3. kontrolna naloga
3. A, 15. 2. 2023

Ime in priimek: Lira Jurkovič Razred: 3. A

dosežene točke	možne točke	odstotki	ocena
34	40	85	4

ČAS PISANJA: 45 minut

1. Dana je funkcija $f(x) = \arcsin(2x-4) + \frac{\pi}{6}$.

   a) Izračunaj ničlo funkcije f. [3t] 3

   $\arcsin(2x-4) + \frac{\pi}{6} = 0$

   $\arcsin(2x-4) = -\frac{\pi}{6}$

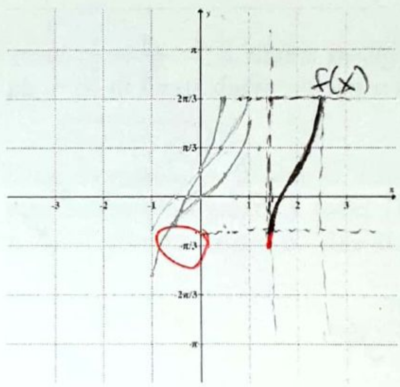
   $-\sin\frac{\pi}{6} = 2x-4$

   $2x-4 = -\frac{1}{2}$

   $2x = -\frac{1}{2} + \frac{8}{2}$ /:2

   $x = \frac{7}{4}$ ✓

   b) Nariši graf funkcije f. [4t] 3



$2(x-2)$

2. Reši enačbi:

   a) $3\sin^2 x + \sin(2x) - 3\cos^2 x + 1 = 0$ [6t] 6

   $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

   $3\sin^2 x + 2\sin x\cos x - 3\cos^2 x + 1 = 0$ ✓ $/ : \cos^2 x$ ✓

   $3\tan^2 x + 2\tan x - 3 + \frac{1}{\cos^2 x} = 0$

   $\tan^2 x + 1 = \frac{1}{\cos^2 x}$

   $3\tan^2 x + 2\tan x - 3 + 1 + \tan^2 x = 0$

   $4\tan^2 x + 2\tan x - 2 = 0$ /:2

   $2\tan^2 x + \tan x - 1 = 0$ ✓

   $(2a+1)(a+1) = 2a^2 + 2a + \dots - 1$

   $(2\tan x - 1)(\tan x + 1) = 0$

   ① $2\tan x - 1 = 0$

   $2\tan x = 1$

   $\tan x = \frac{1}{2}$

   $x = \arctan\left(\frac{1}{2}\right) + k\pi;\ k \in \mathbb{Z}$ ✓

   ② $\tan x + 1 = 0$

   $\tan x = -1$

   $x = -\frac{\pi}{4} + k\pi;\ k \in \mathbb{Z}$ ✓

b) $\sin(8x) + \sin(2x) = \sin(3x - \frac{5\pi}{2})$ [6t] 3

$(2\sin 4x \cos 4x + 2\sin x \cos x = \sin 3x \cos\frac{5\pi}{2} - \cos 3x \sin\frac{5\pi}{2})$

$\sin\frac{5\pi}{2} = \sin\frac{\pi}{2} = 1$

$\cos\frac{5\pi}{2} = \cos\frac{\pi}{2} = 0$

$2\sin\frac{8x+2x}{2}\cos\frac{8x-2x}{2} = \sin\left(\frac{6x-5\pi}{2}\right)$

$2\sin 5x \cos 3x = -\cos 3x$

$2\sin x\cos 3x + \cos 3x = 0$

$2\cos 3x\,(\sin x + \frac{1}{2}) = 0$ $(2\sin(5x) + 1)$

① $\cos 3x = 0$

$3x = \frac{\pi}{2} + k\pi \;/ :3$

$x_3 = \frac{\pi}{6} + \frac{k\pi}{3}\,;\; k \in \mathbb{Z}$

② $\sin 5x = -\frac{1}{2}$ ② $\sin(5x) = 1$

$x_1 = \frac{\pi}{6} + 2k\pi\,;\; k \in \mathbb{Z}$

$x_2 = \frac{5\pi}{6} + 2k\pi\,;\; k \in \mathbb{Z}$

$x_2 = -\frac{\pi}{30} + k\frac{2\pi}{5}$

$x_3 = \frac{7\pi}{30} + k\frac{2\pi}{5}$

3. Dani sta kompleksni števili $z_1 = \frac{\sqrt{3}}{2} - \frac{1}{2}i$ in $z_2 = \text{cis}\frac{\pi}{4}$. Zapiši število z_1 v polarni obliki. Določi **en** par naravnih števil m in n, da bo $z_1^m = (z_2^*)^n$. $= (\bar{z}_1)^n$ [7t] 6

$z_1 = \frac{\sqrt{3}}{2} - \frac{1}{2}i$

$z_2 = \text{cis}\frac{\pi}{4}$

$r_1 = \sqrt{(\frac{\sqrt{3}}{2})^2 + (-\frac{1}{2})^2} = \sqrt{\frac{3}{4} + \frac{1}{4}} = 1$

$(\varphi = \tan\left(\frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right))$ $\tan\varphi = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\varphi = \frac{11\pi}{6}$

$z_1 = \text{cis}\frac{11\pi}{6}$

$\bar{z}_2 = r\,\text{cis}(-\varphi) = \text{cis}(-\frac{\pi}{4}) =$

$z_1^m = (\bar{z}_2)^n$

$r_1^m\,\text{cis}(m\varphi) = r_2^n\,\text{cis}(n\varphi)$

$\text{cis}(m\cdot\frac{11\pi}{6}) = \text{cis}(n\cdot\frac{7\pi}{4})$

$-\frac{\pi}{4} = \frac{7\pi}{4}$

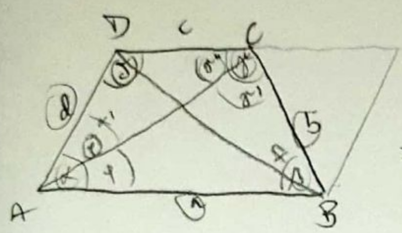
$m\cdot\frac{11\pi}{6} = n\cdot\frac{7\pi}{4}$

$\frac{m\,11}{6} = \frac{7n}{4}$

$m = \frac{7n6}{11\cdot 4} = \frac{21n}{22} \rightarrow$ $n = 22$, $m = 21$

4. V trapezu $ABCD$ z osnovnicama a in c merijo $a = 7$ dm, $b = 5$ dm, $e = 6$ dm in $\delta = 94°$. Izračunaj ploščino trapeza. Rezultat naj bo na eno decimalno mesto natančen.

[8t] 8



$e^2 = a^2 + b^2 - 2ab\cos\beta$

$36 = 49 + 25 - 70\cos\beta$

$70\cos\beta = 38$

$\cos\beta = \frac{38}{70}$

$\beta = 57{,}12°$ ✓

$\gamma = 122{,}88°$

$b^2 = e^2 + a^2 - 2ae\cos\varphi$

$25 = 36 + 49 - 84\cos\varphi$

$84\cos\varphi = 60$

$\varphi = 44{,}42°$

$\gamma' = 180 - \beta - \varphi = 78{,}46°$ ✓

$\gamma'' = 44{,}42°$

$\alpha' = 86 - \varphi = 41{,}58$ ✓

$a = 7dm$

$b = 5dm$

$e = 6dm$

$\delta = 94°$ $\quad \alpha = 180° - \delta = 86°$ ✓

$\frac{d}{\sin\gamma''} = \frac{e}{\sin\delta}$

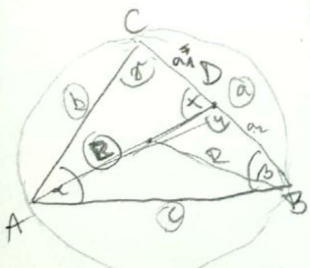
$d = \frac{e \cdot \sin\gamma''}{\sin\delta} = 4{,}21dm$ ✓

$S = ?$ $\quad S = \frac{a+c}{2} \cdot v$

$S = \frac{ab\sin\beta}{2} + \frac{ed\sin\alpha'}{2} = 23{,}1dm^2$ ✓

5. V trikotniku ABC je dana dolžina stranice $a = 27$ cm in velikost kotov $\alpha = 83°$ ter $\beta = 57°$. Simetrala kota α seka nasprotno stranico v točki D. Izračunaj dolžino daljice AD in polmer trikotniku ABC očrtane krožnice. Rezultata zaokroži na eno decimalno mesto.

[6t] 5



$|AD| = ?$

$R = ?$

$|AD|^2 = a_2^2 + c^2 - 2a_2c\cos\beta$

$\frac{|AD|}{\sin\beta} = \frac{a_2}{\sin(\frac{\alpha}{2})}$ $\quad \frac{c}{\sin 81{,}5} = \frac{AD}{\sin\beta}$

$\alpha = 83°$ $\quad \frac{\alpha}{2} = 41{,}5°$

$\beta = 57°$

$\gamma = 40°$

$a = 27cm$

$x = 98{,}5°$

$y = 81{,}5°$ ✓

$AD = \frac{c\sin\beta}{\sin 81{,}5}$

$AD = 14{,}83cm$ ✓

$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$

$b = \frac{a\sin\beta}{\sin\alpha} = 22{,}81cm$ ✓

$c = \frac{a\sin\gamma}{\sin\alpha} = 17{,}49cm$ ✓

$S = 2R^2\sin\alpha\sin\beta\sin\gamma$ $\quad S = \frac{cb\sin\alpha}{2} = 197{,}99cm^2$

$R^2 = \frac{S}{2\sin\alpha\sin\beta\sin\gamma} = \frac{\frac{cb\sin\alpha}{2}}{2\sin\alpha\sin\beta\sin\gamma}$

$R = 13{,}6cm$ ✓

DODATNA NALOGA:

Dokaži, da je v poljubnem trikotniku razmerje med razliko in vsoto dolžin dveh stranic enako razmerju tangensov polovične razlike in vsote stranicama nasprotnih kotov. Na primer:

$$\frac{a-b}{a+b}=\frac{\tan\frac{\alpha-\beta}{2}}{\tan\frac{\alpha+\beta}{2}}$$

Povezava se imenuje tangensni izrek. [3t]

FORMULE:

$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$

$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$\tan\alpha\pm\tan\beta=\frac{\sin(\alpha\pm\beta)}{\cos\alpha\cos\beta}$

$\cot\alpha\pm\cot\beta=\frac{\sin(\beta\pm\alpha)}{\sin\alpha\sin\beta}$

$\sin\alpha\cdot\cos\beta=\frac{1}{2}(\sin(\alpha+\beta)+\sin(\alpha-\beta))$

$\sin\alpha\cdot\sin\beta=-\frac{1}{2}(\cos(\alpha+\beta)-\cos(\alpha-\beta))$

$\cos\alpha\cdot\cos\beta=\frac{1}{2}(\cos(\alpha+\beta)+\cos(\alpha-\beta))$