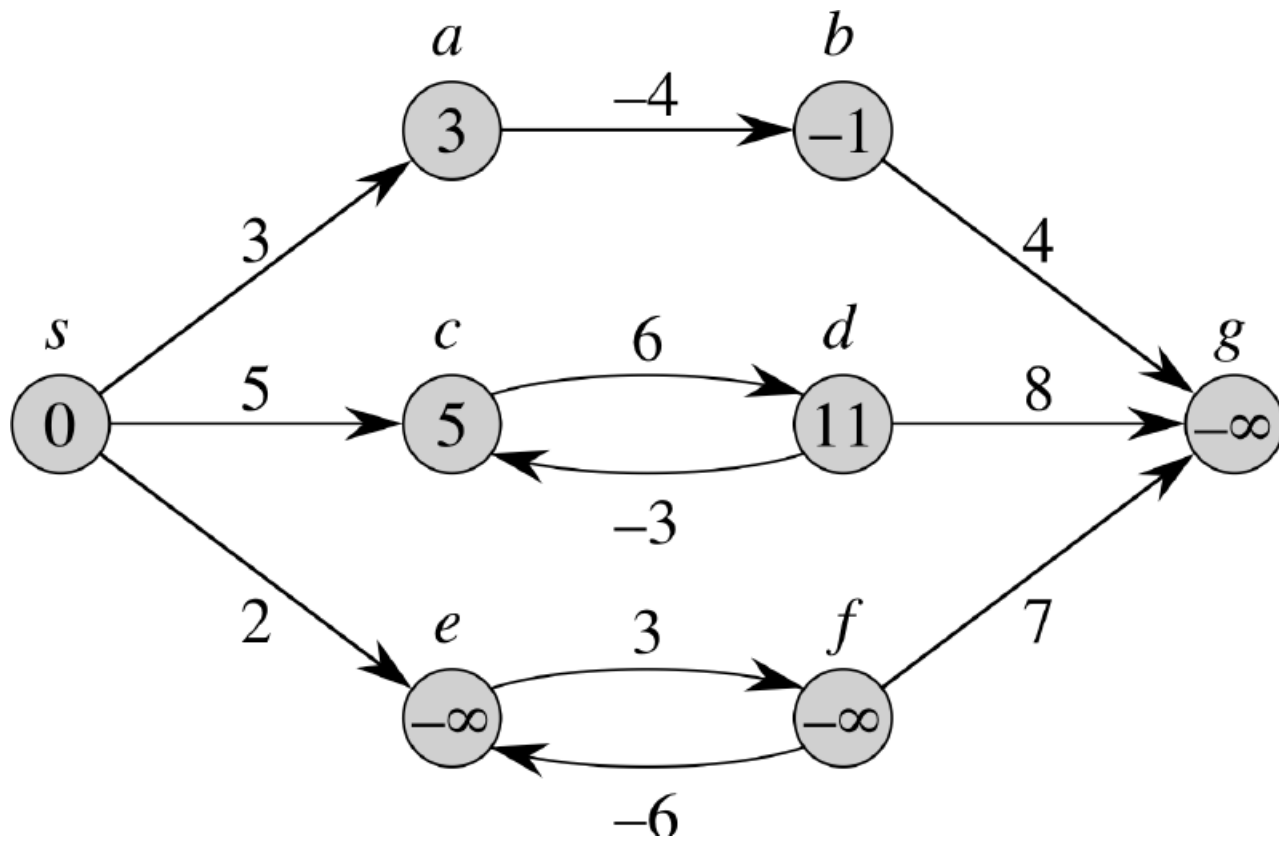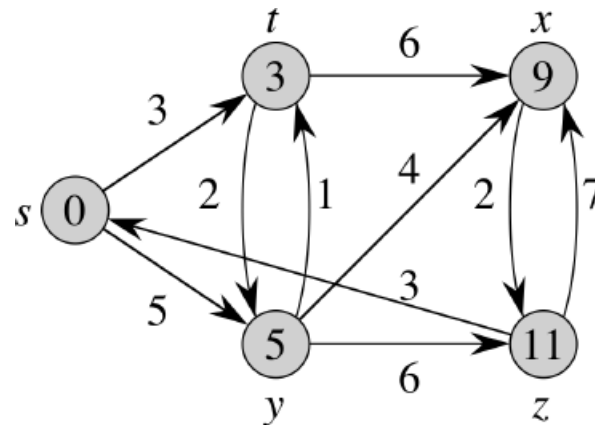# Spring 2012
ICS621 Shortest Paths

Lipyeow Lim
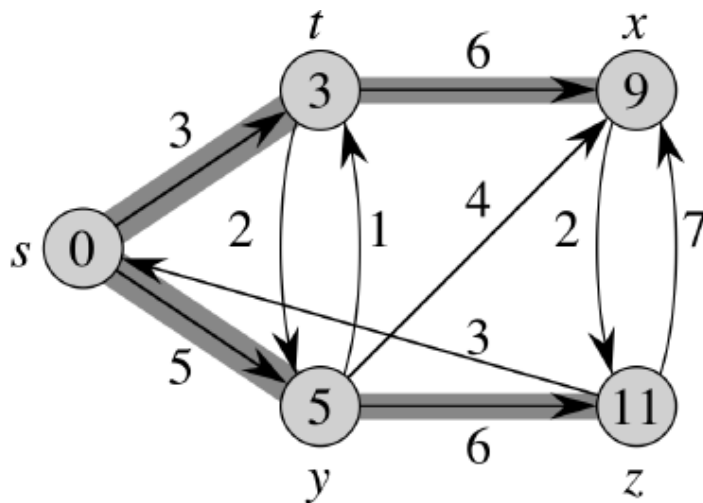
Negative Cycles
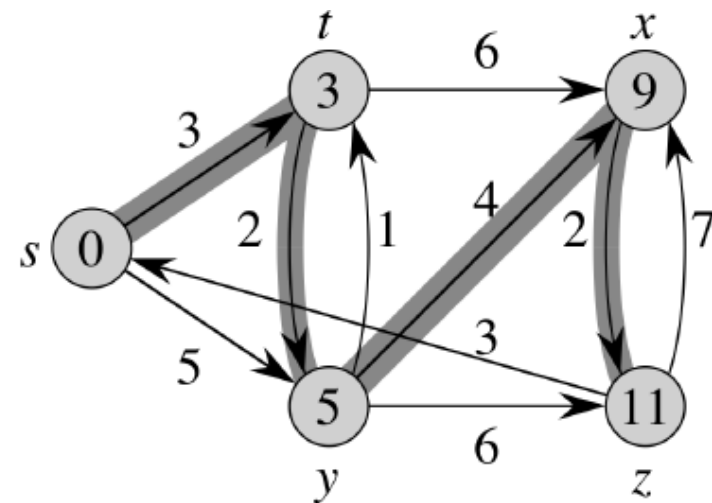
Examples of Shortest Paths

(a)

(b)

(c)

Bellman & Ford

BELLMAN-FORD(G, w, s)

INIT-SINGLE-SOURCE(G, s)
for $i = 1$ to $|G.V| - 1$
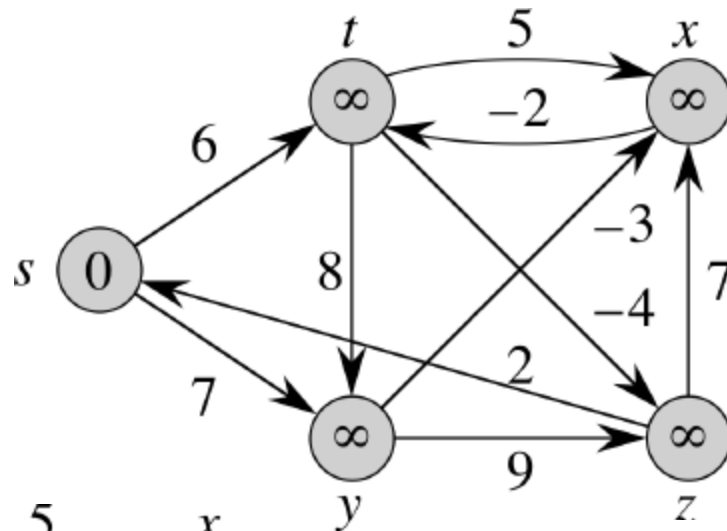   **for** each edge $(u, v) \in G.E$
        RELAX(u, v, w) |E| |V|-1
for each edge $(u, v) \in G.E$
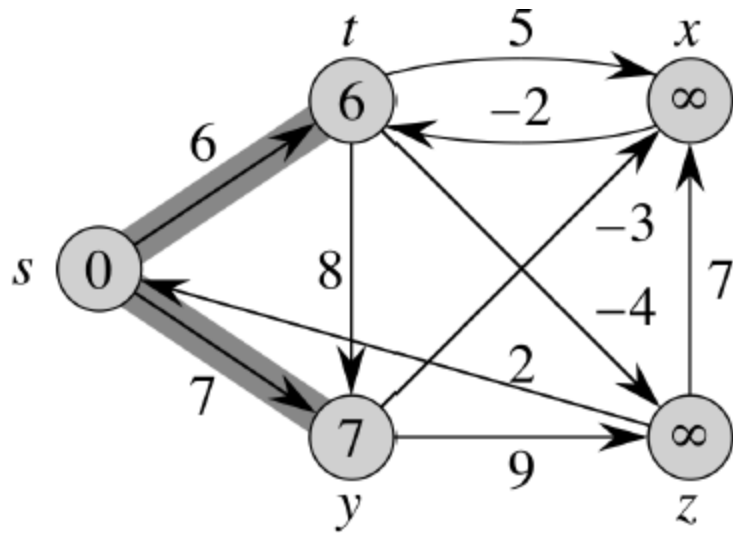   **if** $v.d > u.d + w(u, v)$ |E|
        return FALSE
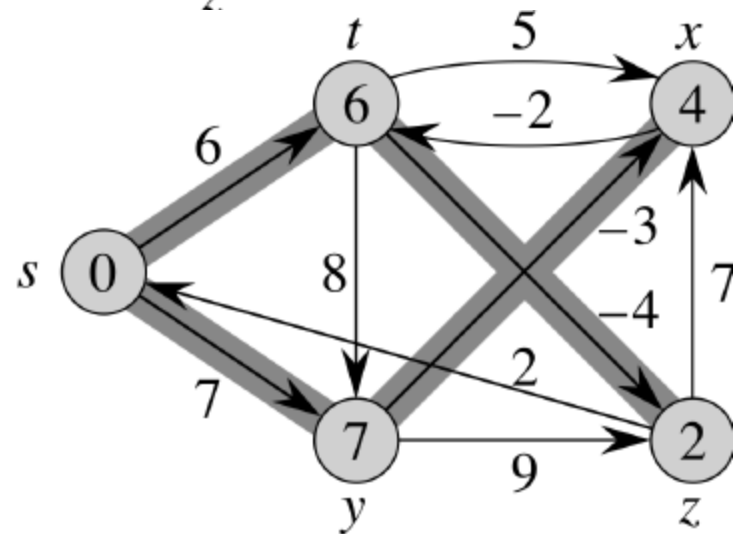**return** TRUE

O(VE) = O(V³)

Bellman-Ford Trace 1/2
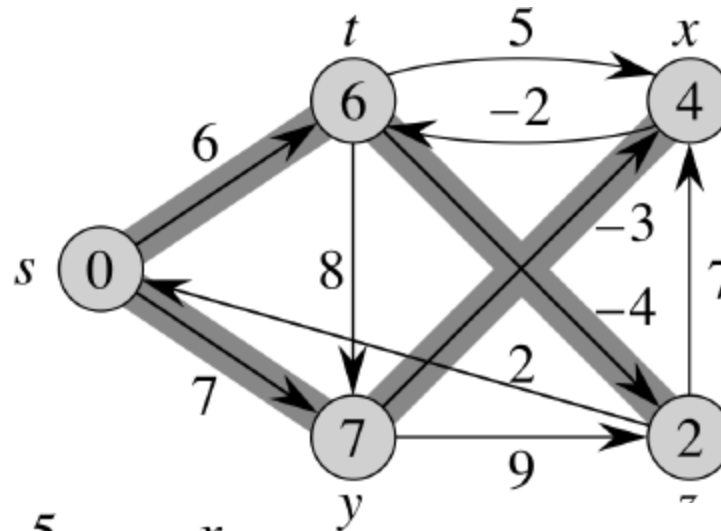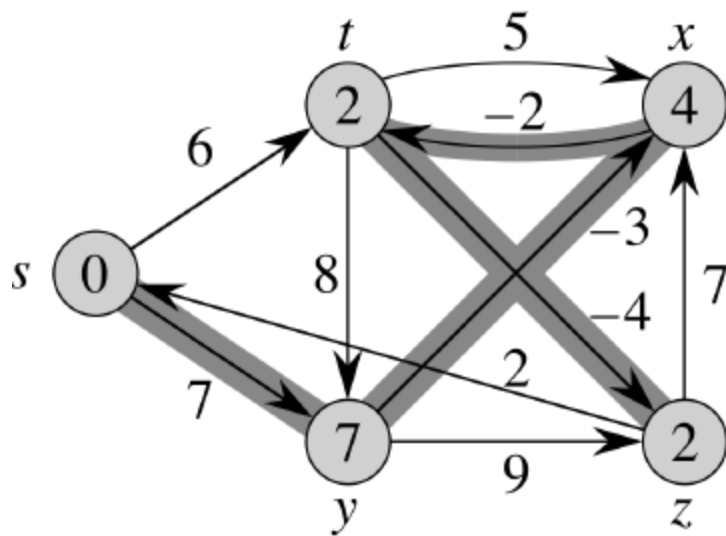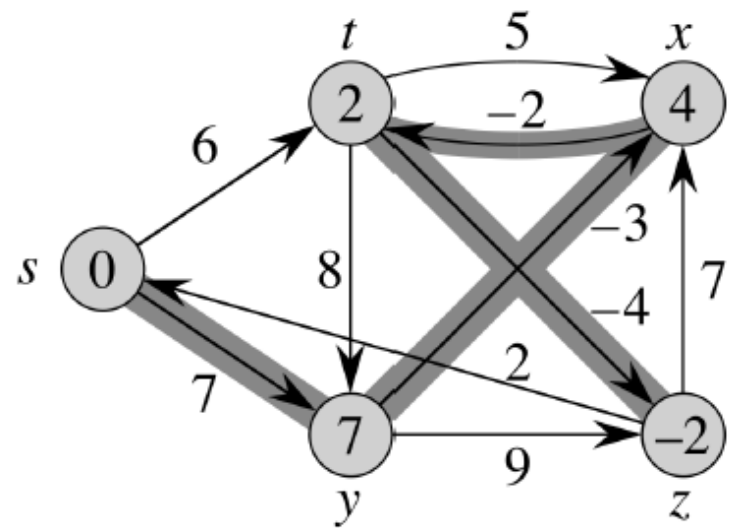

(a)

(b)

(c)

Bellman-Ford Trace 2/2

(c)

(d)

(e)

Shortest Paths for DAGs

DAG-SHORTEST-PATHS(G, w, s)

   topologically sort the vertices   O(V+E)

   INIT-SINGLE-SOURCE(G, s)

   for each vertex u, taken in topologically sorted order

      for each vertex $v \in G.Adj[u]$ |E|

         RELAX(u, v, w)

Dijkstra's Algorithm

DIJKSTRA(G, w, s)

INIT-SINGLE-SOURCE(G, s)

$S = \emptyset$

$Q = G.V$ // i.e., insert all vertices into Q

while $Q \neq \emptyset$

     $u = $ EXTRACT-MIN(Q)

     $S = S \cup \{u\}$

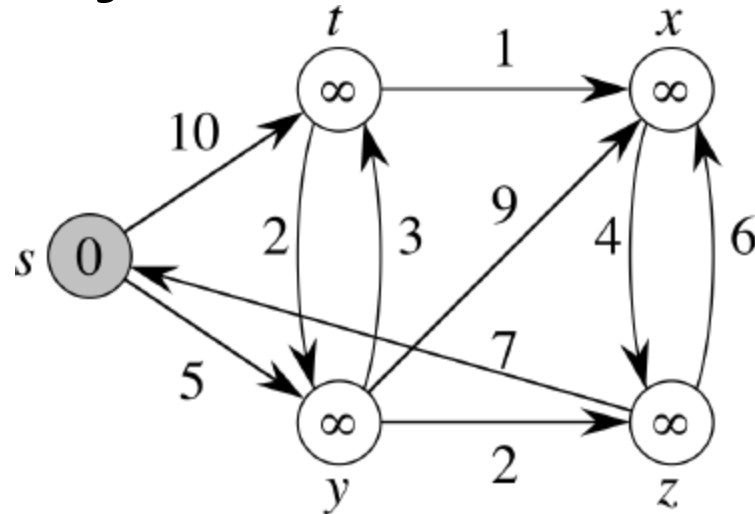   **for** each vertex $v \in G.Adj[u]$

         RELAX(u, v, w)
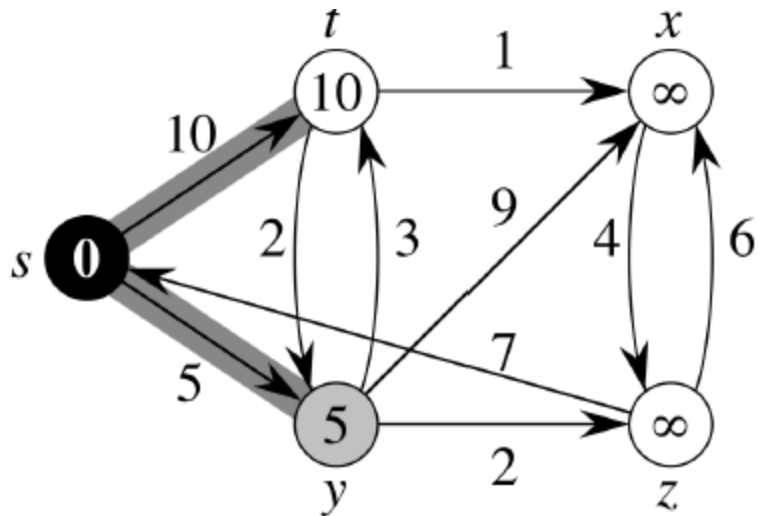
O(V lg V + E) Using Fib Heaps
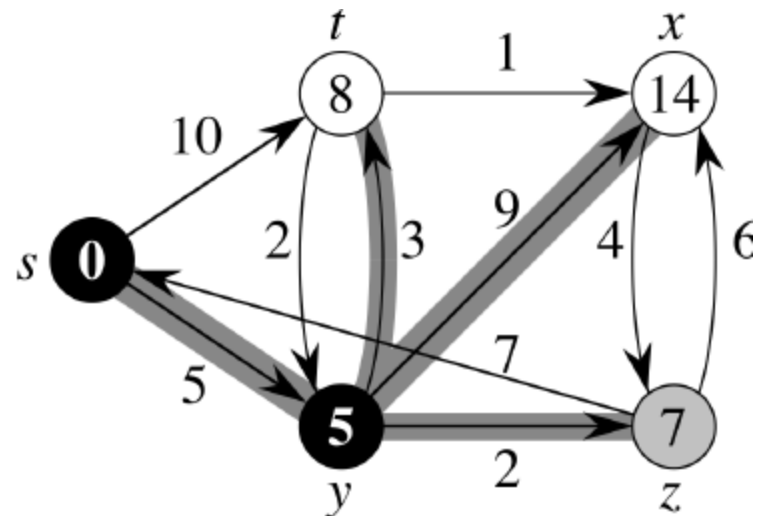
O(lg V)

O(1)
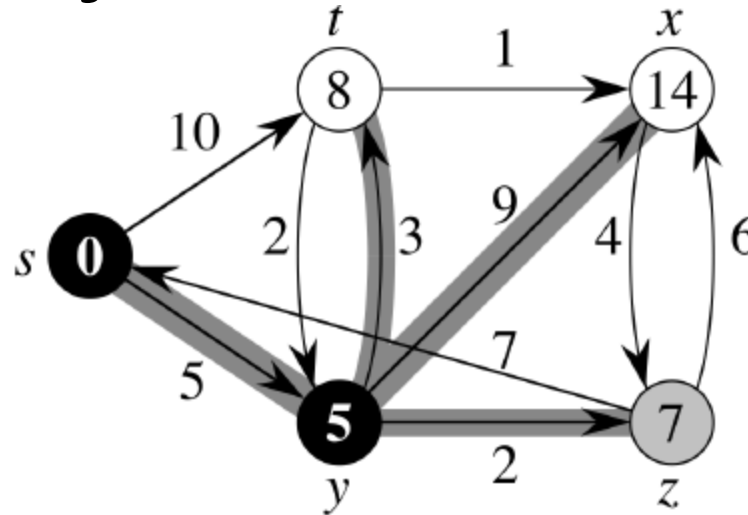
|V|

|E|

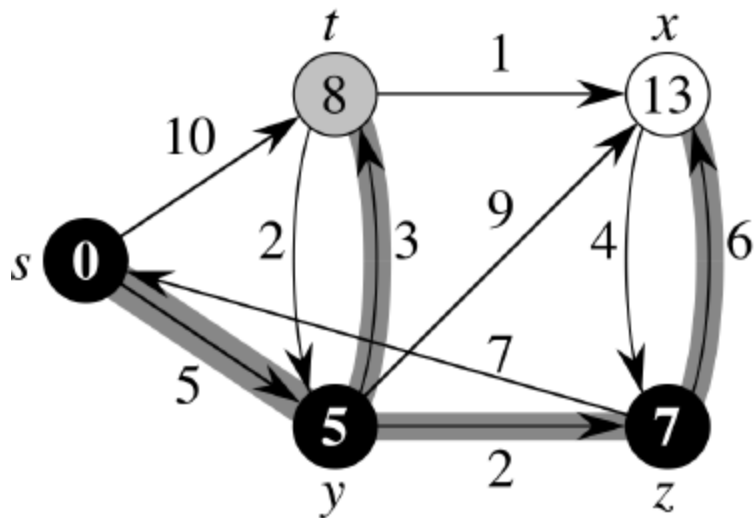Calls **Decrease-Key(Q,v.d,newd)**
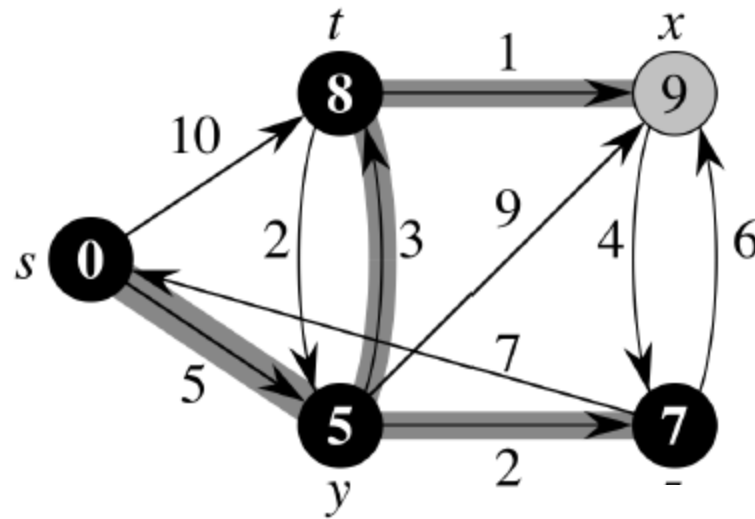
Dijkstra's Trace 1/3

(a)

(b)

(c)

Dijkstra's Trace 2/3

(c)

(d)
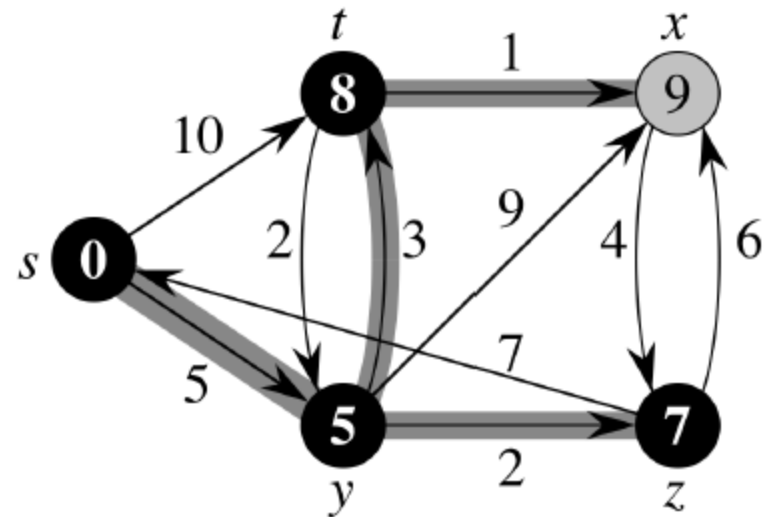
(e)

Dijkstra's Trace 3/3

(e)
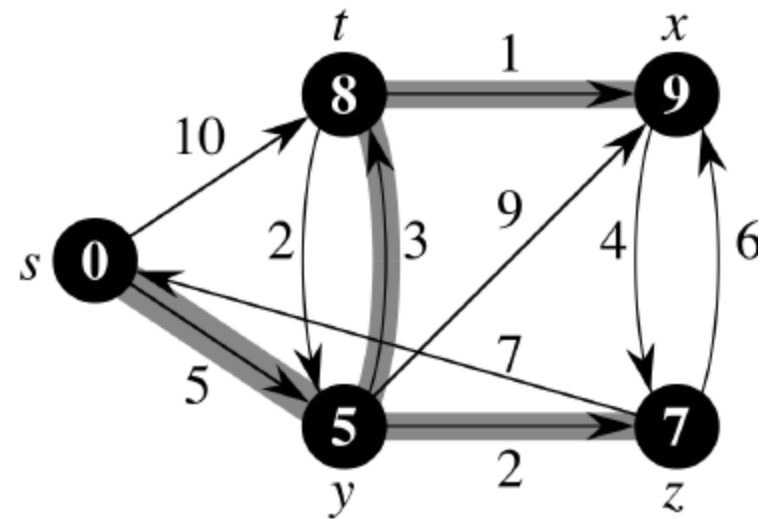
(f)

System of Difference Constraints

$$\overbrace{\begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}}^{n} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \leq \begin{pmatrix} 0 \\ -1 \\ 1 \\ 5 \\ 4 \\ -1 \\ -3 \\ -3 \end{pmatrix}$$

m

$$
\begin{aligned}
x_1 - x_2 &\leq 0, \\
x_1 - x_5 &\leq -1, \\
x_2 - x_5 &\leq 1, \\
x_3 - x_1 &\leq 5, \\
x_4 - x_1 &\leq 4, \\
x_4 - x_3 &\leq -1, \\
x_5 - x_3 &\leq -3, \\
x_5 - x_4 &\leq -3.
\end{aligned}
$$