Proves and Problem Solving

February 6, 2023

1 Upper bounds and Least upper bound

1.1 Bounded Sequence

We say that a sequence (a_n) is **bounded** if the set of values a1,a2,... is a bounded set.
i.e. thereare m,M such that $m \leq a_n \leq M$ for all n. **Proposition 6.1**
Suppose the sequence (a_n) converges. Then it is bounded.

Proposition 6.3
Suppose that $x_n \to L$ as $n \to \infty$ and that $k \in \mathbb{N}$. Then $x_n^k \to L^k$ as $n \to \infty$.

1.2 Application: the existence of roots

Theorem 6.6
Let $x > 0$ and $k \in \mathbb{N}$. Then there is a unique $y > 0$ such that $y^k = x$.

1.3 Infinite limits

Definition 6.4
Let x_n be a sequence of real numbers.

(a) We say (x_n) tends to ∞ or **diverges** to ∞, and write $x_n \to \infty$ as $n \to \infty$ if for all $M > 0$, there exists $N \in \mathbb{N}$ such that $n > N$ implies $x_n \geq M$.

(b) Similar for $x_n \to -\infty$