$$\int \frac{1}{x^2-1}\,dx$$

1. $\ln|x^2-1|+C_1$

2. $$\int \frac{1}{x^2-1}dx = \int \frac{1}{(x+1)(x-1)}dx = \int\left(\frac{a}{x+1}+\frac{b}{x-1}\right)dx$$

$$= \int \left[\frac{a(x-1)+b(x+1)}{(x+1)(x-1)}\right]dx.$$

$\because\ 1 = a(x-1)+b(x+1) \rightarrow\ a=-\frac{1}{2}\quad b=\frac{1}{2}$.

$$\therefore \int\left(\frac{a}{x+1}+\frac{b}{x-1}\right)dx = -\frac{1}{2}\int\frac{1}{x+1}dx + \frac{1}{2}\int\frac{1}{x-1}dx$$

$$= -\frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C_2$$