

ICS621 Homework 6: Dynamic Programming

Choose one of the following.

Problem 15-6 from CLRS. Planning a company party. Professor Stewart is consulting for the president of a corporation that is planning a company party. The company has a hierarchical structure; that is, the supervisor relation forms a tree rooted at the president. The personnel office has ranked each employee with a conviviality rating, which is a real number. In order to make the party fun for all attendees, the president does not want both an employee and his or her immediate supervisor to attend.

Professor Stewart is given the tree that describes the structure of the corporation, using the left-child, right-sibling representation described in Section 10.4. Each node of the tree holds, in addition to the pointers, the name of an employee and that employee's conviviality ranking. Describe an algorithm to make up a guest list that maximizes the sum of the conviviality ratings of the guests. Analyze the running time of your algorithm.

Problem 15-10 from CLRS. Planning an investment strategy. Your knowledge of algorithms helps you obtain an exciting job with the Acme Computer Company, along with a \$10,000 signing bonus. You decide to invest this money with the goal of maximizing your return at the end of 10 years. You decide to use the Amalgamated Investment Company to manage your investments. Amalgamated Investments requires you to observe the following rules. It offers n different investments, numbered 1 through n . In each year j , investment i provides a return rate r_{ij} . In other words, if you invest d dollars in investment i in year j , then at the end of year j , you have dr_{ij} dollars. The return rates are guaranteed, that is, you are given all the return rates for the next 10 years for each investment. You can make investment decisions only once per year. At the end of each year, you can leave the money made in the previous year in the same investments, or you can shift the money to other investments, by either shifting money between existing investments or moving money to a new investment. If you do not move your money between two consecutive years, you pay a fee of f_1 dollars, whereas if you switch your money, you pay a fee of f_2 dollars, where $f_2 > f_1$.

- a) The problem, as stated, allows you to invest your money in multiple investments in each year. Prove that there exists an optimal investment strategy that, in each year, puts all the money into a single investment. (Recall that an optimal investment strategy maximizes the amount of money after 10 years and is not concerned with any other objectives, such as minimizing risk.)
- b) Prove that the problem of planning your optimal investment strategy exhibits optimal substructure.
- c) Design an algorithm that plans your optimal investment strategy. What is the running time of your algorithm?
- d) Suppose that Amalgamated Investments imposed the additional restriction that, at any point, you can have no more than \$15,000 in any one investment. Show that the problem of maximizing your income at the end of 10 years no longer exhibits optimal substructure.