4. kontrolna naloga - 2. rok

3. A, 4. 4. 2023

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dosežene točke	možne točke	odstotki	ocena
26	36	72	3

ČAS PISANJA: 45 minut

1. Pri deljenju polinoma $p(x) = 2x^4 - 3x^3 + bx + a$ s polinomom $q(x) = x + 2$ dobimo ostanek 49. Izračunaj vrednost parametrov a in b, če je $p(2) = 5$. [6t] 6

	2	-3	0	b	a
-2		-4	14	-28	56-2b+a
	2	-7	14	-28+b	56-2b+a

$56 - 2b + a = 49$

$-2b + a = -7$

$2 \cdot 2^4 - 3 \cdot 2^3 + b \cdot 2 + a = 5$

$32 - 24 + 2b + a = 5$

$2b + a = -3$

$2b + a = -3$

$-2b + a = -7$)+

$2a = -10$

$a = -5$

$2b - 5 = -3$

$2b = 2$

$b = 1$

2. Zapiši polinom $p(x) = x^4 - 4x^3 + 8x^2 - 8x + 4$ kot popoln kvadrat nekega polinoma. Prikaži postopek. [5t] 5

$(x^2 - ax + b)^2 = (x^2 - ax + b)(x^2 - ax + b) =$

$= x^4 - ax^3 + bx^2 - ax^3 + a^2x^2 - abx + bx^2 - abx + b^2 =$

$= x^4 - 2ax^3 + (2b + a^2)x^2 - 2abx + b^2$

$b^2 = 4$

$2a = 4$

$a = 2$

$2b + a^2 = 8$

$2b + 4 = 8$

$2b = 4$

$b = 2$

$p(x) = (x^2 - 2x + 2)^2$

3. Izračunaj, pri katerih vrednostih realnega števila a je polinom $p(x) = x^4 + x^2 + a$ deljiv s polinomom $q(x) = x^2 + x + a$. [5t] 5

$r(x) = 0$

$p(x) = k(x) \cdot q(x)$

$k(x) = \frac{p(x)}{q(x)}$

$(x^4 + x^2 + a) : (x^2 + x + a) = x^2 - x + 2 - a$

$-x^4 - x^3 - ax^2$

$-x^3 + (1-a)x^2 + a$

$+x^3 + x^2 + ax$

$(1-a+1)x^2 + ax + a$

$-((2-a)x^2 - (2-a)x - (2-a)a)$

$ax - (2-a)x + a - 2a + a^2 =$

$= (a-2+a)x + a^2 - 2a + a$

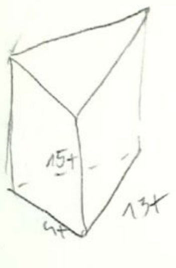
$(2a-2)x + a^2 - 2a + a = 0$

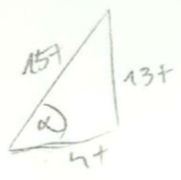
$2a - 2 = 0$ $\quad 1 - 2 + 1 = 0$

$a_1 = 1$

$(2a-2)x = -a^2 - a$

4. Dolžine osnovnih robov pokončne tristrane prizme so v razmerju $4 : 13 : 15$. Ploščina plašča prizme je $112\ m^2$, prostornina prizme pa $168\ m^3$. Izračunaj natančno površino prizme. [6t] 4





$S_\Delta = ?$

$13^2 = 15^2 + 4^2 - 2 \cdot 4 \cdot 15 \cdot \cos\alpha$

$169 = 225 + 16 - 120 \cos\alpha$

$72 = 120 \cos\alpha$

$\cos\alpha = \frac{72}{120} = \frac{12}{20} = \frac{6}{10}$ $\quad \alpha = 53{,}13°$

$O = \frac{4 \cdot 15 \cdot \sin\alpha}{2} = 30 \cdot \sin\alpha$

$pl = 112 m^2 = 13t \cdot v + 4t \cdot v + 15t \cdot v = 32t \cdot v$

$V = 168 m^3$

$V = O \cdot v$

$168 = 30 \cdot \sin\alpha \cdot v$

$v = \frac{168}{30 \cdot \sin\alpha}$

$32t \cdot v = 112$ $\quad v = \frac{112}{32t}$

$24t^2 \cdot v = 168$

$24t^2 \cdot \frac{112}{32t} = 168$

$24t^2 \cdot \frac{3{,}5}{t} = 168$

$84t = 168$ $\quad t = 2$

$S_\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ $\quad s = \frac{(4+13+15)t}{2} = \frac{32t}{2} = 16t$

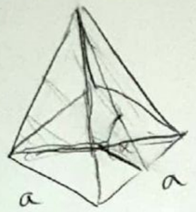
$= \sqrt{\frac{32}{2}\left(\frac{32}{2} - \frac{8}{2}\right)\left(\frac{32}{2} - \frac{26}{2}\right)\left(\frac{32}{2} - \frac{30}{2}\right) t^4} = \sqrt{16 \cdot 12 \cdot 1 \cdot 3 \, t^4} = 24t^2$

$v = \frac{112}{32\sqrt{2}}$

$v = \frac{3{,}5}{\sqrt{2}}$ m

5. Prostornina pravilne štiristrane piramide je $36\sqrt{3}\ \mathrm{dm}^3$. Presek piramide z ravnino, ki poteka skozi vrh piramide in dve nasprotni oglišči osnovne ploskve, je lik s ploščino $9\sqrt{6}\ \mathrm{dm}^2$. Natančno izračunaj površino piramide in kot med stransko in osnovno ploskvijo piramide. [8t] 5

$S_\Delta = 9\sqrt{6}\ dm^2 = \frac{a\sqrt{2}\cdot v}{2}$ ✓



$\frac{a\sqrt{2}\cdot v}{2} = 9\sqrt{6}$ $\quad a\sqrt{2}\cdot v = 18\sqrt{6}$

$v = \frac{18\sqrt{6}}{a\sqrt{2}} = \frac{18\sqrt{3}}{a}$ ✓

$V = 36\sqrt{3}\,dm^3 = \frac{a^2\cdot v}{3}$ $\qquad \frac{a^2\cdot v}{3} = 36\sqrt{3}$

$a^2\cdot\frac{18\sqrt{3}}{a} = 3\cdot 36\sqrt{3}$

$P = ?$

$\alpha = ?$

$18a = 36$

$a = \cancel{2\,dm}$ $\qquad O = a^2 = 4\,dm^2$

$\tan\alpha = \frac{9\sqrt{3}}{1}$

$\alpha \doteq \cancel{86{,}33^\circ}$

$4\cdot v = 36\sqrt{3}$

$v = \cancel{9\sqrt{3}\,dm}$

$x = \sqrt{(9\sqrt{3})^2 + 1} = \sqrt{244}\ dm$

$P = a^2 + a\cdot\sqrt{244}\cdot 2 \cancel{= 8\cdot\sqrt{244}\,dm^2 = 8\cdot 2\cdot\sqrt{61} = 16\sqrt{61}\,dm^2}$

$= (4 + 4\cdot\sqrt{244})\,dm^2 = (4 + \cancel{8\sqrt{61}})\,dm^2$

$t = \sqrt[3]{2}$

$\frac{112\sqrt[3]{2}}{2} = 56\sqrt[3]{2}$

$= \frac{3{,}5\sqrt[3]{2}}{2} = \frac{7\sqrt[3]{2}}{4}\ m$

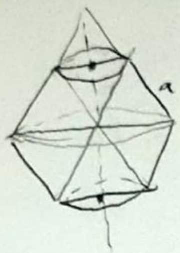
$\cancel{P = 24\cdot(\sqrt[3]{2})^4 + 32\cdot 56\sqrt[3]{2}\cdot\sqrt[3]{2} = 24\cdot 2\sqrt[3]{2} + 1792\sqrt[3]{4} =}$

$\cancel{= (48\sqrt[3]{2} + 1792\sqrt[3]{4})\,m^2}$

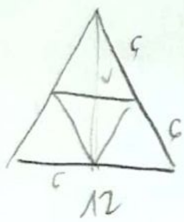
$P = 24(\sqrt[3]{2})^4 + 32\cdot\sqrt[3]{2}\cdot\frac{3{,}5}{\sqrt[3]{2}} =$

$= (48\sqrt[3]{2} + 115)\,m^2$

6. Pravilni šetkotnik z dolžino stranice $a = 6$ cm zavrtimo za 180° okoli simetrijske osi, ki poteka skozi razpolovišči nasprotnih (vzporednih) stranic. Izračunaj natančno prostornino tako nastalega telesa. [6t] 1



$a = 6 cm$



$$\frac{a\sqrt{3}}{2}$$

$$v = \cancel{\frac{a^2\sqrt{3}}{4}} = 36\sqrt{3} cm$$

$$O = \pi a^2 = 36\pi cm$$

$$V_1 = \frac{O \cdot v}{3} = \frac{36\sqrt{3} \cdot 36 \cdot \pi}{3} cm^3 = \cancel{432}\pi\sqrt{3} cm^3$$

prisekana stožec (dvojni)

$$V = 2 \cdot V_1 = \cancel{864}\sqrt{3}\pi cm^3$$