

Armstrong's Axioms and rules for splitting and combining.

$$X \subseteq Y \Rightarrow Y \rightarrow X \quad (\text{Reflexivity})$$

$$X \rightarrow Y \Rightarrow XZ \rightarrow YZ \quad \forall Z \quad (\text{Augmentation})$$

$$X \rightarrow Y \wedge Y \rightarrow Z \Rightarrow X \rightarrow Z \quad (\text{Transitivity})$$

$$X \rightarrow Y \wedge X \rightarrow Z \Rightarrow X \rightarrow YZ \quad (\text{Combining})$$

$$X \rightarrow YZ \Rightarrow X \rightarrow Y \wedge X \rightarrow Z \quad (\text{Splitting})$$

Exercise 3.2.1 from the textbook. Consider a relation with schema $R(A, B, C, D)$ and FD's $AB \rightarrow C, C \rightarrow D, D \rightarrow A$.

1. . What are all the nontrivial FD's that follow from the given FD's? You should restrict yourself to FD's with single attributes on the right hand side.
2. . What are all the keys of R?
3. . What are all the superkeys for R that are not keys ?

Exercise 3.2.2. Repeat the Exercise 3.2.1 for the following schemas and sets of FD's:

1. $S(A, B, C, D)$ with FD's $A \rightarrow B, B \rightarrow C$, and $B \rightarrow D$.
2. $T(A, B, C, D)$ with FD's $AB \rightarrow C, BC \rightarrow D, CD \rightarrow A$, and $AD \rightarrow B$.
3. $U(A, B, C, D)$ with FD's $A \rightarrow B, B \rightarrow C, C \rightarrow D$, and $D \rightarrow A$