5. kontrolna naloga
4. A, 24. 4. 2024

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dosežene točke	možne točke	odstotki	ocena
39	50	78	4

1. Določi enačbo normale na krivuljo z enačbo $3x^2 - y^2 - 36x - 10y + 71 = 0$ v točki $T(10, y_0)$, $y_0 > 0$. [7t] 5

$3x^2 - y^2 - 36x - 10y + 71 = 0$

$6x - 2y \cdot y' - 36 - 10 \cdot y' = 0$ ✓

$6x - 36 = 2y \cdot y' + 10y'$

$y'(2y + 10) = 6x - 36$

$y' = \dfrac{6x - 36}{2y + 10}$ ✓

$T(10, y_0)$

$3 \cdot 100 - y^2 - 360 - 10y + 71 = 0$

$y^2 + 10y = 11$ $\quad y^2 + 10y - 11 = 0$ ✓

$(y + 11)(y - 1) = 0$

$y > 0 \Rightarrow y = 1$ ✓

$T(10, 1)$

$k_t = \dfrac{6 \cdot 10 - 36}{2 \cdot 1 + 10} = \dfrac{24}{12} = 2$ ✓

$y = kx + n$

$1 = 2 \cdot 10 + n$

$n = -19$

$\boxed{\cancel{y = 2x - 19}}$ normala, ne tangenta

2. Izračunaj dane integrale:

a) $\int \frac{\sqrt[3]{x^2} - x \cdot 2^{3x+1} + 5x \sin x}{x}\, dx =$ [4t] 3

$x^{\frac{2}{3}}$

$= \int x^{-\frac{1}{3}} dx - \int 2^{3x+1} dx + \int 5 \sin x \, dx =$

$= \dfrac{x^{\frac{2}{3}}}{\frac{2}{3}} - \dfrac{2^{3x+1}}{3 \ln \cancel{a}} + 5 \cdot (-\cos x) + C$

2

:(

$(a^x)' = a^x \cdot \ln a$

$t = x^2+2x+1$

$dt = 2x+2$

$u=$

$u = x^2+2x+1 \quad du = (2x+2)dx$

$dv = e^{2x}dx \quad v = \frac{e^{2x}}{2}$

b) $\int (x^2+2x+1)e^{2x}\,dx =$ [5t] 3

$= \frac{e^{2x}}{2}(x^2+2x+1) - \frac{1}{2}\int (2x+2)e^{2x}\,dx =$

$u = 2x+2 \quad du = 2dx$

$dv = e^{2x}dx \quad v = \frac{e^{2x}}{2}$

$= \frac{e^{2x}}{2}(x^2+2x+1) - \frac{1}{2}\left(\frac{e^{2x}}{2}(2x+2) - \int e^{2x}dx\right) =$

$= \frac{e^{2x}}{2}(x^2+2x+1) - \frac{e^{2x}}{4}\overset{2(x+1)}{(2x+2)} - \frac{e^{2x}}{4} = \frac{e^{2x}}{2}(x^2+2x+1-2x-2-1) =$

$\frac{1}{2}e^{2x}(x^2+x) + \frac{1}{4}e^{2x} + C$

$= \frac{e^{2x}}{2}(x^2-2) + C$

c) $\int_1^e \frac{\sqrt{\ln x+1}}{x}\,dx =$ [6t] 6

$\sqrt{8} = 2\sqrt{2}$

$= \int_1^2 \sqrt{t}\,dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}}\Big|_1^2 =$

$t = \ln x + 1$

$dt = \frac{1}{x}dx$

$= \frac{2^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3}\cdot\sqrt{8} - \frac{2}{3} =$

$+\frac{1}{2}$

$\ln 1 + 1 = 1$

$= \frac{2}{3}(2\sqrt{2}-1)$

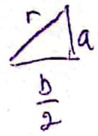
$\ln e + 1 = 2$

3. V polkrog s polmerom r včrtamo pravokotnik, ki ima dve oglišči na premeru, dve pa na loku polkroga. Izračunaj, koliko naj bosta dolgi stranici pravokotnika, da bo njegova ploščina največja. [7t] 5

$S = a\cdot b$

$b < 2r$

$a < 2r$



$r = \dots$

$a = \dots$

$S(a) = \dots$

$S'(a) = 0 \rightarrow S_{max}$

$S_\Delta = \frac{\pi r^2}{2}$

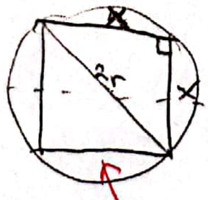
$S_\square = \frac{\pi r^2}{2} - a\cdot b$

$$\boxed{\begin{aligned} b &= r\sqrt{2} \\ a &= \frac{r\sqrt{2}}{2} \end{aligned}}$$

$2x^2 = 4r^2$

$x^2 = 2r^2$

$x = r\sqrt{2}$

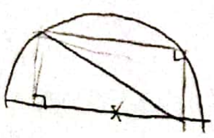


$a = b$

$l = \pi r$

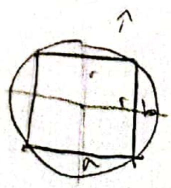
$2a + b = 2r$

$b = \frac{2r}{2a} = \frac{r}{a}$



+dokaz, da je največji možni 4-kotnik včrtan krogu kvadrat

$S_{max}: a = b$



$l = \frac{\alpha}{360°}\cdot 2\pi r = \frac{\alpha\pi r}{180°} = \pi r$

$(\ln x)' = \frac{1}{x}$

4. Dana je funkcija f s predpisom $f(x) = x^2 \ln \frac{x}{3}$.

a) Zapiši definicijsko območje in izračunaj ničle funkcije f. [3t] 3

$\frac{x}{3} > 0$

$D_f = \mathbb{R}^+$

$x^2 \cdot \ln\frac{x}{3} = 0$

$x^2 = 0$ $\quad$ $\ln\frac{x}{3} = 0$

$x_{1,2} = 0$ $\quad$ $\frac{x}{3} = 1$

↳ izven D_f $\quad$ $x_3 = 3$

b) Izračunaj stacionarne točke in določi njihovo vrsto. Zapiši interval naraščanja funkcije. [7t] 7

$f'(x) = 2x \cdot \ln\frac{x}{3} + x^{\cancel{2}} \cdot \frac{3}{\cancel{x}} \cdot \frac{1}{3} = 2x\ln\frac{x}{3} + x$

$I_\nearrow = \left(\frac{3}{\sqrt{e}}, \infty\right)$

$2x\ln\frac{x}{3} + x = 0$

$x(2\ln\frac{x}{3} + 1) = 0$

$x_1 = 0$ → izven D_f ~~ln~~

$2\ln\frac{x}{3} + 1 = 0$

$2\ln\frac{x}{3} = -1$

$\ln\frac{x}{3} = -\frac{1}{2}$

$e^{-\frac{1}{2}} = \frac{x}{3}$

$x_2 = 3 \cdot \frac{1}{\sqrt{e}}$

$-$ L $+$

$\frac{3}{\sqrt{e}}$

$\frac{3}{3\sqrt{e}} = \frac{1}{\sqrt{e}} = e^{-\frac{1}{2}}$

$y = \frac{9}{e} \cdot \ln\frac{\frac{3}{\sqrt{e}}}{\frac{3}{1}} = \frac{9}{e} \cdot \ln e^{-\frac{1}{2}} = \frac{9}{e} \cdot \left(-\frac{1}{2}\right) = -\frac{9}{2e}$

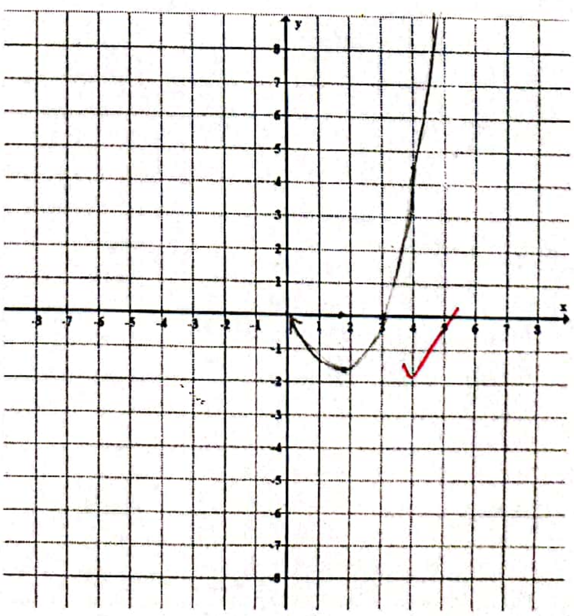
lokalni minimum

$T_1\left(\frac{3}{\sqrt{e}}, -\frac{9}{2e}\right)$

c) Določi $\lim\limits_{x\to\infty} f(x)$ in $\lim\limits_{x\to 0} f(x)$. Nariši graf funkcije. [4t] 4

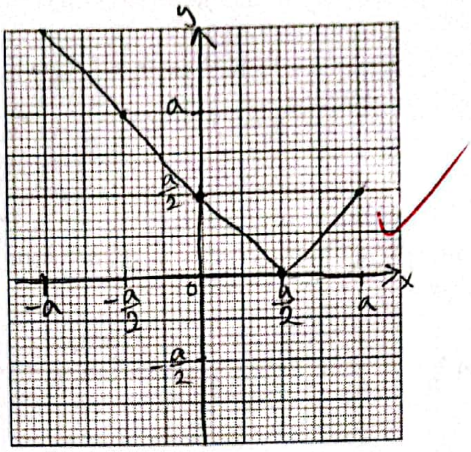
$\lim\limits_{x\to\infty} (f(x)) = \lim\limits_{x\to\infty} (x^2 \cdot \ln\frac{x}{3}) = \infty$

$\lim\limits_{x\to 0} (x^2 \ln\frac{x}{3}) = 0$



0,001

5. a) Naj bo $a > 0$. V spodnjo mrežo nariši graf funkcije $f(x) = |x - \frac{a}{2}|$ za $-a \leq x \leq a$. [3t] 3



b) Izračunaj vrednost k, da bo veljalo: $\int_{-a}^{0} |x - \frac{a}{2}| dx = k \int_{0}^{a} |x - \frac{a}{2}| dx$. [4t] 0

$$\int_{-a}^{0} |x - \tfrac{a}{2}| dx = k \int_{0}^{a} |x - \tfrac{a}{2}| dx$$

$$\int_{-a}^{0} |x - \tfrac{a}{2}| dx = \left(\left|\frac{x^2}{2} - \frac{ax}{2}\right|\right)\Big|_{-a}^{0} =$$

$$= \left|-\frac{a^2}{2} + \frac{a^2}{2}\right| = 0$$

$$\int_{0}^{a} |x - \tfrac{a}{2}| dx = \left(\left|\frac{x^2}{2} - \frac{ax}{2}\right|\right)\Big|_{0}^{a} =$$

$$= \left|\frac{a^2}{2} - \frac{a^2}{2}\right| = 0$$

$k \in \mathbb{R}$

$$\int_{-a}^{0} |x - \tfrac{a}{2}| dx = a^2$$

$$\int_{0}^{a} |x - \tfrac{a}{2}| dx = \frac{a^2}{4} \qquad k = 4$$

$$|x - \tfrac{a}{2}| = \begin{cases} x - \frac{a}{2}; & x \in \ldots \\ -x + \frac{a}{2}; & x \in \ldots \end{cases}$$

→ vzameš vrednost iz domoćja na določenem integralu

DODATNA NALOGA:

Izračunaj nedoločeni integral $\int \sqrt{4 - x^2}\, dx$. Pri izračunu uvedi novo spremenljivko θ, tako da je $x = 2 \sin \theta$. [3t] 0

$$\int \sqrt{4 - x^2}\, dx =$$

$x = 2\sin\emptyset$

$dx = 2\cos\theta\, d\theta$

$$= \int \sqrt{4 - 4 \cdot \sin^2 \emptyset}\, dx =$$

$$= \int \sqrt{4(1 - \sin^2\emptyset)}\, dx = 2 \int \sqrt{\cos^2 \emptyset}\, dx = 2 \int \cos \emptyset\, dx = 2 \sin\emptyset + C =$$

$$= x + C$$

$\int (1 - \sin^2 x) =$

$$\frac{x}{2}\sqrt{4 - x^2} + 2\arcsin\frac{x}{2} + C$$