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7.1-2

所有元素都一样的时候, PARTITION返回r

Algorithm 1 PARTITION(A, p, r)

$x = A[r]$
$i = p - 1$
for $j = p$ to $r - 1$ **do**
  if $A[j] \le x$ and $j \mod 2 == (p+1) \mod 2$ **then**
    $i = i + 1$
    exchange $A[i]$ with $A[j]$
  end if
end for
exchange $A[i+1]$ with $A[r]$
return $i + 1$

7.2-1

假设$T(n) = \Theta(n^2)$,带入$T(n) = T(n-1) + \Theta(n)$, 有：
$T(n) = c^2(n-1)^2 + cn = (cn)^2 + (c-2)n + 1 = \Theta(n^2)$ 得证

7.2-2

此时是最坏情况，每次划分，左右划分最不平衡，时间复杂度$\Theta(n^2)$

7.4-1

将假设 $T(n) \ge cn^2$带入

$$\begin{aligned}
T(n) &= \max_{0\le q\le n-1}(T(q) + T(n-q-1)) + \Theta(n) \\
&\ge \max_{0\le q\le n-1}(cq^2 + c(n-q-1)^2) + \Theta(n) \\
&= c \cdot \max_{0\le q\le n-1}(q^2 + (n-q-1)^2) + \Theta(n) \\
&\ge c \cdot n^2
\end{aligned}$$

所以$T(n) = \Omega(n^2)$

7.4-4

$$
\begin{aligned}
E[X] &= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{2}{j-i+1} \\
&= \sum_{i=1}^{n-1} \sum_{k=1}^{n-i} \frac{2}{k+1} \\
&\geq \sum_{i=1}^{n-1} 2\ln(n-i+1) \\
&= 2\ln(\prod_{i=1}^{n-1} n-i+1) \\
&= 2\ln(n!) \\
&= \frac{2}{\lg e} \lg(n!) \\
&\geq cn\lg n
\end{aligned}
$$

所以期望运行时间是 $T(n) = \Omega(n\lg n)$