Tomography and Control of Aperture Errors: Writeup

Wilmer E. Henao

UT Southwestern

Abstract. Here goes a little abstract

Table of Contents

Introduction

Tomotherapy is a technique that combines the accuracy of on board imaging capability with computer tomography (CT) and the power of intensity modulated radiation therapy. Similar to VMAT, the gantry rotates 360° around the patient delivering radiation. As opposed to VMAT however, the couch does not rotate, instead the couch moves straight on an axis creating a characteristic helicoidal delivery pattern endemic to this type of treatment. Also, secondary differences with respect to IMRT include the absence of a flattening filter, a beam hardener and an electron stopper, This causes the radiation beam to be significantly different from other treatment units.

The advantages over VMAT are not only the possibility to run a wider target area due to the axial movement of the couch which allows treatment of larger regions, but also higher precision and the consequent sparing of healthy tissue. It is for this reason that tomotherapy is appropriate for patients who have reached their maximum tolerance dose of traditional radiation.

Tomotherapy is performed via leaf pulsations, it requires several cycles of the gantry around the couch and a longer delivery time.

1 Description of Tomotherapy

As it was already mentioned, tomotherapy involves the gantry movement around a couch, the couch itself moves along an axis as depicted on figure 1. This movement generates a helical pattern. When properly modulated this pattern conforms to the particular case of each patient.

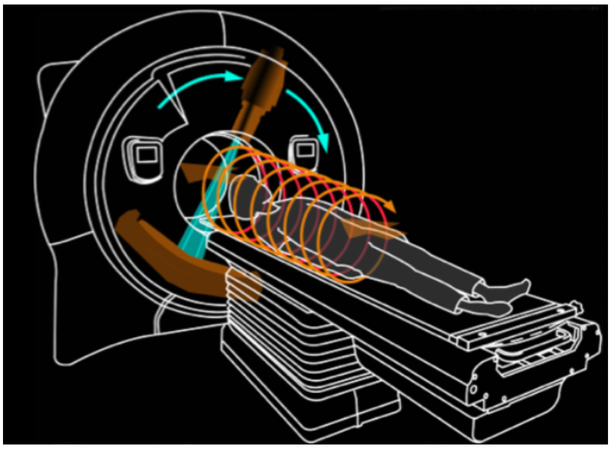

Fig. 1: Tomotherapy Treatment

Among the specifics, the linear accelerator mounted on the rotating gantry uses a treatment field that is modulated with an N-leaf binary multileaf collimator. The source to axis distance at 85 cms as opposed to 100 cms, is shorter than in other types of treatment.

In our case in particular, we focus on the problem of optimal aperture modulation. As it turns out, excessive pulsations in aperture leafs generate errors that are out of control for the practitioner. Worse, this seems to be the standard behavior in this type of treatment if not controlled properly.

1.1 Aperture Modulation

The aperture is usually subdivided into 64 leafs where each leaf can be in one of two states, on or off, the length of each leaf can vary somewhere from 10 mm.

(a) Aperture from Front of Patient (b) Aperture from the right Back

Fig. 2: Aperture Modulation

to 50 mm. and the width is 6 mm. The fan beam continuously rotates around the patient maintaining leaf apertures either open or closed as seen on figures 2a and 2b

1.2 Modelling the problem

The aperture is modelled as an angular distance α degree angle fan beam, each aperture is assumed to remain static during a length equivalent to the time that the aperture spends in the corresponding aperture neighbourhood. There exists a series of K control points from which calculations will be performed. A "fractional" problem description follows:

$$\begin{aligned}
(FP)\ \underset{\substack{z_j\\ k\in\{1,\dots,K\}\\ j\in\mathcal{V}\\ i\in\{1,\dots,N\}}}{\text{minimize}}\ & F(z)\\
\text{subject to } z_j &= \sum_{k=1}^{K}\sum_{i=1}^{N} f_{ik} D_{ijk} \alpha y_k,\ j \in \mathcal{V} \qquad (1)\\
y_k &= Y^*,\ k = 1,\dots,K\\
S_k &= S^*,\ k = 1,\dots,K\\
A_k &\in \mathcal{A},\ k = 1,\dots,K.
\end{aligned}$$

In this version of the problem, the objective function is nothing but the dose-related cost function where the choice variable is the fraction of time that each leaf i remains open in each of the control points k (f_{ik}). In the first constraint, z_j is expressed as the sum of doses delivered to voxel j from all control points K. This is calculated for all voxels in $\mathcal{V}$. The fluence rate (in units of MU deg^{-1}) and the gantry speed (in units of deg s^{-1}) are kept constant during the whole trip at values Y^* and S^* respectively.

Each control point works as seen on figure 3. At each control point some discovery mechanism chooses an aperture configuration from the set of possible apertures $A_k \in \mathcal{A}$ and this combination is added to the problem formulation.

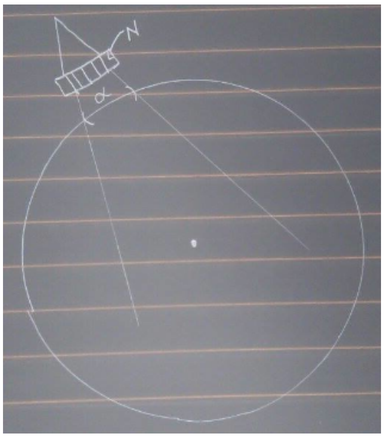

Fig. 3: Modeling of the aperture

The problem with the formulation as it stands here is that leafs may be constantly opening and closing at each control point. If practitioners do not pay attention the leaves will end up opening and closing the aperture at every control point, since there are 51 control points, this means opening and closing 51 times per cicle. We would like to turn each leaf on and off as little as possible.

1.3 Reducing Pulsations

Considering a set ϕ of N leafs[1], we want to account exactly at which points in time I keep the apertures open. This can be seen in figure 4 in which time goes in the horizontal axis and ϕ represents the collection of beams. Notice that each "time box" represents time spent at a particular control point.

2 Modeling the Problem

Assuming that you only need to turn the leaf on and off at most once per control point. The times at which I open and close a leaf can be represented by a new variable t_{ik}, this is the proportional time at control point k at which I change the leaf to be on or off.[2] Since I only flip the switch once inside the control point, I can establish the state of the leaf at each endpoint using the new variable δ_k, which can take values 1 or 0, depending if the leaf is open or closed. With these new variables the formulation of the problem looks like this:

[1] the number of leafs $N = 64$ for most machines

[2] According to problem formulation below, it is possible that at time t_{ik} I actually do nothing. Regardless, this is the time in the interval when I actually make a decision.

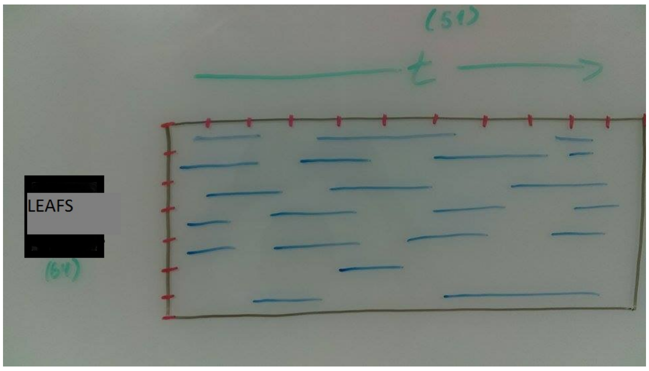

Fig. 4: evolution of leaf openness across time

$$
\begin{aligned}
(GP)\ \underset{\substack{z_j\\ k\in\{1,\ldots,K\}\\ j\in\mathcal{V}\\ i\in\{1,\ldots,N\}}}{\text{minimize}}\ & F(z) + C\sum_{k=1}^{K}\sum_{i=1}^{N}\left|\delta_{i,k}-\delta_{i,k-1}\right| \\
\text{subject to } & z_j = \sum_{k=1}^{K}\sum_{i=1}^{N}\left(t_{ik}\delta_{i,k-1} + (1-t_{ik})\,\delta_{i,k}\right) D_{ijk}\alpha y_k,\ j\in\mathcal{V} \\
& y_k = Y^*,\ k=1,\ldots,K \\
& \delta_{i,k}\in\{0,1\},\ k=0,\ldots,K \\
& t_{ik}\in[0,1]
\end{aligned}
\tag{2}
$$

Notice that the new objective function consists of the usual function $F(z)$ plus a penalization term that inflicts a handicap proportional to the number of times that the switch goes on and off. This penalty can be controlled using the value corresponding to a constant C.

As explained above, the switch can once be turned on, or off, or not changed at all at any time during the interval $[\bar{t}_{k-1}, \bar{t}_k]$. The state of the leaf is defined at the endpoints by the value of the variable δ_k. The variable t_{ik} enters the problem as a variable in the proportional interval $[0,1]$. But this represents a point in the actual time interval $[\bar{t}_{k-1}, \bar{t}_k]$.

At this point this is a complete representation of the problem. However, during the implementation phase two issues need to be corrected. First, the product of a binary and a continuous variable need to be changed to a binary only variable, and second, the absolute value needs to be removed from the objective function.

2.1 Creation of New Binary Variables

First I will introduce the new binary values by the inclusion of new variables ξ and ζ that will represent the amount of time spent at endpoints of the intervals $[\bar{t}_{k-1}, \bar{t}_k]$ in what I call the binary problem (BP):

$$
\begin{aligned}
(BP)\ \underset{\substack{z_j\\ k\in\{1,\dots,K\}\\ j\in\mathcal{V}\\ i\in\{1,\dots,N\}}}{\text{minimize}}\ & F(z)+C\sum_{k=1}^{K}\sum_{i=1}^{N}|\delta_{i,k}-\delta_{i,k-1}| \\
\text{subject to } & z_j=\sum_{k=1}^{K}\sum_{i=1}^{N}(\xi_{ik}+\zeta_{ik})\,D_{ijk}\alpha y_k,\ j\in\mathcal{V} \\
& y_k=Y^*,\ k=1,\dots,K \\
& \delta_{i,k}\in\{0,1\},\ i=1,\dots,N,\ k=0,\dots,K \\
& t_{ik}\in[0,1],\ i=1,\dots,N,\ k=1,\dots,K \\
& \xi_{ik}\le M\delta_{i,k-1},\ i=1,\dots,N,\ k=1,\dots,K \\
& \xi_{ik}\le t_{ik},\ i=1,\dots,N,\ k=1,\dots,K \\
& \xi_{ik}\ge t_{ik}-(1-\delta_{i,k-1})M,\ i=1,\dots,N,\ k=1,\dots,K \\
& \xi_{ik}\ge 0,\ i=1,\dots,N,\ k=1,\dots,K \\
& \zeta_{ik}\le M\delta_{i,k},\ i=1,\dots,N,\ k=1,\dots,K \\
& \zeta_{ik}\le t_{ik},\ i=1,\dots,N,\ k=1,\dots,K \\
& \zeta_{ik}\ge (1-t_{ik})-t_{ik}M,\ i=1,\dots,N,\ k=1,\dots,K \\
& \zeta_{ik}\ge 0,\ i=1,\dots,N,\ k=1,\dots,K
\end{aligned}
\tag{3}
$$

where $M > t_{ik}, 1-t_{ik}$ is a big-M constant that applies to both. A value of $M = 1+\epsilon$ will do. Now in order to understand variable ξ, Whenever $\delta_{i,k-1} = 0$, you would have from restrictions $\xi_{ik} \le M\delta_{i,k-1}$ and $\xi_{ik} \ge 0$ that $\xi_{ik} = 0$. On the other hand, whenever $\delta_{i,k-1} = 1$ you would have that $\xi_{ik} \le t_{ik}$ and that $\xi_{ik} \ge t_{ik} - (1-\delta_{i,k-1})M = t_{ik}$; which means $\xi_{ik} = t_{ik}$. Which is exactly what we want.

The same applies to the constraints relevant to $zeta$.

2.2 Removing absolute Value in Objective Function

Now, in order to create an implementable problem (IP). I have to remove the absolute value from the objective and a final transformation is due:

$$
\begin{aligned}
(IP)\ \underset{\substack{k\in\{1,\ldots,K\}\\ j\in\mathcal{V}\\ i\in\{1,\ldots,N\}}}{\text{minimize}}\ & F(z)+C\sum_{k=1}^{K}\sum_{i=1}^{N}(z_{ik}^{+}+z_{ik}^{-})\\
\text{subject to}\ & z_j=\sum_{k=1}^{K}\sum_{i=1}^{N}(\xi_{ik}+\zeta_{ik})\,D_{ijk}\alpha y_k,\ j\in\mathcal{V}\\
& y_k=Y^*,\ k=1,\ldots,K\\
& \delta_{i,k}\in\{0,1\},\ i=1,\ldots,N,\ k=0,\ldots,K\\
& t_{ik}\in[0,1],\ i=1,\ldots,N,\ k=1,\ldots,K\\
& \xi_{ik}\le M\delta_{i,k-1},\ i=1,\ldots,N,\ k=1,\ldots,K\\
& \xi_{ik}\le t_{ik},\ i=1,\ldots,N,\ k=1,\ldots,K\\
& \xi_{ik}\ge t_{ik}-(1-\delta_{i,k-1})M,\ i=1,\ldots,N,\ k=1,\ldots,K\\
& \xi_{ik}\ge 0,\ i=1,\ldots,N,\ k=1,\ldots,K\\
& \zeta_{ik}\le M\delta_{i,k},\ i=1,\ldots,N,\ k=1,\ldots,K\\
& \zeta_{ik}\le t_{ik},\ i=1,\ldots,N,\ k=1,\ldots,K\\
& \zeta_{ik}\ge (1-t_{ik})-t_{ik}M,\ i=1,\ldots,N,\ k=1,\ldots,K\\
& \zeta_{ik}\ge 0,\ i=1,\ldots,N,\ k=1,\ldots,K\\
& \delta_{i,k}-\delta_{i,k-1}=z_{ik}^{+}-z_{ik}^{-}\\
& z_{ik}^{+},z_{ik}^{-}\ge 0
\end{aligned}
\tag{4}
$$

Notice that the trick here is to change the variable from $\delta_{i,k}-\delta_{i,k-1}$ to $z_{i,k}^{+}-z_{i,k}^{-}$, the only reason to introduce this equation is that whenever both $\delta_{i,k}=1$ and $\delta_{i,k-1}=1$, I would rather have a variable that makes both of them simultaneously 0 instead. This is equivalent because it means that there was no change in the switch during the interval, and it would make it clear during the optimization that I would prefer to have as few flips of the switch as possible.

- If $\delta_{i,k}-\delta_{i,k-1}=1\Rightarrow z_{ik}^{+}=1,\ z_{ik}^{-}=0$
- If $\delta_{i,k}-\delta_{i,k-1}=-1\Rightarrow z_{ik}^{+}=0,\ z_{ik}^{-}=-1$
- If $\delta_{i,k}-\delta_{i,k-1}=0\Rightarrow z_{ik}^{+}=0,\ z_{ik}^{-}=0$. And this could be either because $\delta_{i,k}=0,\delta_{i,k-1}=0$ or because $\delta_{i,k}=1,\delta_{i,k-1}=1$

Also, in the objective function the value should be added, not subtracted; $z_{ik}^{+}+z_{ik}^{-}$.

3 multi-objective modeling of the problem:

The solution presented on model 4 states an exchange between two potentially conflicting goals of an optimal radiation treatment and a smooth opening and closing of aperture leafs. In this original solution a parameter C takes care of how much priority is given to each of the goals. This approach however falls short of explaining the importance and trade-offs between the two goals. For multiobjective optimization the objective in 4 changes to $F(x)=min(f_1(x),f_2(x))$:

$$
\begin{aligned}
(MOP) \quad & \underset{\substack{k\in\{1,\ldots,K\} \\ j\in\mathcal{V} \\ i\in\{1,\ldots,N\}}}{\text{minimize}} \left(F(z), \sum_{k=1}^{K}\sum_{i=1}^{N}(z_{ik}^{+}+z_{ik}^{-}) \right) \\
& \text{subject to } z_j = \sum_{k=1}^{K}\sum_{i=1}^{N}\left(\xi_{ik}+\zeta_{ik}\right) D_{ijk}\alpha y_k,\ j\in\mathcal{V} \\
& \vdots
\end{aligned}
\tag{5}
$$

3.1 Utility Function Approach:

Assuming that there is a utility function $u : Y \mapsto \mathbb{R}$ s.t. $\forall y_1, y_2 \in Y,\ u(y_1) > u(y_2)$ if y_1 is preferred to y_2, that establishes preferences between two solutions. It is possible to find the optimal combination that maximizes the utility.

The utility function in this case should be...

3.2 Goal Programming:

The approach of goal programming involves trying to satisfy certain conditions, without making them strict restrictions.

Bibliography

Jeraj, R., Mackie, T. R., Balog, J., Olivera, G., Pearson, D., Kapatoes, J., Ruchala, K. et Reckwerdt, P. (2004). Radiation characteristics of helical tomotherapy. *Medical Physics*, 31(2):396–404.