

$$\boxed{\int \frac{1}{x^2-1} dx}$$

$$1. \underline{\ln|x^2-1| + C_1}$$

$$\begin{aligned} 2. \int \frac{1}{x^2-1} dx &= \int \frac{1}{(x+1)(x-1)} dx = \int \left( \frac{a}{x+1} + \frac{b}{x-1} \right) dx \\ &= \int \left[ \frac{a(x-1) + b(x+1)}{(x+1)(x-1)} \right] dx. \end{aligned}$$

$$\therefore 1 = a(x-1) + b(x+1) \rightarrow a = -\frac{1}{2} \quad b = \frac{1}{2}$$

$$\begin{aligned} \therefore \int \left( \frac{a}{x+1} + \frac{b}{x-1} \right) dx &= -\frac{1}{2} \int \frac{1}{x+1} dx + \frac{1}{2} \int \frac{1}{x-1} dx \\ &= \underline{\underline{-\frac{1}{2} \ln|x-1| - \frac{1}{2} \ln|x+1| + C_2}} \end{aligned}$$