$\tan = \frac{\sin}{\cos}$ $\quad \tan^2\alpha + 1 = \frac{1}{\cos^2\alpha}$ $\quad \sin^2\alpha + \cos^2\alpha = 1$

$\cot = \frac{\cos}{\sin}$ $\quad \cot^2\alpha + 1 = \frac{1}{\sin^2\alpha}$

$\tan \cdot \cot = 1$

2. kontrolna naloga
2. A, 8. 12. 2021

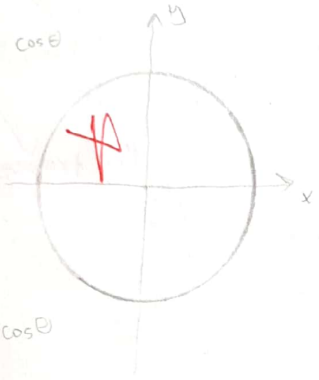
PIŠI S KEMIČNIM SVINČNIKOM!!! 😊

Ime in priimek: Lira Jurkovič

dosežene točke	možne točke	odstotki	ocena
23	35	66	3

ČAS PISANJA: 45 minut

1. Naj bo x kot, za katerega velja $0° < x < 360°$. V enotskem krogu predstavi in nato izračunaj velikosti vseh kotov x, za katere velja $\cos x = -0,8$. Velikosti kotov zapiši na minuto natančno. [5t] 3t



$\cos x = -0,8$

$\cos\alpha = 0,8$

$\alpha = 36,87° = 36°52'$ ✓

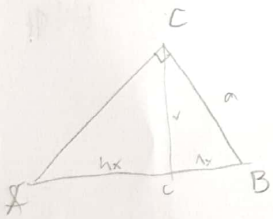
$\cos(180° - 36°52') = -0,8$

$x_1 = 143°8'$ ✓

$\cos(180° + 36°52') = -0,8$

$x_2 = 216°52'$ ✓

2. Dan je pravokotni trikotnik ABC s hipotenuzo AB. Izračunaj natančni dolžini hipotenuze c in katete a, če je $v = 24$ cm in $a_1 : b_1 = 1 : 4$. [6t] 5t



$v^2 = 1x \cdot 4x$

$576 = 4x^2$

$x^2 = 144$

$x = 12cm$ ✓

$v = 24cm$

$a_1 : b_1 = 1 : 4$

$a_1 = 12cm$,

$b_1 = 48cm$

$c = a_1 + b_1 = 60cm$ ✓

$a^2 = a_1 c$

$a^2 = 12cm \cdot 60cm = 720cm$

$a = 26,83cm$

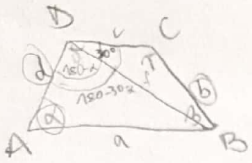
↑ to je približek!

$v^2 = a_1 b_1$

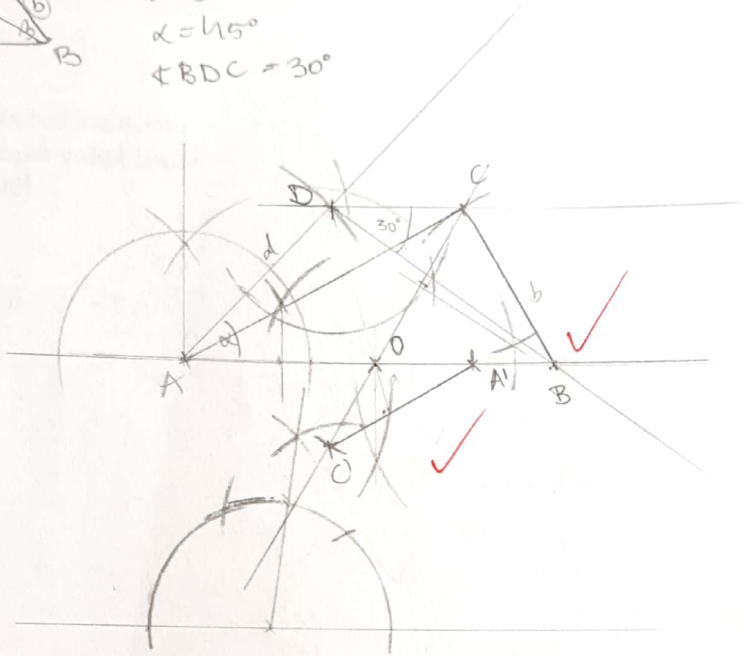
$a^2 = a_1 c$

$b^2 = b_1 c$

3. Konstruiraj trapez $ABCD$ z ostrima kotoma α in β s podatki: $b = 2,5$ cm, $d = 3$ cm, $\alpha = 45°$ in $\angle BDC = 30°$.
Nato konstruiraj daljico $A'C'$ (brez zapisa poteka konstrukcije), ki se s središčnim raztegom s središčem v razpolovišču osnovnice a trapeza in koeficientom raztega $k = -2$ preslika v diagonalo trapeza AC. [6t] 6t



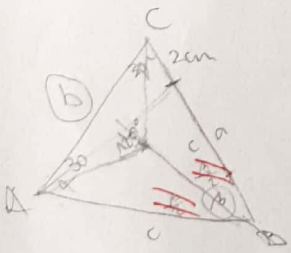
b = 2,5cm
d = 3cm
α = 45°
∠BDC = 30°

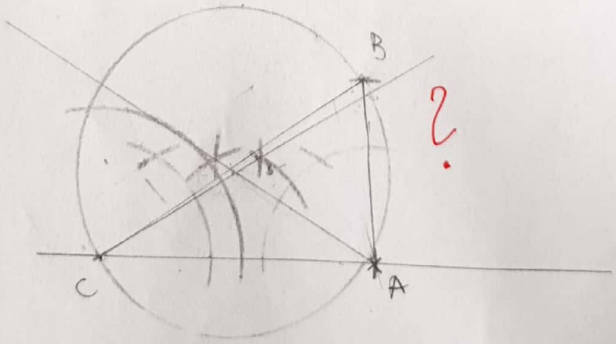


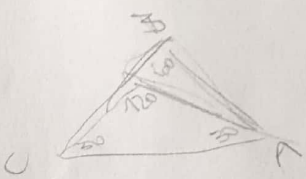
① α → A
② k(A, r = 3cm) → D
③ konst. ∠ADB
↳ 180° − α − 30°
P(F, a) → B
④ c ∥ a, k(B, r = 2,5cm)
→ C
⑤ trapez ABCD

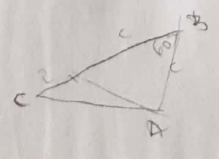
4. Konstruiraj trikotnik ABC s podatki: $a - c = 2$ cm, $\beta = 60°$ in $b = 4$ cm. Zapis poteka konstrukcije ni obvezen. [5t] 1t

ΔABC
a − c = 2cm
β = 60°
b = 4cm





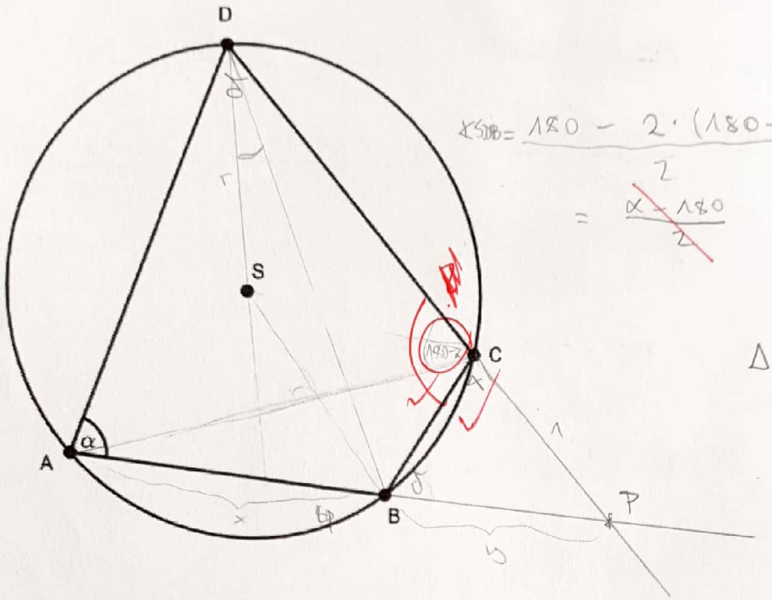




5. Na sliki je štirikotnik $ABCD$.

a) S kotom α izrazi kot $\angle SDB$. [3t] 0t

b) Nosilki stranic CD in AB se sekata v točki P. Zapiši podobna trikotnika in utemelji, zakaj sta si podobna (lahko s krajšimi zapisi ob sliki). Izrazi ploščino S trikotnika BPC s ploščino S_{ABCD} štirikotnika $ABCD$, če je $|PA| = 4$ in $|PC| = 1$. [5t] 3t



$\angle SDB = \frac{180 - 2\cdot(180-\alpha)}{2} = \frac{180 - 360 + \alpha}{2} = \frac{\alpha - 180}{2}$

$\Delta BPC \sim \Delta DPA$ ✓

skladna kota: α in $\angle BCP$ ($\angle DCB = 180°$ -)

β in $\angle PBC$ ($\angle CBA = 180°$ - (ker je tetivni štirikotnik))

$\Rightarrow$ trikotnika sta podobna

$S_{ABCD} =$

$S_{\Delta BPC} = \frac{S_{ABCD}}{\quad}$

$4 : 1 =$

$a = h$

$a' = 1$

$a' = ka$

$1 = k \cdot h$

$k = \frac{1}{h}$

$k^2 S_{\Delta DPA} = S_{\Delta BPC}$

$\frac{1}{16} \cdot S_{\Delta ABD} = S_{\Delta BPC}$

$S_{\Delta BPC} = \frac{S_{\Delta DPA}}{16}$ ✓

30	45	60
$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$

6. Poenostavi izraz: $\cot x + (1 - 2(1 + \tan^2 x)^{-1}) \cdot \sin^{-1} x : \cos x$. [5t] 5t

$\tan = \frac{\sin}{\cos}$

$\cot = \frac{\cos}{\sin}$

$\tan \cdot \cot = 1$

$\sin^2 + \cos^2 = 1$

$\tan^2 + 1 = \frac{1}{\cos^2}$

$\cot^2 + 1 = \frac{1}{\sin^2}$

$\cot x + (1 - 2(1 + \tan^2 x)^{-1}) \cdot \sin^{-1} x : \cos x =$

$= \cot x + \left(1 - \frac{2}{1 + \tan^2 x}\right) \cdot \frac{1}{\sin x} : \cos x =$

$= \cot x + (1 - 2\cos^2 x) \cdot \frac{1}{\sin x} \cdot \frac{1}{\cos x} =$

$= \cot x + \frac{1 - 2\cos^2 x}{\sin x \cos x} = \frac{\frac{\cos x}{\sin x} \cdot \frac{\sin x \cos x}{1} + 1 - 2\cos^2 x}{\sin x \cos x} =$

$= \frac{\cos^2 x + 1 - 2\cos^2 x}{\sin x \cos x} = \frac{1 - \cos^2 x}{\sin x \cos x} = \frac{\sin^2 x}{\sin x \cos x} = \tan x$

DODATNA NALOGA:
Dokaži, da so razpolovišča stranic poljubnega konveksnega štirikotnika $ABCD$ oglišča paralelograma. [3t]