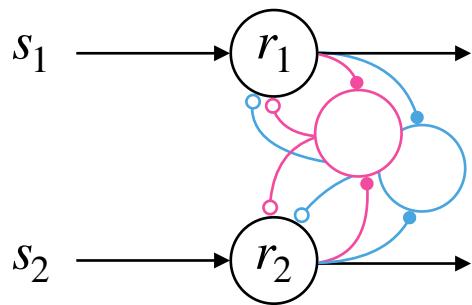
Multi-timescale model intuition



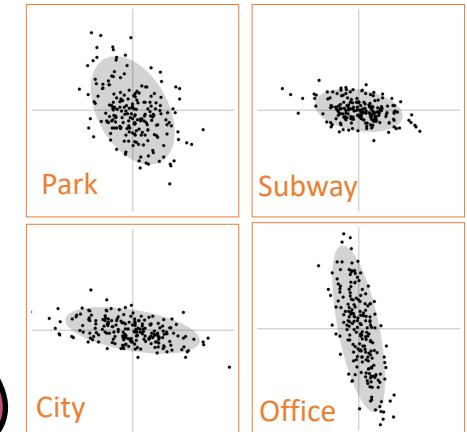
Multi-timescale model intuition



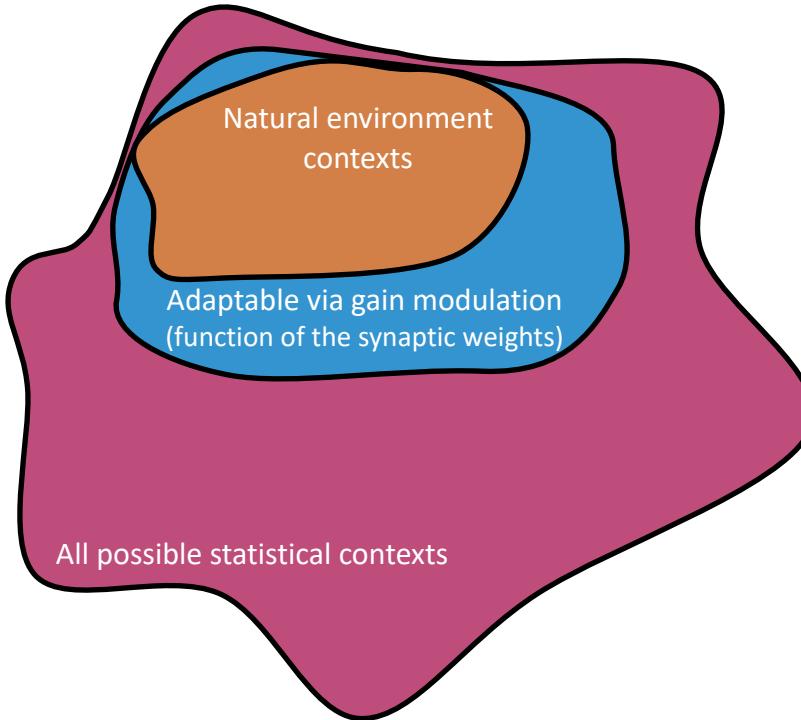
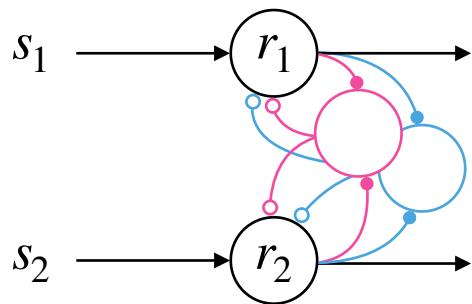
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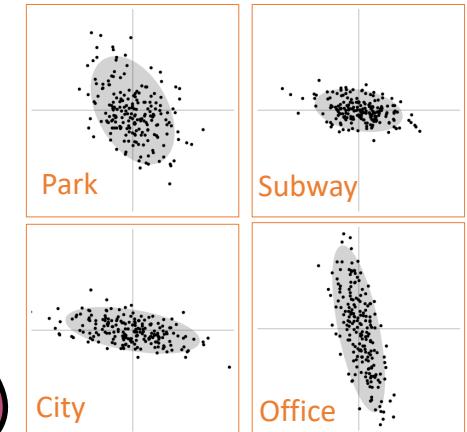
Natural context examples



Multi-timescale model intuition



Natural context examples



Adaptive whitening via gain modulation

Adaptation objective

Adaptive whitening via gain modulation

Adaptation objective

$$\max_{\mathbf{W}} \mathbb{E}_{c \in p(c)} \left[\max_{\mathbf{g}} \mathbb{E}_{\mathbf{s} \sim p(\mathbf{s}|c)} \left[\min_{\mathbf{r}} \ell(\mathbf{W}, \mathbf{g}, \mathbf{s}, \mathbf{r}) \right] \right]$$

$$\ell(\mathbf{W}, \mathbf{g}, \mathbf{s}, \mathbf{r}) = \|\mathbf{r} - \mathbf{s}\|^2 + \sum_{i=1}^K g_i \left\{ (\mathbf{w}_i^\top \mathbf{r})^2 - 1 \right\}$$

Adaptive whitening via gain modulation

Adaptation objective

$$\max_{\mathbf{W}} \mathbb{E}_{c \in p(c)} \left[\max_{\mathbf{g}} \mathbb{E}_{\mathbf{s} \sim p(\mathbf{s}|c)} \left[\min_{\mathbf{r}} \ell(\mathbf{W}, \mathbf{g}, \mathbf{s}, \mathbf{r}) \right] \right]$$

synapses are
optimized
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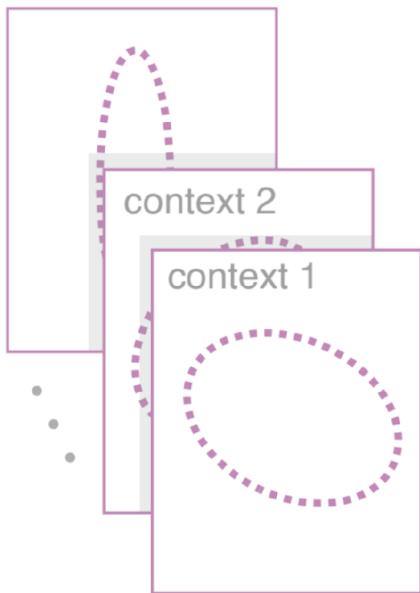


Adaptation algorithm

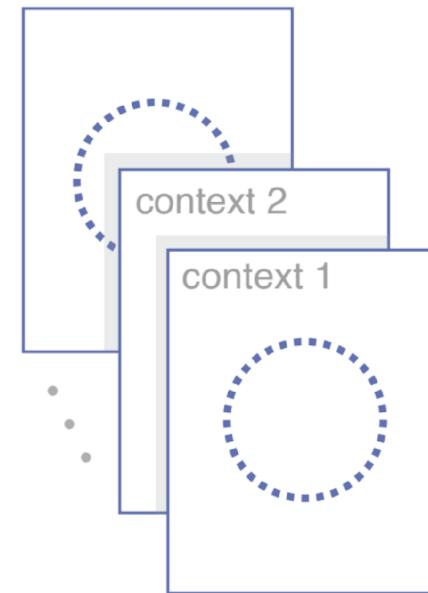
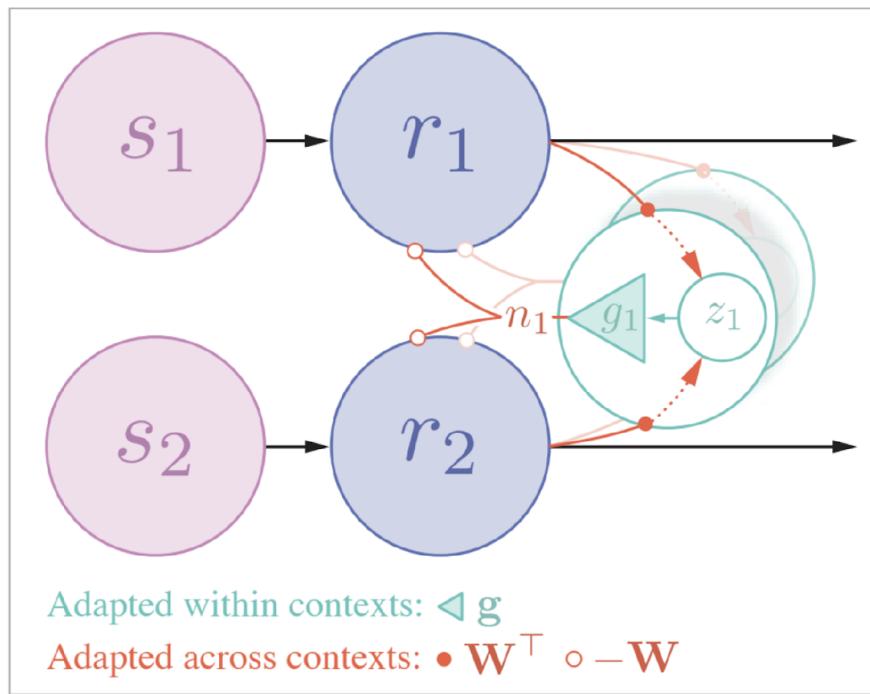
Algorithm 1: Multi-timescale adaptive whitening

```
1: Input:  $\mathbf{s}_1, \mathbf{s}_2, \dots \in \mathbb{R}^N$ 
2: for  $t = 1, 2, \dots$  do
3:    $\mathbf{r}_t \leftarrow \mathbf{0}$ 
4:   while not converged do
5:      $\mathbf{z}_t \leftarrow \mathbf{W}^\top \mathbf{r}_t$ 
6:      $\mathbf{n}_t \leftarrow \mathbf{g} \circ \mathbf{z}_t$ 
7:      $\mathbf{r}_t \leftarrow \mathbf{r}_t + \eta_r (\mathbf{s}_t - \mathbf{W}\mathbf{n}_t - \alpha\mathbf{r}_t)$ 
8:   end while
9:    $\mathbf{g} \leftarrow \mathbf{g} + \eta_g (\mathbf{z}_t \circ \mathbf{z}_t - \text{diag}(\mathbf{W}^\top \mathbf{W}))$ 
10:   $\mathbf{W} \leftarrow \mathbf{W} + \eta_w (\mathbf{r}_t \mathbf{n}_t^\top - \mathbf{W} \text{diag}(\mathbf{g}))$ 
11: end for
```

Multi-timescale adaptive RNN architecture



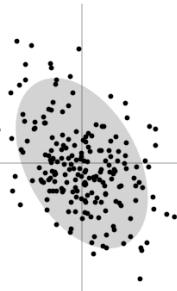
Inputs (s_1, s_2)



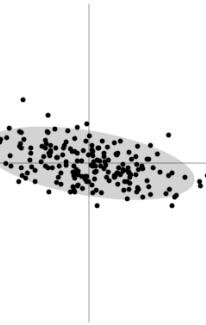
Responses (r_1, r_2)

Learning to adapt across and within contexts

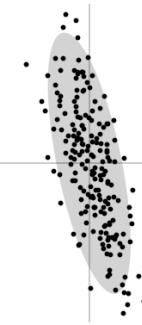
Context 1



Context 2



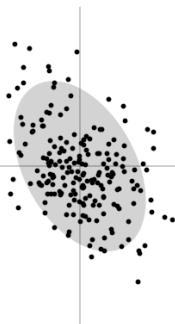
Context 100



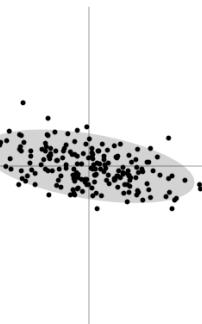
...

Learning to adapt across and within contexts

Context 1



Context 2



Context 100



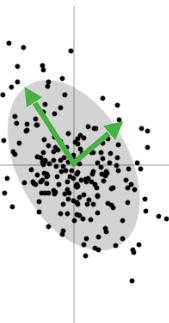
...

Training procedure:

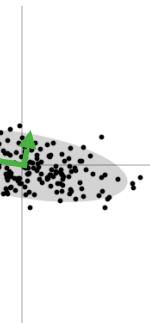
1. Sample context from all possible contexts
2. Sample stimulus within context 1000x

Learning to adapt across and within contexts

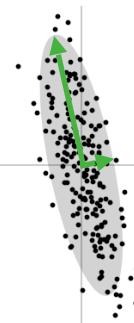
Context 1



Context 2



Context 100



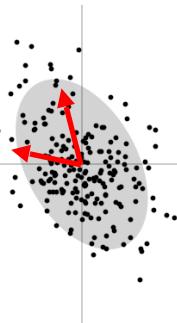
...

Training procedure:

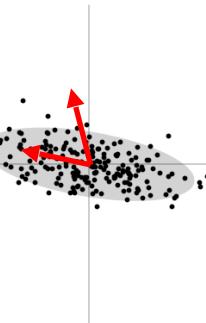
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Learning to adapt across and within contexts

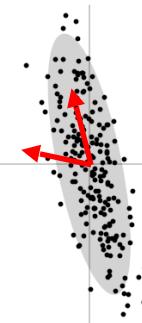
Context 1



Context 2



Context 100



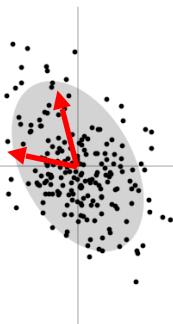
...

Training procedure:

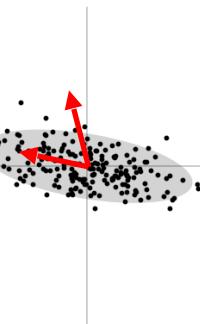
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Learning to adapt across and within contexts

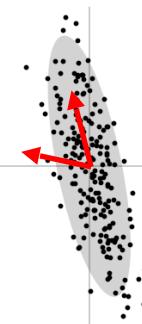
Context 1



Context 2



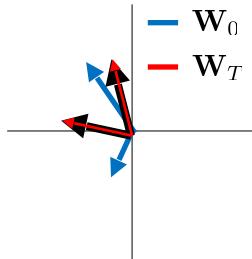
Context 100



Training procedure:

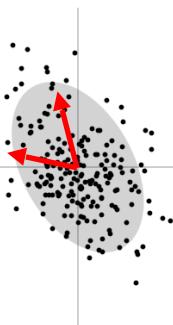
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Weights before/after training

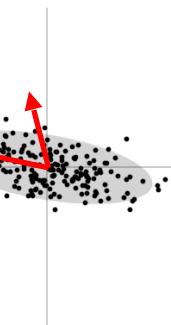


Learning to adapt across and within contexts

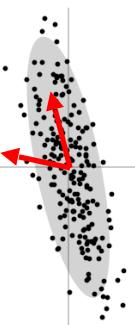
Context 1



Context 2



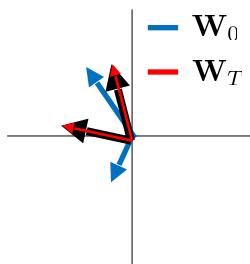
Context 100



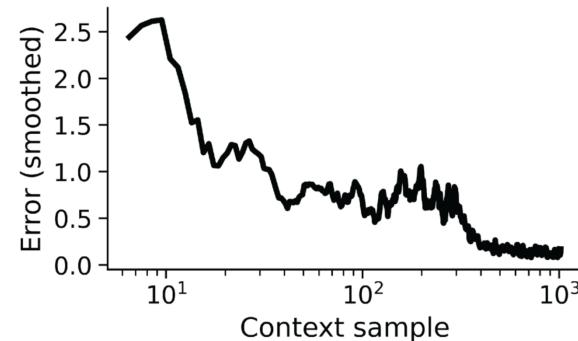
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Weights before/after training

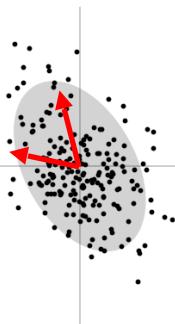


Error through training

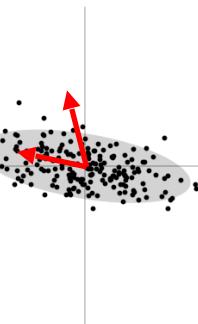


Learning to adapt across and within contexts

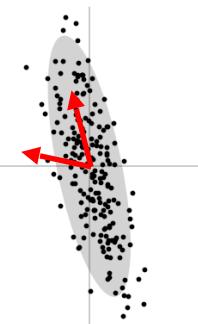
Context 1



Context 2



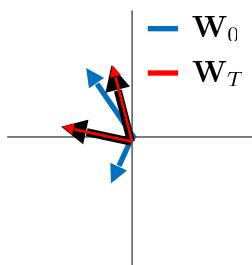
Context 100



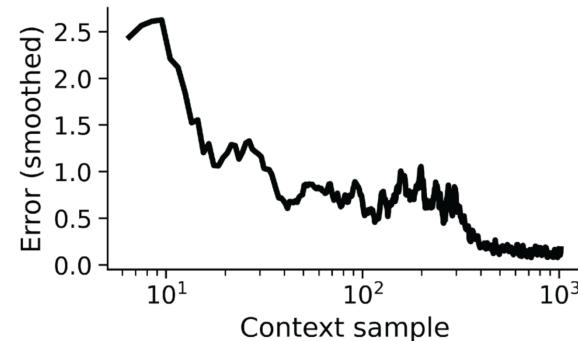
Training procedure:

1. Sample context from all possible contexts
2. Sample stimulus within context 1000x

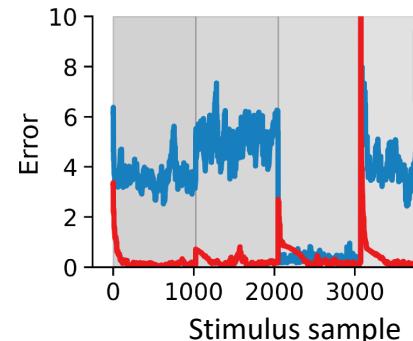
Weights before/after training



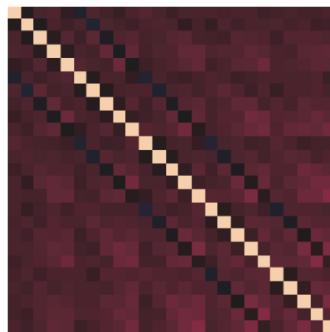
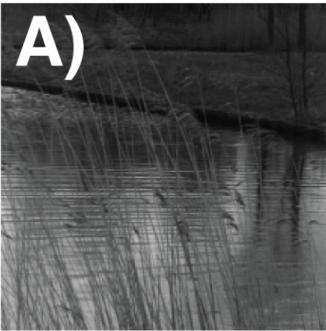
Error through training



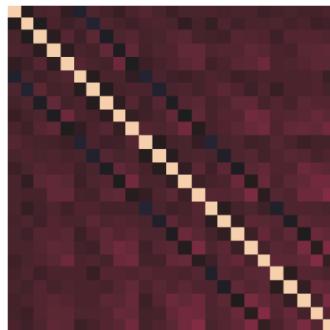
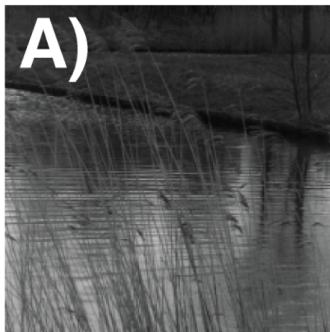
Within-context error before/after training



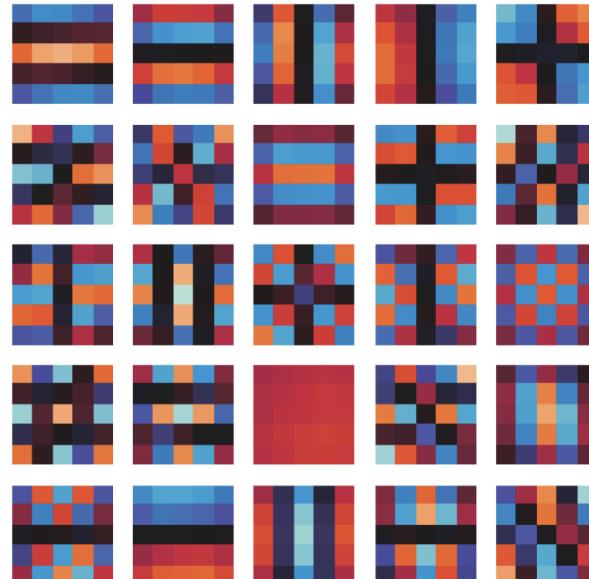
Adaptive whitening of natural images



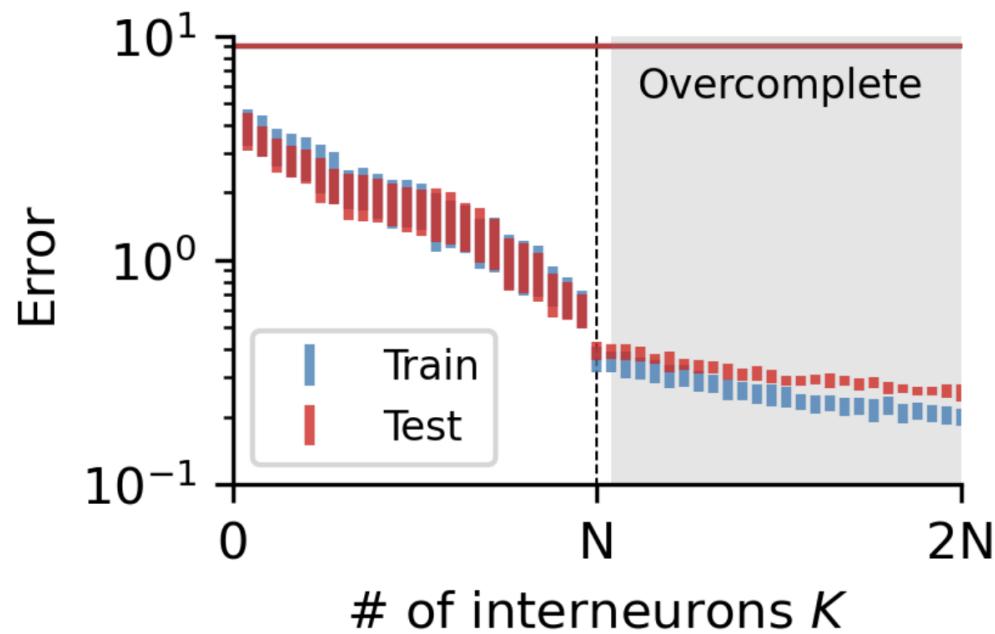
Adaptive whitening of natural images



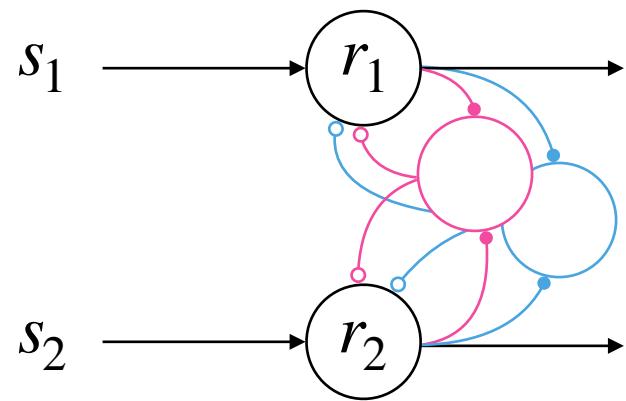
synapses learn 2D sinusoidal filters



Dependence on the # of interneurons

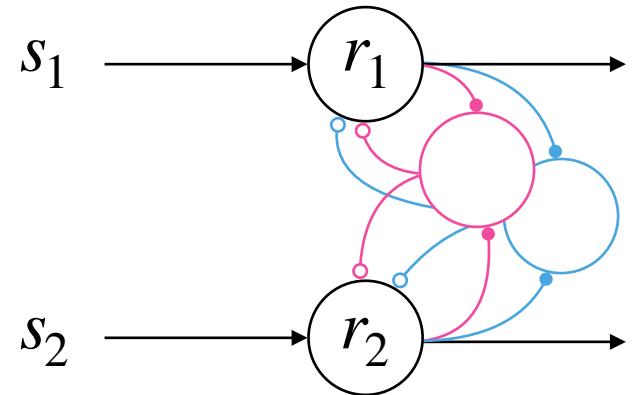


Summary



Summary

Circuit with **fast** gain modulation and
slow synaptic plasticity

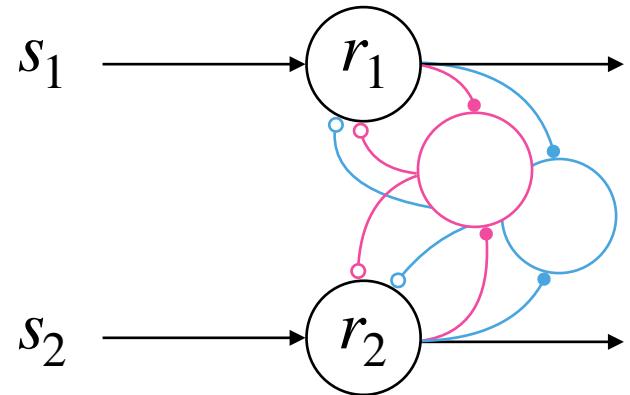


Summary

Circuit with **fast** gain modulation and
slow synaptic plasticity

Complementary computations:

- **gains** adapt within each context to whiten responses

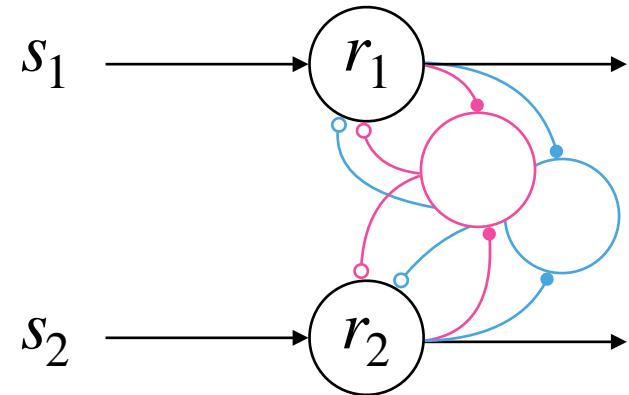


Summary

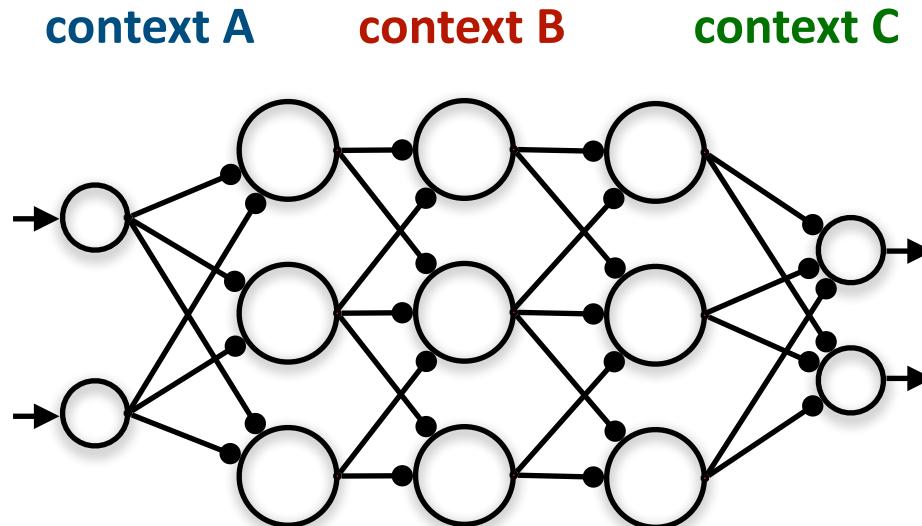
Circuit with **fast** gain modulation and
slow synaptic plasticity

Complementary computations:

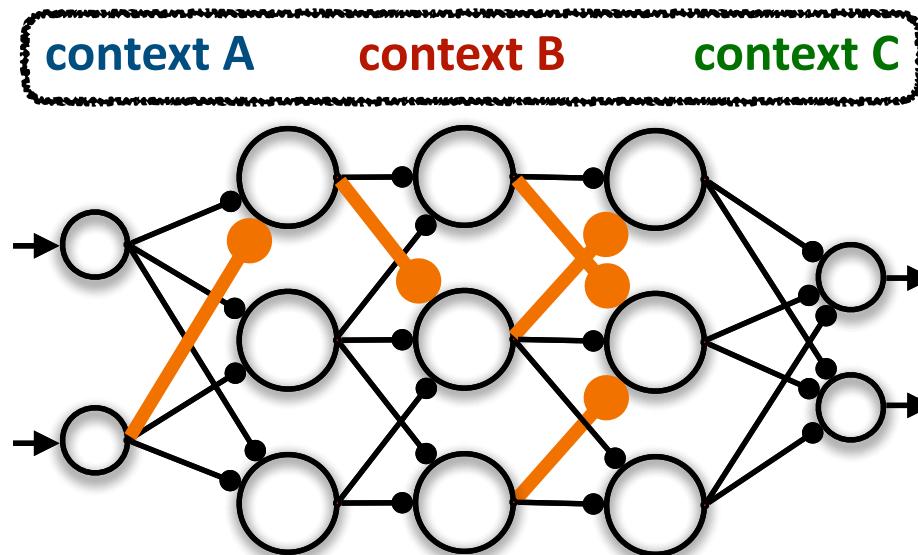
- **gains** adapt within each context to whiten responses
- **synapses** adapt across contexts to learn structural properties of the inputs



Extension: general networks

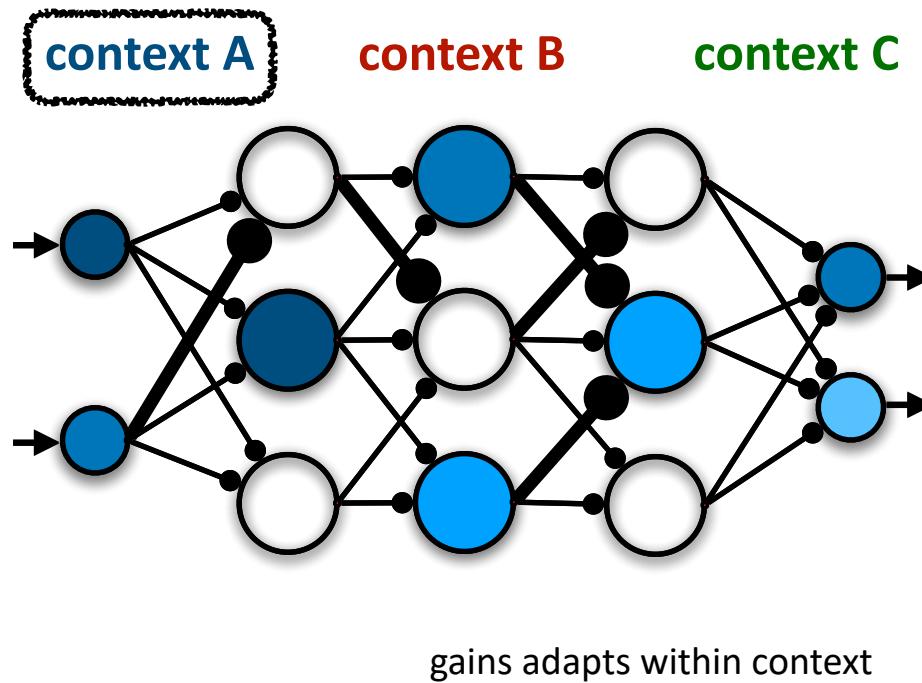


Extension: general networks

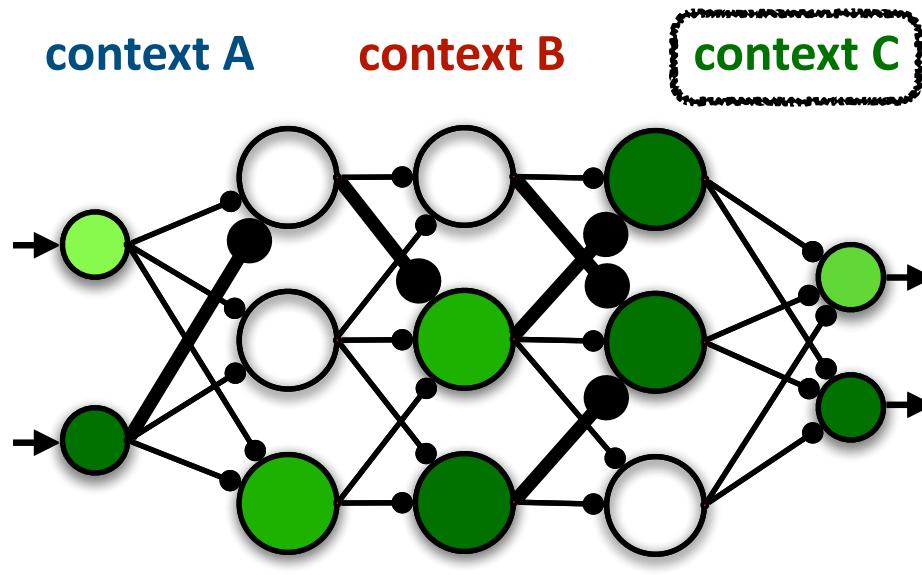


synapses learn
across contexts

Extension: general networks



Extension: general networks



gains adapts within context

Thank you



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Eero Simoncelli
Flatiron Institute / NYU



Duong*, Lipshutz*, Heeger, Chklovskii & Simoncelli, Adaptive whitening in neural populations with gain-modulating interneurons. *ICML*, 2023

Duong, Simoncelli, Chklovskii & Lipshutz, Adaptive whitening with fast gain modulation and slow synaptic plasticity. *arXiv preprint*, 2023