2. kontrolna naloga
3. A, 7. 12. 2022

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dosežene točke	možne točke	odstotki	ocena
27	42	64	3

ČAS PISANJA: 45 minut

1. Izračunaj natančno vrednost izrazov $\sin(2x + \frac{\pi}{6})$ in $\tan\frac{x}{2}$, če je $\cot x = -\frac{3}{4}$, $\frac{3\pi}{2} \leq x < 2\pi$. [9t] 6

$\cot x = -\frac{3}{4}$

$\tan x = -\frac{4}{3}$

$1 + \cot^2 x = \frac{1}{\sin^2 x}$

$1 + \frac{9}{16} = \frac{1}{\sin^2 x}$

16 + 9

$\sin^2 x = \frac{1}{\frac{25}{16}}$

$\sin^2 x = \frac{16}{25}$

$\sin x = -\frac{4}{5}$

$\tan x = \frac{\sin x}{\cos x}$

$\cos x = \frac{\sin x}{\tan x} = \frac{-\frac{4}{5}}{-\frac{4}{3}} = \frac{12}{20} = \frac{3}{5}$

$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$

$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\frac{3}{5}}{1+\frac{3}{5}}} = \pm\sqrt{\frac{\frac{2}{5}}{\frac{8}{5}}} = \pm\sqrt{\frac{10}{40}}$

$\tan\frac{x}{2} = \frac{\sqrt{10}}{2\sqrt{10}} = -\frac{1}{2}$

$\sin(2x + \frac{\pi}{6}) = \sin 2x \cos\frac{\pi}{6} + \cos 2x \sin\frac{\pi}{6} =$

$= \frac{\sqrt{3}}{2}\sin 2x + \frac{1}{2}\cos 2x =$

$= \frac{\sqrt{3}}{2} 2 \sin x \cos x + \frac{1}{2}(\cos^2 x - \sin^2 x) =$

$= -\frac{\sqrt{3}}{2} \cdot \frac{4}{5} \cdot \frac{3}{5} + \frac{1}{2} \cdot \left(\frac{9}{25} - \frac{16}{25}\right) =$

$-\frac{7}{25} \cdot \frac{1}{2} = -\frac{7}{50}$

$= -\frac{6\sqrt{3}}{25} + \frac{7}{50} = -\frac{12\sqrt{3}}{50} - \frac{7}{50} = -\frac{12\sqrt{3}-7}{50}$

$-\frac{24\sqrt{3}-7}{50}$

2. Dana je funkcija $f(x) = -2\cos\left(2x - \frac{\pi}{2}\right)$. $= -2\cos(2(x-\frac{\pi}{4}))$

a) Izračunaj ničle in maksimume funkcije f. [6t] 5

točka

$f(x)=0$

$-2\cos(2x-\frac{\pi}{2})=0$

$\cos(2x-\frac{\pi}{2})=0$ ✓

$2x-\frac{\pi}{2}=\frac{\pi}{2}$

$2x=\pi$

$x=\frac{\pi}{2}+\frac{k\pi}{2};\ k\in\mathbb{Z}$ ✓

$\cos(2x-\frac{\pi}{2})=-1$ ✓

$2x-\frac{\pi}{2}=\pi$

$2x=\frac{3\pi}{2}$

$x=\frac{3\pi}{4}+k\pi;\ k\in\mathbb{Z}$ ✓

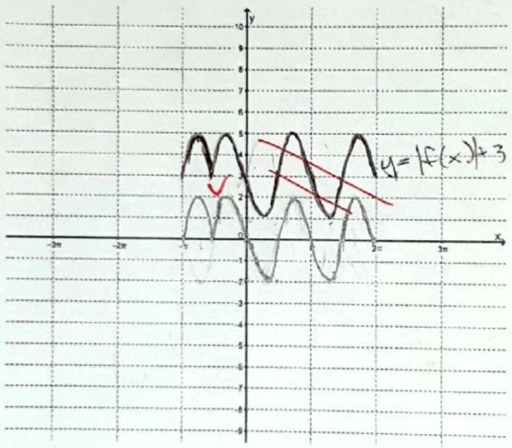
$T_{max}(\frac{3\pi}{4}+k\pi, -1)$

perioda: $2\pi\cdot\frac{1}{2}=\frac{2\pi}{2}=\pi$

b) Na intervalu $[-\pi, 2\pi]$ nariši graf krivulje $y = |f(x)| + 3$. [4t] 2

cos:

$\cos\pi=-1$



3. Poenostavi izraz (za vse vrednosti x, za katere je izraz definiran). [5t] 3

$$\frac{\tan x + \sin x}{\tan x \cdot \sin(2x)} \cdot (1 - \cos x) =$$

$\sin(2x)=2\sin x\cos x$

$$= \frac{\frac{\sin x}{\cos x}+\sin x}{\frac{\sin x}{\cos x}\cdot 2\sin x\cos x}\cdot(1-\cos x)=$$

$$= \frac{\frac{\sin x}{\cos x}+\frac{\sin x\cos x}{\cos x}}{2\sin^2 x}\cdot(1-\cos x)=$$

$$= \frac{\sin x+\sin x\cos x}{2\sin^2 x\cdot\cos x}(1-\cos x)=$$

$$= \frac{\sin x(1+\cos x)(1-\cos x)}{2\sin^2 x} = \frac{1-\cos^2 x}{2\sin x} = \frac{\sin^2 x}{2\sin x\cos x} =$$

$$= \frac{\cancel{\sin x}}{2}$$

$\frac{1}{2}\tan x$

4. Pokaži, da je vrednost danega izraza enaka $\sin 20°$. [6t] 5

$$\frac{2\sin^2 550°}{\cot(-460°)} + \frac{\cos(-810°)}{\sin(2525°)} =$$

$$= \frac{2\sin^2(180°+10°)}{-\cot(180°-80°)} + \frac{-\cos 90°}{\sin 5°} =$$

$$= -\frac{2\cdot\sin^2(-10°)}{\cot(-80°)} =$$

$$= +\frac{2\cdot\sin^2(10°)}{\cot 80°} =$$

$$= \frac{2\cdot\sin^2 10°}{\frac{\sin 10°}{\sin 80°}} = 2\cdot\sin 10°\sin 80° = 2\cdot\sin 10°\cos 10° = \sin 20°$$

$\cot 80 = \frac{\cos 80}{\sin 80} = \frac{\sin 10}{\sin 80}$

550 − 360 = 190

460 − 360 = 100

360 · 2 = 720, + 90 = 810

2525

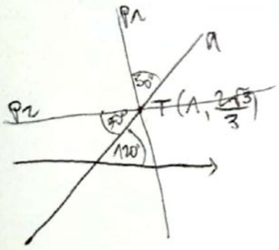
360 · 5 = 1800

360 · 7 = 2520

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

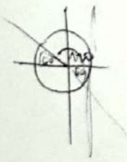
$\sin 20° =$

5. Dana je premica q z naklonskim kotom 120°. Zapiši enačbi premic p_1 in p_2, ki potekata skozi točko $T(1, \frac{2\sqrt{3}}{3})$ in s premico q oklepata kot 30°. [7t] 6



$\boxed{T(1, \frac{2\sqrt{3}}{3})}$

$\tan 120° = k_q = -\tan 60° = -\sqrt{3}$



$\tan 60° = \frac{\sin 60°}{\cos 60°} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

$\tan 30° = \frac{\sin 30°}{\cos 30°} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\tan\alpha = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$

$y = kx + n$

$\frac{2\sqrt{3}}{3} = -\frac{\sqrt{3}}{3} + n$

$n = \frac{3\sqrt{3}}{3} = \sqrt{3}$

1) $\frac{\sqrt{3}}{3} = \left|\frac{k_q - k_1}{1 + k_q k_1}\right|$

$\frac{\sqrt{3}}{3} = \frac{-\sqrt{3} - k_1}{1 - \sqrt{3}k_1}$

$\sqrt{3}(1 - \sqrt{3}k_1) = 3(-\sqrt{3} - k_1)$

$\sqrt{3} - 3k_1 = -3\sqrt{3} - 3k_1$

$\frac{\sqrt{3}}{3} = \frac{-\sqrt{3} - k_1}{\sqrt{3}k - 1}$

$\sqrt{3}(\sqrt{3}k - 1) = 3(-\sqrt{3} - k)$

$3k - \sqrt{3} = -3\sqrt{3} - 3k$

$6k = -2\sqrt{3}$

$k = -\frac{\sqrt{3}}{3}$

2) $\frac{\sqrt{3}}{3} = \frac{k_1 + \sqrt{3}}{1 - \sqrt{3}k_1}$

$\sqrt{3}(1 - \sqrt{3}k_1) = 3(k_1 + \sqrt{3})$

$\sqrt{3} - 3k_1 = 3k_1 + 3\sqrt{3}$

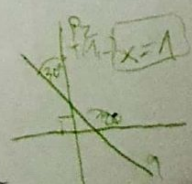
$\sqrt{3} - 3\sqrt{3} = 6k_1$

$6k_1 = -2\sqrt{3}$

$k_1 = -\frac{\sqrt{3}}{3}$

$y_1 = -\frac{\sqrt{3}}{3}x + \sqrt{3}$

$y_2 = \cancel{2\sqrt{3}x + -\frac{\sqrt{3}}{3}}$



6. Faktoriziraj. [5t] 0

$2\alpha + \alpha$

$$\frac{\cos\alpha + \cos(2\alpha) + \cos(3\alpha)}{\cos(2\alpha)} =$$

$$= \frac{2\cos\frac{3\alpha}{2}\cos\frac{\alpha}{2} + \cos 3\alpha}{\cos 2\alpha} =$$

$$= \cancel{2\cos\frac{3\alpha}{2}\, 2\cos\sqrt{\frac{1-\cos\alpha}{2}}}$$

DODATNA NALOGA:

Izračunaj $\tan^4 x + \cot^4 x$, če je $\tan x + \cot x = a$. [3t]

FORMULE:

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$

$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$\sin\alpha\cdot\cos\beta = \frac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$

$\sin\alpha\cdot\sin\beta = -\frac{1}{2}(\cos(\alpha+\beta) - \cos(\alpha-\beta))$

$\cos\alpha\cdot\cos\beta = \frac{1}{2}(\cos(\alpha+\beta) + \cos(\alpha-\beta))$

$\tan\alpha \pm \tan\beta = \frac{\sin(\alpha\pm\beta)}{\cos\alpha\cos\beta}$

$\cot\alpha \pm \cot\beta = \frac{\sin(\beta\pm\alpha)}{\sin\alpha\sin\beta}$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\tan(2\alpha) = \frac{2\tan\alpha}{1-\tan^2\alpha}$

$\tan\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin(2\alpha) = 2\sin\alpha\cos\alpha$

$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$

$\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos\alpha}{2}}$

$\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos\alpha}{2}}$

$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha}$