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Project Report

Simulation of BER for OFDM Variant in a Fading Channel

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A Final Year Project Report submitted to the Department of Electrical and Electronics Engineering in partial fulfillment of the requirements for the award of a Bachelor of Science Degree in Electrical and Electronics Engineering.

November 17, 2020

Declaration

This project report is our original work and to the best of our knowledge has not been submitted to Jomo Kenyatta University of Agriculture and Technology or any other institution for the award of an academic accolade.

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TITLE OF PROJECT
Simulation of BER for OFDM Variant in a Fading Channel

Supervisor Confirmation

This project report has been submitted to the department of Electrical and Electronics Engineering, Jomo Kenyatta University of Agriculture and Technology, with my approval as the university supervisor:

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Abstract

Design and analysis of an Orthogonal Frequency Division Multiplexing (OFDM) based system necessitates access to powerful computing hardware, costly proprietary software and programming proficiency on the specialist's part. This is because the performance metrics of an OFDM system are unavailable for analytical evaluation. These metrics include: Bit Error Rate (BER) which is a function of Signal-to-Noise Ratio (SNR) and Peak-to-Average Power Ratio (PAPR). BER can only be numerically determined by rigorous calculations involving stochastic processes within a simulation. Without the aforementioned resources, the design process is greatly impeded in the best case, completely hampered in the worst. Whereas a powerful computer can make short work of a complex simulation, the same computer can solve an analytic model even faster. Regardless, even with unencumbered access to computational power, simulation is significantly time-consuming for elaborate models such as an accurate simulation of an OFDM variant.

This project aims to come up with an analytic model for BER performance of an OFDM variant within a fading channel. A mathematical model is a powerful tool in the hands of a system designer and analyst, and even more powerful when used with a computer.

The project will be implemented by creation of a MATLAB simulation model of an OFDM variant operating over fading channels that have both Rayleigh and Rician distributions. System performance can be obtained from simulation models in form of BER curves. Using regression and continuous variation of the simulation model parameters, an expression for a curve of best fit will be obtained.

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Abbreviations

ADC Analog-to-Digital Converter. 24

ASK Amplitude Shift Keyed. 8, 10

AWGN Additive White Gaussian Noise. 29, 32, 33

BER Bit Error Rate. 2–4, 6, 24, 29, 37, 39

BPF Band-pass Filter. 8, 14, 17

DAC Digital-to-Analog Converter. 10, 15, 16, 24

DFT Discrete Fourier Transform. 9, 13, 15–17

DSL Digital Subscriber Loop. 6

DSSS Direct Sequence Spread Spectrum. 7

FDM Frequency Division Multiplexing. 1, 5

FEC Forward Error Correction. 6

FFT Fast Fourier Transform. 2, 5, 9, 15, 35

HF High Frequency. 5, 15

HPA High Power Amplifier. 24

ICI Intercarrier interference. 1, 2

IDFT Inverse Discrete Fourier Transform. 9, 13, 15, 16

IF Intermediate Frequency. 16, 17

IFFT Inverse Fast Fourier Transform. 2, 9, 15, 16, 30

ISI Intersymbol interference. 1, 2, 8, 9, 16, 20, 25

LPF Low-pass Filter. 14

LSB Lower Side Band. 12, 14, 17

MASK M-ary Amplitude Shift Keyed (ASK). 10, 11

MCM Multicarrier Modulation. 1, 2, 5, 8

MIMO Multiple Input Multiple Output. 8

MPSK M-ary Phase Shift Keyed (PSK). 11

OFDM Orthogonal Frequency Division Multiplexing. 2–6, 8, 10, 16, 17, 24, 29, 37–39, 46, 48

PAPR Peak-to-Average Power Ratio. 2, 6, 24, 25, 29, 37, 38, 41

PDF Probability Density Function. 21–23

PSD Power Spectral Density. 10–12

PSK Phase Shift Keyed. 2, 8, 10, 11, 27, 30, 35

QAM Quadrature Amplitude Modulation. 2, 10, 11

RF Radio Frequency. 9, 11, 12, 15, 17, 18, 30, 35

SCM Single Carrier Modulation. 1, 2

SNR Signal-to-Noise Ratio. 2, 3, 20, 25–27

USB Upper Side Band. 12, 14

WLAN Wireless Local Area Network. 7

Chapter 1

Introduction

1.1 Background Information

Ceaseless research into existing and prospective communication technologies has resulted in numerous permutations of fundamental standard formats. Thus, for a novel communication system to be considered for standardization and widespread adoption, it has to have an edge over existing implementations. It is for this reason that thorough analyses are performed on communication systems even before they are brought to public light.

In digital communication, information is conveyed in bits which are grouped into symbols. In order to avoid Intersymbol interference (ISI), symbol duration must be greater than delay time, by Nyquist's criterion for ISI avoidance[1]. Naturally, data rate being inversely proportional to symbol duration implies that longer symbol periods translate to low data rates, and thus diminished communication efficiency. This limitation is especially salient in a Single Carrier Modulation (SCM) system. The system designer has to compromise data rate so as to mitigate against ISI.

However, in a Multicarrier Modulation (MCM) system, total available bandwidth is divided into sub-channels over which multiple sub-carriers can transmit in parallel. Reducing the spectral spacing between the sub-carriers facilitates a higher data rate as more sub-channels can be accommodated. Nevertheless, Intercarrier interference (ICI) occurs when carrier spacing is too small. In practical Frequency Division Multiplexing (FDM) implementation, a spectral distance between adjacent sub-carriers called a guard band is introduced. While a guard band introduces spectral inefficiencies, it reduces the influence of ICI.

OFDM is an MCM that addresses both the ISI problem of high data rate SCM schemes and the ICI problem of overcrowded-spectrum MCM systems. OFDM uses a large number of low data rate sub-carriers to accomplish a cumulatively high data rate. The sub-carriers are orthogonally spaced on the spectrum which allows them to overlap without suffering from ICI. As each sub-carrier has a relatively low data rate, ISI is greatly diminished.

Whereas OFDM technology had already been patented in 1966 by *Chang* of *Bell Labs*[1] and earned strong interest, mainstream adoption only just took hold in the 21st century. This is because at the time, implementation was extremely costly. Thanks to the Fast Fourier Transform (FFT) and Inverse Fast Fourier Transform (IFFT) algorithm pairs alongside an exponential increase in computational power over the years, large scale implementation became viable. This is why in a modern world with an oppressive and stringently regulated spectrum budget, OFDM remains to be the preferred multiplexing scheme.

OFDM symbols are generated based on the modulation technique used, for example:

- M-ary PSK modulation
- Quadrature Amplitude Modulation (QAM)

Some important OFDM system parameters are:

- The number of sub-carriers.
- Symbol duration, hence sub-carrier spacing.

Both parameters are commensurate with the performance and complexity of the OFDM system[2]. In designing an OFDM system, coherence bandwidth is to be considered:

$$B_c > \frac{1}{T_s} \tag{1.1}$$

B_c : Coherence Bandwidth
T_s : Symbol Duration

Under this condition, the OFDM signal only suffers from slow, flat fading.

1.2 Problem Statement

OFDM became the most popular multiplexing scheme for the following reasons:

- Robustness to multi-path fading.
- High spectral efficiency.

The performance of an OFDM system is quantified in its BER. This measure is heavily grounded in statistics and probability since analytic estimation would require simultaneous solution of a near-infinite number of sets of Maxwell's equations, and that's before multi-path propagation is considered[2].

As a result, computer modelling and physical implementation remain to be the only ways to obtain a BER measurement for a specified system. This means that design and testing of OFDM systems is only accessible to an elite few who have access to powerful computing hardware, to run simulations or costly lab equipment for the same purpose.

Even having access to the aforementioned equipment, simulation is still quite computationally intensive when the model is accurate in its complexity[2]. Consequently, even modern powerful processors running highly optimized code take a significant amount of time to complete the simulation. This delay, while bearable for a single pass, is completely unacceptable in an iterative design process.

1.3 Problem Justification

It has been shown that OFDM system analysis is time consuming and costly, even with the right gear. This project aims to come up with an analytic model for the BER performance of an OFDM system over both *Rayleigh* and *Rician* fading channel models.

The analytic expressions so obtained should allow a communication system designer to establish a rough estimate for the parameters they expect their system to have and from that data, to know the value of the resulting BER for various SNR levels. Being an analytic technique, this would permit system analysis on literal paper with relatively inexpensive computational tools such as scientific calculators.

Moreover, where possible, the expression can be evaluated on a computer at relatively negligible cost in processor time, allowing a ludicrously high count of iterations with extraordinary time savings compared to the conventional route.

1.4 Objectives

1.4.1 Main Objective

To come up with an analytic expression for BER performance of an OFDM variant over a fading channel.

1.4.2 Specific Objectives

`EN271-0496/2015` — **Manyara Peter Njuguna**

1. To simulate a standard OFDM communication system compliant to IEEE Wireless standard 802.11 over a Rician fading channel.
2. To simulate an OFDM variant over a Rician fading channel model.
3. To simulate PAPR performance of the standard system and of the variant.

`EN271-6661/2015` — **Litunya Rosemary Omuoma**

1. To simulate a standard OFDM system conforming to IEEE Wireless standard 802.11 over a Rayleigh fading channel.
2. To simulate an OFDM variant over a Rayleigh fading channel.
3. To simulate PAPR performance of the standard and of the OFDM variant.

Chapter 2

Literature Review

2.1 Orthogonal Frequency Division Multiplexing

OFDM is a modulation technique widely used in high speed wireless communication systems. It divides a channel into a number of equally spaced frequency bands.

2.1.1 The History of OFDM

MCM was initially used for military High Frequency (HF) radios in the years 1950–1960. The use of orthogonal frequencies for transmission first appears in a 1966 patent by *Robert W. Chang* of Bell Laboratories[1]. The proposal to use FFT to generate orthogonal signals originally surfaced in 1969[3]. Parallel data streams and FDM would be used, but with overlapping sub-channels. The merits of this method include:

- Avoiding the use of high speed equalization.
- Mitigation of impulsive noise and multi-path distortion.
- Maximal utilization of available bandwidth.

The earliest adoption of OFDM was in military communication.

Thereafter, some fundamental improvements were incorporated into OFDM, most notably the inclusion of a *cyclic prefix* in the early 1980s[3]. The technique started to be considered for practical adoption in the mid 80s. In 1987, particularly,

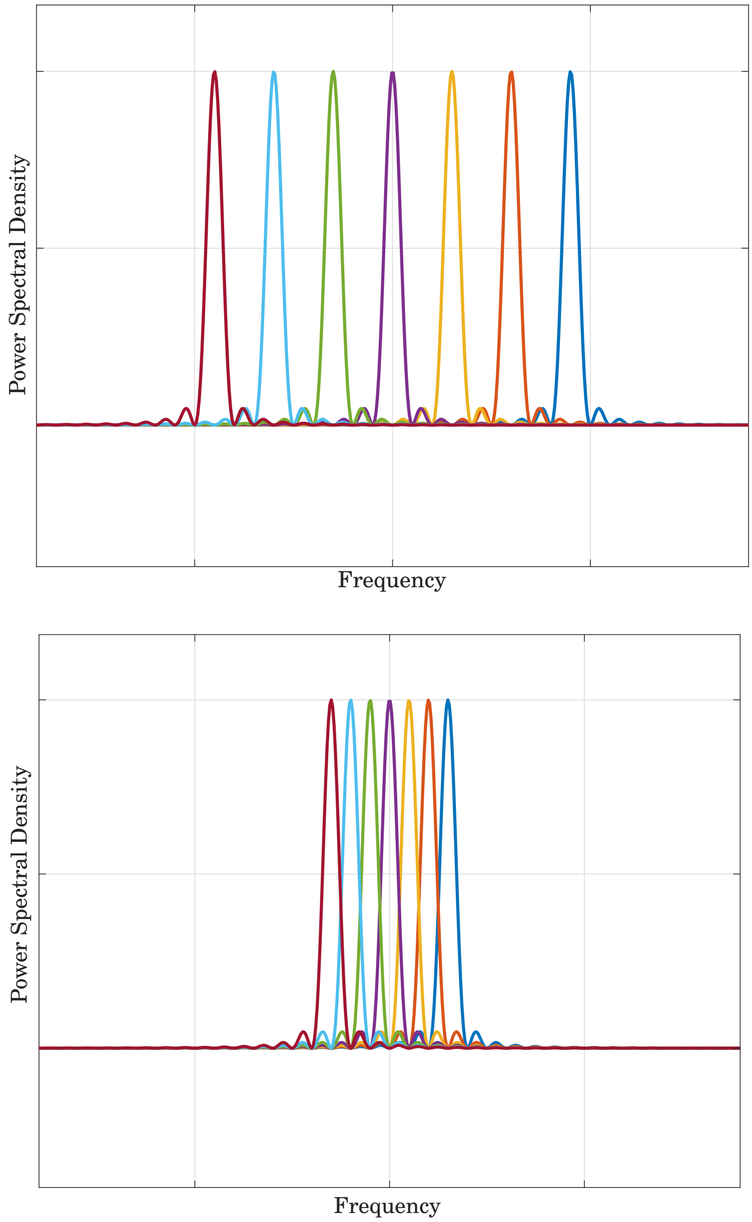

Figure 2.1: Comparison of OFDM and FDM spectral efficiency

Lassalle and *Alard*, based in France, considered the use of OFDM for radio broadcasting. They noted the necessity of combining Forward Error Correction (FEC) with OFDM.

A novel application of OFDM was pioneered by *Cioffi* and others at *Stanford*, who showed its potential as a modulation technique for Digital Subscriber

Loop (DSL) applications. In 1999, the first OFDM Wireless Local Area Network (WLAN) standard *802.11a* was published, followed in succession by *802.11n* and *802.16d*. However, the most widely deployed WLAN standard is *802.11b*, which uses Direct Sequence Spread Spectrum (DSSS). OFDM is the basis of many telecommunication standards.

IEEE 802.11 Standard

The *IEEE 802.11* specification is a standard that specifies the set requirements for the physical layer and a medium access control layer. The standard provides two definitions for physical layers– *802.11b* for 2.4 GHz operation and *802.11a* for 5 GHz operation[4].

802.11a and *802.11b* are not inter-operable unless the equipment in use has dual band capability. These are the standard's specifications:

Parameter	*Value*
Modulation technique	BPSK, QPSK, 16-QAM, 64-QAM
Coding rate	1/2, 2/3, 3/4
Number of sub-carriers	52
Number of pilots	4
Symbol duration	4 µs
Guard interval	800 ns
Sub-carrier spacing	312.5 kHz
3 dB bandwidth	16.56 MHz
Channel spacing	20 Hz

Table 2.1: IEEE 802.11 specifications

All data sub-carriers use the same modulation format within a given burst but this can vary from burst to burst.

OFDM Variants

Variants of OFDM follow the standard implementation but possess additional augmentative attributes. Some of these include:

Coded OFDM This is a variant of OFDM in which error correcting code is incorporated into the signal.

Flash OFDM This is a fast-hopped version of OFDM which utilized multiple tones and fast hopping to spread signals over a given spectrum band.

Vector OFDM This variant uses Multiple Input Multiple Output (MIMO) techniques. It is a proprietary variant being developed by CISCO Systems. MIMO techniques involve the use of multiple antennas to transmit and receive such that multi-path effects can be utilized to enhance signal reception and transmission speeds.

Wideband OFDM A form of OFDM that uses such a large degree of spacing between its sub-channels that frequency errors between transmitter and receiver do not affect communication performance. It is particularly applicable to Wi-Fi systems.

2.1.2 Principles of OFDM

OFDM is a type of MCM, with the most apparent advantage of being that simultaneous transmission of N symbols over N sub-carriers reduces symbol rate $1/N$ times the original. This also means that symbol duration is increased N times, which extenuates ISI, consequently reducing the need for channel equalization.

Recovering the signals of the sub-channels at the receiver can be achieved through:

- Spacing sub-carrier frequencies such that the spectra of N sub-channels do not overlap. N Band-pass Filter (BPF)s are then used to separate these sub-channels, each requiring a sharp frequency response. This is the method used in FDM.
- Allowing the sub-carriers which are orthogonally spaced by $1/T$ to overlap. The signals in the sub-carriers are recovered from the sub-carriers using correlators in the receiver. This method is used in OFDM.

The advantages of OFDM over single carrier modulation are:

1. The Nyquist's rate for a given channel can be approached without the use of sharp cutoff filters.

2. It elongates symbol period, countering the effects of ISI due to channel dispersion and multipath interference.

3. OFDM divides the frequency band into narrow bands, reducing sensitivity to wide-band impulse noise and frequency selective fading. The complex fading coefficient on each sub-band signal can be removed by multiplying the signal with the conjugate of the fading coefficient.

4. Different modulation formats can be used on different sub-carriers depending on sub-channel noise.

5. OFDM is digitally implementable using Inverse Discrete Fourier Transform (IDFT)/Discrete Fourier Transform (DFT) pair through their IFFT/FFT algorithm pair, greatly reducing system complexity.

Wide adoption of OFDM in recent years has been commensurate with increasing computing power.

OFDM Signal and Spectrum

The final form of an OFDM signal can be a *baseband* or a *passband* signal. It is always baseband for wired systems due to limited bandwidth. In wireless systems, baseband signals are up-converted to the Radio Frequency (RF) band for transmission.

Baseband OFDM Signal Its general form is expressed as:

$$s(t) = \sum_{i=0}^{N-1} s_i(t) = \sum_{i=0}^{N-1} A_i \cos(2\pi f_i t + \phi_i) \qquad 0 \le t \le T \tag{2.1}$$

$s(t)$:	Baseband OFDM signal
A_i :	Amplitude of i^{th}sub-carrier
f_i :	Frequency of i^{th}sub-carrier
ϕ_i :	Phase of i^{th}sub-carrier
N :	Number of sub-carriers
T :	Symbol period of the data

Depending on the modulation format (ASK or PSK), A_i or ϕ_i respectively are determined by data while the others remain constant.

Proof of orthogonality For the sub-carriers to be orthogonal, f_i must be integer multiples of $1/2T$ and the minimum frequency separation between them must be $1/T$. The sub-carrier frequencies are taken to be:

$$f_i = iR_s = \frac{i}{T} \qquad\qquad i = 0, 1, \ldots, N-1 \tag{2.2}$$

f_i : i^{th}sub-carrier frequency
R_s : Symbol duration

If we take the lowest sub-carrier frequency as f_0, the sub-carrier frequencies are: $f_0, f_0 + R_s, f_0 + 2R_s, \ldots, f_0 + (N-1)R_s$. In practice, f_0 isn't 0 Hz to avoid a DC offset in the Digital-to-Analog Converter (DAC). With the sub-carrier frequencies in equation (2.2), orthogonality is verified by:

$$\int_0^T s_i(t)s_j(t)\cdot dt = \begin{cases} {A_0}^2 T\cos^2\phi_0 & i = j = 0 \\ \frac{1}{2}{A_i}^2 T & i = j \neq 0 \\ 0 & i \neq j \end{cases} \tag{2.3}$$

This is the proof of orthogonality between sub-carriers and it holds for all values of A_i, A_j and ϕ_i, ϕ_j for the frequency allocation of equation (2.2)[1].

The frequency spectrum of the OFDM signal is characterized by its Power Spectral Density (PSD). With the assumption that the data on each sub-carrier is independent of others, overall PSD is a superposition of the PSDs of all sub-channel signals[5]:

$$S(f) = \sum_{i=0}^{N-1} S_i(f) \tag{2.4}$$

$S(f)$: Overall PSD of the OFDM signal.
$S_i(f)$: PSD of the i^{th}sub-channel signal $s_i(t)$

For $i \neq 0$, where the modulation format is M-ary ASK (ASK) or QAM with symmetrical constellation, the PSD of the i^{th}sub-channel's signal is:

$$S_i(f) = \frac{1}{2}{A_{avg}}^2 T\left[\left(\frac{\sin\pi(f-f_i)T}{\pi(f-f_i)T}\right)^2 + \left(\frac{\sin\pi(-f-f_i)T}{\pi(-f-f_i)T}\right)^2\right] \qquad i \neq 0 \tag{2.5}$$

For QAM, ${A_{avg}}^2 = 2{\sigma_a}^2$ whereas it is ${\sigma_a}^2$ for MASK.

${\sigma_a}^2$: Amplitude variance of *in-phase* and *quadrature* components.

For M-ary PSK (PSK) modulation, the PSD function is the same, except that A_{avg} is equal to the PSK amplitude.

When $i = 0$, $s_i(t) = A_i \cos\phi_i$ is a baseband signal which is equivalent to the in-phase channel data for QAM or PSK. Its PSD is given by:

$$S_0(f) = {\sigma_a}^2 T(\text{sinc } \pi f T)^2 = \frac{1}{2}{A_{avg}}^2 T(\text{sinc } \pi f T)^2 \tag{2.6}$$

Combining (2.4), (2.5) and (2.6) then normalizing the PSD by its maximum and only showing the positive frequency part:

$$S(f) = \sum_{i=0}^{N-1} \left(\frac{\sin \pi(f - f_i)T}{\pi(f - f_i)T} \right)^2 \qquad f \geq 0 \tag{2.7}$$

Each member of PSD has the shape of a squared sinc function. The first PSD's null is at $(f - f_i) = 1/T$. As separation between sub-carriers is $1/T$, this means that the first null point coincides with the peak of the PSDs of immediately adjacent sub-channel signals.

The null bandwidth of the composite PSD of an N sub-carrier baseband OFDM signal is:

$$B_{null} = \frac{N}{T} = \frac{N}{R_s} \tag{2.8}$$

$$B_{null-to-null} = \frac{2N}{T} = 2NR_s \tag{2.9}$$

Passband OFDM Signal When an RF bandwidth is allocated to an OFDM signal, sub-carrier frequencies are chosen to be symmetrically distributed around a nominal carrier frequency. Thus, the sub-carrier frequencies are:

$$f_i = f_c - \frac{N-1}{2T} + \frac{i}{T} \qquad i = 0, 1, \ldots, N-1 \tag{2.10}$$

f_c : Nominal RF carrier frequency.

This means:

$$f_c - \frac{N-1}{2T} \leq f_i \leq fc + \frac{1}{2T} \tag{2.11}$$

The passband OFDM signal can be expressed:

$$s(t) = \sum_{i=0}^{N-1} A_i \cos\left[2\pi\left(f_c - \frac{N-1}{2T} + \frac{i}{T}\right)t + \phi_i\right] \qquad 0 \le t \le T \tag{2.12}$$

The sub-channels at RF frequencies need not be orthogonal to each other since demodulation is not performed at RF frequencies[5]. The RF-band OFDM signal is first down-converted to baseband frequencies where sub-carrier frequencies are multiples of $1/2T$[5]. At this point, it's then demodulated.

The passband OFDM signal is a frequency shifted version of the baseband signal. When data on each sub-channel is independent of the others, the total PSD is a sum of individual PSDs:

$$S(f) = \sum_{i=0}^{N-1} \left(\frac{\sin\left[\pi(f - f_c{}' - \frac{i}{T})T\right]}{\pi(f - f_c{}' - \frac{i}{T})T} \right)^2 \tag{2.13}$$

$f_c{}'$: Frequency shift of passband OFDM signal

The frequency shift can be found by mixing the baseband OFDM signal with an RF signal of frequency $f_c{}'$ while keeping the Upper Side Band (USB) but discarding the Lower Side Band (LSB). The mixing frequency is the nominal carrier frequency with an offset:

$$f_c{}' = f_c - \frac{N-1}{2T} \tag{2.14}$$

The null-to-null bandwidth is given by:

$$B_{null-to-null} = \frac{N+1}{R_s} \tag{2.15}$$

Transition bands of passband PSDs get sharper and side-lobes get lower as N increases. Normalized null-to-null bandwidth approaches 1 as $N \to \infty$[5].

The Nyquist rate of a channel with a rectangular frequency response is equal to its passband bandwidth. This is achievable with OFDM since its normalized passband frequency bandwidth is 1[5].

2.1.3 OFDM Modulator and Demodulator

There are two differentiated implementations of the OFDM modem:

- *Analog modem.* Modulation by multipliers and demodulation by correlators.

- *Digital modem.* Modulator uses IDFT and demodulator uses DFT.

Analog OFDM Modem

This implementation is a scaled version of single carrier communication system. Therefore, for a large number of sub-carriers, it becomes prohibitively complex and impractical.

Transmitter side The process by which the passband OFDM signal is generated:

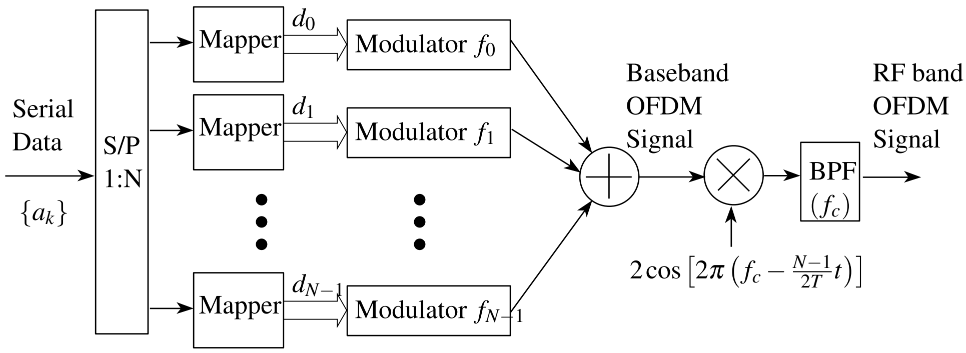

Figure 2.2: Analog OFDM Modulator

- Serial data bits $\{a_k\}$ are converted by the $1 : N$ serial-to-parallel converter into N data streams.

- The bits in each stream are then mapped to a symbol, typically denoted by a complex number.

$$\begin{aligned} d_1 &= A_i \exp(j\phi_i) = I_i + jQ_i \qquad\qquad i = 0, 1, \ldots, N-1 \qquad (2.16) \\ I_i &= A_i \cos\phi_i \\ Q_i &= A_i \sin\phi_i \end{aligned}$$

I_i : Symbol in-phase component.
Q_i : Symbol quadrature component.

- d_i is modulated onto the i^{th}sub-carrier in the modulator. Sub-channels may have different modulation formats. There are N modulators, each comprising a sub-carrier frequency oscillator. The separate sub-channel signals are then added by the adder to generate the baseband OFDM signal.

- To get a passband OFDM signal, the baseband signal is fed into an RF mixer which has a reference signal of frequency $f_c{}'$ and a BPF which rejects the resulting LSB of the mixer output. The passband signal's spectrum is centered at f_c.

Receiver side The processing in this stage reverses the transmitter's work[1].

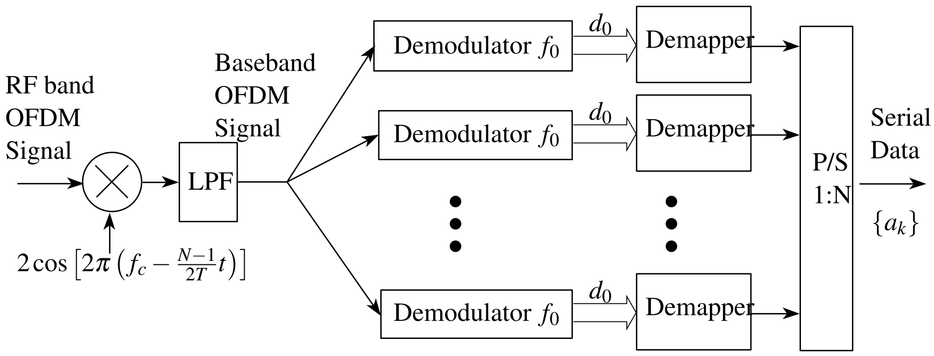

Figure 2.3: Analog OFDM Demodulator

- The passband signal gets translated to baseband through a down-converter with a reference frequency $f_c{}'$.

- The USB is rejected by the Low-pass Filter (LPF) prior to the baseband signal proceeding to the N demodulators. Each demodulator comprises a local oscillator, multiplier, integrator and threshold detector.

DFT-based OFDM Modem

The passband OFDM signal can be written as:

$$s(t) = \Re\left\{ \left(\sum_{i=0}^{N-1} d_i \exp(j2\pi \frac{i}{T}) \right) \exp\left[f_c - \frac{N-1}{2T} \right] \right\} \qquad 0 \leq t \leq T \qquad (2.17)$$

d_i : Complex data symbol

With respect to the lowest carrier frequency ${f_c}'$, the complex envelope of the passband OFDM signal is:

$$\tilde{s}(t) = \sum_{i=0}^{N-1} d_i \exp(j2\pi \frac{i}{T} t) \qquad 0 \le t \le T \tag{2.18}$$

Out of this, the baseband OFDM signal can be written as:

$$s(t) = \Re\left[\sum_{i=0}^{N-1} d_i \exp(j2\pi \frac{i}{T} t)\right] = \Re[\tilde{s}(t)] \qquad 0 \le t \le T \tag{2.19}$$

That is to say that the baseband OFDM signal is the real part of the complex envelope of the passband OFDM signal. Sampling the complex envelope with a sampling period $\Delta t = T/N$ with a normalizing factor $1/N$ gives:

$$s_n = \frac{1}{N} \sum_{i=0}^{N-1} d_i \exp\left(j2\pi \frac{in}{N}\right) \qquad n = 0, 1, \ldots, N-1 \tag{2.20}$$

The equation (2.20) is the IDFT. *Samples of the complex envelope of an OFDM signal can be generated by IDFT[5].* The input complex symbols are in the frequency domain while the output samples which are also complex are in the time domain.

If these samples arrive at the receiver without distortion, the original data symbols d_i can be recovered through DFT as given by:

$$d_i = \sum_{n=0}^{N-1} s_n \exp\left(-j2\pi \frac{in}{N}\right) \qquad i = 0, 1, \ldots, N-1 \tag{2.21}$$

Thus, in summary, OFDM modulation and demodulation can be implemented through IDFT and DFT respectively. IDFT and DFT can be efficiently calculated using their fast algorithms: IFFT and FFT which reduce computational complexity to $O(n \log n)$ from $O(n^2)$.

In practice, complex signal samples $\{s_n\}$ must be separated into a real and imaginary component in the form of an in-phase and quadrature channel. They're each converted to an analog signal before transmission or modulated onto HF carriers for RF band transmission.

Passband sampling Distortion-less *in-phase* and *quadrature* analog signals cannot be recovered through the DAC as the sampling frequency is finite. The sampled signals are:

$$I_n(n) = \frac{1}{N}\sum_{i=0}^{N-1} A_i \cos\left(2\pi\frac{in}{N} + \phi_i\right) \qquad n = 0, 1, \ldots, N-1 \tag{2.22}$$

$$Q_n(n) = \frac{1}{N}\sum_{i=0}^{N-1} A_i \sin\left(2\pi\frac{in}{N} + \phi_i\right) \qquad n = 0, 1, \ldots, N-1 \tag{2.23}$$

Null-to-null bandwidth is $2N/T$ therefore sampling frequency should at least be the same to avoid aliasing. However, the sampling frequency is only N/T as there are N samples within a symbol period T. Thus, the solution is to increase the number of samples to $2N$. N zeros are appended to the data set:

$$d_0, d_1, \ldots, d_{N-1}, 0, 0, \ldots, 0 \tag{2.24}$$

A $2N$-point IDFT is used to generate the OFDM signal while an N-point DFT is used to demodulate the signal. The zero data sub-carriers are called *dummy/virtual* sub-channels. This way there will be no significant aliasing.

DFT-based Digital Modulator This is the process by which an OFDM signal is generated:

- Serial data $\{a_k\}$ is made parallel then mapped onto the symbols $\{d_i\}_{-N/2}^{N/2-1}$.
- The IFFT block transforms the symbols into complex time domain samples $\{s_n{}'\}_{-N/2}^{N/2-1}$.
- The complex samples are separated into real and imaginary parts which are converted from parallel format to serial.
- A cyclic prefix is added to the in-phase and quadrature samples to mitigate ISI, as well as a guard interval so as to reduce the side lobes in the spectrum. The samples are then sent to the DAC.
- The continuous-time *in-phase* and *quadrature* signals are modulated onto an Intermediate Frequency (IF) carrier in quadrature format which shifts both

baseband OFDM signals to the IF band.

$$f_{IF}' = f_{IF} + \frac{1}{2T} \tag{2.25}$$

$$f_{IF}: \quad \text{Center IF}$$

- A second up-conversion stage shifts the IF signal into the passband. A BPF with center frequency f_c and bandwidth $(N+1)/T$ rejects the LSB component of center frequency $f_c - 2f_{IF}$.

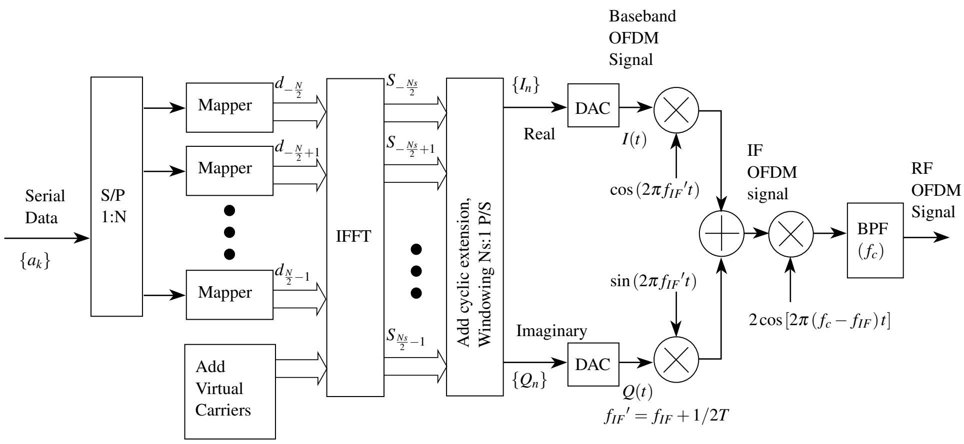

Figure 2.4: Digital OFDM Modulator

DFT-based OFDM Demodulator The demodulator reverses the transmitter's processes:

- The RF signal is down-converted to the IF band.
- The IF signal is then demodulated to the baseband *in-phase* and *quadrature* OFDM signals.
- The demodulated signals are sampled. Their guard interval and cyclic extensions are removed.
- An N-point DFT is performed on the sampled signals to obtain the original signals which are de-mapped back into a binary data stream.

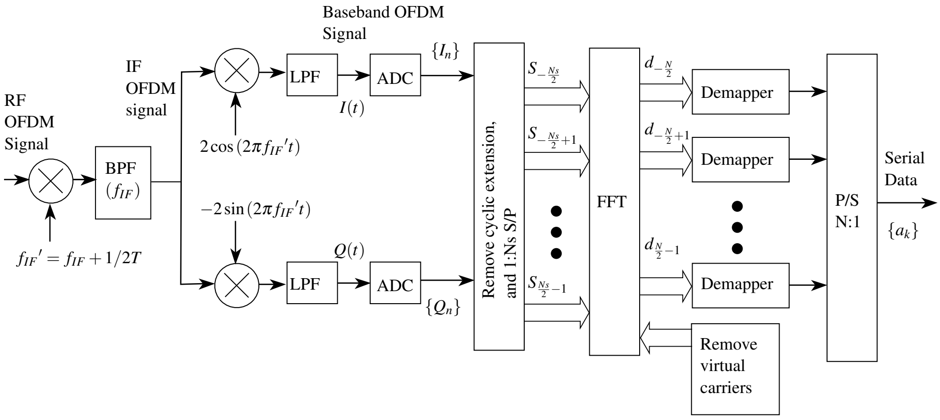

Figure 2.5: Digital OFDM Demodulator

2.2 Multi-path Fading

Fading is the phenomenon that occurs when the amplitude and phase of a radio signal change rapidly over a short period of time or travel distance[5]. Sky wave propagation, which involves passband transmitted signals being refracted by the ionosphere, tends to result in signals arriving at the receiver via different propagation paths and at varying time delays. These are *multi-path components*. These components have different carrier-phase offsets and may add destructively at times, resulting in fading[6].

The free space signal propagation model treats the region between the transmitting and receiving antennas as being free of obstacles. It also assumes that within the region, the atmosphere behaves as a uniform and non-absorbing medium. In this idealized model, the attenuation of RF energy behaves according to an inverse square law[7].

$$L_s(d) = \left(\frac{4\pi d}{\lambda}\right)^2 \tag{2.26}$$

$L_s(d)$: Attenuation loss
d : Distance from transmitter
λ : Wavelength of the propagating signal

For practical channels in which signal propagation takes place in the (non-uniform) atmosphere and close to the ground, the free space model is inadequate. Multi-path propagation causes fluctuations in the received signal's amplitude, phase and angle of arrival, all of which contribute to multi-path fading[7].

In a multi-path channel, the received signal comprises a large number of plane waves whose complex low-pass signal can be modelled as a Gaussian random process[5]:

$$\tilde{r}(t) = r_I(t) + jr_Q(t) \tag{2.27}$$

r : Gaussian Random variable whose $\mu = 0$ and $\sigma^2 = 1$

2.2.1 Fading Channel Characteristics

A fading channel is characterized by several parameters:

Delay Spread In a multi-path channel, signal power at the receiver spreads over a certain period of time. The i^{th}component's delay from the arrival of the 1st component is called *excess delay*, denoted τ_i[5]. Excess delay spread is defined as the longest time delay during which multi-path energy falls to X dB below the maximum:

$$\tau_X - \tau_0 \tag{2.28}$$

τ_0 : The delay of the first arriving signal component.
τ_X : Maximum delay for which a component is within X dB of the strongest.

The *r.m.s. delay spread* $\sqrt{\sigma_\tau}$ is the standard deviation of the delay of multi-path components, weighted proportional to their energy.

Coherence Bandwidth This is the range of frequencies over which the channel can be considered frequency-flat[8] and can be determined using the reciprocal of delay spread[9].

$$B_i = \frac{1}{\tau_i} \tag{2.29}$$

Doppler Spread The *Doppler spectrum* is the bandwidth of fluctuations of received signal strength. *Doppler spread* is a measure of spectrum broadening caused by relative movement of the mobile and base station or of obstacles on the ground.

$$B_D = f_M \tag{2.30}$$

B_D : Doppler Spread
f_M : Maximum Doppler Frequency

The total bandwidth of the received signal is determined by the bandwidth of the baseband signal and the Doppler spread. If baseband $BW \gg B_D$ the effects of Doppler spread are negligible at the receiver.

Coherence Time This is the time interval between the arrival of the first and last multi-path components in a transmitted signal[8].

2.2.2 Channel Classification

Fading channels are classified based on frequency-selectivity and coherence time:

Flat fading

This is also called *non-frequency selective fading*. It is found in wireless channels with a constant gain and linear phase response over a bandwidth greater than signal bandwidth[5]. All of the received multi-path components of a symbol arrive within a single symbol duration. This means that the separate components are not resolvable.

There is no channel-induced ISI. There is still performance degradation, as the unresolvable components may still add destructively to cause a significant reduction in SNR. The fading channel is characterized by:

$$\begin{aligned} B_c &> B_s \\ \sigma_\tau &< T_s \end{aligned} \tag{2.31}$$

B_c : Coherence Bandwidth
B_s : Signal Bandwidth
T_s : Symbol Duration

Frequency Selective Fading

A channel has frequency selective fading if multi-path delay time is greater than symbol time. The signal suffers ISI due to time dispersion. Some frequency components have greater gains than others and thus the channel is considered a linear filter[5]. This channel is characterized by:

$$\begin{aligned} B_c &< B_s \\ \sigma_\tau &> T_s \end{aligned} \tag{2.32}$$

Fast Fading

This is a channel whose impulse response changes within a single symbol duration. Such rapid changes are attributed to motion or Doppler spreading. In fast fading, there is low data rate and high speed of the mobile unit[5].
For a fast fading channel:

$$\begin{aligned} T_s &> T_c \\ B_s &< B_D \end{aligned} \tag{2.33}$$

Slow Fading

In slow fading, the channel impulse response changes at a slower rate than symbol rate. It corresponds to high data rate and low speed of the mobile unit. Characterized by:

$$\begin{aligned} T_s &\ll T_c \\ B_s &\gg B_D \end{aligned} \tag{2.34}$$

2.2.3 Fading Envelope Distribution

Rayleigh Fading

In a channel with a large number of reflective paths but no line-of-sight component, the received signal's envelope is statistically described by a Rayleigh Probability Density Function (PDF)[7].

The Rayleigh distribution describes the statistical time-varying nature of the received envelope of a flat-fading signal[10]. In considering a complex low-pass signal without a dominant multi-path component, $r_I(t)$ and $r_Q(t)$ are Gaussian processes with zero mean and a variance of σ^2 [5]. The Rayleigh envelope PDF is given by:

$$p(z) = \begin{cases} \frac{z}{\sigma^2} \exp\left(-\frac{z^2}{2\sigma^2}\right) & z \geq 0 \\ 0 & z < 0 \end{cases} \tag{2.35}$$

$z:$ Gaussian-distributed envelope magnitude

Maximum probability $p(z)$ occurs at $z = \sigma$: It is assumed that all multi-path com-

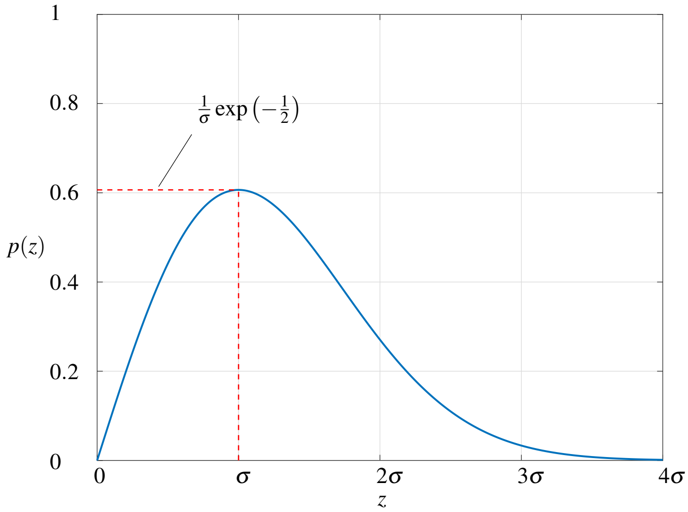

Figure 2.6: Rayleigh probability distribution function

ponents suffer the same attenuation but arrive at different phases, the amplitude corresponding to the Gaussian random variable z[8].

Rician Fading

When there is a dominant (line-of-sight) propagation path, the fading envelope is described by a Rician PDF[7]. Random multi-path components arrive at different angles and are superimposed on a stationary dominant signal. The PDF of the Rician envelope is given by:

$$p(z) = \begin{cases} \frac{z}{\sigma^2} \exp\left(-\frac{z^2+A^2}{2\sigma^2}\right) I_0\left(\frac{Az}{\sigma^2}\right) & z \geq 0 \\ 0 & z < 0 \end{cases} \tag{2.36}$$

A : Peak amplitude of the dominant signal.

$I_0(\cdot)$: 0^{th} modified Bessel function of the 1st kind[5]

K (in dB) is the ratio of the power of the specular signal to the power of the scattered components[8].

$$K = 10\log\frac{A^2}{2\sigma^2} \tag{2.37}$$

Average power is given by:

$$E\{z^2\} = \Omega = A^2 + 2\sigma^2 = 2\sigma^2(1+K) \tag{2.38}$$

The Rician PDF in terms of specular gain and average power:

$$p(z) = \begin{cases} \frac{2z(K+1)}{\Omega}\exp\left(-K - \frac{(K+1)z^2}{\Omega}\right) I_0\left(2z\sqrt{\frac{K(K+1)}{\Omega}}\right) & z \geq 0 \\ 0 & z < 0 \end{cases} \tag{2.39}$$

The figure below shows the Rician distribution density for various K at fixed average power $\Omega = 2$.

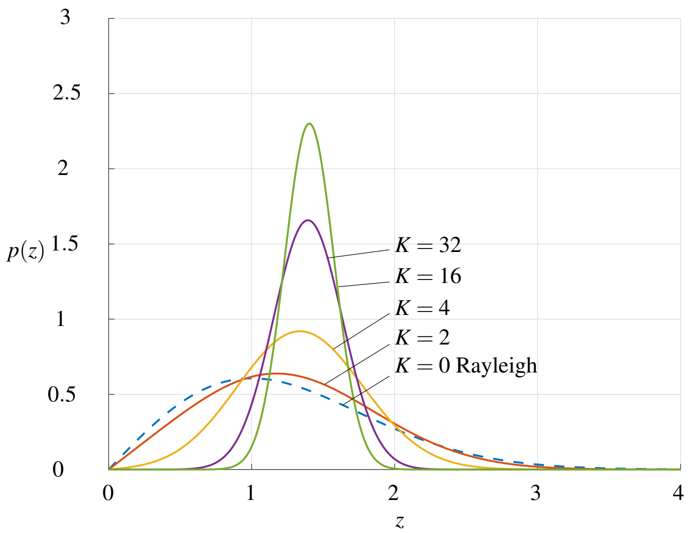

Figure 2.7: Rician probability distribution function

When $A = 0$ and therefore $K = -\infty$, implying the absence of a specular component, the Rician distribution becomes a Rayleigh distribution.

2.2.4 Q function

The Gaussian Q function, or, equivalently, the error function $\mathrm{erf}(\cdot)$ is the tail distribution function of the standard normal distribution. $Q(z)$ is defined as the probability that a normal random variable will obtain a value which is larger than z standard deviations[11]. The Q function is tabulated, and often made available as a built in function in mathematical software tools, for example as `qfunc(x)` in MATLAB.

The Gaussian process is the underlying model for an AWGN channel and its PDF is given by:

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{2.40}$$

μ : Population mean

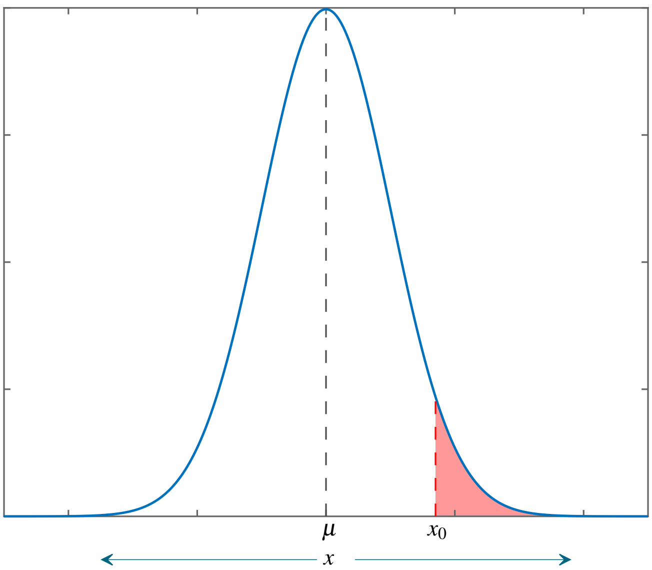

Figure 2.8: Gaussian PDF and Q function

The shaded region is evaluated as:

$$P(X \geq x_0) = \int_{x_0}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{2.41}$$

Substituting $y = \frac{x-\mu}{\sigma}$, this can be re-written as:

$$P\left(y > \frac{x_0 - \mu}{\sigma}\right) = \int_{\frac{x_0-\mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \tag{2.42}$$

The right-hand-side integral is termed as the Q function[12], given by:

$$Q(z) = \int_{\frac{x_0-\mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \tag{2.43}$$

Thus the Q function gives the area of shaded part of the curve with the transformation $y = \frac{x-\mu}{\sigma}$ applied to the Gaussian PDF.

2.2.5 Digital Modulation in slow, flat fading channels

A signal undergoes multiplicative variation in a flat fading channel. This means that both amplitude and phase are affected since the multiplying coefficient is a complex one. In the worst case, amplitude attenuation and phase shift may be considered constant over a symbol duration. For a transmitted low-pass signal $\tilde{s}(t)$, the received low-pass complex signal may be written as:

$$\tilde{r}(t) = ze^{-j\phi}\tilde{s}(t) + \tilde{n}(t) \tag{2.44}$$

$ze^{-j\phi}$: Complex channel gain

$\tilde{n}(t)$: Low-pass complex additive Gaussian noise.

The channel effect is corrected by dividing the received signal by the channel gain.

$$\tilde{r}_1(t) = \frac{\tilde{r}(t)}{ze^{-j\phi}} = \tilde{s}(t) + \frac{\tilde{n}(t)}{ze^{-j\phi}} \tag{2.45}$$

Complex channel gain is estimated through *equalization/channel estimation*.

2.3 Communication Link Parameters

The main consideration when designing a digital communication system is its ability to exceed certain measurable thresholds when functioning over a non-ideal channel[13]. These parameters are discussed here.

2.3.1 Peak-to-Average Power Ratio

PAPR is the relation between maximum power of a sample in a given OFDM transmit symbol divided by the average power of the symbol[14]. An OFDM signal is a sum of N independent, complex random variables, each of which is a signal of a different sub-carrier frequency. In an extreme case, different sub-carriers may line up in phase at the same time instance, producing an amplitude peak which is equal to the sum of the amplitudes individual sub-carriers.

A transmitted OFDM signal (that tends to) have large peaks suffers significant degradation in performance when the signal passes through a saturated High Power Amplifier (HPA). Due to the saturation of HPAs, in-band distortion is introduced which causes out-of-band radiation, interfering with adjacent channels[15] and consequently increasing BER.

High PAPR requires an expensive HPA, Analog-to-Digital Converter (ADC) and DAC. Without large power back-offs, out of band power cannot be kept within specified limits, causing inefficiencies in amplification and requiring expensive (powerful) transmitter amplifiers.

The problem of PAPR is nost acutely felt at the transmitter output where to transmit the peaks without clipping, the following requirements must be met[1]:

- The DAC must have enough bits to accommodate the peaks.
- The power amplifier must remain linear over an amplitude range that includes peak amplitudes.

According to [16], for an OFDM system with N sub-carriers, the baseband with normalized power from an IFFT operation is:

$$s(t) = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} S_i e^{jk\Delta f t} \tag{2.46}$$

Δf : Sub-carrier spacing
S_i : Spectrum of the i^{th}sub-carrier.

From this expression, PAPR of an OFDM signal can be derived as:

$$\text{PAPR} = \frac{\left(|x(t)|^2\right)_{\max}}{E\left\{|x(t)|^2\right\}} \tag{2.47}$$

$E\{\cdot\}$: Average

2.3.2 Average Signal-to-Noise Ratio

SNR is defined as the ratio of signal power to background noise power *within the same bandwidth*[17]. For an effective transmission technology, retrieval of bits from a received waveform should be as error-free as possible in spite of non-idealities in the communication system. This can be done by achieving the best possible SNR free of ISI.

SNR degrades through cumulative losses (absorption, scattering, reflection of a portion of signal) or increasing noise/interference power. According to [7], the primary causes of error performance degradation are:

- The effect of filtering at the transmitter, channel, and receiver for non-ideal system transfer with ISI.
- Electrical noise and interference produced by a variety of sources such as *atmospheric noise, switching transients* and *intermodulation noise*.

Average SNR is the most appropriate measure in a communication system subject to fading.

$$\bar{\gamma} \triangleq \int_0^\infty \gamma P_\gamma(\gamma) d\gamma \tag{2.48}$$

$$\begin{aligned} \gamma : &\quad \text{Instantaneous SNR} \\ P_\gamma(\gamma) : &\quad \text{Probability density function of } \gamma. \end{aligned}$$

It is the statistical averaging of instantaneous SNR over the probability distribution of the fading.

2.3.3 Outage Probability

When the SNR level at the receiver *falls below a certain threshold value*, the system is said to be in outage. Accurate detection may not be possible below such a threshold, resulting in reception of many bits in error, rendering the system unreliable[18].

Outage probability is defined as the probability that instantaneous SNR falls below a certain specified threshold:

$$P_{\text{out}} = \int_0^{\gamma_{\text{th}}} P_\gamma(\gamma) d\gamma \tag{2.49}$$

$$\gamma_{\text{th}} : \quad \text{SNR threshold}$$

Outage probability is the cumulative distribution function of SNR.

2.3.4 Complexity

System Complexity

Refers to the circuit implementation and the technical difficulty associated with the system. This usually reflects upon the cost of manufacturing which is a key consideration when selecting a communication system.

For most communication systems, the demodulator is usually more complex than the modulator, with coherent modulators being more complex because of their need for carrier recovery. Implementation of sophisticated algorithms adds to the system complexity.

Computational Complexity

Computational complexity is a measure of the number of communication bits that the participants of a communication system need to exchange to perform certain tasks.

The study of computational complexity focuses on *problems that model typical computational needs of communication scenarios* and attempts to discern boundaries on the requisite amount of communication between processors for these scenarios.

For a collection of communication problems f, a lower bound L_f can be found for their computational complexity under various protocols such as:

- One-way
- Multi-way
- Multi-party

The value L_f points to possible algorithmic improvements.

2.3.5 Average Bit Error Probability

This is the probability that the reconstructed symbol at the receiver output differs from the transmitted binary symbol. This measure is the most revealing about a system's reliability. It most prominently features in all system performance evaluations[13].

The conditional bit error probability of a system is a non-linear function of instantaneous SNR. The nature of this non-linearity is a function of the modulation and detection scheme being used[17]. Suppose that conditional bit error probability is:

$$P_b(E|\gamma) = C_1 \exp(a_1 \gamma) \tag{2.50}$$

$$C_1, a_1 : \quad \text{Constants}$$

This would be the case for coherent detection of PSK. The average bit error probability can be written as:

$$\begin{aligned} P_b(E|\gamma) &\triangleq \int_0^\infty P_b(E|\gamma) P_\gamma d\gamma \\ &= \int_0^\infty C_1 \exp(-a_1 \gamma) P_\gamma d\gamma \end{aligned} \tag{2.51}$$

The bit error rate of a communication system can be calculated for each SNR value as:

$$\text{BER} = \frac{\text{Number of error bits}}{\text{Number of transmit bits}} \tag{2.52}$$

2.3.6 Symbol Rate

This is also known as *baud rate*. It represents the number of signalling events across the transmission medium per unit time using a digitally modulated signal. Symbols of various sizes are quantized and referenced to an M-ary alphabet set, each symbol representing one of all the possible discrete amplitude levels. For a symbol duration of T seconds, the baud rate is $\frac{1}{T}$.

For a communication system where M symbols are used, $k = \log_2 M$ bits get transmitted during each T. M-ary symbols enable a k-fold increase in data rate within the same bandwidth. Thus their use reduces bandwidth requirements by a factor of k[7].

2.3.7 Spectral Efficiency

It is also called bandwidth efficiency and is defined by [13]:

$$\rho = \frac{R_b}{B} \tag{2.53}$$

$$\begin{aligned} \rho &: \quad \text{Spectral Efficiency} \\ R_b &: \quad \text{Bit Rate} \\ B &: \quad \text{Bandwidth} \end{aligned}$$

It is advisable to use spectrum-efficient modulation techniques that minimize bandwidth utilization and thus reduce spectral congestion[7].

2.3.8 Latency

This is the time interval between source stimulation and destination symbol change. Latency is a consequence of:

- Limited velocity with which a transmission can propagate.
- Delays due to data processing stages along the channel.

The lower boundary of latency is fixed by the medium being used for communication. Latency also limits the rate of information transfer due to the inherent limit on the amount of information that can be in transit at any time instance.

Latency can be mitigated by:

- Synchronization between transmitter and receiver. Both are synchronized to a master clock which enables the receiver to compensate for the delay offset.
- Embedding clock signal in the data stream[6]. This permits asynchronous operation and facilitates higher data transmission rates as there is no limit on the amount of information that can be in transit at a time.

Chapter 3

Methodology

The project was to be executed in the following sequence:

i. Simulate IEEE 802.11 standard OFDM on MATLAB over an Additive White Gaussian Noise (AWGN) channel.

ii. Obtain average PAPR performance of the simulated system.

iii. Simulate the OFDM system over a Rayleigh fading channel.

iv. Simulate the OFDM system over a Rician fading channel.

v. Obtain BER performance of all simulated systems.

vi. Use MATLAB's regression tools to derive an expression for the BER curves generated.

3.1 OFDM Simulation

The communication system was to be modeled in MATLAB. The blocks in the diagrams that follow would have analogous objects programmed to represent them.

3.1.1 OFDM Transmitter

The transmitter:

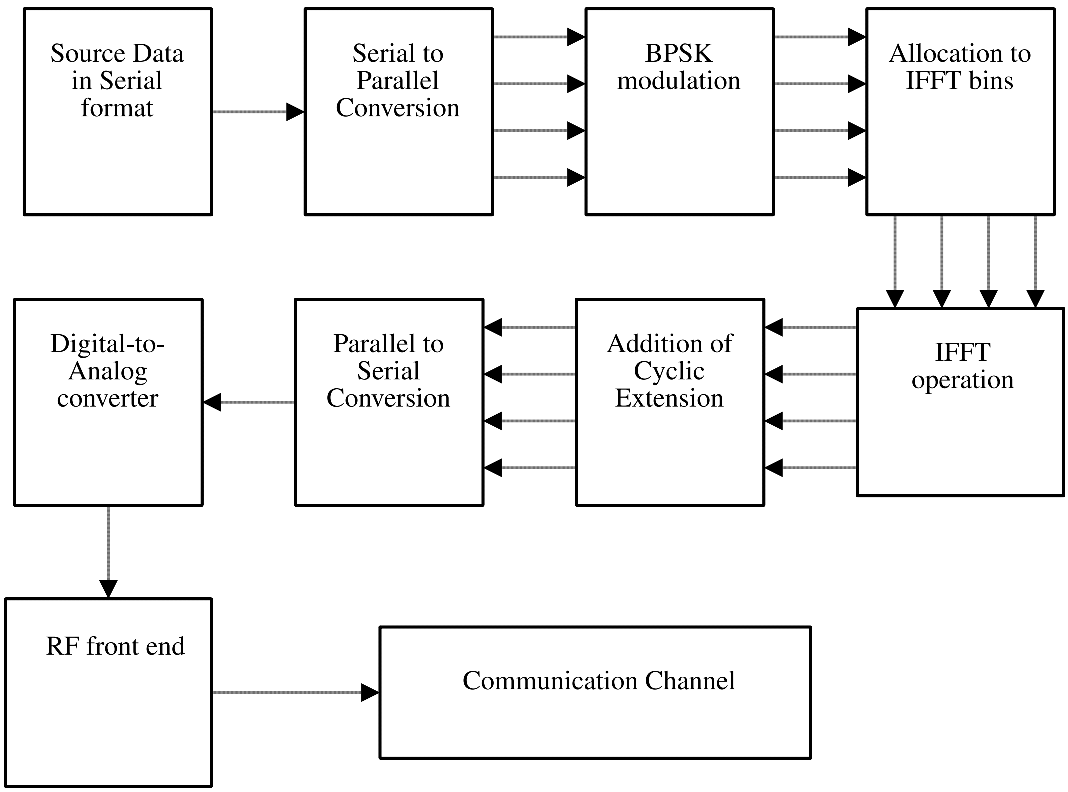

Figure 3.1: OFDM Transmitter Block Diagram

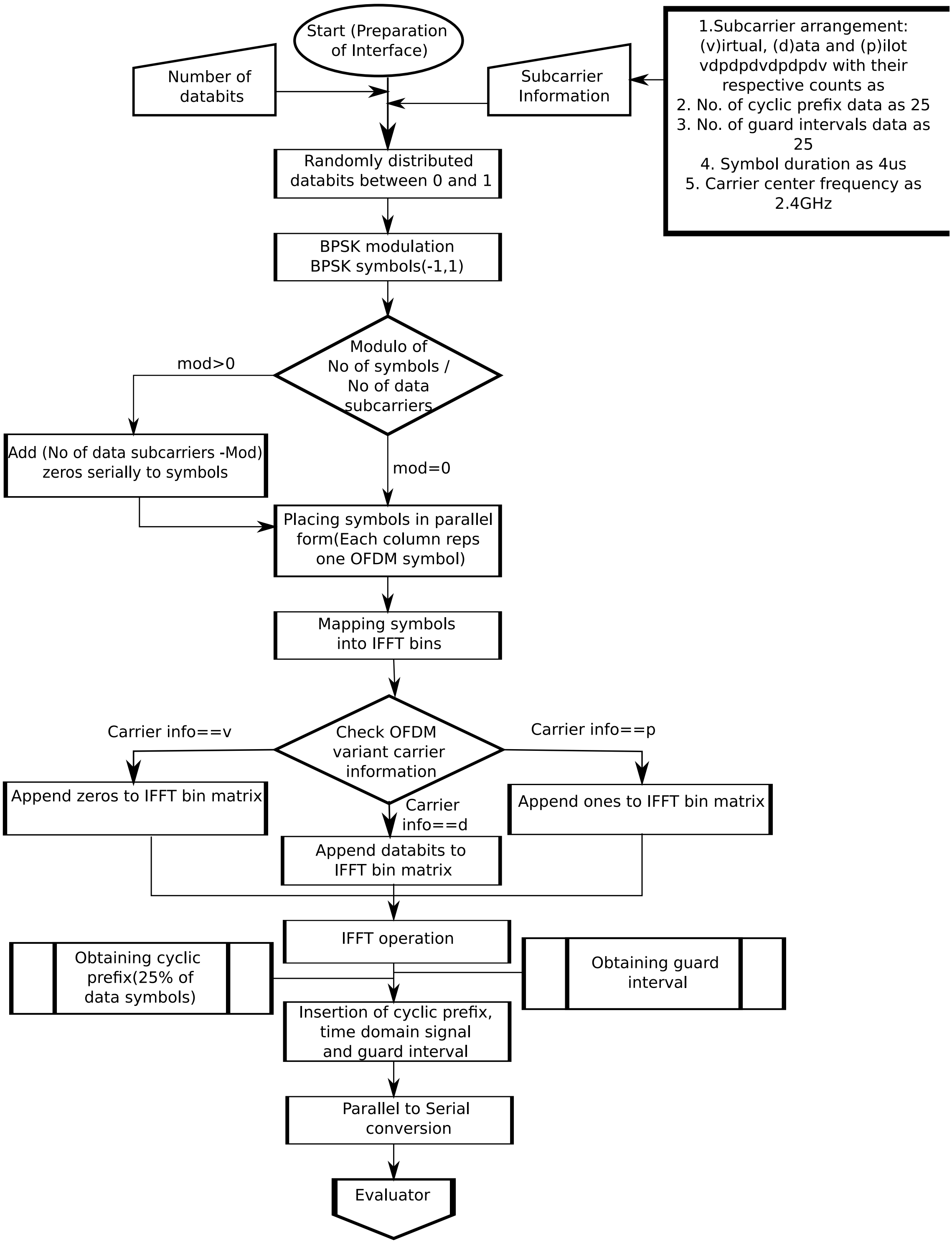

Figure 3.2: Transmitter Flowchart

3.1.2 Channel

The channel models that will be simulated are:

AWGN Channel

This channel adds white Gaussian noise to the signal as it is propagated through.

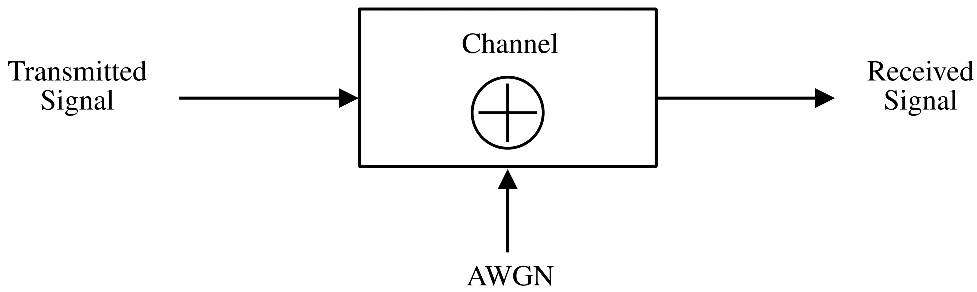

Figure 3.3: AWGN channel Block Diagram

Rayleigh Fading Channel

The Rayleigh fading channel assumes no specular components in the transmitted signal. The signal Rayleigh fading has a Rayleigh Distribution.

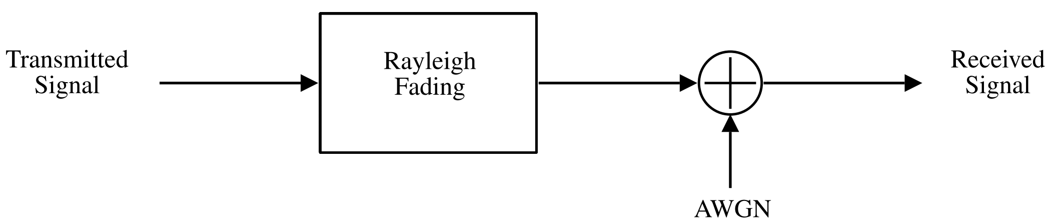

Figure 3.4: Rayleigh Fading Channel Block Diagram

Rician Fading Channel

The Rician fading channel has multipath components as with the Rayleigh fading channel but has, in addition, a dominant line-of-sight component. The strength of the specular component determines the shape of the Rician distribution.

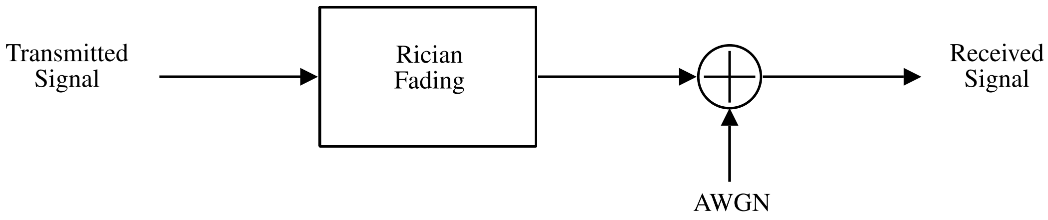

Figure 3.5: Rician Fading Channel Block Diagram

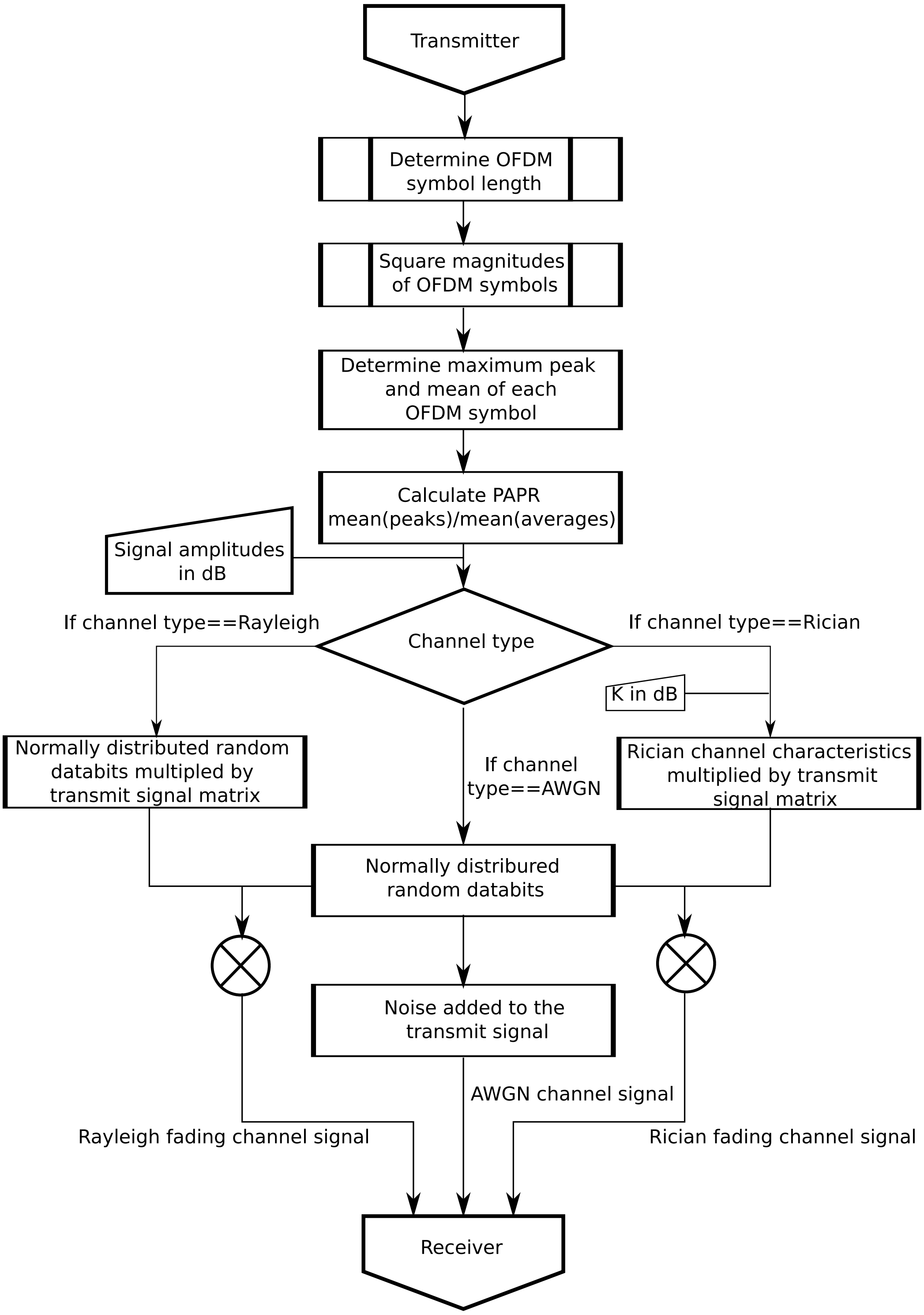

Figure 3.6: Channel Flowchart

3.1.3 OFDM Receiver

The receiver:

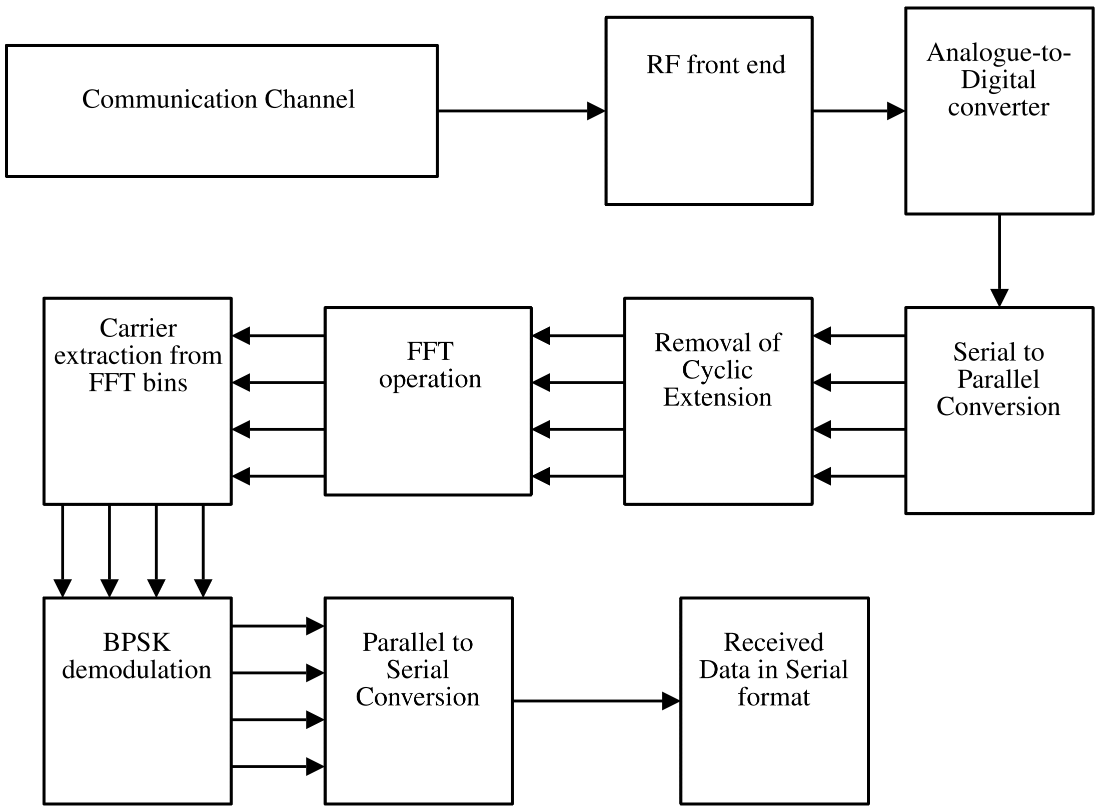

Figure 3.7: Receiver Block Diagram

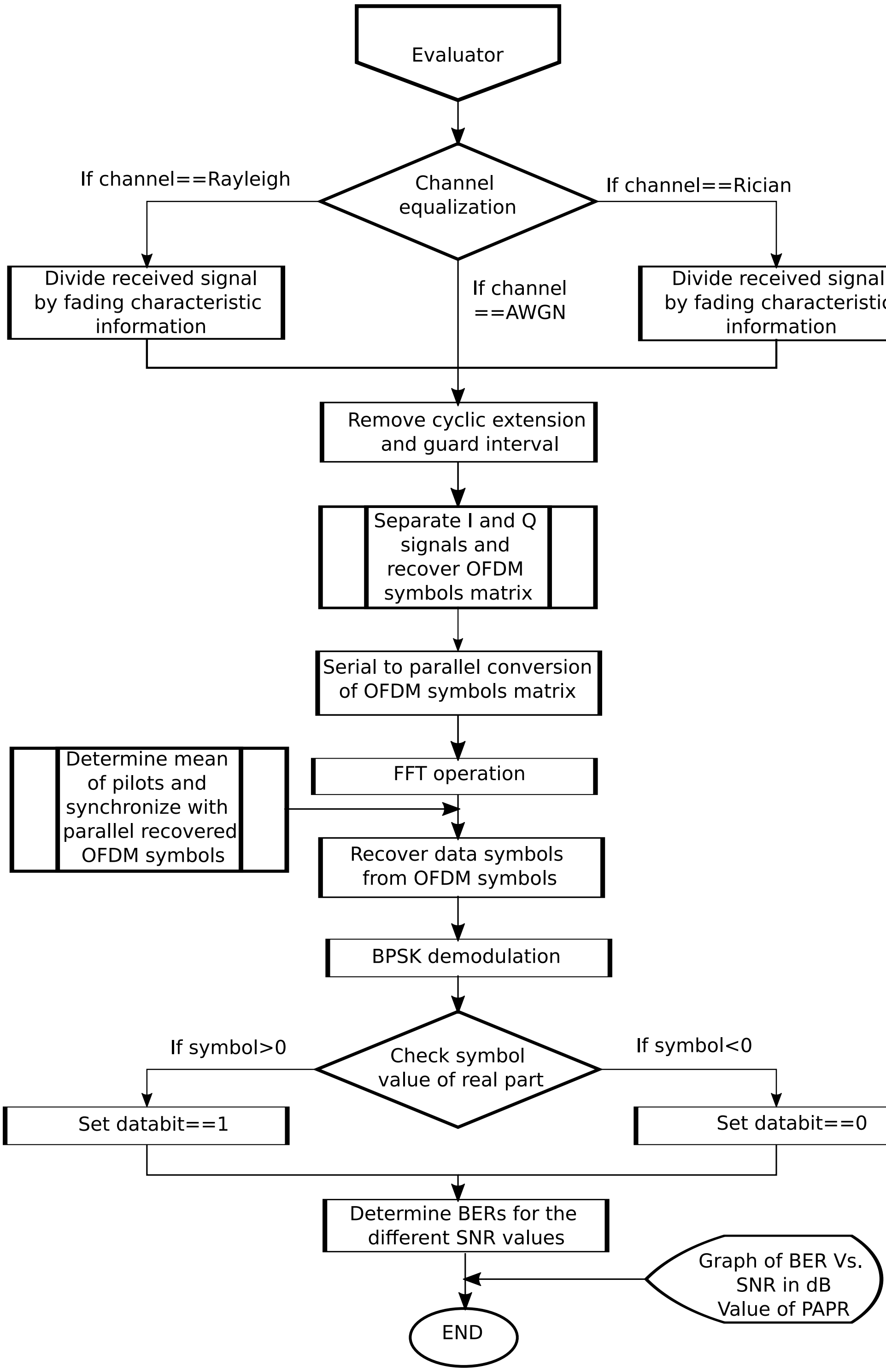

Figure 3.8: Receiver Flowchart

3.1.4 BER Evaluation

The system's BER performance is to be evaluated within Simulink as well. The equivalent implementation would be as in figure 3.9

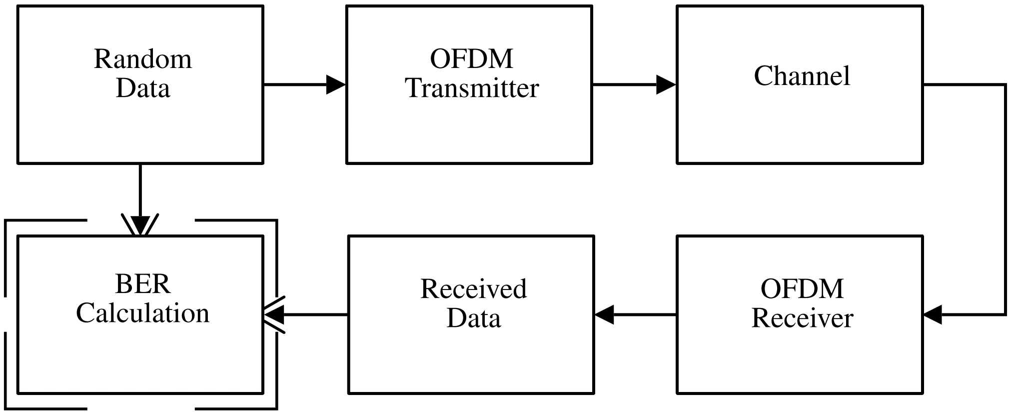

Figure 3.9: BER Evaluation

3.2 PAPR Evaluation

PAPR is evaluated at the transmitter output. To achieve this measurement, the peak amplitude is sampled over several symbol durations then its square divided by the square of the average signal amplitude.

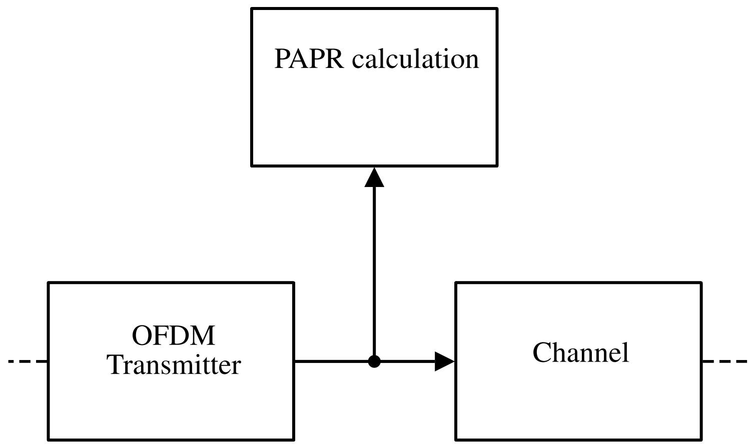

Figure 3.10: PAPR Evaluation

3.3 Curve fitting

To determine the expressions for the resulting BER curves from the system's simulation, Q function curves are fitted onto a line of best fit from the resulting BER data points:

3.3.1 Rayleigh and Gaussian Models Curve Fitting

From empirical observation over multiple simulation instances, see figure 5.2, it was determined that the positions of the Gaussian and Rayleigh BER curves are constant. To fit them to a curve, the Q function was scaled and shifted until it aligned perfectly with the curve. For higher SNRs, a different Q function applied for both models.

3.3.2 Rician Model Curve Fitting

Obtaining a general expression for Rician fading followed these steps:

- The system was simulated with Rician fading model over values of K ranging from 0 to 10 in increments of 1, as in figure 5.2.

- For each curve, a scaled and shifted Q function was fitted. See figure 4.5.

$$y(x) = aQ\left(\frac{x-b}{c}\right) \tag{3.1}$$

$$\begin{aligned} a: &\quad y \text{ axis scaling} \\ b: &\quad x \text{ axis shift} \\ c: &\quad x \text{ axis scaling} \end{aligned}$$

- The values obtained are:

K dB	a	b	c
10	0.4	8.5	1.9
9	0.4	8.7	1.95
8	0.4	8.9	2
7	0.4	9.1	2.1
6	0.41	9.4	2.2
5	0.41	9.8	2.35
4	0.41	10.1	2.53
3	0.42	10.5	2.55
2	0.425	10.8	2.6
1	0.43	11	2.69
0	0.44	11.1	2.72

- These values are then plotted as in figure 3.11 and fitted to a curve which relates their value to K. This yields the expressions:

$$a = \begin{cases} 0.0004K^2 - 0.0081K + 0.4392 & 0 \leq K < 5 \\ 0.4 & 5 \leq K \end{cases} \tag{3.2}$$

$$b = -0.2855K + 11.522 \tag{3.3}$$

$$c = -0.0917K + 2.785 \tag{3.4}$$

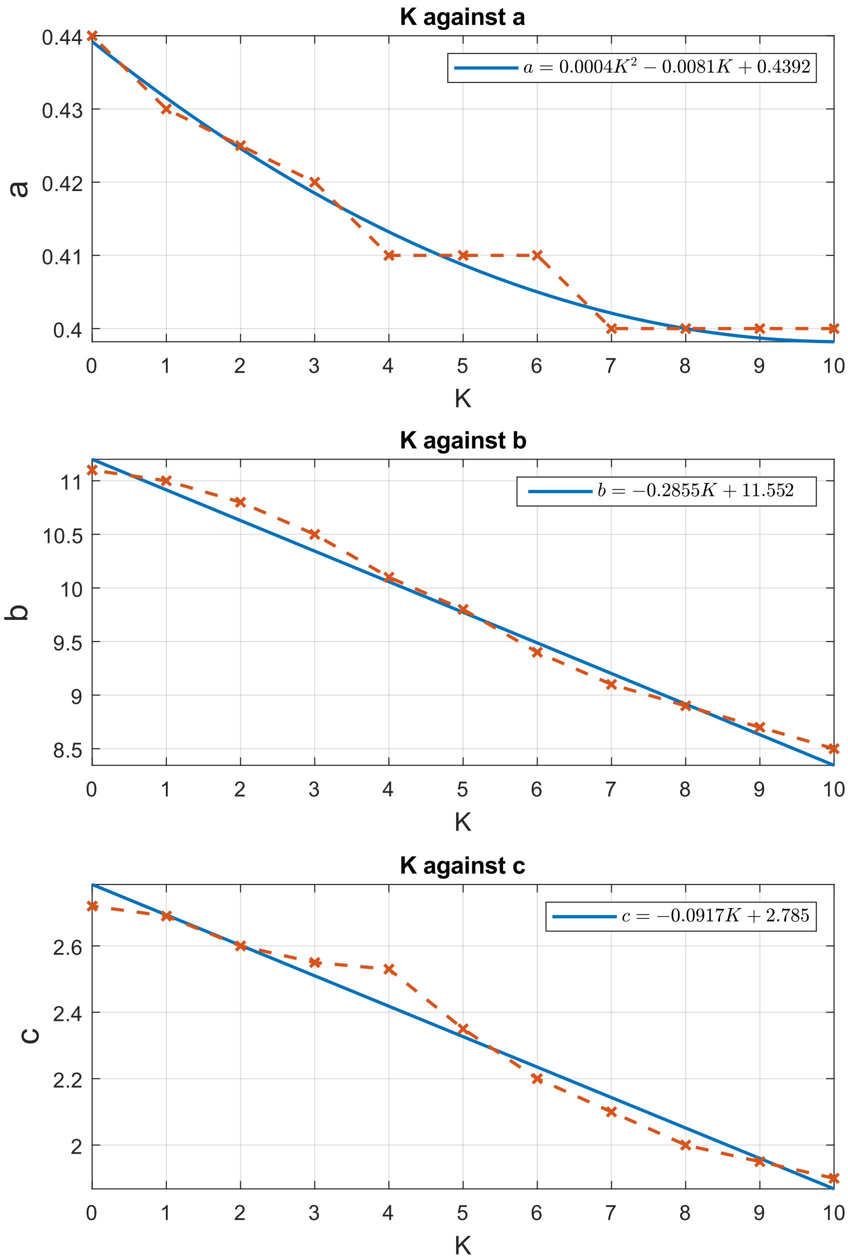

Figure 3.11: Relation of a, b and c to K

Chapter 4

Results

An OFDM-based system was modeled according to the IEEE 802.11 standard. *One million* simulated bits were used to evaluate the system. A derivative variant was designated from the standard, having the following properties:

	IEEE 802.11	*Project Variant*
Data sub-carriers	48	60
Pilot sub-carriers	4	4
Virtual subcarriers	12	16
Cyclic extension	25%	25%
Guard Interval	25%	25%

Table 4.1: Standard OFDM vs the Project Variant

IEEE 802.11 standard sub-carrier arrangement.

. . . . . | | | | | ↑ | | | | | | | | | | | | | | ↑ | | | | | | . | | | | | | ↑ | | | | | | | | | | | | | ↑ | | | | | . . . . . .

Project Variant sub-carrier arrangement.

. . . . | | | | | | | | | | | | ↑ | | | | | | | | | | | | | | ↑ | | | | | | . | | | | | | ↑ | | | | | | | | | | | | | | | ↑ | | | | | | | | | | | . . . .

4.1 BER Curves

The standard OFDM was simulated to obtain its BER performance curves over the three channel models *(Rayleigh, Rician and AWGN.)* These were compared to the variant's.

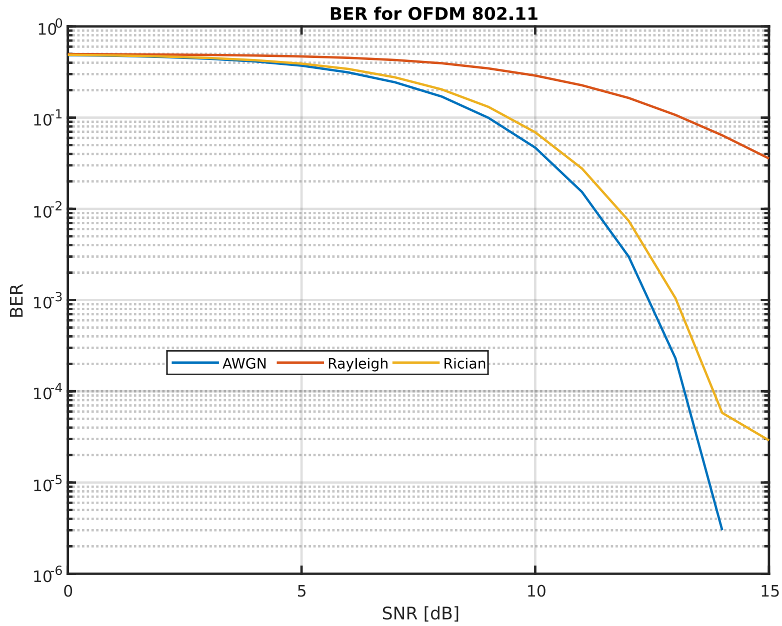

Figure 4.1: IEEE 802.11 BER Performance Curves

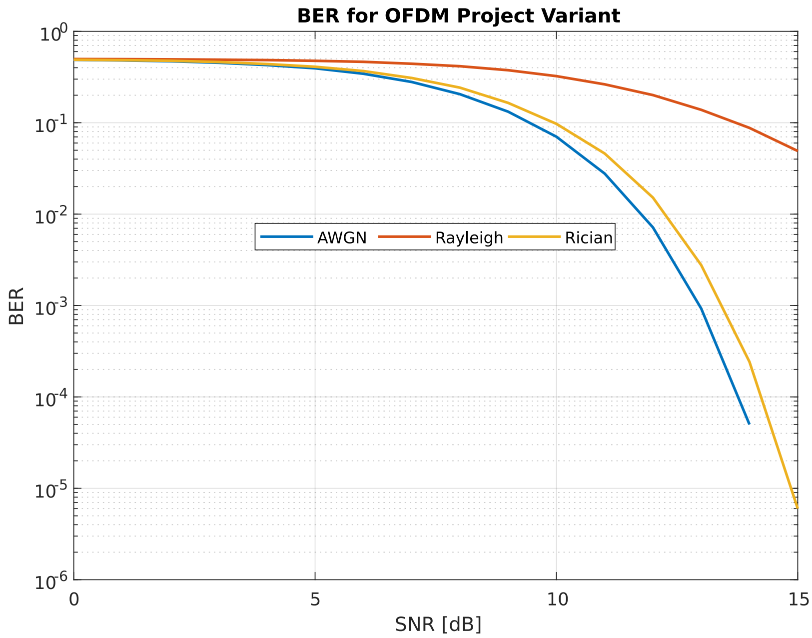

Figure 4.2: Project Variant BER Performance Curve

4.2 PAPR Performance

The PAPR was determined according to the definition stated in the literature review: the relation between maximum sample power in an OFDM symbol divided by the symbol's average power. For consistency, individual symbols' PAPRs were averaged to get each variant's equivalent value.

The values obtained are:

- *IEEE 802.11* standard OFDM implementation: 7.25 dB
- *OFDM Variant* 7.35 dB

The increase in average PAPR from the standard OFDM implementation to the variant is expected since there are more data sub-carriers in the variant. This means that more signalling energy is expended in transmitting a symbol.

4.3 Curve Fitting

4.3.1 Gaussian Channel Model

The equation representing BER for the project's OFDM variant in a Gaussian channel is:

$$y = f(x) = \begin{cases} 0.47Q\left(\frac{x-7.5}{2.5}\right) & -\infty \leq x \leq 9.5 \\ 0.15Q\left(\frac{x-10.5}{1.17}\right) & 9.5 < x \leq 15 \\ 0 & \text{otherwise} \end{cases} \tag{4.1}$$

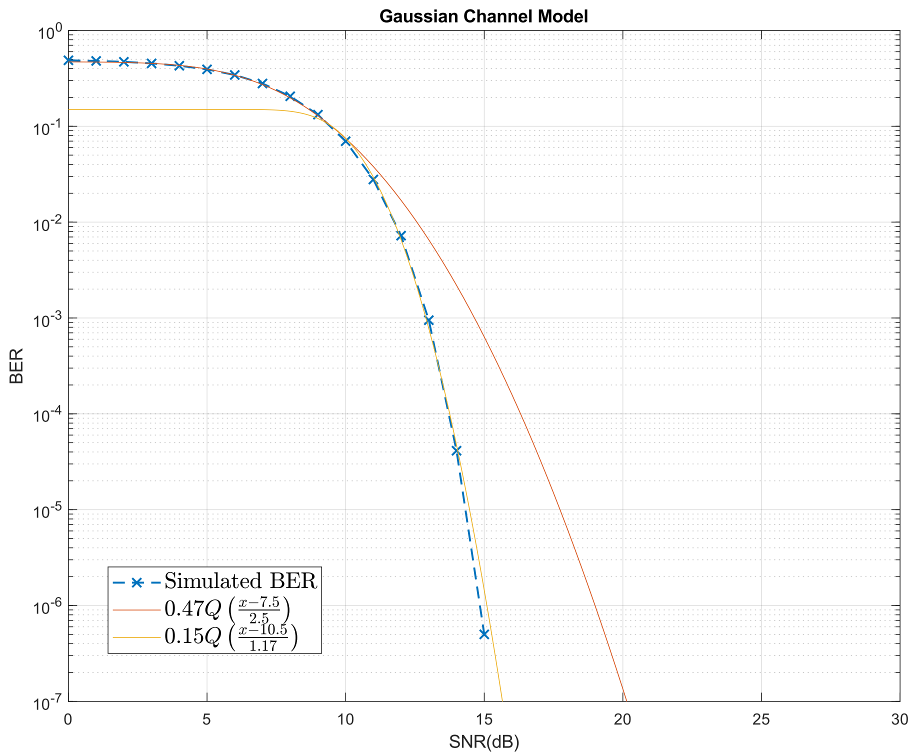

Figure 4.3: AWGN Channel curve fitting

4.3.2 Rayleigh Channel Model

The equation representing BER for the project's OFDM variant in a Rayleigh fading channel is:

$$y = f(x) = \begin{cases} 0.5Q\left(\frac{x-11.1}{3.05}\right) & -\infty \leq x \leq 17 \\ 1_E 4Q\left(\frac{x-35}{11}\right) & \text{otherwise} \end{cases} \tag{4.2}$$

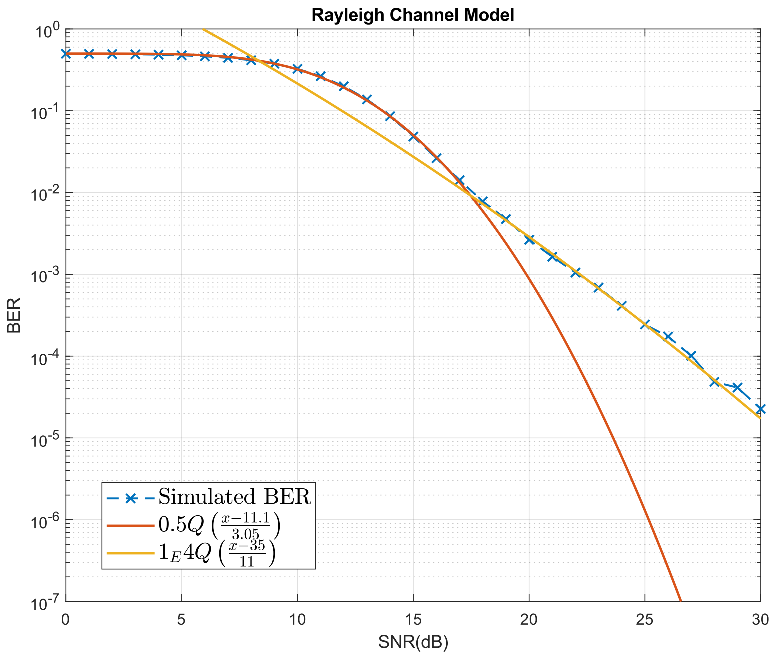

Figure 4.4: Rayleigh Channel Curve Fitting

4.3.3 Rician Channel Model

The equation representing BER for the project's OFDM variant in a Rician fading channel is:

$$y = f(x) = \begin{cases} 0.5 & -\infty < x \leq 0 \\ aQ\left(\frac{x-b}{c}\right) & 0 < x \end{cases} \tag{4.3}$$

$$x: \quad \text{SNR}$$

The values of a, b and c are determined by K according to equations (3.2), (3.3) and (3.4) respectively. The range of K for which equation (4.3) is valid is:

$$0 \leq K \leq 10.5 \tag{4.4}$$

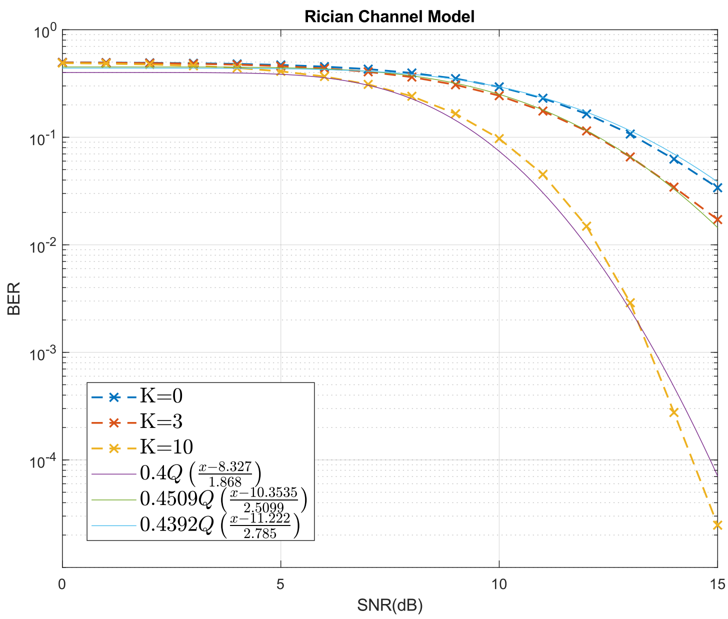

Figure 4.5: Rician Channel Curve Fitting

Chapter 5

Conclusion

5.1 Challenges

5.1.1 Computational complexity

For the simulated communication system to approximate the actual physical implementation, it would require passband elements, which also implies digital to analog conversion. The problem with simulating analog signals is that in reality they require infinite bandwidth and therefore an infinite number of samples to achieve fidelity.

That said, with limitations in memory capacity and processing power, true analog simulation is in reality infeasible with the technology available. Consequently, passband fidelity can only at best be roughly approximated. Thankfully, it has been found that the error performance of various modulation schemes is the same regardless of transmission band.

Thus, while a true passband communication system could not be simulated, its error performance was still obtained through the successfully modelled and simulated baseband model.

5.1.2 Randomness

The simulation depends on random processes for the following vital functions:

- The data source
- Additive White Gaussian Noise

- Fading channel models

It's factual that the consistency of results in the face of such entropy underlines the simulation's reliability. However, due to the intrinsic chaotic nature of randomness, on some occasions the results in unexpected instances suffer indeterministic offsets, invalidating outcomes.

This is mitigated by using high bit counts. The effect of outliers in the large population is effectively eliminated, and the accuracy of results restored.

5.1.3 FFT/IFFT induced errors

In the early stages of the simulation's modelling, prior to implementation of OFDM, the system's error performance was found to fully conform to expected outcomes, particularly in AWGN channel models. However, with implementation of OFDM through the fast IDFT/DFT algorithm pairs, it was found that the error performance curves shifted right, indicating an decline in error performance.

The authors of this report theorize that the finer resolution of the orthogonal modulation product suffers channel degradation more severely than single-carrier modulated signals. This remains unproven and is grounds for further study.

5.1.4 Model variation from real world performance

The channel model's used can only approximate real world transmission on a macro scale. Under closer scrutiny, the models are insufficient for specific cases which require unique models of their own.

The only way around this issue is to develop unique models for each specific use case which again fails the requirement for generalizability.

5.2 Observations

5.2.1 Outcome reliability and simulated bits

It was found through innumerable iterations that the resulting bit error performance curves' consistency was commensurate with the number of simulated bits being transmitted through the system. This is due to the following:

- Statistically, large populations eliminate the spurious effects of randomness, as highlighted in the challenges section.

- Higher bit counts allow for smaller BER values, which in turn facilitate continuous, deeper and more accurate BER performance curves.

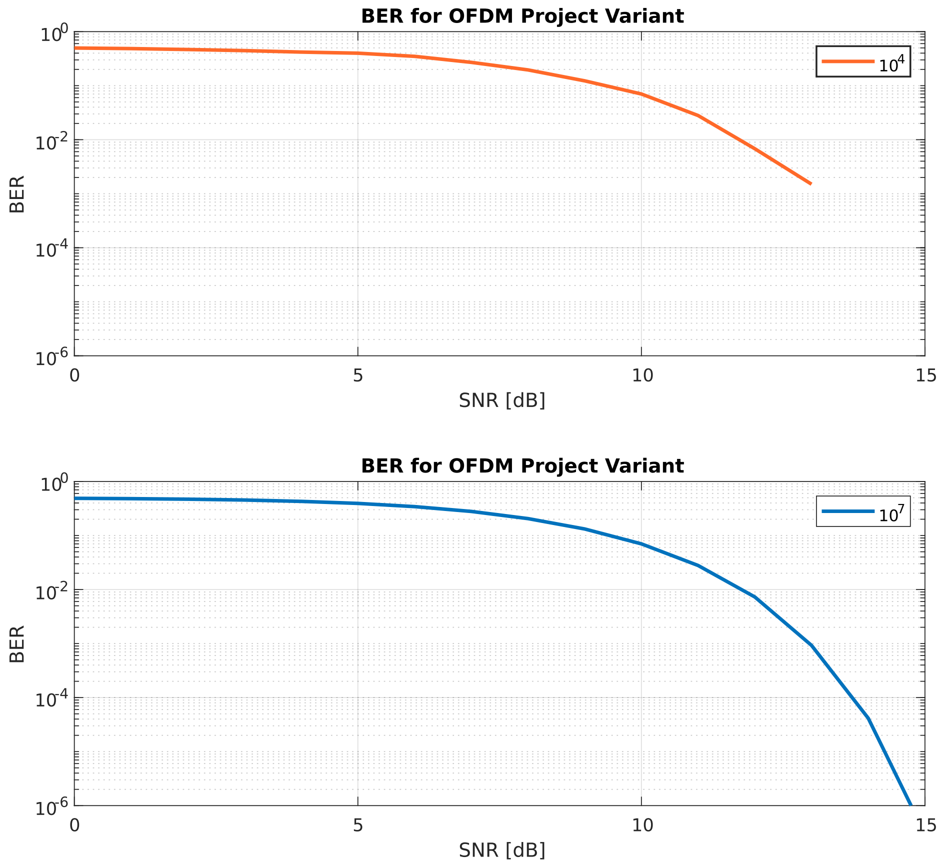

Figure 5.1: Effect of data-bits on BER performance

5.2.2 Effect of K on BER curve

It was observed that as $K \to \infty$, the BER performance curve for the Rician channel model approximates an AWGN channel while as $K \to -\infty$, the Rician channel's error performance approximates a Rayleigh channel model.

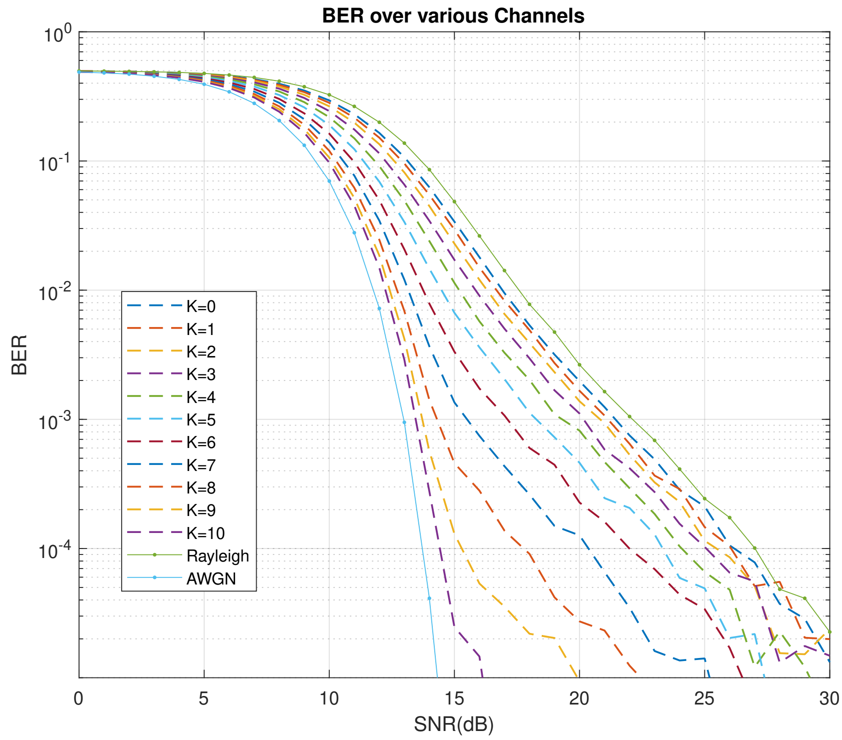

Figure 5.2: Effect of K on BER performance

5.3 Recommendations

5.3.1 Case specific modelling

For the simulation to truly represent real world performance, the channel model has to be modelled to the specific intended use case for it. The generalized models implemented in this project, while fair approximations, are inadequate for micro-scale modelling.

5.3.2 Various modulation schemes for each sub-carrier

Real-world communication systems using OFDM implement different modulation schemes for different sub-carriers depending on the signal degradation at specific frequencies. This technique is mostly used in frequency-selective fading channels.

This couldn't be implemented here as it lay beyond this project's scope.

Bibliography

[1] Ahmad R. S. Bahai, Burton R. Saltzberg, and Mustafa Ergen. *Multi-carrier Digital Communications*. Springer US, 2nd edition, 2004.

[2] Haesik Kim. *Wireless Communications Systems Design*, chapter 7. Wiley & Sons, Ltd., The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, United Kingdom, 1st edition, 2015.

[3] S. B. Weinstein. The history of orthogonal frequency-division multiplexing [history of communications]. *IEEE Communications Magazine*, 47(11):26–35, 2009.

[4] Abdul-Salam Mohammed Abood. A performance enhancement of ieee 802.11a OFDM standard model. *Journal of Telecommunications*, 10(2):27 – 30, September 2011.

[5] Fuqin Xiong. *Digital Modulation Techniques*, chapter 12. Artech House, Inc., 2nd edition, 2006.

[6] John G. Proakis and Masoud Salehi. *Fundamentals of Communication Systems*. Pearson Education, 2nd edition, 2014.

[7] Bernard Sklar and Pabitra Kumar Ray. *Digital Communications: Fundamentals and Applications*. Pearson Education, 2nd edition, 2014.

[8] Kaveh Pahlavan and Prashant Krishnamurthy. *Principles of wireless networks*. Prentice Hall PTR, 2002.

[9] John G. Proakis and Masoud Salehi. *Communication Systems Engineering*. Prentice-Hall, Inc., USA, 1994.

[10] Theodore S. Rappaport. *Wireless Communications*. Prentice Hall PTR, 1996, 2nd edition, 2007.

[11] Chiani Marco, Dardari Davide, and Simon Marvin K. New exponential bounds and approximations for the computation of error probability in fading channels. *IEEE Transactions on Wireless Communications*, page 1, July 2003.

[12] Robert S. Witte and John S. Witte. *Statistics*. John Wiley & Sons, Inc., 2017.

[13] Simon Haykin. *Communication Systems*. John Wiley & Sons, Inc., 2001.

[14] Sajjad Hussain. *PEAK TO AVERAGE POWER RATIO ANALYSIS AND REDUCTION OF COGNITIVE RADIO SIGNALS*. Theses, Université Rennes 1, October 2009.

[15] Paredes Martha C. and M. Julia Fernández-Getino García. The problem of peak-to-average power ratio in OFDM systems. *CoRR*, abs/1503.08271, 2015.

[16] SHAN Weifeng, MENG Baohong, LIU Ningning, LI Hui, and ZHANG Hongwei. Simulation on peak-to-average power ratio for orthogonal frequency division multiplexing. *Wuhan University Journal of Natural Sciences*, 6:1873 – 1876, May 2006.

[17] Marvin K. Simon and Mohammed-Slim Alouini. *Digital Communication over Fading Channels: A Unified Approach to Performance Analysis*, chapter 1, pages 4–12. John Wiley & Sons, Inc., 2000.

[18] Brijesh Kumbhani, Kshetrimayum, and Rakhesh Singh. *MIMO Wireless Communications over Generalized Fading Channels*. CRC Press, Inc., USA, 1 edition, 2017.

Appendix

The code in its entirety can be found by cloning the git repository at:

https://signalsproject@bitbucket.org/signalsproject/simulation.git

Program Flow Modules

main.m

```
clc;clear;
%% Interface instance
CLI = Interface();
% Interface() properties
bitCount = CLI.bitCount;
rfFlag = CLI.rfFlag;
Ts = CLI.Ts;
fc = CLI.fc;
KdB = CLI.KdB;
%channelType = CLI.channelType;
channelType = ["gauss", "rayl", "rice"];
variant = CLI.variant;
sigAmp = 0:1:30;

%% Simulation of Communication
dataSource = ofdm.DataSource(...);
transmitter = ofdm.Transmitter(...);
% Evaluator: Gets PAPR from transmitter
commCount = length(sigAmp);
% Channel and reception for different SNRs
for j = 1:length(channelType)
    eval = ofdm.Evaluator(transmitter);
    for i = 1:commCount
        comm = ofdm.Transmission(...);
        CLI.showStatus(...);
        eval = eval.getBer(...);
        clear comm;
    end
```

```
% BER plot and show PAPR
    CLI.showReport(eval, sigAmp);
end
```

Transmitter.m

```
classdef Transmitter
    % All transmitter operations
    properties
        rfFlag
        baseBandOfdmSig
        centerFreq
        passBandAnalog
        nTs
        symbolTime
        variant
        samplingInterval
    end
    methods
        function trans = Transmitter(...)
            trans.rfFlag = rf;
            trans.symbolTime = Ts;
            trans.centerFreq = fc;
            trans.variant = ofdmVariant;
            trans.samplingInterval = 0.49*(fc)^-1;
            % Convert bit stream into symbol stream
            serBauds = mapBits(...);
            % Serial -- parallel according to variant
            parBauds = makeParallel(...);
            binData = binBauds(...);
            trans.baseBandOfdmSig = ofdmMux(...);
            if (trans.rfFlag)
                [baseBandAnalogI, ...] = dac(...);
                % Upscale frequency to RF
                trans.passBandAnalog = freqUpScale(...);
            end
        end
    end
end

%% S - P Conversion
function parBauds = makeParallel(serBauds,ofdmVariant)
    ofdmVariant = ofdmVariant.subCarriers;
    dataSubs = sum(ofdmVariant(:) == 'd');
    if (mod(length(serBauds),dataSubs) ~= 0)
        serBauds = [serBauds, ...];
    end
```

```
    parBauds = reshape(serBauds, dataSubs, []);
end

%% BPSK Modulation
 function bauds = mapBits(bitArray)
    % BPSK Modulation
    bauds = 2*bitArray - 1;
 end

%% Map symbols to IFFT bins
 function bins = binBauds(baudMatrix, ofdmVariant)
    ofdmVariant = ofdmVariant.subCarriers;
    [~, symbCount] = size(baudMatrix);
    % Total subcarriers x number of OFDM symbols
    bins = int8(zeros(length(ofdmVariant),symbCount));
    j = 1;
    for i=1:length(ofdmVariant)
        if ofdmVariant(i) == 'v'
            bins(i,:) = zeros(1,symbCount);
        elseif ofdmVariant(i) == 'd'
            bins(i,:) = baudMatrix(j,:);
            j = j + 1;
        elseif ofdmVariant(i) == 'p'
            bins(i,:) = ones(1,symbCount);
        end
    end
 end

%% Operate on binned symbols, give baseband signal
 function serOfdmSig = ofdmMux(binData, ofdmVariant)
    cp = ofdmVariant.cycPrefix/100;
    gi = ofdmVariant.guardInt/100;
    binData = binData';
    [symbCount, ofdmSize] = size(binData);
    symbLength = ofdmSize+floor(cp*ofSiz)+floor(gi*ofSiz);
    cycData = (zeros(symbCount,symbLength));
    prefixStart = ofdmSize-floor(cp*ofdmSize)+1;
    guardSize = floor(gi*ofdmSize);
    for i = 1:symbCount
        % ifft per symbol
        ifftData = ifft(binData(i,:));
        % cyclic prefix and guard-interval
        cycData(i,:) = [ifftData, ... ];
    end
    serOfdmSig = reshape(cycData', 1, []);
 end

%% Digital to analog conversion
 function [baseBandAnalogI,...] = dac(...)
```

```
    % In-phase component
    baseBandSigI = real(baseBandSig);
    % Quadrature component
    baseBandSigQ = imag(baseBandSig);
    % baseband signal samples
    n = 0:length(baseBandSig)-1;
    % distribute samples over multiples
    nTs = ifftBinSize(2)*(n*Ts)/length(baseBandSig);
    nTsMax = max(nTs);
    % Interpolation intervals for DAC up to nTs
    t = 0:Dt:nTsMax;
    baseBandAnalogI = spline(nTs, baseBandSigI, t);
    baseBandAnalogQ = spline(nTs, baseBandSigQ, t);
 end

%% Frequency upscaling for pass band
 function bandPassSig = freqUpScale(...)
    % Mixing to get cos(fc+fm) + cos(fc-fm)
    mixedSig = baseBAgI.*(cos(fc*t)) + baseBAgQ.*(sin(fc*t));
    % High pass filter to get RF signal
    fs = Dt^-1;
    % Applying amplification
    bandPassSig = 1000*highpass(mixedSig, fc, fs);
 end
```

Channel.m

```
classdef Channel
    properties
        noisySignal
        channelCharacterization
    end
    methods
        function link = Channel(...)
            rfFlag = transmitter.rfFlag;
            if (rfFlag)
                transmitSig = transmitter.passBandAnalog;
            else
                transmitSig = transmitter.baseBandOfdmSig;
            end
            t = transmitter.nTs;
            Ts = transmitter.symbolTime;
            No = 10^-(sigAmp/10);
            if type == "gauss"
                [link.noisySignal,...] = addGaussianNoise(...);
            elseif type == "rayl"
                [link.noisySigna;,...] = rayleighFading(...);
```

```
            elseif type == "rice"
                [link.noisySignal,...] = ricianFading(...);
            end
        end
    end
end

%% Function to make AWGN Channel
function [noisySignal,...] = addGaussianNoise(...)
    n = length(transmitSig);
    channelChar = 1;
    noisySignal = transmitSig+sqrt(No^2/2)*(randn(1,n)+...;
end

%% Function to implement Rayleigh Fading
function [fadedSignal,...] = rayleighFading(...)
    n = length(transmitSig);
    if rfFlag
        symbCount = ceil(max(t)/Ts);
        fading = randn(1,symbCount) + 1i*randn(1,symbCount);
        channelChar = (1/sqrt(2))*repelem(fading, floor(n/symbCount));
        m = length(channelChar);
        if n ~= m
            channelChar = [channelChar repelem(channelChar(m), n - m)];
        end
    else
        channelChar = (1/sqrt(2))*(randn(1,n) + 1i*randn(1,n));
    end
    fadedSignal=transmitSig.*channelChar + ...;
end

%% Function to implement Rician Fading
function [fadedSignal,...] = ricianFading(...)
    n = length(transmitSig);
    K = 10^(speculardB/10);
    if rfFlag
        symbCount = ceil(max(t)/Ts);
        fading = randn(1,symbCount) + ...;
        channelChar = sqrt(K/(K+1)) + sqrt(1/(K+1))*(1/sqrt(2))* ...;
        m = length(channelChar);
        if n ~= m
            channelChar = [channelChar, ...)];
        end
    else
        channelChar = sqrt(K/(K+1)) + sqrt(1/(K+1))*...;
    end
    fadedSignal = transmitSig.*channelChar + sqrt(No^2/2)*...;
end
```

Receiver.m

```
classdef Transmitter
    % All transmitter operations
    properties
        rfFlag
        baseBandOfdmSig
        centerFreq
        passBandAnalog
        nTs
        symbolTime
        variant
        samplingInterval
    end
    methods
        function trans = Transmitter(...)
            trans.rfFlag = rf;
            trans.symbolTime = Ts;
            trans.centerFreq = fc;
            trans.variant = ofdmVariant;
            trans.samplingInterval = 0.49*(fc)^-1;
            % Convert bit stream into symbol stream
            serBauds = mapBits(...);
            % Serial -- parallel according to variant
            parBauds = makeParallel(...);
            binData = binBauds(...);
            trans.baseBandOfdmSig = ofdmMux(...);
            if (trans.rfFlag)
                [baseBandAnalogI, ...] = dac(...);
                % Upscale frequency to RF
                trans.passBandAnalog = freqUpScale(...);
            end
        end
    end
end

%% S - P Conversion
function parBauds = makeParallel(serBauds,ofdmVariant)
    ofdmVariant = ofdmVariant.subCarriers;
    dataSubs = sum(ofdmVariant(:) == 'd');
    if (mod(length(serBauds),dataSubs) ~= 0)
        serBauds = [serBauds, ...];
    end
    parBauds = reshape(serBauds, dataSubs, []);
end

%% BPSK Modulation
 function bauds = mapBits(bitArray)
    % BPSK Modulation
    bauds = 2*bitArray - 1;
```

```
 end

%% Map symbols to IFFT bins
 function bins = binBauds(baudMatrix, ofdmVariant)
    ofdmVariant = ofdmVariant.subCarriers;
    [~, symbCount] = size(baudMatrix);
    % Total subcarriers x number of OFDM symbols
    bins = int8(zeros(length(ofdmVariant),symbCount));
    j = 1;
    for i=1:length(ofdmVariant)
        if ofdmVariant(i) == 'v'
            bins(i,:) = zeros(1,symbCount);
        elseif ofdmVariant(i) == 'd'
            bins(i,:) = baudMatrix(j,:);
            j = j + 1;
        elseif ofdmVariant(i) == 'p'
            bins(i,:) = ones(1,symbCount);
        end
    end
 end

%% Operate on binned symbols, give baseband signal
 function serOfdmSig = ofdmMux(binData, ofdmVariant)
    cp = ofdmVariant.cycPrefix/100;
    gi = ofdmVariant.guardInt/100;
    binData = binData';
    [symbCount, ofdmSize] = size(binData);
    symbLength = ofdmSize+floor(cp*ofSiz)+floor(gi*ofSiz);
    cycData = (zeros(symbCount,symbLength));
    prefixStart = ofdmSize-floor(cp*ofdmSize)+1;
    guardSize = floor(gi*ofdmSize);
    for i = 1:symbCount
        % ifft per symbol
        ifftData = ifft(binData(i,:));
        % cyclic prefix and guard-interval
        cycData(i,:) = [ifftData, ... ];
    end
    serOfdmSig = reshape(cycData', 1, []);
 end

%% Digital to analog conversion
 function [baseBandAnalogI,...] = dac(...)
    % In-phase component
    baseBandSigI = real(baseBandSig);
    % Quadrature component
    baseBandSigQ = imag(baseBandSig);
    % baseband signal samples
    n = 0:length(baseBandSig)-1;
    % distribute samples over multiples
```

```
    nTs = ifftBinSize(2)*(n*Ts)/length(baseBandSig);
    nTsMax = max(nTs);
    % Interpolation intervals for DAC up to nTs
    t = 0:Dt:nTsMax;
    baseBandAnalogI = spline(nTs, baseBandSigI, t);
    baseBandAnalogQ = spline(nTs, baseBandSigQ, t);
 end

%% Frequency upscaling for pass band
 function bandPassSig = freqUpScale(...)
    % Mixing to get cos(fc+fm) + cos(fc-fm)
    mixedSig = baseBAgI.*(cos(fc*t)) + baseBAgQ.*(sin(fc*t));
    % High pass filter to get RF signal
    fs = Dt^-1;
    % Applying amplification
    bandPassSig = 1000*highpass(mixedSig, fc, fs);
 end
```

Evaluator.m

```
classdef Evaluator
    properties
        bitErrors
        papr
    end
    methods
        function eval = Evaluator(transmitter)
            eval.bitErrors = [];
            eval.papr = [eval.papr, findPapr(transmitter)];
        end
        % Add commInst BER
        function eval = getBer(eval, dataSource, commInst)
            % BER
            txData = dataSource.serialBits;
            rxData = commInst.rece.serRecBits(1:length(txData));
            ber = sum(rxData ~= txData)/length(txData);
            eval.bitErrors = [eval.bitErrors ber];
        end
    end
end

%% PAPR Determination
function papr = findPapr(transmitter)
    ofdmSize = length(transmitter.variant.subCarriers);
    cp = transmitter.variant.cycPrefix/100;
    gi = transmitter.variant.guardInt/100;
    symbLength = ofdmSize+floor(cp*ofdmSize)+floor(gi*ofdmSize);
```

```
    queryWave = abs(transmitter.baseBandOfdmSig).^2;
    symbCount = length(queryWave)/symbLength;
    peaks = zeros(1,symbCount); avgs = peaks;
    for i = 0:symbCount-1
        thisSymb = queryWave(i*symbLength+1:(i+1)*symbLength);
        peaks(i+1) = max(thisSymb);
        avgs(i+1) = mean(thisSymb);
    end
    papr = max(peaks)/mean(avgs);
end
```