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17.4-2

当删除后，如果$\alpha_i \geq 1/2$

$$\begin{aligned}\hat{c}_i &= c_i + \Phi_i - \Phi_{i-1} \\ &= 1 + (2num_i - size_i) - (2num_{i-1} - size_{i-1}) \\ &= 1 + 2num_i - size_i - (2num_i + 2 - size_i) \\ &= -1\end{aligned}$$

当删除后，如果$\alpha_i < 1/2$

$$\begin{aligned}\hat{c}_i &= c_i + \Phi_i - \Phi_{i-1} \\ &= 1 + (\frac{1}{2}size_i - num_i) - (2num_{i-1} - size_{i-1}) \\ &= 1 + (\frac{1}{2}size_i - num_i) - (2num_i + 2 - size_i) \\ &= -1 + \frac{3}{2}size_i - 3num_i\end{aligned}$$

因为$\alpha_i = num_i/size_i < 1/2$

$$\begin{aligned}\hat{c}_i &= -1 + \frac{3}{2}size_i - 3num_i \\ &\leq 1\end{aligned}$$

所以摊还代价的上界是一个常数

17.4-3

首先，显然势函数$\Phi(T) > 0$

当删除后，如果$\alpha_i \geq 1/3$

$$\begin{aligned}\hat{c}_i &= c_i + \Phi_i - \Phi_{i-1} \\ &= 1 + |2num_i - size_i| - |2num_{i-1} - size_{i-1}| \\ &= 1 + |2num_i - size_i| - |2num_i + 2 - size_i| \\ &\leq 1 + |2num_i - size_i| - |2num_i - size_i| + 2 \\ &= 3\end{aligned}$$

当删除后，如果$\alpha_i < 1/3$ ，收缩

此时有$num_i = num_{i-1} - 1$，$size_i = \frac{2}{3}size_{i-1}$，$2num_i = size_i$

$$
\begin{aligned}
\hat{c}_i &= c_i + \Phi_i - \Phi_{i-1} \\
&= num_i + 1 + |2num_i - size_i| - |2num_{i-1} - size_{i-1}| \\
&= num_i + 1 + |2num_i - size_i| - |2num_i + 2 - \frac{3}{2}size_i| \\
&\le num_i + 1 + |2num_i - size_i| - |2num_i - \frac{3}{2}size_i| + 2 \\
&= num_i + 1 - |size_i - \frac{3}{2}size_i| + 2 \\
&= 3 + num_i - \frac{1}{2}size_i \\
&= 3
\end{aligned}
$$

所以摊还代价的上界是常数