

Par1

$$1 \ P(X=0, Y=0) = P(X) * P(Y|X) \\ = 0.3 \times 0.3 \\ = 0.09 \text{ — use product rule}$$

$$2 \ P(X=0, Y=1) = P(X) * P(Y|X) \\ = 0.3 \times 0.7 \\ = 0.21 \text{ — use product rule}$$

$$3 \ P(X=1, Y=0) = P(X) * P(Y|X) \\ = P(X=1) * P(Y=0|X=1) \\ = 0.7 \times 0.8 \\ = 0.56 \text{ — use product rule}$$

$$4 \ P(X=1, Y=1) = P(X) * P(Y|X) \\ = P(X=1) * P(Y=1|X=1) \\ = 0.7 \times 0.2 \\ = 0.14 \text{ — use product rule}$$

2

X	Y	Z	P(X,Y,Z)
0	0	0	0.054
0	0	1	0.036
0	1	0	0.168
0	1	1	0.042
1	0	0	0.0336
1	0	1	0.224
1	1	0	0.112
1	1	1	0.028

Conditional independent rule:

X and Z are conditionally independent given Y

$$P(X|Z,Y) = P(X|Y)$$

$$P(Z|X,Y) = P(Z|Y)$$

$$P(X,Z|Y) = P(X|Y) * P(Z|Y)$$

$$P(X,Y,Z) = P(X,Z,Y)$$

$$= P(X|Z,Y) * P(Z|X,Y)$$

$$= P(X|Y) * P(Z|Y)$$

$$= P(X|Z,Y) * P(Z,Y) \text{ — conditional independent rule}$$

$$= P(X,Y) * P(Z|Y) \text{ — Product rule}$$

$$= P(X) * P(Y|X) * P(Z|Y)$$

The Formula I use is $P(X) * P(Y|X) * P(Z|Y)$

The rule I use Product rule and conditional independent rule

3

i)

$$\text{sum rule : } P(Z=0) = P(0,0,0) + P(0,1,0) + P(1,0,0) + P(1,1,0) = 0.67$$

ii)

$$\text{If there are independent: } P(Z|X) = P(Z) \text{ when } P(Z) = 0.67. \ P(Z=0|X=0) = P(Z=0, X=0) / P(X) = 0.222 / 0.3 = 0.74$$

Therefore X and Z are not independent.

4

$$i) \ P(A,B,C) = P(C) * P(A,B|C) \text{ using chain rule}$$

$$\begin{aligned}
P(X=1, Y=0, Z=1) &= P(Z=1) * P(X=1, Y=0|Z=1) \\
P(X=1, Y=0|Z=1) &= P(X=1, Y=0, Z=1)/P(Z=1) \\
P(X=1, Y=0|Z=1) &= 0.224/0.33 \\
&= 0.679
\end{aligned}$$

$$\begin{aligned}
\text{ii) } P(A, B, C) &= P(C) * P(B|C) * P(A|B, C) \\
P(X=0, Y=0, Z=0) &= P(Z=0) * P(Y=0|Z=0) * P(X=0|Y=0, Z=0) \\
P(X=0|Y=0, Z=0) &= P(X=0, Y=0, Z=0) / P(Z=0) * P(Y=0|Z=0) \\
P(Y=0|Z=0) &= P(X=0, Y=0, Z=0) + P(X=1, Y=0, Z=0) \\
&= 0.054 + 0.336 = 0.39 \\
P(X=0|Y=0, Z=0) &= 0.054 / 0.67 * 0.39 = 0.207
\end{aligned}$$