4. kontrolna naloga - 2. rok
4. A, 8. 3. 2024

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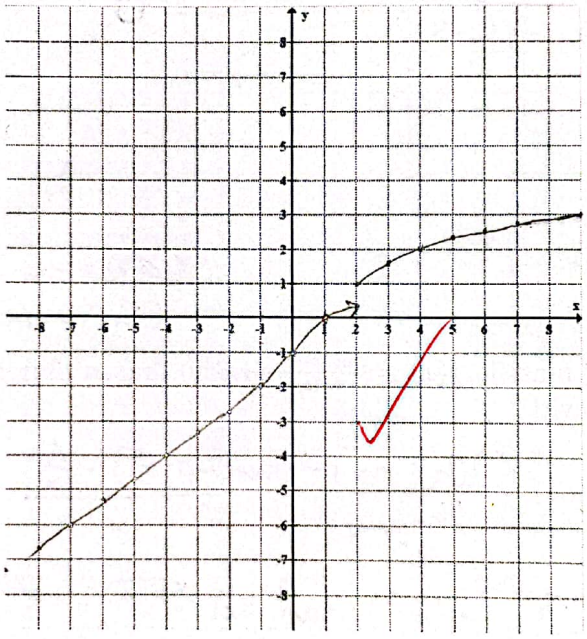
dosežene točke	možne točke	odstotki	ocena
30	44	68	3

1. Dana je funkcija f s predpisom $f(x) = \begin{cases} \frac{2}{3}x - \frac{4}{3}; & x < -1 \\ \sqrt[3]{x} - 1; & -1 \leq x < 2 \\ \log_2 x; & x \geq 2 \end{cases}$.

a) Nariši graf funkcije f in določi točke nezveznosti. [4t] 4



točke nezveznosti:
$x = 2$

$-\frac{2}{3} \cdot 3 - \frac{4}{3}$
$-\frac{6}{3} - \frac{4}{3} = -\frac{10}{3}$

$\sqrt[3]{(-1)} - 1 = -1 - 1 =$

$\log_2 2 = \frac{\log 2}{\log 2} =$

b) Zapiši predpis inverzne funkcije f^{-1}. [5t] 4

① $x = \frac{2}{3}y - \frac{4}{3}$
$x + \frac{4}{3} = \frac{2}{3}y \;/ : \frac{2}{3}$
$\frac{3x}{2} + 2 = y$

② $x = \sqrt[3]{y} - 1$
$x + 1 = \sqrt[3]{y} \;/()^3$
$y = (x+1)^3$

③ $x = \log_2 y$
$y = 2^x$

$\frac{4}{3} \cdot \frac{3}{2} =$

$f^{-1}(x) = \begin{cases} \frac{3}{2}x + 2; & x < -1 \\ (x+1)^3; & -1 \leq x < 2 \\ 2^x; & x \geq 2 \end{cases}$

$(\operatorname{arctg} x)' = -\frac{1}{x^2+1}$

$\frac{-3x+6}{x^3-8x^2+16x} = \frac{-3(x+2)}{x(x^2-8x+16)} = \frac{-3(x+2)}{x(x-4)^2}$

2. Izračunaj limiti:

$(-3x-6)(x^2+8x+16) = -3x^3-24x^2-48x-6x^2-48x-96 =$ [3t] 1

a) $\lim_{x\to 4} \frac{-3x+6}{x^3-8x^2+16x} = \lim_{x\to 4} \frac{-3}{3x^2-16x+16}$ $\frac{0}{0}$

$= \lim_{x\to 4} \frac{-3(x+2)'}{x(x-4)^2} = \lim_{x\to 4} \frac{-3(x+2)(x+4)^2}{x\cdot(x^2-16)} =$

$= \lim_{x\to 4} \frac{-3x^3-30x^2-96x-96}{x^3-16x} =$

$= \lim_{x\to 4} \frac{-9x^2-60x-96}{3x^2-16} = \frac{-480}{32} = -15$

$(x^3-8x^2+16x):(x-4) = x^2-4x$

$-x^3+4x^2$

$-4x^2+16x$

$+4x^2-16x$

0

$x^3-16x \to 3x^2-16$

b) $\lim_{x\to 0} \frac{4x^2\cot(3x)}{\sqrt[3]{x+8}-2} = \frac{0}{0} =$ [4t] 1

$= \lim_{x\to 0} \frac{8x\cdot\cot(3x) + 4x^2\cdot\frac{-3}{\sin^2 3x}}{\frac{1}{3}(x+8)^{-\frac{2}{3}}} = 0$

$(\cot 3x)' = -\frac{1}{\sin^2 3x}$

$\cot x = \frac{1}{\sin^2 x}$

$8^{-\frac{2}{3}} = \frac{1}{4}$

$(x+8)^{\frac{1}{3}} \to \frac{1}{3}(x+8)^{-\frac{2}{3}}$

$(\ln x)' = \frac{1}{x}$

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$(\sqrt[3]{x+8}-2)^3 = x+8-8$

$\frac{1}{12}$

$(\cos x)' = -\sin x$

$(\sin x)' = \cos x$

$(\tan x)' = \frac{1}{\cos^2 x}$

$x^{\frac{2}{5}}$

3. Odvajaj funkcijo $f(x) = \frac{\sin(2x)-x^5}{4-\ln x} + 3^{2x^4-5}\cdot(\tan^2(3x) - \sqrt[5]{x^2})$. Odvoda ni potrebno poenostaviti. [5t] 4

$2\cos 2x$

$f'(x) = \frac{(2\cos 2x - 5x^4)(4-\ln x) - (\sin(2x)-x^5)(-\frac{1}{x})}{(4-\ln x)^2} +$

$(\sin x)' = \frac{1}{\sqrt{x^2+1}}$

$+ (2x^4-5)\cdot 3^{2x^4-6}\cdot(8x^3)\cdot(\tan^2(3x) - \sqrt[5]{x^2}) + 3^{2x^4-5}\cdot\left(2\tan(3x)\cdot\frac{1}{\cos^2 3x}\cdot 3 - \frac{2}{5}x^{-\frac{3}{5}}\right)$

$\frac{1}{\cos^2 3x}$

4. Dani sta funkciji $f(x) = \frac{1-x}{x}$ in $g(x) = \frac{x+1}{x^2-4x+5}$.

a) Izračunaj in poenostavi funkcijo $f \circ g$. [3t] 3

$(f\circ g)(x) = \frac{1-\left(\frac{x+1}{x^2-4x+5}\right)}{\frac{x+1}{x^2-4x+5}} = \frac{\frac{x^2-4x+5-x-1}{x^2-4x+5}}{\frac{x+1}{x^2-4x+5}} =$

$= \frac{x^2-5x+4}{x+1} = \frac{(x-4)(x-1)}{x+1}$

b) Nariši graf funkcije $h(x) = f \circ g$. [6t] 6

pol: $x = -1$ (L) ✓

ničle: $x^2 - 5x + 4 = 0$

$x_1 = 4$

$x_2 = 1$ ✓

asimptota: $y = x - 6$

$(x^2 - 5x + 4) : (x + 1) = x - 6$ ✓

$-x^2 - x$

$-6x + 4$

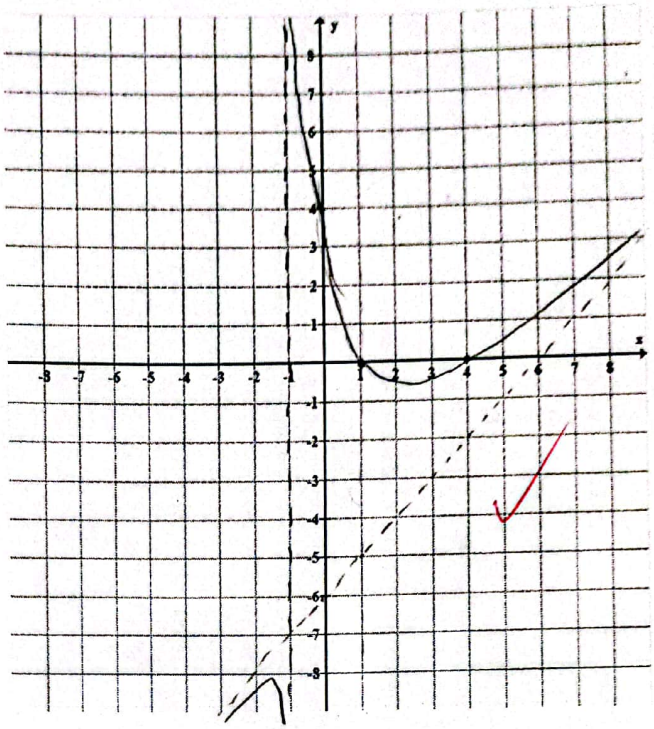
$+6x + 6$

10

$\lim_{x \to \infty} = \infty$

$\lim_{x \to -\infty} = \infty$

$f(0) = 4 \quad P_y(0, 4)$ ✓



5. Zapiši enačbo tiste tangente na graf funkcije $f(x) = 4 \ln \frac{1}{x+7}$, ki ima naklonski kot $\alpha = \arctan(-4)$. [6t] 3

$\alpha = \arctan(-4)$

$f'(x) = -4 \cdot (x+7) = -4x - 28 = k_t$

$k_t = \tan \alpha = -4$ ✓

$y - y_0 = k_t (x - x_0)$

$y = -4x + 24$

$-4x - 28 = -4 \quad / : (-4)$

$x + 7 = 1$

$x = -6$

$y = 4 \cdot \ln \frac{1}{-6+7} = 4 \cdot \ln 1 = 0$

sončasma enaka rezultat (kljub napaki)

$\frac{1}{\sqrt{x-1}}$ $\cos^2 x = 1 - \sin^2 x$

6. Na minuto natančno izračunaj kot med krivuljama $y = \sin^2 x$ in $y = \cos^2 x - \cos x$ v tistem presečišču, ki ima od vseh presečišč najmanjšo pozitivno absciso. [8t] 4

$k_{t_1} = (\sin^2 x)' = \cancel{2\sin x \cos x} \quad \cos^2 x = \cos^2(k\pi) = 1 \rightarrow \cos^2\left(\frac{2\pi}{3}\right) = \left(-\frac{1}{2}\right)^2$

$k_{t_2} = \cancel{-2\sin x \cos x} - \sin^2 x + \sin x = 0 \rightarrow -\sin^2\left(\frac{\pi}{3}\right) + \sin\frac{\pi}{3} = -\frac{3}{4} + \frac{\sqrt{3}}{2} =$

$= \frac{3}{4} + \frac{2\sqrt{3}}{4} = 3+$

$\cancel{\sin^2 x = 1 - \sin^2 x - \cos x}$

$\cancel{2\sin^2 x = 1 - \cos x}$

$1 - \cos^2 x = \cos^2 x - \cos x$

$1 + \cos x = 2\cos^2 x$

$2\cos^2 x - \cos x - 1 = 0 \qquad a = \cos x$

$2a^2 - a - 1 = 0$

$a_1 = \frac{1 + \sqrt{1+8}}{4} = \frac{1+3}{4} = 1$

$a_2 = -\frac{1}{2}$ ✓

$\cos x = 1$

$x = k\pi \rightarrow y = \sin^2(k\pi) = 0$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3} + 2k\pi \rightarrow y = \sin^2\left(\frac{2\pi}{3}\right) = \sin^2\left(\frac{\pi}{3}\right) = \frac{3}{4}$

$x = \frac{2\pi}{3}$

$\frac{\sqrt{3}}{2}$

$\tan\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right| = \left|\frac{1}{1+0}\right| = 1$ ✓

$\varphi = 45°$ ✓

$\tan\varphi = \left|\frac{-\frac{1}{2} - \frac{3+2\sqrt{3}}{4}}{1 + \frac{3+2\sqrt{3}}{4} \cdot \left(-\frac{1}{2}\right)}\right|$

$\varphi = \ldots$