

3. kontrolna naloga
4. A, 20. 12. 2023

Ime in priimek: Lira Jurkovič Razred: 4 a



dosežene točke	možne točke	odstotki	ocena
27	42	64	3

1. V aritmetičnem zaporedju je $a_3 = 4 - a_1$ in $a_2 \cdot a_4 = -4$. Najmanj koliko členov zaporedja moramo sešteti, da bo vsota manjša od -10000 ? Zapiši odgovor. [7] 5

$$\frac{n}{2}(a_1 + a_n)$$

$$\frac{n}{2}(a_1 + (n-1)d)$$

$$a_3 = 4 - a_1 = a_1 + 2d$$

$$a_2 \cdot a_4 = -4$$

$$4 - a_1 = a_1 + 2d$$

$$(a_1 + d)(a_1 + 3d) = -4$$

$$2a_1 = 4 - 2d \quad | :2$$

$$a_1^2 + a_1 \cdot 3d + a_1 d + 3d^2 = -4$$

$$S < -10000$$

$$a_1 = 2 - d \quad \checkmark$$

$$a_1^2 + 4a_1 d + 3d^2 = -4$$

$$a_n = 4 + (n-1)d \quad a_1 = 4 \quad \checkmark$$

$$\frac{n}{2}(4 + (4 + (n-1)d)) < -10000 \quad \checkmark$$

$$d = -2 \quad \checkmark$$

$$4, 2, 0, -2, -4, -6, \dots$$

$$110$$

$$240$$

$$420$$

$$650$$

$$930$$

$$110$$

$$130$$

$$150$$

$$170$$

$$190$$

$$210$$

$$230$$

$$250$$

$$270$$

$$290$$

$$310$$

$$330$$

$$350$$

$$370$$

$$390$$

$$410$$

$$430$$

$$450$$

$$470$$

$$490$$

$$510$$

$$530$$

$$550$$

$$570$$

$$590$$

$$610$$

$$630$$

$$650$$

$$670$$

$$690$$

$$710$$

$$730$$

$$750$$

$$770$$

$$790$$

$$810$$

$$830$$

$$850$$

$$870$$

$$890$$

$$910$$

$$930$$

$$950$$

$$970$$

$$990$$

$$1010$$

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$$1950$$

$$1970$$

$$1990$$

$$2010$$

$$2030$$

$$2050$$

$$2070$$

$$2090$$

$$2110$$

$$2130$$

$$2150$$

$$2170$$

$$2190$$

$$2210$$

$$2230$$

$$2250$$

$$2270$$

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$$4070$$

$$4090$$

$$4110$$

$$4130$$

$$4150$$

$$4170$$

$$4190$$

$$4210$$

$$4230$$

$$4250$$

$$4270$$

$$4290$$

$$4310$$

$$4330$$

$$4350$$

$$4370$$

$$4390$$

$$4410$$

$$4430$$

$$4450$$

2. Dano je zaporedje s splošnim členom $a_n = \left(\frac{4n+3}{4n+1}\right)^{n+2}$.

[4t] 1

a) Izračunaj limito zaporedja.

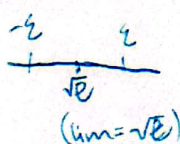
$$\lim_{n \rightarrow \infty} \left(\frac{4n+3}{4n+1}\right)^{n+2} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{4n+1}\right)^{n+2} =$$

$$= \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2n+\frac{1}{2}}\right)^{n+2} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{\frac{1}{2}x+1} = e^{\frac{1}{2}}$$

$$2n \cdot \frac{1}{2} = n$$

b) Najmanj za koliko bi morali zmanjšati vrednost tretjega člena zaporedja, da bi ležal v ε -okolici limite zaporedja, če je $\varepsilon = 0,19$? Rezultat naj bo na dve decimalni mesti natančen.

[3t] 1

$$\varepsilon = 0,19$$


(lim = \sqrt{e})

$$a_3 = \left(\frac{4 \cdot 3 + 3}{4 \cdot 3 + 1}\right)^{3+2} = \left(\frac{15}{13}\right)^5 = 2,045$$

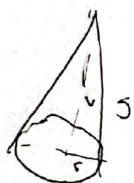
$$2,045 - 0,19 = 1,86$$

$$2,045 - x = \sqrt{e} + 0,19$$

3. V pokončnem krožnem stožcu je ploščina trikotnika, ki predstavlja osni presek stožca, enaka 16. Prostornina stožca je 64π . Polmer osnovne ploskve r , dolžina stranice s in višina stožca v so, v tem vrstnem redu, zaporedni členi geometrijskega zaporedja. Izračunaj površino stožca.

[7t]

(7)^N



$$S_{\Delta} = 16$$

$$V = 64\pi = \frac{\pi r^2 \cdot v}{3}$$

$$r, s, v$$

$$s^2 = r \cdot v$$

$$64\pi = \frac{\pi r^2 \cdot v}{3}$$

$$r^2 v = 192$$

$$r^2 = \frac{192}{v}$$

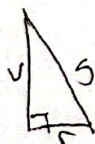
$$r = \sqrt{\frac{192}{v}} = \frac{4\sqrt{3}}{\sqrt{v}}$$

$$r = \frac{8\sqrt{3}}{\sqrt{v}}$$

$$192 = 16 \cdot 12$$

$$v = \frac{192}{r^2}$$

$$P = \pi r^2 + \pi r s$$



$$r^2 + v^2 = s^2$$

$$r^2 + v^2 = r \cdot v$$

$$\frac{192}{v} + v^2 = \frac{8\sqrt{3}}{\sqrt{v}} \cdot \sqrt{v} \quad \cdot \sqrt{v}$$

$$v^2 + 192 = 8\sqrt{3}v$$

$$r^2 + \left(\frac{192}{r^2}\right)^2 = r \cdot \frac{192}{r^2}$$

$$\frac{36864}{r^4} + r^2 - \frac{192}{r} = 0 \quad \cdot r^4$$

$$36864 + r^6 - 192r^3 = 0 \quad \cdot 192 \quad r^3 = y$$

$$y^2 - y + 192 = 0$$

$$y = \frac{1 \pm \sqrt{1 - 768}}{2} = \sqrt{192}$$

$$\pi r s$$

$$S = \frac{a_1(q^n - 1)}{q - 1}$$

$$q^3 - 3q^2 + q - \frac{q^3 - 1}{13} = 0$$

4. Tri števila tvorijo geometrijsko zaporedje z vsoto 52. Če prvo število povečamo za 1, drugo število povečamo za 5 in tretje število zmanjšamo za 7, dobimo novo tričleno aritmetično zaporedje. Izračunaj člene geometrijskega zaporedja. [7t] 3

$$a_1, a_2, a_3$$

$$a_1, a_1q, a_1q^2$$

$$\frac{a_1(q^3 - 1)}{q - 1} = 52$$

$$a_1(q^3 - 1) = 52(q - 1)$$

$$a_1 = \frac{52(q - 1)}{(q^3 - 1)}$$

$$a_1(q^3 - 1) = 52q - 52$$

$$x, y, z$$

$$a_1 + 1, a_2 + 5, a_3 - 7$$

$$\frac{a_1 + 1 + a_3 - 7}{2} = a_2 + 5$$

$$\frac{a_1 + 1 + a_1q^2 - 7}{2} = a_1q + 5$$

$$\frac{a_1(q^2 + 1) - 6}{2} = a_1q + 5$$

$$a_1(q^2 + 1) - 6 = 2a_1q + 10$$

$$a_1q^2 + a_1 = 2a_1q + 16$$

$$\frac{16(q^3 - 1)}{(q - 1)^2} = 52(q - 1) \quad \checkmark \quad / : (q - 1)^2$$

$$16(q^3 - 1) = 52(q - 1)^3$$

$$4q^3 - 4 = 13(q^3 - 2q^2 + 2q - 1)$$

$$4q^3 - 4 = 13q^3 - 26q^2 + 26q - 13$$

$$9q^3 - 26q^2 + 26q - 9 = 0$$

$$q^2(9q - 26) - 9 = 0$$

$$a_1(q^2 + 1) - 2a_1q = 16$$

$$a_1(q^2 + 1 - 2q) = 16$$

$$a_1 = \frac{16}{(q - 1)^2}$$

$$x_1 = 36 \quad y_1 = 12 \quad z_1 = 4$$

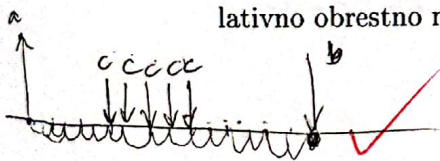
$$x_2 = 4 \quad y_2 = 12 \quad z_2 = 3$$

$$a_1 = \frac{16}{(q^2 - 2q + 1)}$$

$$a_1 = \frac{16}{(q - 1)(q - 1)}$$

5. Za nakup stanovanja smo pri banki najeli kredit v vrednosti 20000 evrov. Dolg smo začeli odplačevati 3 leta po najemu kredita, in sicer s petimi enakimi polletnimi obroki. Za tem smo za tri leta prenehali z izplačevanjem kredita in nato preostanek dolga izplačali v celoti, z enkratnim zneskom 12000 evrov. Izračunaj, kolikšna je bila vrednost posameznega polletnega obroka. Zapiši odgovor.

Banka pri poslovanju uporablja obrestno obrestovanje, polletno kapitalizacijo in relativno obrestno mero. Letna obrestna mera je 4 odstotna. [6t] 4



$$a = 20\,000 \text{ €}$$

$$b = 12\,000 \text{ €}$$

$$p = 4\%$$

$$r = 1,02$$

$$16$$

$$\frac{ar^6(r^5 - 1)}{r - 1}$$

$$12\,000 + \frac{c}{r^6} + \frac{c}{r^4} + \frac{c}{r^2} + c = 20\,000 \cdot r^{16} \quad \checkmark$$

$$12\,000 + \frac{c(1,02^5 - 1)}{0,02} = 20\,000 \cdot 1,02^{16}$$

$$c(1,02^5 - 1) = 309,11$$

$$c = 2969,95 \text{ €}$$

Vrednost posameznega obroka je bila 2969,95 €.

$$|q| < 1$$

$$\sin x \cos x + \cos x \sin x$$

$$2 \sin x \cos x$$

6. Dana je neskončna geometrijska vrsta $\sin(2x) + 2 \sin^2 x + \frac{2 \sin^3 x}{\cos x} + \dots$ za $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$

a) Za katere vrednosti x z intervala $(-\frac{\pi}{2}, \frac{\pi}{2})$ je vrsta konvergentna? [3t] 3

$$q = \frac{\sin x}{\cos x} \checkmark$$

$$\frac{\sin(2x)}{1 - \frac{\sin x}{\cos x}}$$

$$x \in (-\frac{\pi}{4}, \frac{\pi}{4}) \checkmark$$

$$|\frac{\sin x}{\cos x}| < 1 \checkmark \quad \cos x > \sin x$$

b) Reši enačbo: $\sin(2x) + 2 \sin^2 x + \frac{2 \sin^3 x}{\cos x} + \dots = 2 \cos^2 x$. [5t] 3

$$\frac{\sin x}{\cos x} = 1$$

$$\frac{2 \sin x \cos x}{1 - \frac{\sin x}{\cos x}} = 2 \cos^2 x$$

$$\frac{\cos - \sin}{\cos}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

$$1 + \tan x = \frac{1}{\cos^2 x}$$

$$\frac{\sin x \cos^2 x}{\cos x - \sin x} = \cos^2 x \quad / : \cos^2 x$$

$$\frac{\sin x (1 - \sin^2 x)}{\cos x - \sin x} = (1 - \sin^2 x)$$

$$\cos x - 2 \sin x = 0$$

$$\frac{\sin x}{\cos x - \sin x} = 1 \quad / : (\cos x - \sin x)$$

$$\sin x = \cos x - \sin x$$

$$2 \sin x = \cos x$$

$$\sin x = \frac{\cos x}{2} \checkmark$$

$$2 \sin x \cos x + \cos^2 x = 0$$

$$\cos x (2 \sin x - 1) = 0$$

DODATNA NALOGA:

Izračunaj vsoto neskončne vrste: $\sum_{k=1}^{\infty} \frac{k}{2^{k+1}} = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \frac{5}{64} + \dots$

[3t] ~~3~~