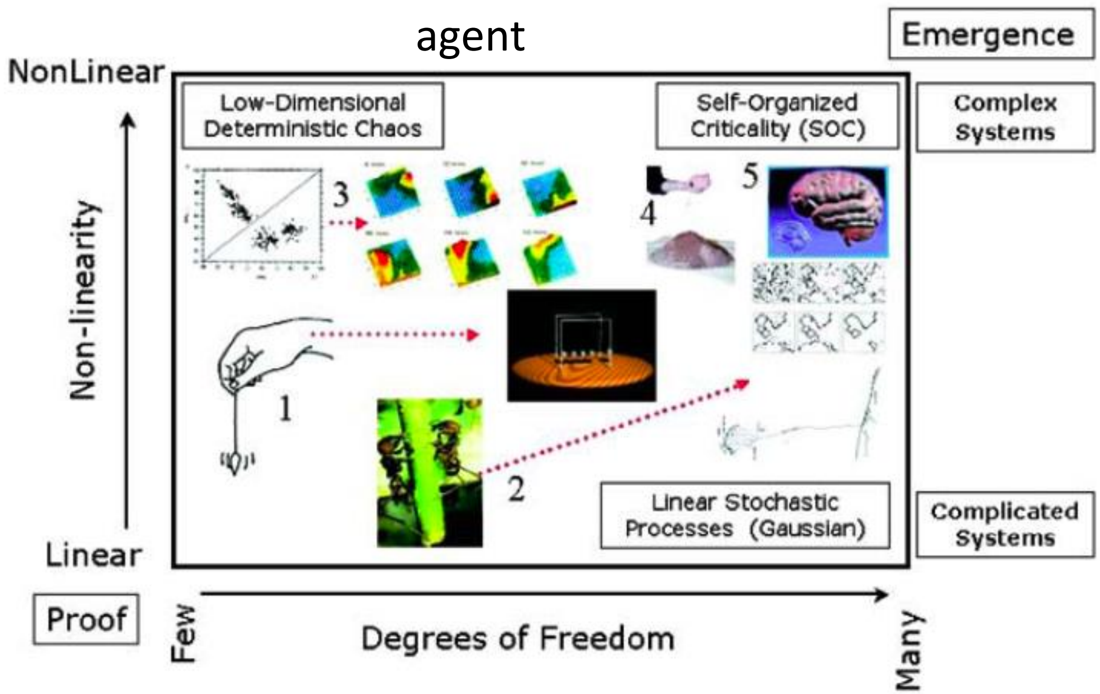
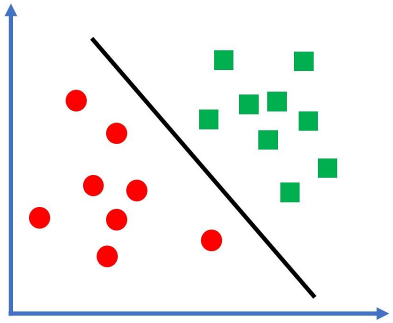
线性模型

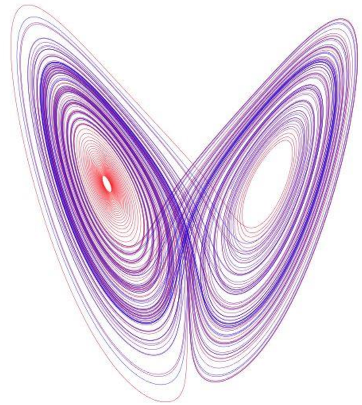
CSL

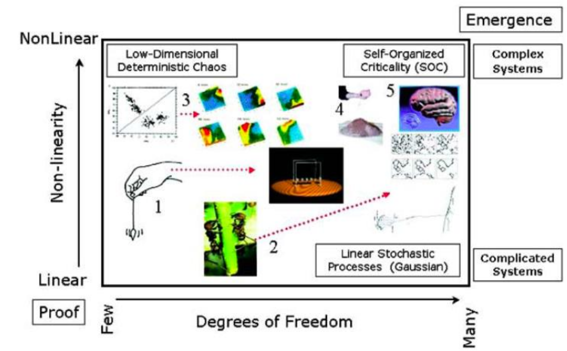
复杂系统

I think the next century will be the century of complexity. — Stephen Hawking （2000）









线性系统 → 非线性系统 → 复杂系统

1+1=2

1+1＞2

Linear model

机器学习：数据驱动的方法论 （工具）

程序员：能通过经验（数据）自动改进算法的计算机程序

应用数学：逼近论，函数拟合 （神经网络, ...）

统计学：不需要解释的简易统计学应用 （线性回归， ...）

CSL

数据挖掘：数据->信息->知识->智慧 （目标）

Linear model

经验主义：强调知识源于经验、强调归纳推理

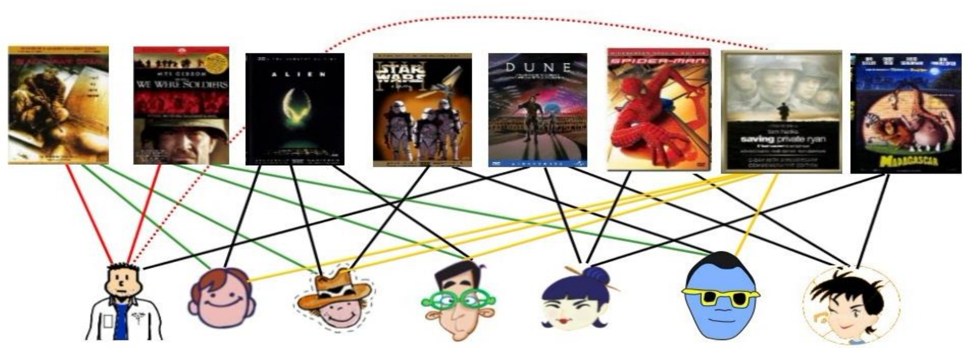
两类任务

- 机器学习人的认知能力 （理性）： speech, NLP，CV

CSL

- 机器学习/理解人的行为 （有限理性、非理性）

model



LEARNING = REPRESENTATION + EVALUATION +OPTIMIZATION

机器学习 = 模型 + 评估 + 优化

Representation	Evaluation	Optimization
Instances	Accuracy/Error rate	Combinatorial optimization
K-nearest neighbor	Precision and recall	Greedy search
Support vector machines	Squared error	Beam search
Hyperplanes	Likelihood	Branch-and-bound
Naive Bayes	Posterior probability	Continuous optimization
Logistic regression	Information gain	Unconstrained
Decision trees	K-L divergence	Gradient descent
Sets of rules	Cost/Utility	Conjugate gradient
Propositional rules	Margin	Quasi-Newton methods
Logic programs		Constrained
Neural networks		Linear programming
Graphical models		Quadratic programming
Bayesian networks		
Conditional random fields		

Domingos P M. *A few useful things to know about machine learning*[J]. Communications of The ACM, 2012, 55(10): 78-87.

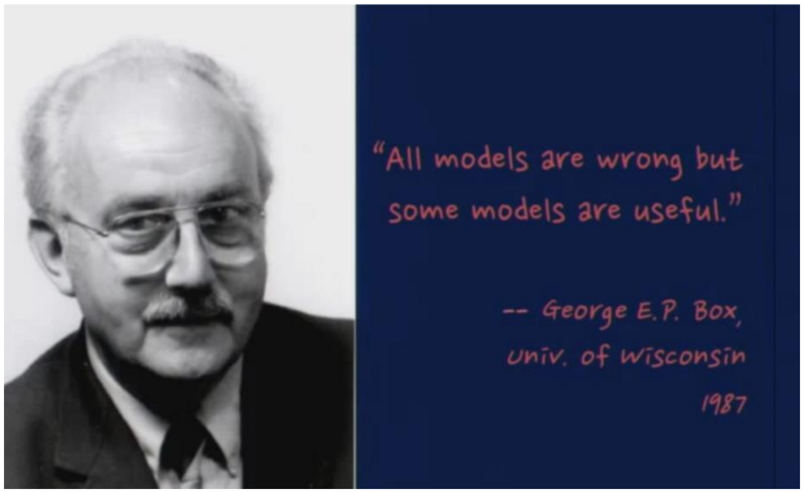
机器学习 = **模型** + 评估 + 优化

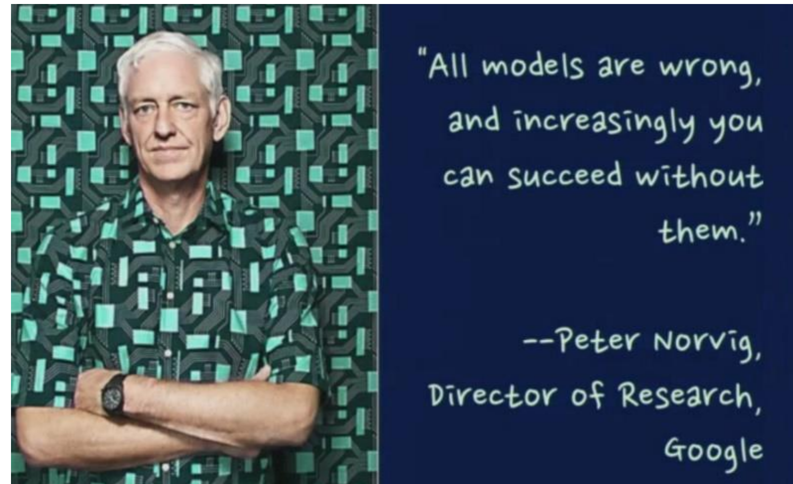
假设空间

$$\mathcal{F} = \{f \mid Y = f(X)\} \qquad \mathcal{F} = \{f \mid Y = f_\theta(X), \theta \in \mathbf{R}^n\}$$

$$\mathcal{F} = \{P \mid P(Y \mid X)\} \qquad \mathcal{F} = \{P \mid P_\theta(Y \mid X), \theta \in \mathbf{R}^n\}$$

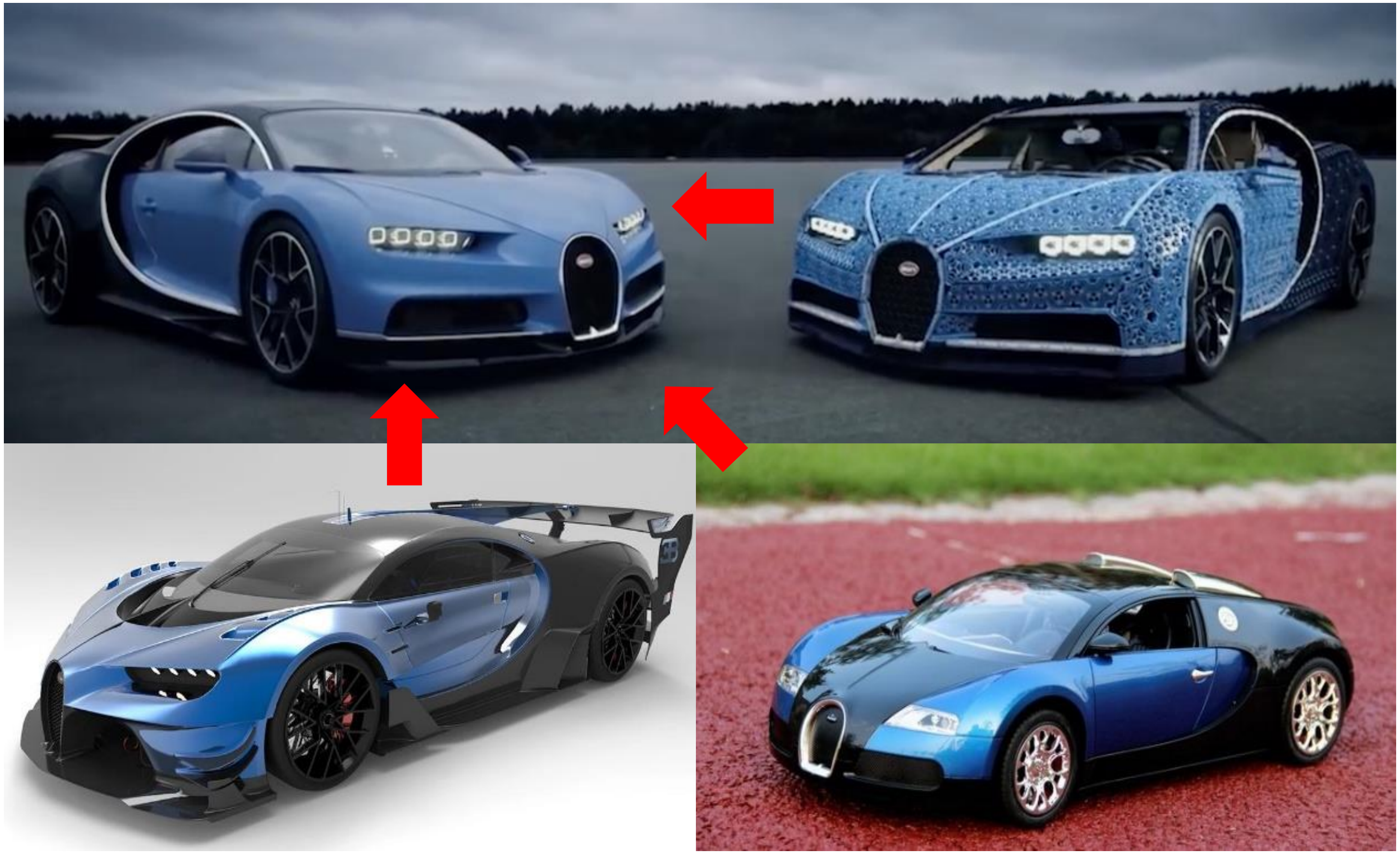
All models are wrong ?





机器学习 = 模型 + 评估 + 优化

度量模型与真实世界的相似性



机器学习 = 模型 + **评估** + 优化

度量模型与真实世界的相似性

Y_i 损失函数 $L(y_i, f(x_i))$ $f(x_i)$

Linear model

经验风险（Empirical risk） (in-sample error)

$$R_{emp}(f) = 1/N \sum_{i=1}^{N} L(y_i, f(x_i))$$

泛化误差（General error） (out-of-sample error)

$$R_{gen}(f) = E_{p(x,y)} L(y_i, f(x_i))$$

机器学习 = **模型** + **评估** + 优化

度量模型与真实世界的相似性

$$Y_i \longrightarrow L(y_i, f(x_i)) \longleftarrow f(x_i)$$

0/1 损失:

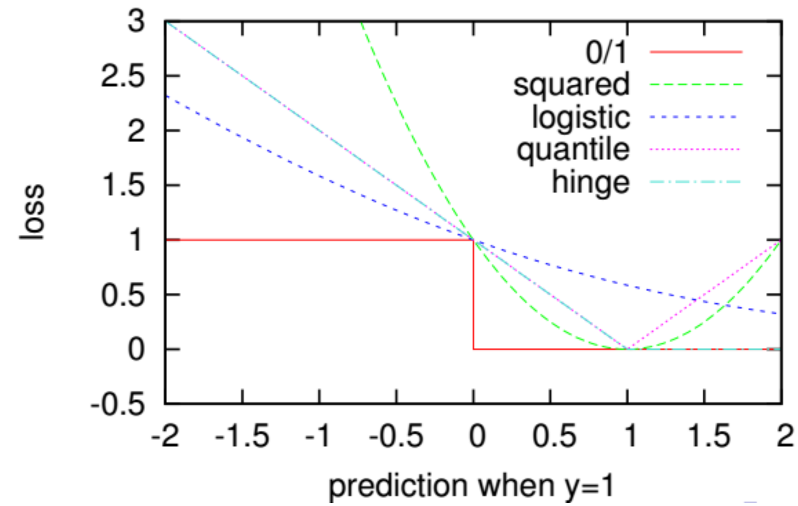
$$L\big(Y, f(x)\big) = \begin{cases} 1, Y \neq f(X) \\ 0, Y = f(X) \end{cases}$$

平方损失（L2, MSE）：

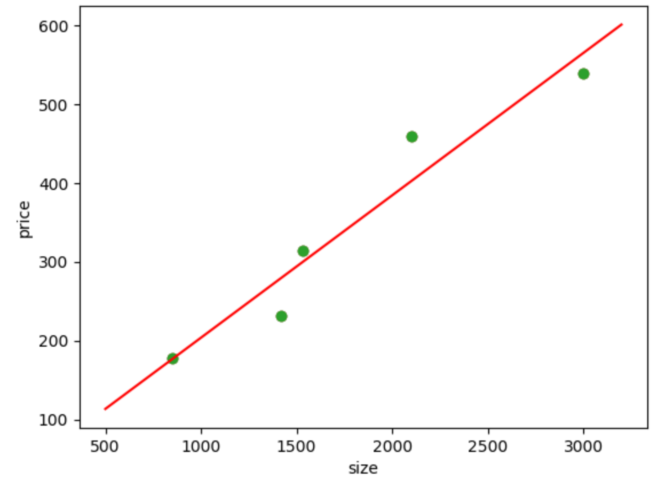
$$L\big(Y, f(x)\big) = \big(Y - f(X)\big)^2$$

对数损失（交叉熵损失）：

$$L\big(Y, f(x)\big) = -\log P(Y|X)$$



机器学习 = 模型 + 评估 + 优化



经验风险（Empirical risk）（in-sample error)

min $$R_{emp}(f) = 1/N \sum_{i=1}^{N} L(y_i, f(x_i))$$

X = (2104, 1416, 1534, 852, 3000), Y = (460, 232, 315,178, 540)

$$f(x) = ax + b$$

$$J(a, b) = \frac{1}{N} \sum_i (ax_i + b - y_i)^2$$

Linear

$$X = \begin{pmatrix} 2104,1 \\ 1416,1 \\ 1534,1 \\ 852,1 \\ 3000,1 \end{pmatrix}, \quad w = \begin{pmatrix} a \\ b \end{pmatrix}, \quad Y = \begin{pmatrix} 460 \\ 232 \\ 315 \\ 178 \\ 540 \end{pmatrix}$$

normal equation

$$\min J(\mathbf{w}) = \frac{1}{2} \sum_i \left(\mathbf{Y}_i - \mathbf{w}^T X_i\right)^2$$

$$X = \begin{pmatrix} X_1^T \\ X_2^T \\ \dots \\ X_N \end{pmatrix} \qquad J = \frac{1}{2} (Y - Xw)^T (Y - Xw)$$

$$X^T(Y - Xw) = 0$$

$$X^T X w = X^T Y$$

$$\widehat{w}_{OLS} = \left(X^T X\right)^{-1} X^T Y$$

array([[0.18078802],
[22.98037744]])

机器学习 = **模型** + **评估** + **优化**

经验风险（Empirical risk） (in-sample error)

min $$R_{emp}(f) = 1/N\sum_{i=1}^{N} L(y_i, f(x_i)) \quad f?$$

CSL

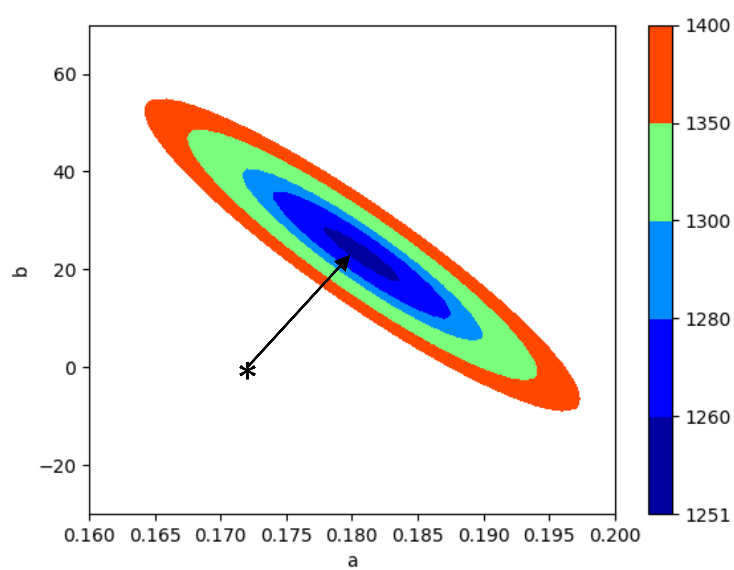
X = (2104, 1416, 1534, 852, 3000), Y = (460, 232, 315,178, 540)

模型： $f(x) = ax + b$

评估： $J(a,b) = \dfrac{1}{N}\sum_i (ax_i + b - y_i)^2$

Linear

优化： $\min J(a,b)$



机器学习 = 模型 + 评估 + 优化

Taylor展开

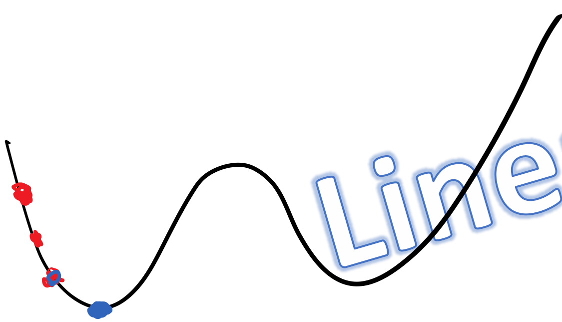
$$f(x+\Delta x)=f(x)+f'(x)\Delta x+1/2\,f''(x)\Delta x^2+o(\Delta x^2)$$

梯度下降法 (Gradient decent)

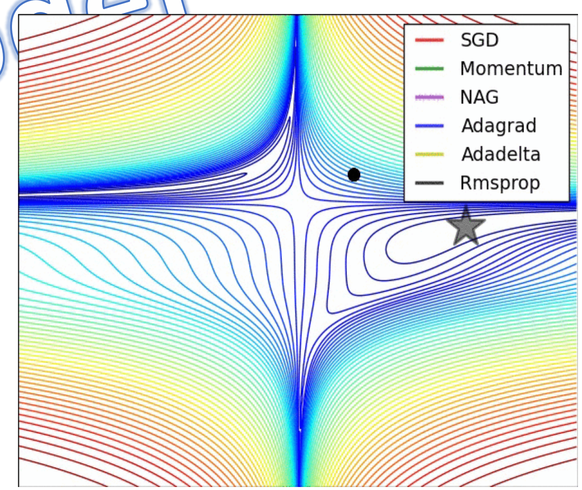
$$f(x+\Delta x)=f(x)+\nabla f(x)\Delta x+\frac{1}{2\alpha}\Delta x^2+o(\Delta x^2)$$

$$min\ f(x_0+\Delta x)=f(x_0)+\nabla f(x_0)\Delta x+\frac{1}{2\alpha}\Delta x^2+o(\Delta x^2)$$

$$\Delta x=-\alpha\nabla f(x_0) \qquad x_{k+1}=x_k-\alpha\nabla f(x_k)$$



$$f(x_0)\to f(x_1)\to f(x_2)\to\cdots$$



梯度下降法: 收敛率 $O(\frac{1}{K})$　　**近似 （已知->未知）**

机器学习 = 模型 + 评估 + 优化

Taylor展开

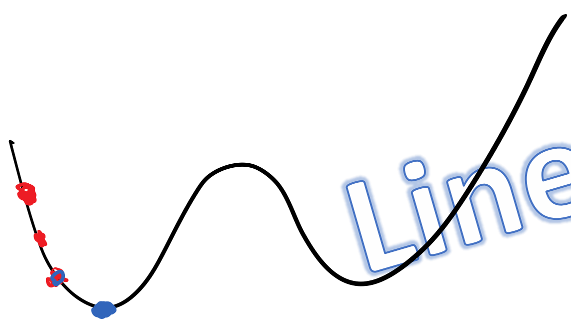
$$f(x+\Delta x) = f(x) + f'(x)\Delta x + 1/2\, f''(x)\Delta x^2 + o(\Delta x^2)$$

Newton法

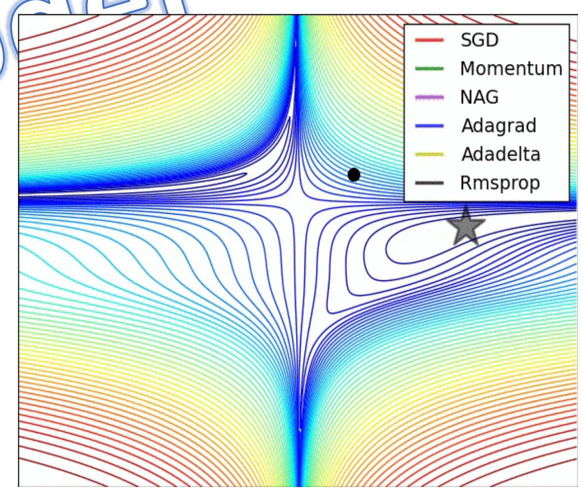
$$f(x+\Delta \mathrm{x}) = f(x) + \nabla f(x)\Delta x + \frac{1}{2}\nabla^2 f(x)\Delta x^2 + o(\Delta x^2)$$

$$min\, f(x_0+\Delta \mathrm{x}) = f(x_0) + \nabla f(x_0)\Delta x + \frac{1}{2}\nabla^2 f(x_0)\Delta x^2 + o(\Delta x^2)$$

$$\Delta x = -\nabla f(x_0)/\nabla^2 f(x_0) \qquad x_{k+1} = x_k - \nabla f(x_0)/\nabla^2 f(x_0)$$



$$f(\mathbf{x}_0) \rightarrow \mathbf{f}(\mathbf{x}_1) \rightarrow f(x_2) \rightarrow \cdots$$



Newton法：收敛率 $O(\frac{1}{K^2})$

$$\min J(\mathbf{w}) = \frac{1}{2N}\sum_i \left(\mathbf{Y}_i - \mathbf{w}^T \boldsymbol{X}_i\right)^2$$

$$J(a,b) = \frac{1}{2N}\sum_i (ax_i + b - y_i)^2$$

while k > 最大步数 or $\nabla J(a_k, b_k) \approx 0$

$$a_{k+1} = a_k - \alpha \frac{\partial \mathrm{J}(a_k, b_k)}{\partial a}$$

$$b_{k+1} = b_k - \alpha \frac{\partial \mathrm{J}(a_k, b_k)}{\partial b_k}$$

$$J'(\mathbf{w}) = \frac{1}{N}\sum \boldsymbol{X}_i^T\left(\mathbf{w}^T \boldsymbol{X}_i - \mathbf{Y}_i\right)$$

$$\frac{1}{N}\sum_i x_i(ax_i + b - y_i)$$

$$\frac{1}{N}\sum_i 1\,(ax_i + b - y_i)$$

批量梯度下降法

(Batch Gradient Descent)

Repeat {

$$w_{k+1} = w_k - \frac{1}{N}\sum \boldsymbol{X}_i^T\left(\mathbf{w}^T\boldsymbol{X}_i - Y\right)$$

}

随机梯度下降法

(Stochastic Gradient Descent)

1. Randomly shuffle dataset;
2. **repeat**{
   for i=1, ... , N
   {
   $$w_{k+1} = w_k - \boldsymbol{X}_i^T\left(\mathbf{w}^T\boldsymbol{X}_i - Y\right)$$
   }
   }

Mini-batch Gradient Descent: batch size

批量梯度下降法
(Batch Gradient Descent)

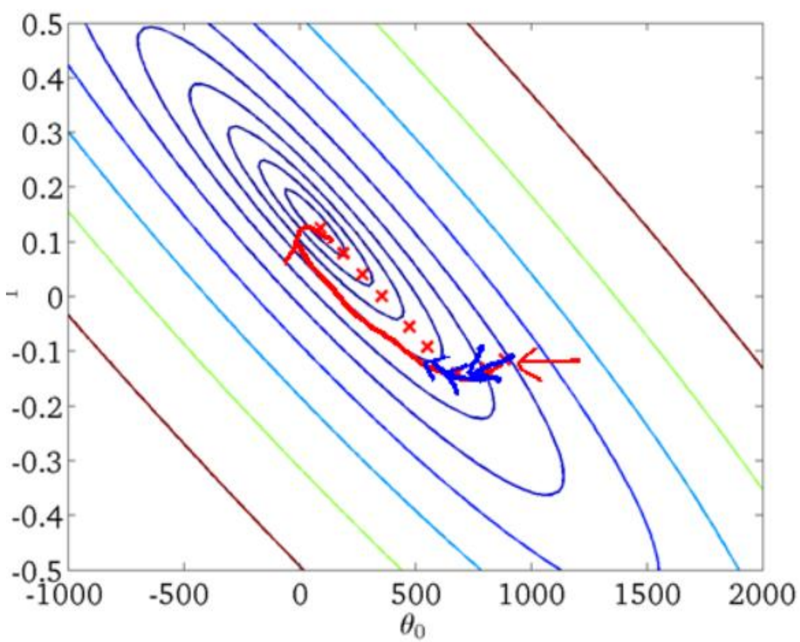
Repeat {

$$w_{k+1} = w_k - \frac{1}{N}\sum \boldsymbol{X}_i^T(\mathbf{w}^T\boldsymbol{X}_i - \boldsymbol{Y})$$

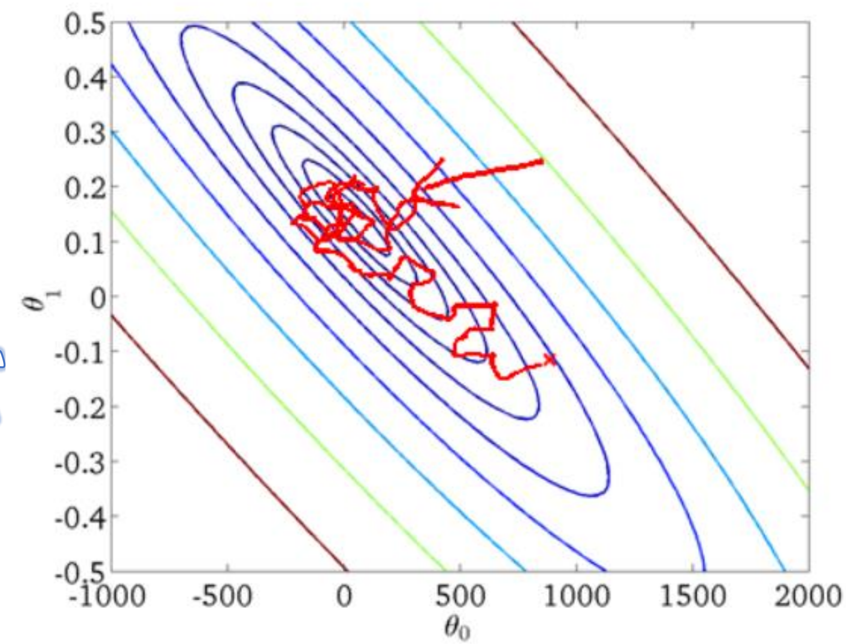
}

随机梯度下降法
(Stochastic Gradient Descent)

1. Randomly shuffle dataset;
2. **repeat**{
   for i=1, … , N
   {
   $$w_{k+1} = w_k - \boldsymbol{X}_i^T(\mathbf{w}^T\boldsymbol{X}_i - \boldsymbol{Y})$$
   }
   }



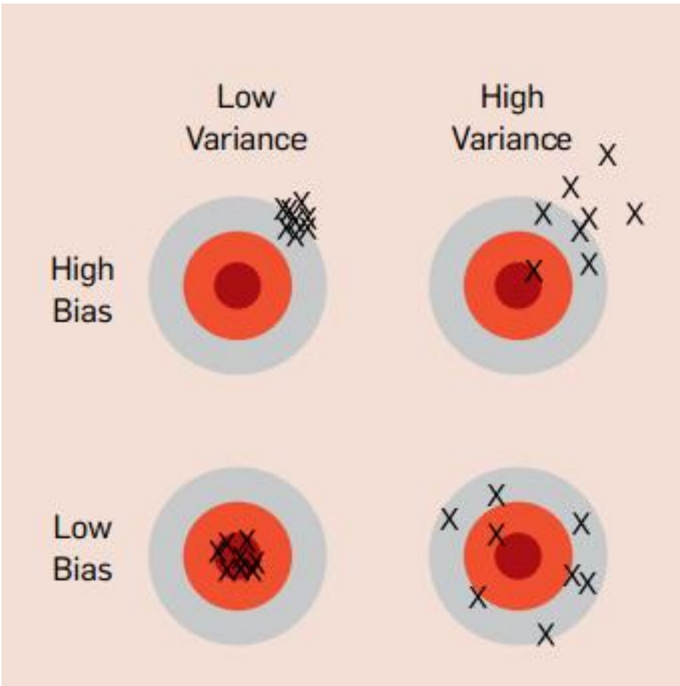
收敛率 $O(\frac{1}{K})$



收敛率 $O(\frac{1}{\sqrt{K}})$

偏差与方差

$$R_{emp}(f) = 1/N\sum_{i=1}^{N} L(y_i, f(x_i)) \qquad R_{gen}(f) = E_{p(x,y)}L(y_i, f(x_i))$$



$$\begin{aligned}
\mathrm{E}\big[(y-\hat{f})^2\big] &= \mathrm{E}[y^2 + \hat{f}^2 - 2y\hat{f}] \\
&= \mathrm{E}[y^2] + \mathrm{E}[\hat{f}^2] - \mathrm{E}[2y\hat{f}] \\
&= \mathrm{Var}[y] + \mathrm{E}[y]^2 + \mathrm{Var}[\hat{f}] + \mathrm{E}[\hat{f}]^2 - 2f\mathrm{E}[\hat{f}] \\
&= \mathrm{Var}[y] + \mathrm{Var}[\hat{f}] + (f^2 - 2f\mathrm{E}[\hat{f}] + \mathrm{E}[\hat{f}]^2) \\
&= \mathrm{Var}[y] + \mathrm{Var}[\hat{f}] + (f - \mathrm{E}[\hat{f}])^2 \\
&= \mathrm{Var}[y] + \mathrm{Var}[\hat{f}] + \mathrm{E}[f - \hat{f}]^2 \\
&= \sigma^2 + \mathrm{Var}[\hat{f}] + \mathrm{Bias}[\hat{f}]^2
\end{aligned}$$

$$Err(x) = \mathrm{Bias}^2 + \mathrm{Variance} + \mathrm{Irreducible\ Error}$$

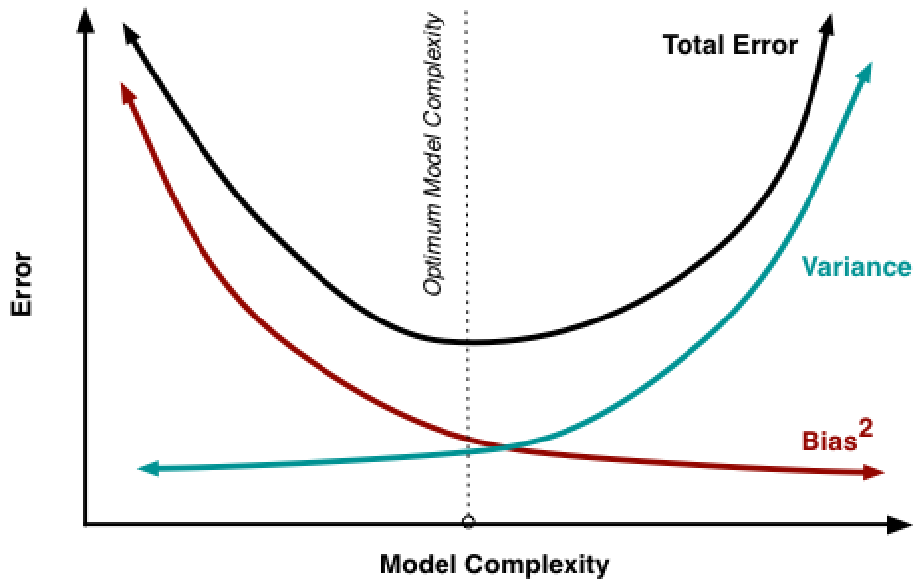
Bias: $\left(E[\hat{f}(x)] - f(x)\right)^2$ 度量了学习算法的平均估计结果能逼近学习目标的程度。

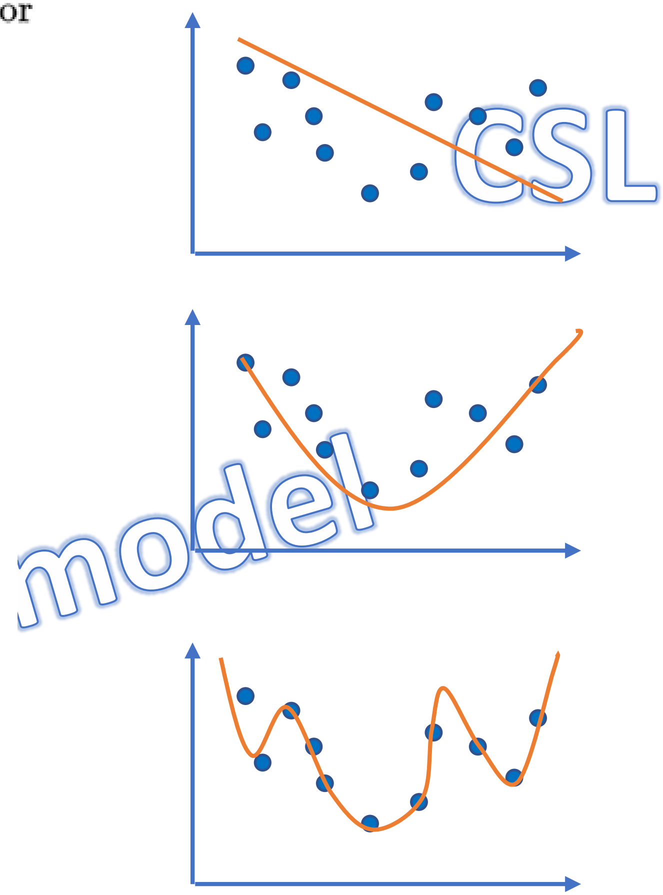
Variance: $E\left[\left(\hat{f}(x) - E[\hat{f}(x)]\right)^2\right]$ 度量了面对不同训练集时，学习算法的估计结果发生变动的程度。

偏差与方差

$$R_{emp}(f) = 1/N \sum_{i=1}^{N} L(y_i, f(x_i)) \qquad R_{gen}(f) = E_{p(x,y)} L(y_i, f(x_i))$$

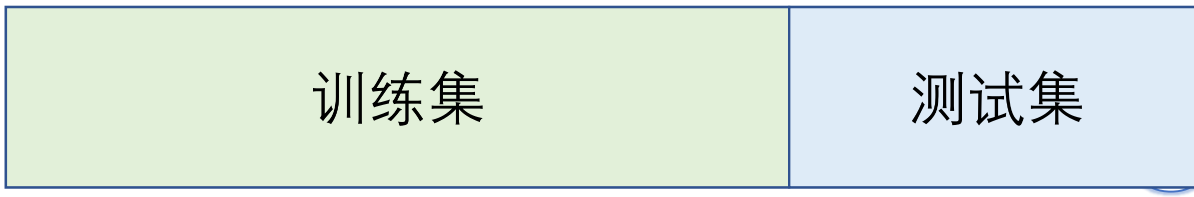
$$Err(x) = \mathrm{Bias}^2 + \mathrm{Variance} + \mathrm{Irreducible\ Error}$$

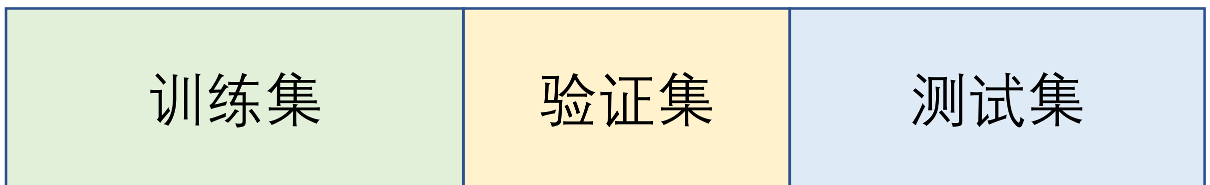




交叉验证（cross validation)

全部数据

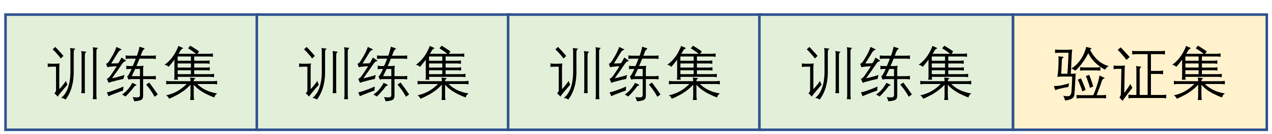


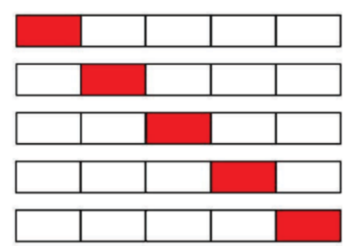


训练集 （training set）： 用于训练模型

验证集 （validation set）： 用于模型选择

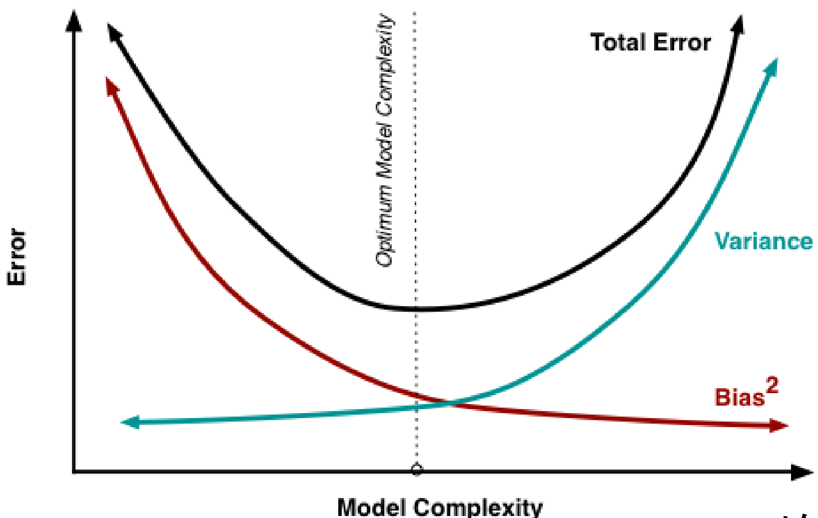
测试集 （test set）： 用于最终对学习方法的评估





正则化（regularization）

偏差与方差 $Err(x) = \text{Bias}^2 + \text{Variance} + \text{Irreducible Error}$

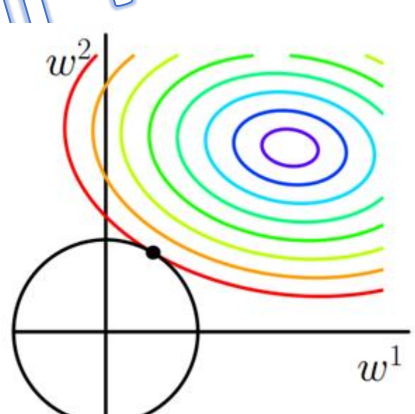


L_p 正则化

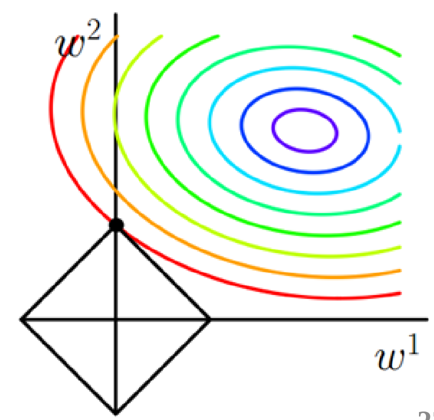
$$\hat{w}^{L_p} = \arg\min_{w\in R^d} \sum_{i=1}^{n}(w^T X_i - Y_i)^2 \text{ subject to } \sum_{j=1}^{d}|w_j|^p \le A.$$

$$\hat{w}^{L_p} = \arg\min_{w\in R^d} \sum_{i=1}^{n}(w^T X_i - Y_i)^2 + \lambda\sum_{j=1}^{d}|w_j|^p$$

岭回归(Ridge Regression)



LASSO



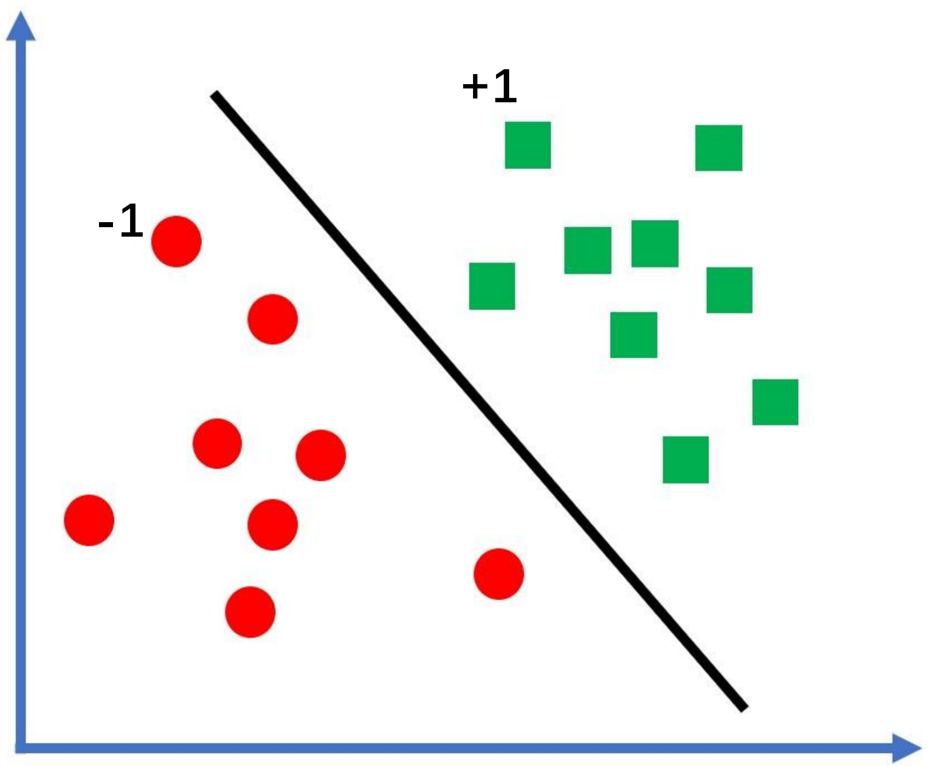
L_2正则化：$\Omega(w) = \lambda||w||_2^2 = \lambda\sum_1^d w_j^2$

L_1正则化：$\Omega(w) = \lambda||w||_1 = \lambda\sum_1^d |w_j|$

L_0正则化：$\Omega(w) = \lambda||w||_0$

线性分类器

$$f(x_i) = h(w^T x_i + b)$$



感知机（Perceptron）

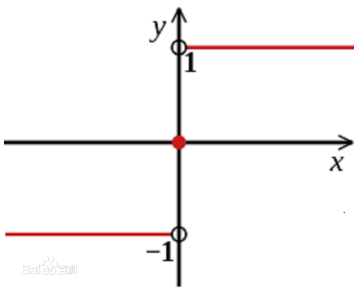
$$f(x_i) = sign(w^T x_i + b)$$

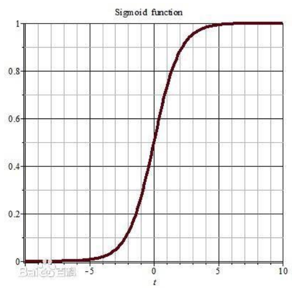
逻辑回归（Logistic regression）

$$f(x_i) = sigmoid(w^T x_i + b)$$

SVM（Support vector machine）

$$f(x_i) = sign(w^T x_i + b)$$

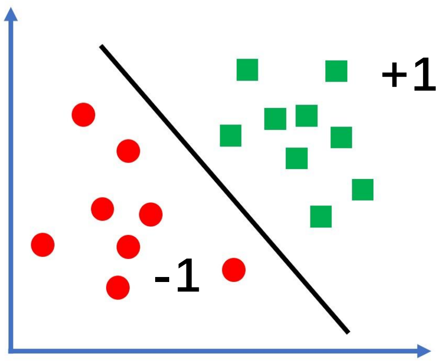


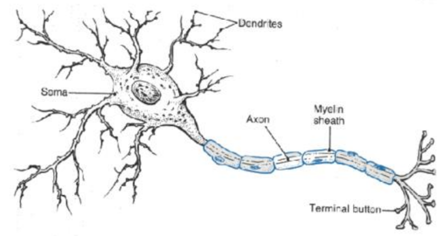


线性分类器

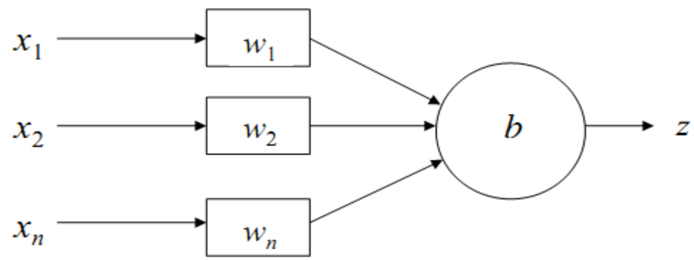
感知机

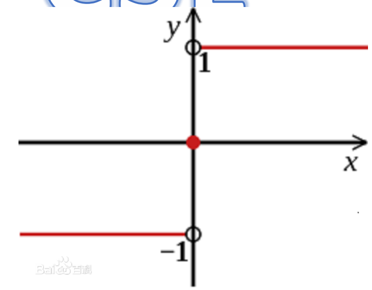
1957年由Rosenblatt提出





McCulloch-Pitts模型 $\boldsymbol{z = sign(w^T x_i + b)}$





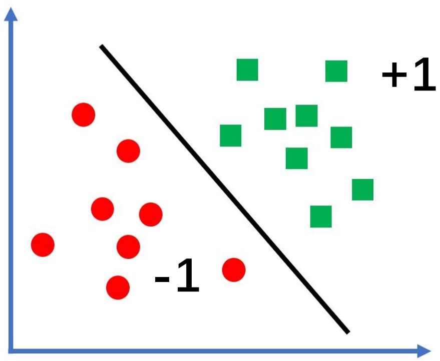
模型： $\boldsymbol{\hat{y} = w^T x_i + b}$

评估： $\boldsymbol{L(y_i, \hat{y}_i) = -y_i(w^T x_i + b)}$

优化： $\min \boldsymbol{J(w)} = \sum \boldsymbol{L(y_i, \hat{y}_i)} = -\sum_{\boldsymbol{i}} \boldsymbol{y_i(w^T x_i + b)}$

线性分类器

感知机



模型： $\hat{y} = w^T x_i + b$

评估： $L(y, \hat{y}) = -y_i(w^T x_i + b)$

优化： $\min J(w) = -\sum_i y_i(w^T x_i + b)$

PLA (Perception Learning Algorithm)

（1）选取初值w_0, b_0
（2）训练集中选取数据（xi, yi）
（3）if $y_i\left(w^T x_i + b\right) \le 0$:
$w = w + \alpha y_i x_i$
$b = b + \alpha y_i$
(4) 转至(2)，直到训练集中没有误分类点

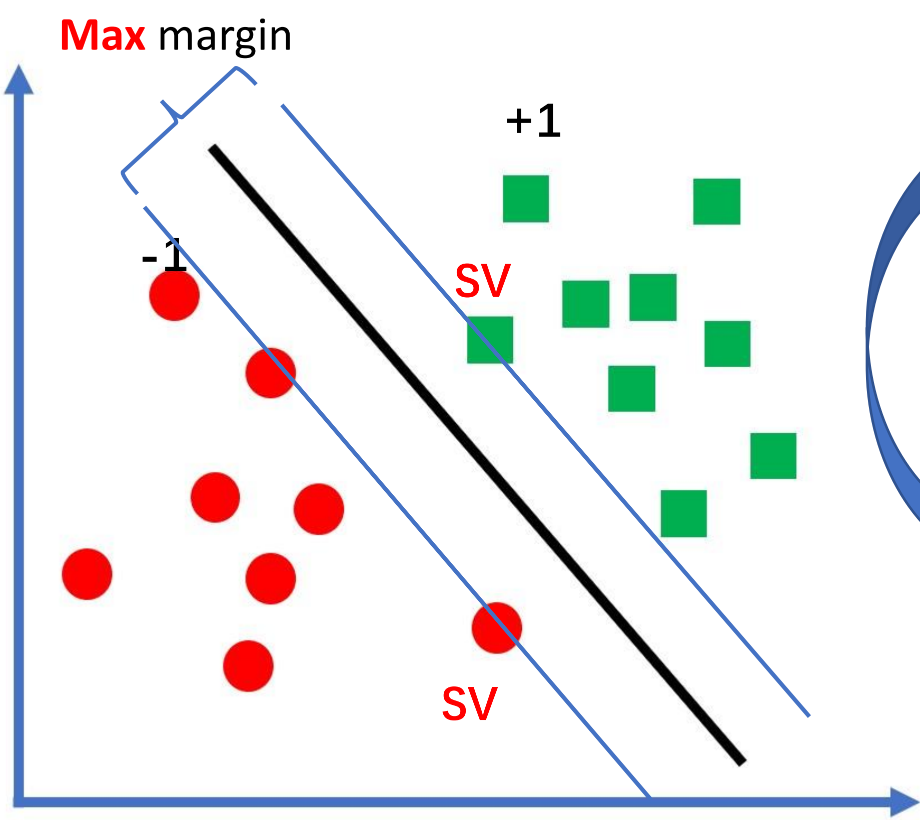
SGD

$$w_{k+1} = w_k - \alpha \frac{\partial J(w_k, b_k)}{\partial a}$$

$$b_{k+1} = b_k - \alpha \frac{\partial J(w_k, b_k)}{\partial b_k}$$

线性分类器

$$f(x_i) = h(w^T x_i + b)$$



感知机（Perceptron）

$$f(x_i) = sign(w^T x_i + b)$$

逻辑回归（Logistic regression）

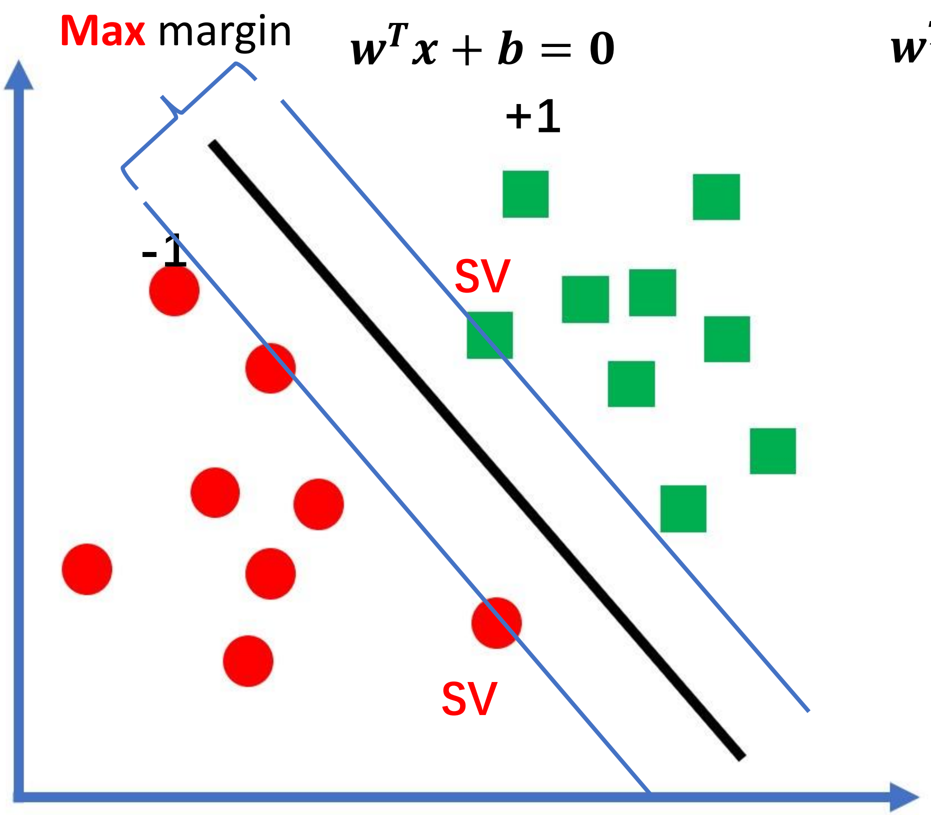
$$f(x_i) = sigmoid(w^T x_i + b)$$

SVM（Support vector machine）

$$f(x_i) = sign(w^T x_i + b)$$

线性分类器 SVM

$$\boldsymbol{f(x_i) = h(w^T x_i + b)}$$



$$\boldsymbol{w^T x + b = 0} \qquad \boldsymbol{d} = \frac{\boldsymbol{w^T x + b}}{||w||_2}$$

$$\max_{w,b} \min_i d_i = \frac{\boldsymbol{y_i(w^T x_i + b)}}{||w||_2}$$

$$\gamma = \min_i d_i = \frac{\boldsymbol{y_i(w^T x_i + b)}}{||w||_2}$$

$$\max_{w,b} \gamma \qquad \text{s.t. } \frac{y_i(w^T x_i + b)}{||w||_2} \geq \gamma$$

$$\boldsymbol{y_i}\left(\frac{\boldsymbol{w^T}}{||\boldsymbol{w}||_2 \boldsymbol{\gamma}}\boldsymbol{x} + \frac{\boldsymbol{b}}{||w||_2 \boldsymbol{\gamma}}\right) \geq 1$$

$$\max_{w,b} \frac{1}{||w'||}$$

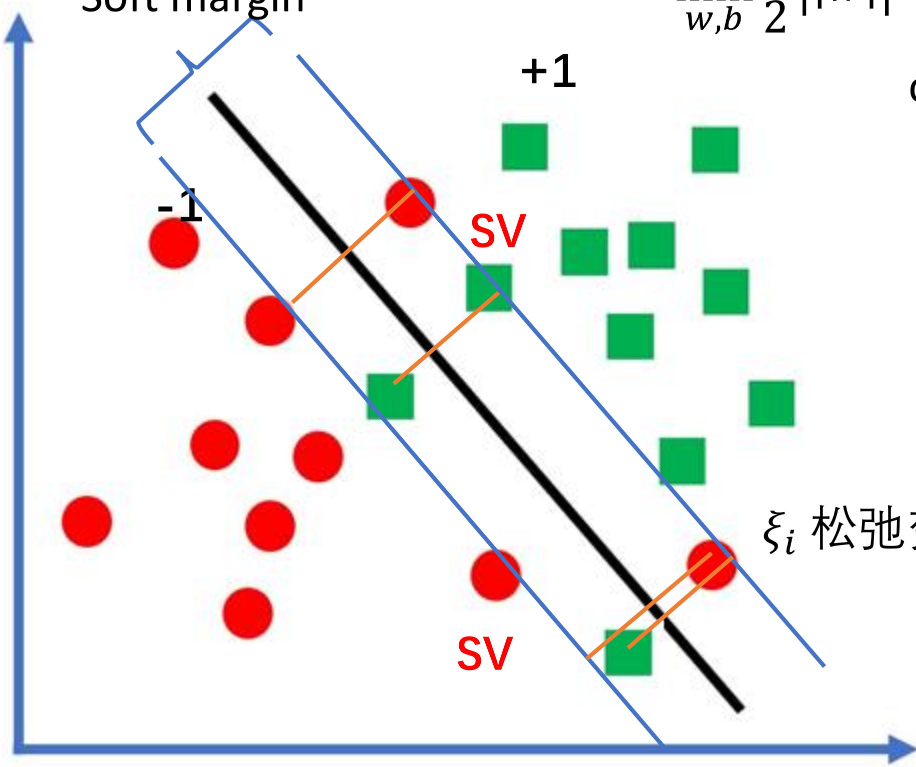
$$\boldsymbol{s.t.}\ \ \boldsymbol{y_i(w'x + b')} \geq 1$$

$$\min_{w,b} \frac{1}{2} ||w||^2$$

$$\boldsymbol{s.t.}\ \ \boldsymbol{y_i(wx_i + b)} \geq 1$$

线性分类器 SVM

Soft margin



$$\min_{w,b} \frac{1}{2}||w||^2 \qquad \boldsymbol{s.t.}\ \ \boldsymbol{y_i(wx_i + b)} \geq 1$$

convex quadratic programming

$$\min_{w,b} \frac{1}{2}||w||^2 + C\sum_i \xi_i$$

$$\boldsymbol{s.t.}\ \ \boldsymbol{y_i(wx_i + b)} \geq 1 - \xi_i$$

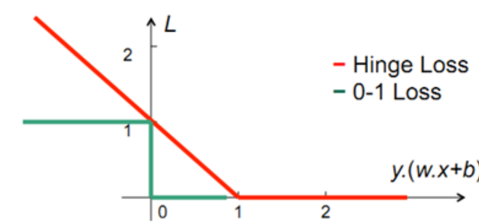
$$\xi_i \geq 0$$

ξ_i 松弛变量

$$\min_{w,b} \frac{1}{2C}||w||^2 + \sum_i \max(1 - y_i(wx_i + b), 0)$$

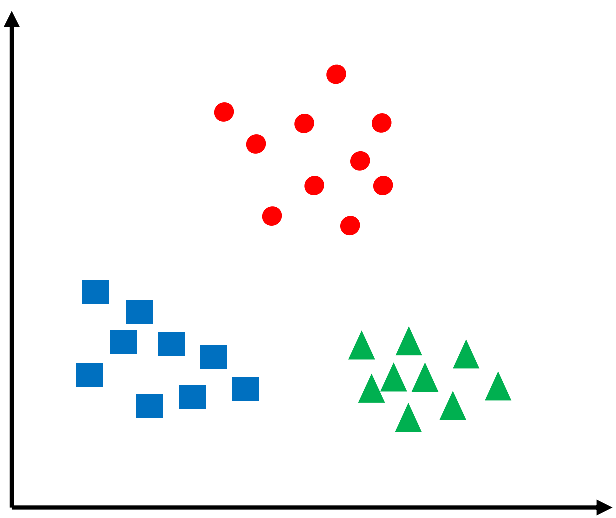
$$\min_{w,b} \sum_i \max(1 - y_i(wx_i + b), 0) + \lambda||w||^2$$

Hinge Loss: $\mathrm{L}(\mathrm{y_i}, \mathrm{wx_i} + \mathrm{b}) = \max(1 - y_i(wx_i + b), 0)$

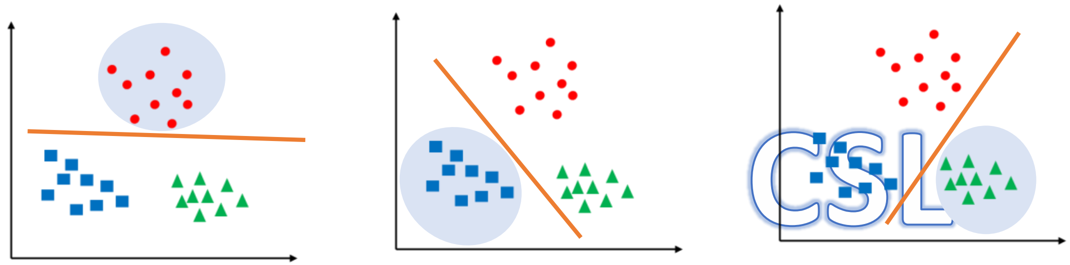


线性分类器

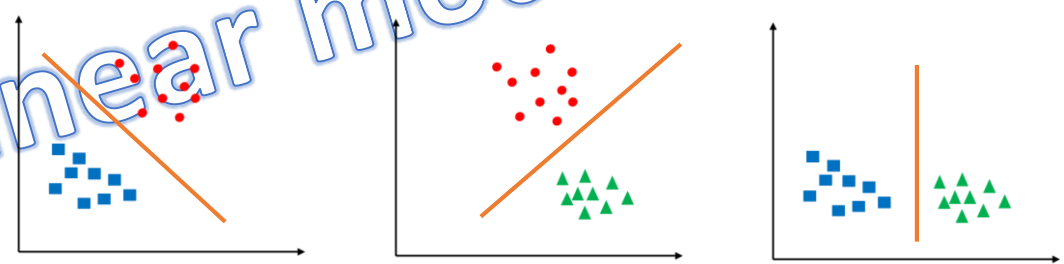
2分类 -> 多分类



One-versus-all （one-versus-rest）

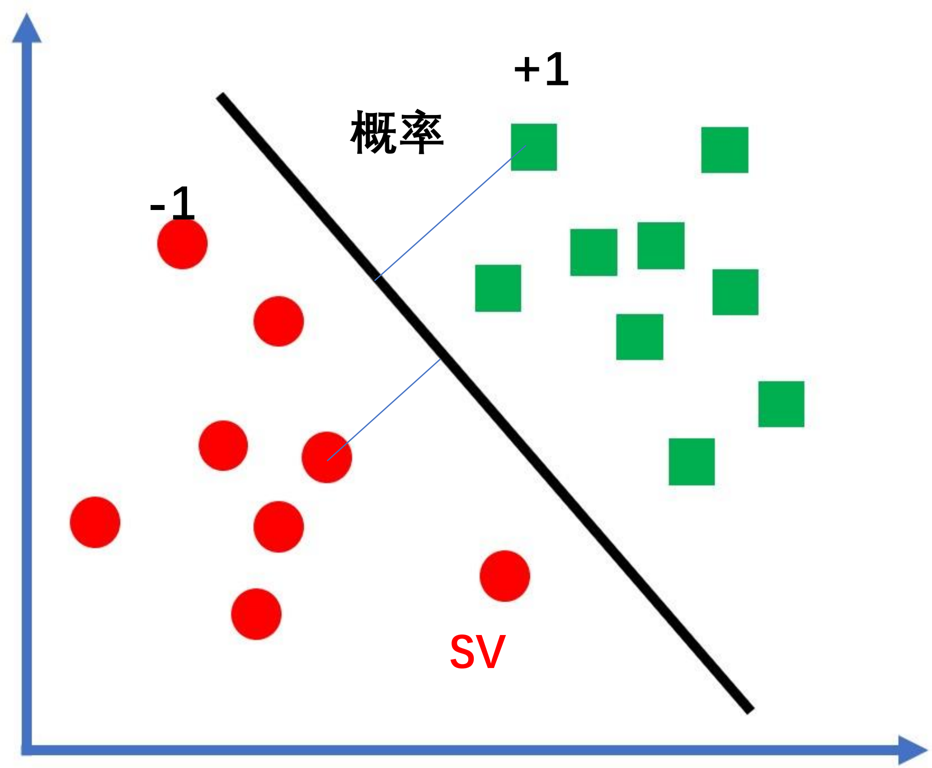


One-versus-one



线性分类器

$$f(x_i) = h(w^T x_i + b)$$



感知机（Perceptron）

$$f(x_i) = sign(w^T x_i + b)$$

逻辑回归（Logistic regression）

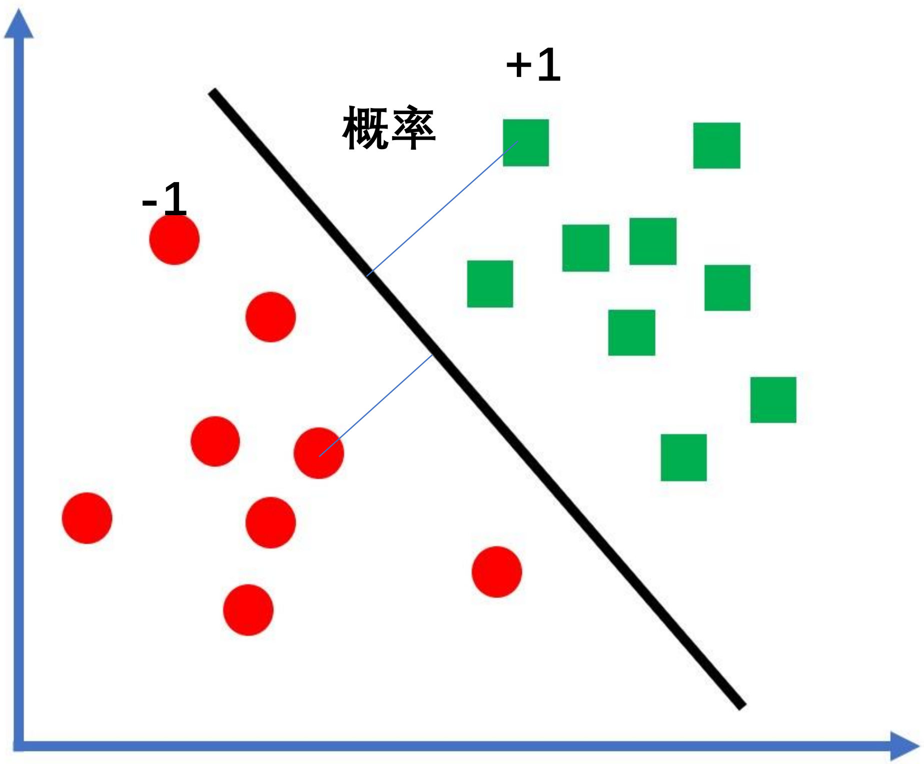
$$f(x_i) = sigmoid(w^T x_i + b)$$

SVM（Support vector machine）

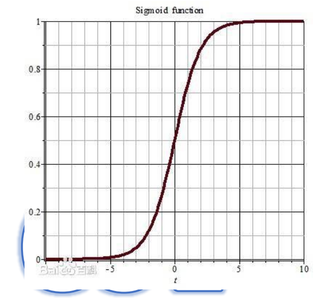
$$f(x_i) = sign(w^T x_i + b)$$

线性分类器　LR

$$f(x_i) = h(w^T x_i + b)$$



$$h(x) = \frac{1}{1 + e^{-x}}$$



(1) 认知神经科学
(2) 物理学（Boltzmann分布）
(3) 经济学（离散选择模型）

$$P(y_i = 1 | x_i) = \frac{1}{1 + e^{-(w^T x_i + b)}}$$

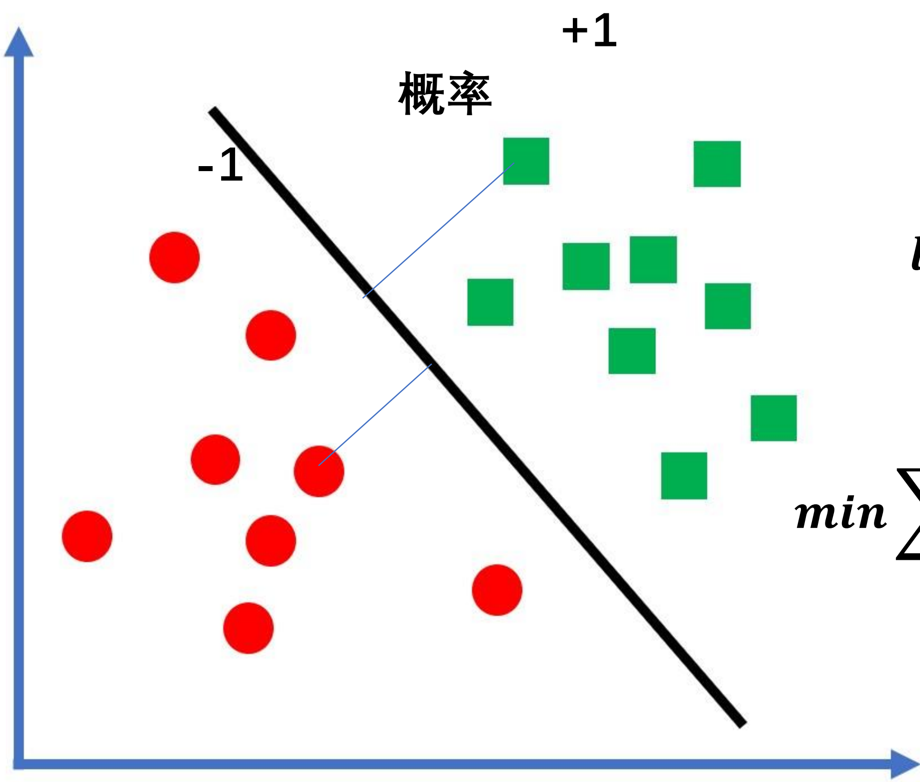
$$P(y_i = -1 | x_i) = \frac{1}{1 + e^{(w^T x_i + b)}}$$

$$\prod_i P(y_i | x_i) = \prod_i \frac{1}{1 + e^{-y_i (w^T x_i + b)}}$$

线性分类器 LR

$$P(y_i = 1|x_i) = \frac{1}{1 + e^{-(w^T x_i + b)}}$$

$$P(y_i = -1|x_i) = \frac{1}{1 + e^{(w^T x_i + b)}}$$



$$\prod_i P(y_i|x_i) = \prod_i \frac{1}{1 + e^{-y_i(w^T x_i + b)}}$$

$$log \prod_i P(y_i|x_i) = log \prod_i \frac{1}{1 + e^{-y_i(w^T x_i + b)}}$$

$$min \sum_i -log\, P(y_i|x_i) = \sum_i log(1 + e^{-y_i(w^T x_i + b)})$$

$$\min_{w,b} log(1 + \mathrm{e}^{-y_i(w^T x_i + b)}) + \lambda||w||^2$$

Logistic Loss: $L(y_i, wx_i + b) = log(1 + e^{-y_i(w^T x_i + b)})$

线性分类器 LR

多项逻辑回归（multi-nominal logistic regression）

Softmax: $\sigma(\mathbf{z})_j = \frac{e^{z_j}}{\sum_{k=1}^{K} e^{z_k}}$

$$P(y_i = 1|x_i) = \frac{1}{1 + e^{-(w^T x_i + b)}}$$

$$P(y_i = -1|x_i) = \frac{e^{-(w^T x_i + b)}}{1 + e^{-(w^T x_i + b)}}$$

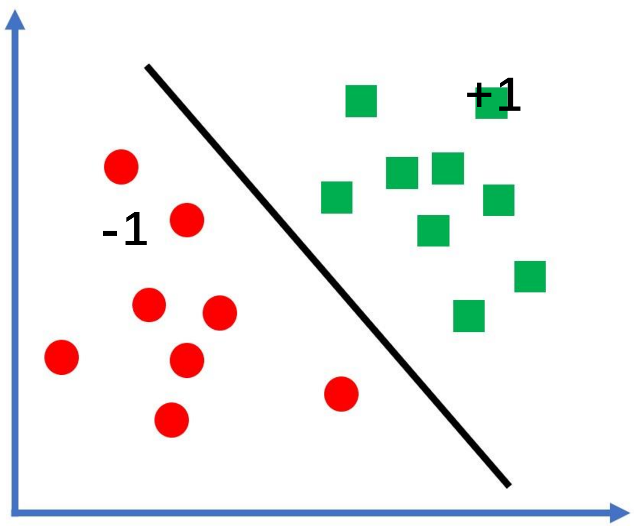
$$P(y_i = k|x_i) = \frac{e^{w_k^T x_i + b_k}}{\sum_k e^{w_k^T x_i + b_k}} \propto e^{w_k^T x_i + b_k}$$

$$log \prod_i \prod_k P(y_i = k|x_i)^{I(y_i = k)} = \sum_i \sum_k I(y_i = k) log \frac{e^{w_k^T x_i + b_k}}{\sum e^{w_k^T x_i + b_k}}$$

Cross entropy Loss: $L = \sum_k I(y_i = k) log \frac{e^{w_k^T x_i + b_k}}{\sum e^{w_k^T x_i + b_k}}$

线性分类器

$$f(x_i) = h(w^T x_i + b)$$



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SVM（Support vector machine）

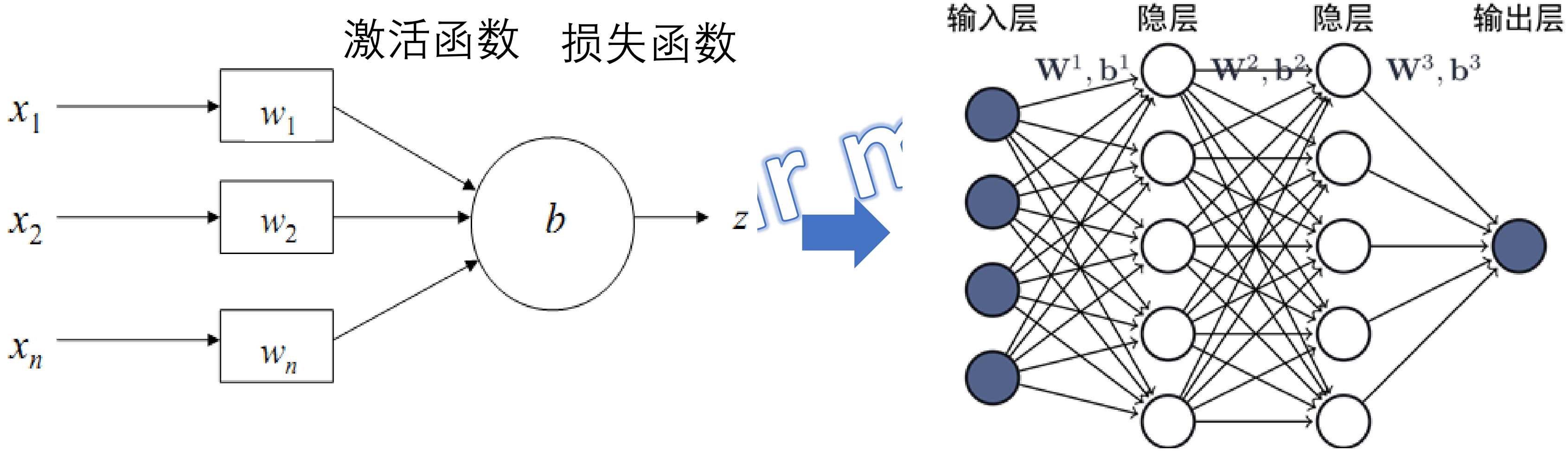
$$f(x_i) = sign(w^T x_i + b)$$

	模型	评价	优化
感知机	$\hat{y}_i = w^T x_i + b$	$L = -\hat{y}_i y_i$	SGD
逻辑回归	$\hat{y}_i = w^T x_i + b$	$L = log(1 + e^{-\hat{y}_i y_i})$	SGD
SVM	$\hat{y}_i = w^T x_i + b$	$L = max(1 - y_i \hat{y}_i, 0)$	SGD

线性分类器

线性分类器 = 浅层神经网络？

	模型	评价	优化
感知机	$\hat{y}_i = w^T x_i + b$	$L = -\hat{y}_i y_i$	SGD
逻辑回归	$\hat{y}_i = w^T x_i + b$	$L = log(1 + e^{-\hat{y}_i y_i})$	SGD
SVM	$\hat{y}_i = w^T x_i + b$	$L = max(1 - y_i \hat{y}_i, 0)$	SGD



由浅入深

总结

机器学习 = 模型 + 评估 + 优化

梯度下降法：

```
while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad
```


正则化： $\hat{w}^{L_p} = \arg\min_{w \in R^d} \sum_{i=1}^{n} (w^T X_i - Y_i)^2 + \lambda \sum_{j=1}^{d} |w_j|^p$ 　　交叉验证

线性分类器：

	模型	评价	优化
感知机	$\hat{y}_i = w^T x_i + b$	$L = -\hat{y}_i y_i$	SGD
逻辑回归	$\hat{y}_i = w^T x_i + b$	$L = log(1 + e^{-\hat{y}_i y_i})$	SGD
SVM	$\hat{y}_i = w^T x_i + b$	$L = max(1 - y_i \hat{y}_i, 0)$	SGD


统计学习方法
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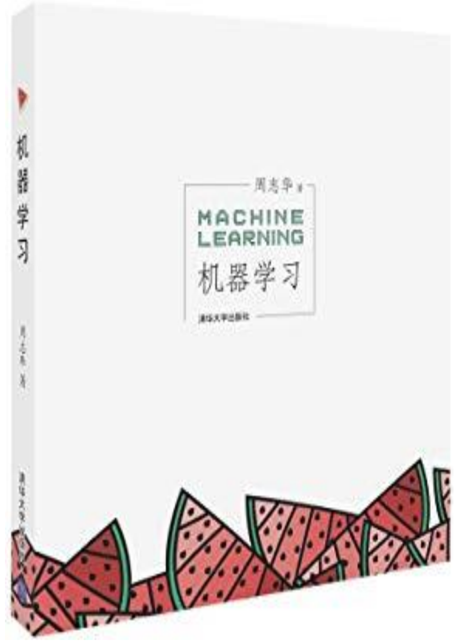
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MACHINE LEARNING
机器学习
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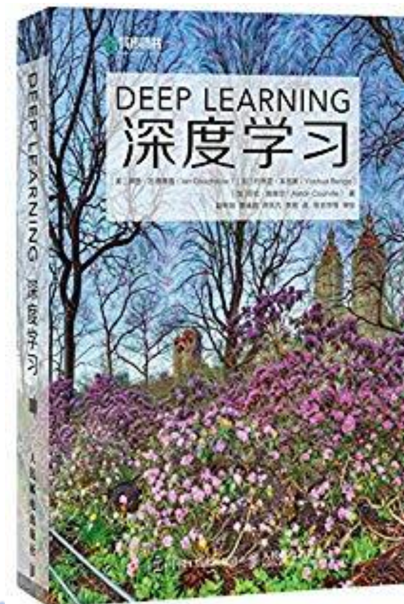
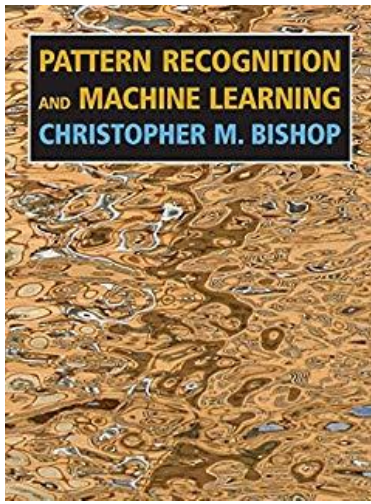

DEEP LEARNING
深度学习


PATTERN RECOGNITION
AND MACHINE LEARNING
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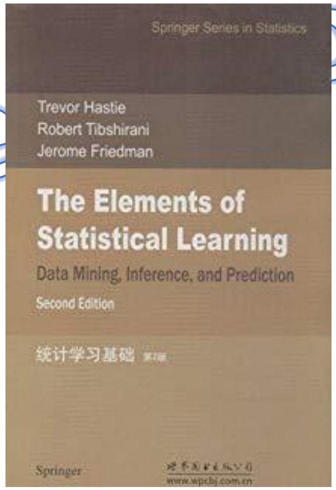
Springer Series in Statistics
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Robert Tibshirani
Jerome Friedman
The Elements of Statistical Learning
Data Mining, Inference, and Prediction
Second Edition
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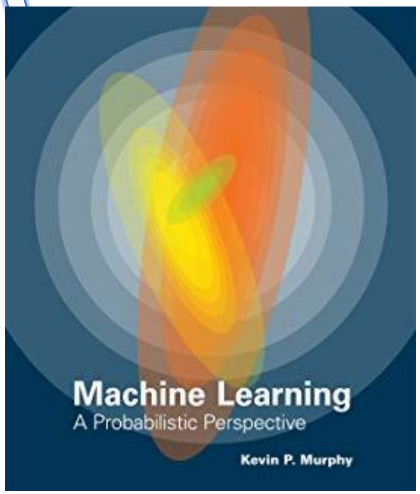
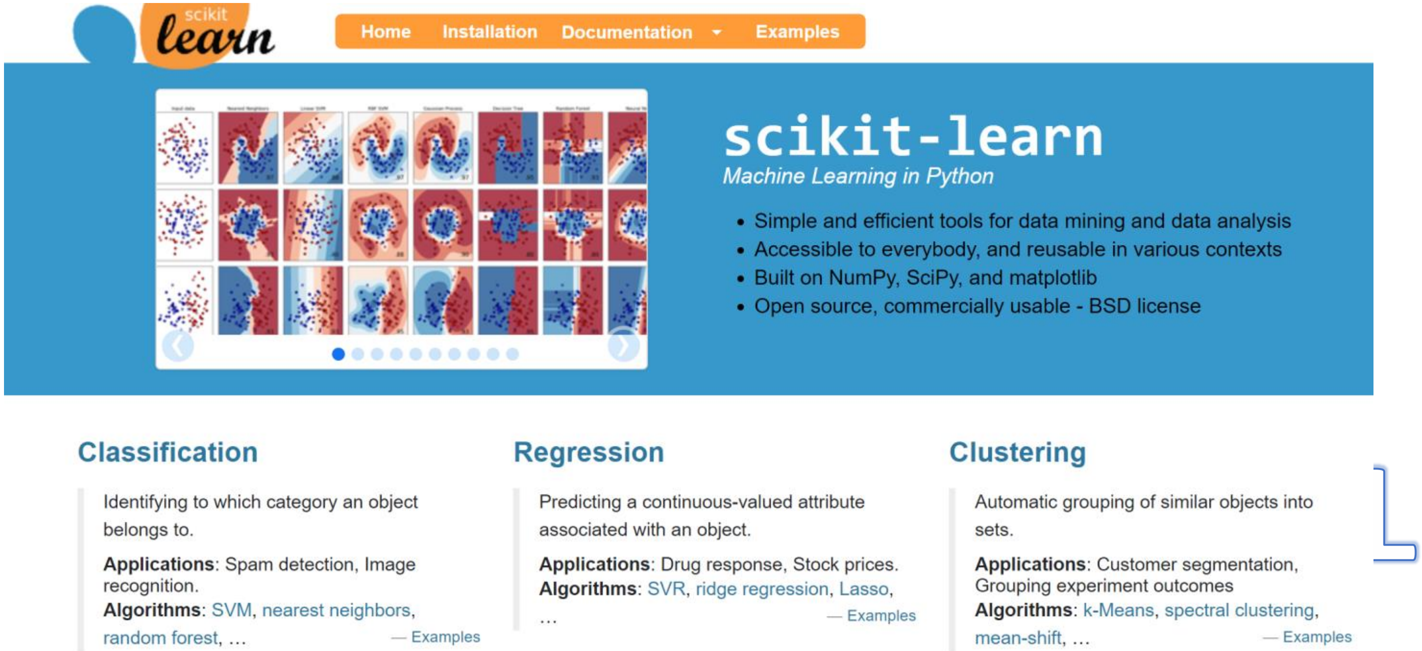

Machine Learning
A Probabilistic Perspective
Kevin P. Murphy

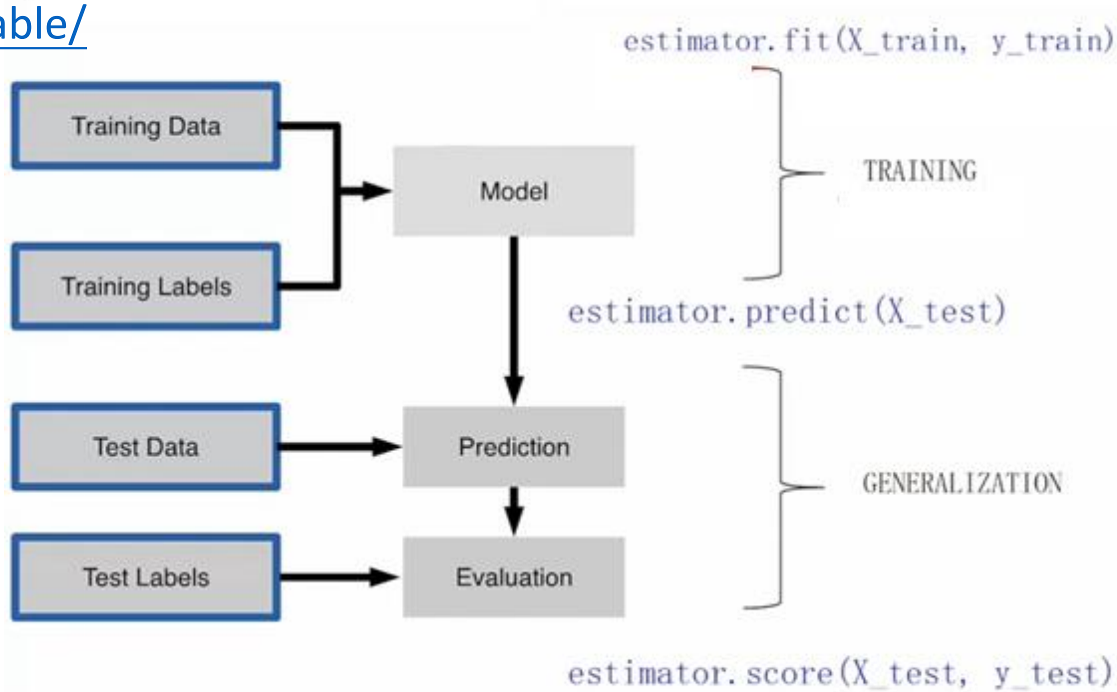
谢 谢！

CSL

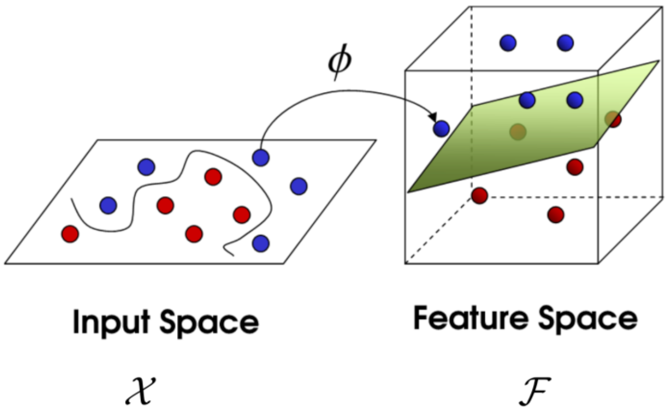
Linear model



https://scikit-learn.org/stable/



核方法 (Kernel trick)



$$\min J(w) = 1/N \sum_{i=1}^{N} L\left(y_i, w^T \phi(x_i)\right) + \lambda \left|\left|w\right|\right|^2$$

表示定理（Representer theory）：

$L(w)$对于w是凸函数，则满足$\min J(w)$的w可以表示为$w = \sum \alpha_j \phi(x_j)$

核函数： $k\left(x_i, x_j\right) = \langle \phi(x_i), \phi(x_j) \rangle$

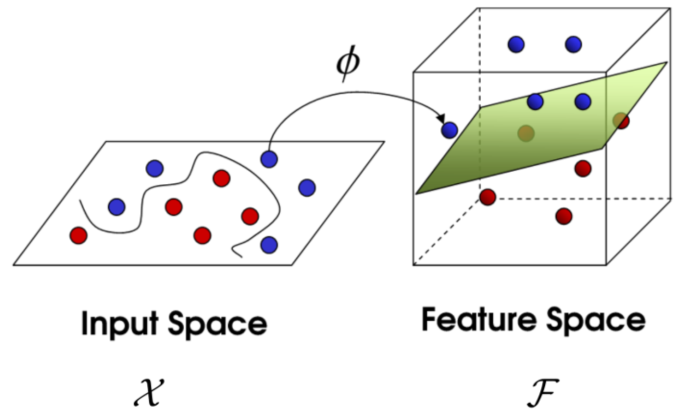
$$\min_\alpha J(w) = 1/N \textstyle\sum_{i=1}^{N} L\left(y_i, \sum \alpha_j < \phi\left(x_j\right), \phi\left(x_i\right) >\right) + \lambda < \sum \alpha_i \phi(x_i), \sum \alpha_i \phi(x_i) >$$

Linear model

$$\min_\alpha J(w) = 1/N \textstyle\sum_{i=1}^{N} L\ \left(y_i, \sum \alpha_j k(x_j, x_i)\ \right)\ \ + \lambda < \sum \alpha_i \phi(x_i), \sum \alpha_i \phi(x_i) >$$

$$\min_\alpha J(w) = 1/N \sum_{i=1}^{N} L\left(y_i, \sum \alpha_j k\left(x_j, x_i\right)\right) + \lambda \alpha^T K a$$

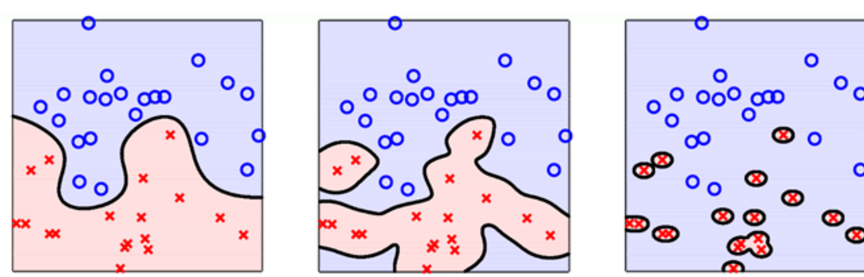
核方法 (Kernel trick)



$$\boldsymbol{min_\alpha\, J(w) = 1/N \sum_{i=1}^{N} L\left(y_i, \sum \alpha_j k(x_j, x_i)\right)} + \lambda \boldsymbol{\alpha^T K \alpha}$$

$$k(x_i, x_j) = \exp\left(-\gamma \left\|x_i - x_j\right\|^2\right) = \exp\left(-\gamma(\left\|x_i\right\|^2 + \left\|x_j\right\|^2 - 2x_i^T x_j)\right)$$

Kernel	Function
Linear	$k(x,x') = \langle x, x' \rangle$
Hom. Polynomial	$k(x,x') = \langle x, x' \rangle^d$
Inho. Polynomial	$k(x,x') = (\langle x, x' \rangle + c)^d$
Gaussian RBF	$k(x,x') = \exp\left(-\frac{\|x-x'\|^2}{2\sigma^2}\right)$
Laplacian	$k(x,x') = \exp\left(-\frac{\|x-x'\|}{\sigma}\right)$
Sigmoid	$k(x,x') = \tanh(\alpha\langle x, x' \rangle + c)$



$\exp(-1\|\mathbf{x}-\mathbf{x}'\|^2)$ $\exp(-10\|\mathbf{x}-\mathbf{x}'\|^2)$ $\exp(-100\|\mathbf{x}-\mathbf{x}'\|^2)$

Lagrange对偶

$$\min_{w,b} \frac{1}{2}||w||^2 \qquad \boldsymbol{s.t.}\ \ \boldsymbol{y_i}(\boldsymbol{w x_i} + \boldsymbol{b}) \geq 1$$

$$L(w,b,\alpha) = \frac{1}{2}||w||^2 + \alpha(1 - y_i(w x_i + b))$$

$$\min_{w,b} \max_{\alpha} L(w,b,\alpha) \ = \max_{\alpha} \min_{w,b} L(w,b,\alpha)$$

$$\nabla_w L(w,b,\alpha) = \mathrm{w} - \sum \alpha_i y_i x_i = 0$$

$$\nabla_b L(w,b,\alpha) = \sum \alpha_i y_i = 0$$

$$\min_{w,b} L(w,b,\alpha) = -\frac{1}{2} \sum\sum \alpha_i \alpha_j y_i y_j < x_i, x_j > + \sum \alpha_i$$

Some models are easy to interpret

Linear/Logistic regression

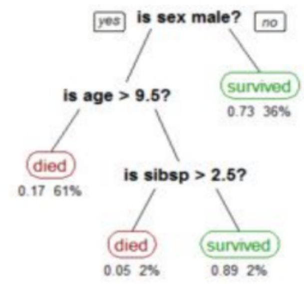
- Weight on each feature
- Know the exact contribution of each feature, negative or positive

$$Y = 3 * X1 - 2 * X2$$

Increasing X1 by 1 unit increases Y by 3 units

Single Decision Tree

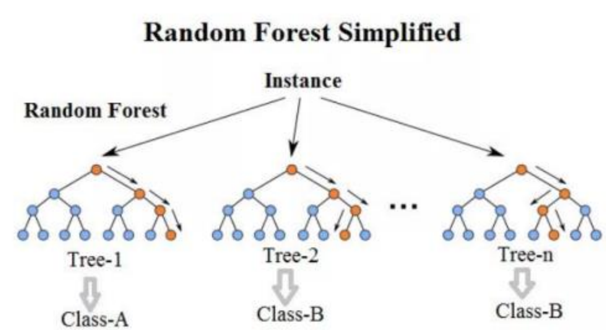
- Easy to understand how a decision was made by reading from top to bottom



Some models are harder to interpret

Ensemble models (random forest, boosting, etc…)

- Hard to understand the role of each feature
- Usually comes with **feature importance**
- Doesn't tell us if feature affects decision positively or negatively

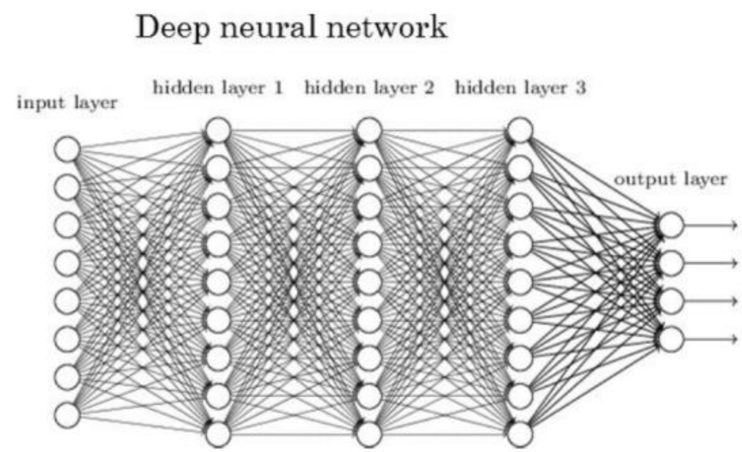


可解释性机器学习

Some are really hard to interpret

Deep Neural Networks

- No straightforward way to relate output to input layer
- “Black-box”



个体的行为选择

- **决策者**（Decision Makers），即做出选择行为的主体；

CSL

- **备选方案**（Alternatives），通常会有多个方案供决策者选择
- **选择方案的属性**（Attributes of Alternatives）。每一种考虑因素称之为一个属性（Attributes），以q_j表示
- **决策准则**（Decision Rules） Model （选择函数）

  基本假设：人类行为是理性的（偏好是完备的并具有传递习性）

Linear model

Utility-based agent

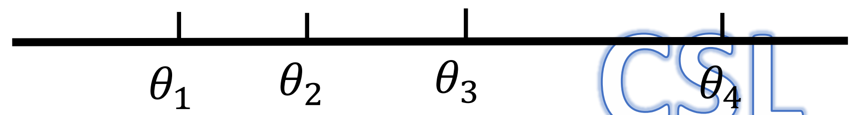
- 定义（效用函数）： p_i是定义在特征向量空间上的线性泛函$v_{p_i} = p_i(q)$。由Riesz表示定理,偏好可以表示为p_i,则有$v_{p_i}(q) =< p_i, q >$.

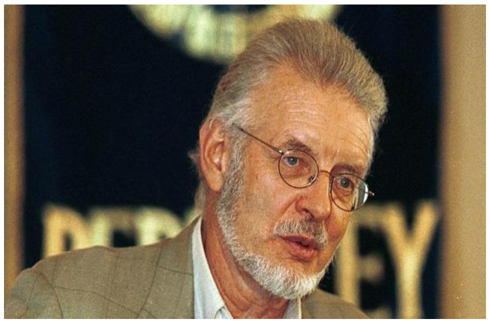
Linear model

- 定义（行为选择）： $\pi_k: S \times A \to R$,其中U为个体i的效用空间 $\pi(a|s) = P(A_t = a|S_t = s)$

离散选择模型

$$u_{k,j} = v_{k,j} + \varepsilon_k = \langle p_k, q_j \rangle + \varepsilon_k$$





Daniel L. McFadden(2000诺奖)

当 ε_i 满足Gumbel 分布（type I extreme value）时，

Utility Maximization

$$\begin{aligned} P_{ni} &= \text{Prob}(V_{ni} + \varepsilon_{ni} > V_{nj} + \varepsilon_{nj} \ \forall j \neq i) \\ &= \text{Prob}(\varepsilon_{nj} < \varepsilon_{ni} + V_{ni} - V_{nj} \ \forall j \neq i). \end{aligned}$$

$$P_{ni} = \int \left(\prod_{j \neq i} e^{-e^{-(\varepsilon_{ni} + V_{ni} - V_{nj})}} \right) e^{-\varepsilon_{ni}} e^{-e^{-\varepsilon_{ni}}} \, d\varepsilon_{ni}. \quad \Rightarrow \quad P_{ni} = \frac{e^{V_{ni}}}{\sum_j e^{V_{nj}}},$$

当只有2种行为时，上述模型退化为binary logit模型.

$$p(Y=1|x)=\exp\big(f_w(x)\big)/\big(1+\exp\big(f_w(x)\big)\big)$$

- Logistic model

$$f_w(\boldsymbol{x}) = w_{k,0} + \sum_{i=1}^{n} w_{k,i}x_i = w_{k,0} + \langle p_{k}, q_{j}\rangle$$

Utility-based agent: $u_{k,j} = w_{k,0} + \langle p_{k}, q_{j}\rangle + \varepsilon_k$

CSL

- Factorization Machine model

$$f_w(\boldsymbol{x}) = w_{k,0} + \sum_{i=1}^{n} w_{k,i}x_i + \sum_{i=1}^{n}\sum_{j=i+1}^{n} \langle v_i, v_j\rangle x_i x_j$$

Utility-based agent: $u_{k,j} = w_{k,0} + \langle p_{k}, q_{j}\rangle + \sum_{i<j}^{n} \langle v_i, v_j\rangle x_i x_j + \varepsilon_k$

Linear model

- 一般化

Utility-based agent: $u_{k,j}(t) = w_{k,0}(t) + h\big(p_k(t), q_j(t)\big) + \varepsilon_k(t)$

$$h\left(p_k, q_j\right) = F_{GBDT}\left(p_k,\ q_j\right) \qquad h\left(p_k, q_j\right) = F_{DNN}\left(p_k,\ q_j\right)$$

- 安装python （Anaconda）
  - 熟悉python的基本数据结构， tuple, list和dict.
  - 熟悉基本numpy操作
- 阅读sklearn中有关LR和SVM的文档， 并尝试使用
- 找到非零x满足1.0+x==1.0?
- 尝试实现GD（SGD） 求解LR
- 尝试实现算法求解LASSO的线性回归问题
- 比较GD和Newton法的收敛速度